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Steady Representation of the Homogeneous Cooling State of a Granular Mixture

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Abstract. The study of a freely evolving granular mixture using particle simulations presents the inherent limitation that, as a consequence of the inelasticity of collisions, the system is continuously cooling. Therefore, the typical particle velocities become very small rather soon, and numerical uncertainties very large. This difficulty can be solved by mapping the properties of the freely evolving granular mixture into those of a stationary state by means of a change in the time scale, similarly as it is done in one-component systems. This steady state representation will be used to study some properties of dilute granular mixtures by using the Direct Simulation Monte Carlo method. We will focus on the temperature ratio of the two components of the mixture, as well as on the fluctuations of the total energy of the mixture, as well as of each of the components.

Keywords: Dilute granular flows; granular mixture; kinetic theory
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INTRODUCTION

A granular fluid is a collection of particles, usually of macroscopic size, whose collisions are inelastic. In the rapid flow regime, or granular gas state, particles move in a ballistic way between collisions. In this regime the behavior of the system resembles that of a molecular fluid, although with many significant differences [1]. The most obvious reason for them is that, as energy is continuously lost in collisions, a steady state of a granular gas can only be sustained if energy is injected into the system by some external driving. Therefore, granular gases are always in a non-equilibrium state. The simplest possible state of a freely evolving granular gas is the homogeneous cooling state (HCS), an homogeneous, isotropic state whose energy decays monotonically in time. This state plays a relevant role in order to investigate the transport properties of granular fluids, since it provides the reference state when applying the Chapman-Enskog procedure to derive hydrodynamics starting from a given kinetic equation [2, 3, 4]. Also, linear response around the HCS has been studied, and expressions for the transport coefficients of a granular gas, generalization to the inelastic case of the Green-Kubo formulas, have been derived [5, 6].

Real granular materials are usually polydisperse. For that reason, the study of granular mixtures is important and has attracted a lot of interest [7, 4, 8, 9]. Also, granular mixtures exhibit some effects that are specific to them, such as segregation properties that are yet not well understood [10], or the violation of energy equipartition, both in homogeneous [8, 11] and inhomogeneous states [12, 13, 14, 15, 16]. The coexistence of different temperatures in a granular mixture raises some fundamental questions such as whether they have to be incorporated as independent variables in a macroscopic description of the system. Finally, it is evident that the theoretical description of granular mixtures is much more complicated than that of a one component system. A clear example of this is the determination of the Navier-Stokes transport coefficients [4, 9].

Particle computer simulations have proven to be a very useful tool to investigate the properties of systems of particles at the most fundamental level of description. That is also true in the case of granular systems. Nevertheless, the simulation of a freely evolving granular system, even in the case of the HCS, has some intrinsic technical difficulties. In particular, as the system is continuously cooling, the typical velocities of the particles become very small after some time, and the numerical errors very large. In the case of one-component systems, this problem was solved by mapping the dynamics of a system in the HCS onto the dynamics around a steady state, by means of a change in the time scale [17, 18]. Another difficulty in the simulation of the HCS is that it is unstable with respect to long wavelength perturbations [19, 20], so only small enough systems stay in the HCS.

In this work, the steady representation of the HCS will be generalized to the case of granular mixtures. It will be done for the case of a binary system, but the arguments are directly extended to a mixture with any number of components. The plan of the paper is as follows. In Sec. the peudo-Liouville dynamics for a mixture of inelastic hard particles will be presented. A solution of the Liouville equation, the HCS, will be described in Sec. , and some of its

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properties will be summarized. In Sec. the previous results will be used to construct a steady representation of the HCS. Finally, in Sec. particle simulations in the new dynamics will be presented, and some properties of the HCS of a dilute granular mixture will be discussed, and compared with theoretical predictions in the gaussian approximation.

PSEUDO-LIOUVILLE DYNAMICS FOR HARD SPHERICAL PARTICLES

We consider a binary mixture of smooth, inelastic hard spheres ($d = 3$) or disks ($d = 2$). The mass and diameter of particles of species γ ($\gamma = 1,2$) are m_γ and σ_γ respectively. The inelasticity of collisions is described by means of constant coefficients of restitution, α_{11}, α_{22}, and $\alpha_{12} = \alpha_{21}$, where $\alpha_{\gamma\lambda}$ refers to collisions between a particle of species γ and a particle of species λ. These coefficients are defined in the interval $0 < \alpha_{\gamma\lambda} \leq 1$, corresponding the value $\alpha_{\gamma\lambda} = 1$ to elastic collisions. The position and velocity of a particle i of component γ will be denoted by $\boldsymbol{r}_{\gamma i}$, $\boldsymbol{v}_{\gamma i}$, with $i = 1,\ldots,N_\gamma$, and N_γ is the number of particles of species γ. The dynamics of a particle consists of free streaming until it collides with a particle j of species λ. The effect of the collision is to instantaneously change the velocities of the two involved particles according to

$$\begin{aligned} \boldsymbol{v}_{\gamma i} \longrightarrow \boldsymbol{v}'_{\gamma i} &= \boldsymbol{v}_{\gamma i} - \frac{m_\lambda}{m_\gamma + m_\lambda}(1+\alpha_{\gamma\lambda})(\boldsymbol{g}_{\gamma i,\lambda j}\cdot\hat{\boldsymbol{\sigma}})\hat{\boldsymbol{\sigma}}\,, \\ \boldsymbol{v}_{\lambda j} \longrightarrow \boldsymbol{v}'_{\lambda j} &= \boldsymbol{v}_{\lambda j} + \frac{m_\gamma}{m_\gamma + m_\lambda}(1+\alpha_{\gamma\lambda})(\boldsymbol{g}_{\gamma i,\lambda j}\cdot\hat{\boldsymbol{\sigma}})\hat{\boldsymbol{\sigma}}\,, \end{aligned} \tag{1}$$

where $\boldsymbol{g}_{\gamma i,\lambda j} = \boldsymbol{v}_{\gamma i} - \boldsymbol{v}_{\lambda j}$ is the relative velocity, and $\hat{\boldsymbol{\sigma}}$ is the unit vector pointing from the center of particle j to the center of particle i at contact. The above collision rule conserves the total momentum,

$$m_\gamma \boldsymbol{v}_{\gamma i} + m_\lambda \boldsymbol{v}_{\lambda j} = m_\gamma \boldsymbol{v}'_{\gamma i} + m_\lambda \boldsymbol{v}'_{\lambda j}\,, \tag{2}$$

while the relative velocity is modified as

$$\boldsymbol{g}'_{\gamma i,\lambda j} = \boldsymbol{g}_{\gamma i,\lambda j} - (1+\alpha_{\gamma\lambda})(\boldsymbol{g}_{\gamma i,\lambda j}\cdot\hat{\boldsymbol{\sigma}})\hat{\boldsymbol{\sigma}}\,. \tag{3}$$

Then, there is an energy loss at each collision that is given by:

$$\Delta E_{\gamma i,\lambda j} = -\frac{1}{2}\frac{m_\gamma m_\lambda}{m_\gamma + m_\lambda}(1-\alpha_{\gamma\lambda}^2)(\boldsymbol{g}_{\gamma i,\lambda j}\cdot\hat{\boldsymbol{\sigma}})^2\,. \tag{4}$$

Let $A(\Gamma)$ be some function of the dynamical state of the system representing a given observable. Here, Γ is a point in the phase space defined by the positions and velocities of the particles, i.e., $\Gamma \equiv \{\boldsymbol{r}_{\gamma i}, \boldsymbol{v}_{\gamma i}; \gamma = 1,2; i = 1,\ldots,N_\gamma\}$. The average of $A(\Gamma)$ at time t for a given initial distribution $\rho(\Gamma,0)$ is given by:

$$\langle A(t)\rangle \equiv \int d\Gamma\, \rho(\Gamma,0) A\left[\Gamma_t(\Gamma)\right]\,, \tag{5}$$

where $\Gamma_t(\Gamma)$ is the point in phase space representing the state of the system at time t. given that the initial phase point was Γ. The generator $\mathscr{L}$ for the representation of the dynamics in Eq. (5) is defined through the expression

$$\langle A(t)\rangle \equiv \int d\Gamma\, \rho(\Gamma,0) e^{t\mathscr{L}} A(\Gamma)\,. \tag{6}$$

Due to the singular character of hard particle interactions, $\mathscr{L}$ is not the usual generator of Hamilton's equations for continuous interaction potentials, but somewhat more complex. It has been extensively studied for the case of elastic collisions [21], and the analysis is easily extended to include the more general case of inelastic collisions [22]. The result reads,

$$\mathscr{L} = \sum_{\gamma=1,2}\sum_{i=1}^{N_\gamma} \boldsymbol{v}_{\gamma,i}\cdot\frac{\partial}{\partial \boldsymbol{r}_{\gamma,i}} + \sum_{1\leq i<j\leq N_1} T^{(11)}(\boldsymbol{x}_{1i},\boldsymbol{x}_{1j}) + \sum_{1\leq i<j\leq N_2} T^{(22)}(\boldsymbol{x}_{2i},\boldsymbol{x}_{2j}) + \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} T^{(12)}(\boldsymbol{x}_{1i},\boldsymbol{x}_{2j})\,. \tag{7}$$

Here $\boldsymbol{x}_{\gamma i} \equiv \{\boldsymbol{r}_{\gamma i}, \boldsymbol{v}_{\gamma i}\}$, and the operator $T^{\gamma\lambda}(\boldsymbol{x}_{\gamma i},\boldsymbol{x}_{\lambda j}) = T^{\lambda\gamma}(\boldsymbol{x}_{\lambda j},\boldsymbol{x}_{\gamma i})$ describes the binary collision between particle i of species γ and particle j of species λ. Its expression is

$$T^{\gamma\lambda}(\boldsymbol{x}_{\gamma i},\boldsymbol{x}_{\lambda j}) = \sigma_{\gamma\lambda}^{d-1}\int d\hat{\boldsymbol{\sigma}}\,\Theta(-\boldsymbol{g}_{\gamma i,\lambda j}\cdot\hat{\boldsymbol{\sigma}})|\boldsymbol{g}_{\gamma i,\lambda j}\cdot\hat{\boldsymbol{\sigma}}|\delta(\boldsymbol{r}_{\gamma i,\lambda j} - \boldsymbol{\sigma}_{\gamma\lambda})\left(b_{\gamma i,\lambda j} - 1\right)\,, \tag{8}$$

where $\sigma_{\gamma\lambda} = (\sigma_\gamma + \sigma_\lambda)/2$, $\boldsymbol{\sigma}_{\gamma\lambda} = \sigma_{\gamma\lambda}\hat{\boldsymbol{\sigma}}$, and $d\hat{\boldsymbol{\sigma}}$ is the solid angle element for the unit vector $\hat{\boldsymbol{\sigma}}$. $\boldsymbol{r}_{\gamma i,\lambda j} = \boldsymbol{r}_{\gamma i} - \boldsymbol{r}_{\lambda j}$ is the relative position vector of the involved particles, and Θ the Heaviside step function. Finally, $b_{\gamma i,\lambda j}$ is a substitution operator, that changes the velocities $\boldsymbol{v}_{\gamma i}$, $\boldsymbol{v}_{\lambda j}$ to its right into their post-collisional values, $\boldsymbol{v}'_{\gamma i}$, $\boldsymbol{v}'_{\lambda j}$ given by Eqs. (1).

The average in Eq. (6) can be expressed in an alternative, equivalent way by integrating over $\Gamma_t(\Gamma)$, rather than over Γ. Then, the time dependence is translated to the probability,

$$\langle A(t)\rangle \equiv \int d\Gamma\, \rho(\Gamma,t) A(\Gamma)\,. \tag{9}$$

The time dependent distribution $\rho(\Gamma,t)$ can be represented as

$$\rho(\Gamma,t) = e^{t\overline{\mathscr{L}}}\rho(\Gamma,0)\,, \tag{10}$$

where $\overline{\mathscr{L}}$ is defined by requiring that Eqs. (6) and (9) are equivalent. The pseudo-Liouville equation for the evolution of the distribution function follows directly from the above equation,

$$\frac{\partial \rho(\Gamma,t)}{\partial t} = \overline{\mathscr{L}}\rho(\Gamma,t)\,. \tag{11}$$

THE HOMOGENEOUS COOLING STATE

There is no stationary state of an isolated granular system due to the energy dissipation in collisions. Instead, it is assumed that there is an homogeneous particular solution of the pseudo-Liouville equation, known as the HCS, for which all the time dependence occurs through a scaling of the velocities with the thermal velocity, $v_0(t)$

$$\rho_{HCS}(\Gamma,t) = [v_0(t)l]^{-d(N_1+N_2)}\,\rho^*\left(\left\{\frac{\boldsymbol{r}_{\gamma i,\lambda j}}{l}, \frac{\boldsymbol{v}_{\gamma i}}{v_0(t)}\right\}\right), \tag{12}$$

where the function ρ^* is invariant under space translations, l is some characteristic length like the mean free path,

$$v_0(t) = \left[\frac{2T_{HCS}(t)}{\mu}\right]^{1/2} \tag{13}$$

with $\mu = m_1 m_2/(m_1+m_2)$, and $T_{HCS}(t)$ the granular temperature of the system,

$$T_{HCS}(t) = \sum_{\gamma=1,2}\frac{N_\gamma}{N_1+N_2}T_{\gamma,HCS}(t)\,. \tag{14}$$

Here, $T_{\gamma,HCS}(t)$ is the partial temperature of component γ, defined as

$$T_{\gamma,HCS}(t) = \frac{m_\gamma}{d}\langle v_{\gamma 1}^2(t)\rangle\,, \tag{15}$$

where the ensemble average has been taken with the HCS distribution defined by Eq. (12).

Equations for the time evolution of the partial temperatures can be easily derived using the Liouville dynamics to get

$$\frac{\partial T_{\gamma,HCS}(t)}{\partial t} = -\zeta_\gamma(t) T_{\gamma,HCS}(t)\,, \tag{16}$$

with the partial cooling rate $\zeta_\gamma(t)$ given by

$$\zeta_\gamma(t) = -\frac{m_\gamma}{dT_{\gamma,HCS}(t)}\int d\Gamma\left[\mathscr{L}v_{\gamma 1}^2\right]\rho_{HCS}(\Gamma,t)\,. \tag{17}$$

Also, the total temperature obeys the equation

$$\frac{\partial T_{HCS}(t)}{\partial t} = -\zeta(t) T_{HCS}(t)\,, \tag{18}$$

where the total cooling rate is

$$\zeta(t) = \frac{N_1 T_{1,HCS}(t)\zeta_1(t) + N_2 T_{2,HCS}(t)\zeta_2(t)}{N T_{HCS}(t)}. \tag{19}$$

A relevant consequence of the assumed scaling for $\rho_{HCS}(\Gamma,t)$ is that both partial temperatures $T_{1,HCS}(t)$ and $T_{2,HCS}(t)$ are proportional to $T_{HCS}(t)$, so their ratio, $T_{1,HCS}(t)/T_{2,HCS}(t)$ is constant, i.e.

$$\frac{\partial}{\partial t}\left[\frac{T_{1,HCS}(t)}{T_{2,HCS}(t)}\right] = 0. \tag{20}$$

This requires that

$$\zeta_1(t) = \zeta_2(t) = \zeta(t). \tag{21}$$

Another consequence of the scaling of the *HCS* distribution function is that $\zeta_\gamma(t) \propto T_{\gamma,HCS}(t)^{1/2}$. This leads directly to the Haff's law for the evolution of the partial temperatures,

$$T_{\gamma,HCS}(t) = T_{\gamma,HCS}(0)\left[1 + \frac{\zeta_\gamma(0)}{2}t\right]^{-2}. \tag{22}$$

In the long time limit, the above equation reduces to

$$T_{\gamma,HCS}(t) \sim 4\left(\overline{\zeta_\gamma}t\right)^{-2}, \tag{23}$$

where

$$\overline{\zeta_\gamma} = \frac{\zeta_\gamma(t)}{T_{\gamma,HCS}(t)^{1/2}}, \tag{24}$$

does depend neither on time nor on the initial temperature. Therefore, in the long time limit both the partial and the global temperatures become independent of the initial values. This is an exact property that follows from the existence of the HCS.

STEADY REPRESENTATION OF THE HCS

Let us define a new time scale τ by

$$\omega_0\tau = \ln\frac{t}{t_0}, \tag{25}$$

where ω_0 and t_0 are constants with dimensions of frequency and time, respectively. The velocity $\boldsymbol{w}_{\gamma i}$ of a particles in the new scale is

$$\boldsymbol{w}_{\gamma i} = \omega_0 t_0 e^{\omega_0\tau}\boldsymbol{v}_{\gamma i} = \omega_0 t\boldsymbol{v}_{\gamma i}. \tag{26}$$

The corresponding dynamics in these variables consists of an accelerating streaming between collisions,

$$\begin{aligned}\frac{\partial \boldsymbol{r}_{\gamma i}}{\partial\tau} &= \boldsymbol{w}_{\gamma i},\\ \frac{\partial \boldsymbol{w}_{\gamma i}}{\partial\tau} &= \omega_0\boldsymbol{w}_{\gamma i},\end{aligned} \tag{27}$$

while the effect of a collision between particle i of species γ and particle j of species λ is to instantaneously change their velocities accordingly to the same rules as given in Eq. (1), only changing the velocities $\boldsymbol{v}_{\gamma i}$ by the scaled ones $\boldsymbol{w}_{\gamma i}$. Of course, the invariance of the collision rule is a consequence of the instantaneous character of hard particles collisions.

The pseudo-Liouville equation in the scaled phase space $\tilde{\Gamma} \equiv \{\boldsymbol{r}_{\gamma i}, \boldsymbol{w}_{\gamma i}; \gamma = 1,2; i = 1,\ldots,N_\gamma\}$ becomes

$$\frac{\partial\tilde{\rho}(\tilde{\Gamma},\tau)}{\partial\tau} + \omega_0\sum_{\gamma=1,2}\sum_{i=1}^{N_\gamma}\frac{\partial}{\partial\boldsymbol{w}_{\gamma i}}\cdot\left[\boldsymbol{w}_{\gamma i}\tilde{\rho}(\tilde{\Gamma},\tau)\right] = \overline{\mathscr{L}}(\tilde{\Gamma})\tilde{\rho}(\tilde{\Gamma},\tau). \tag{28}$$

Here, $\overline{\mathscr{L}}(\tilde{\Gamma})$ is the same operator as in Eq. (10), but substituting velocitites $\boldsymbol{v}_{\gamma i}$ by $\boldsymbol{w}_{\gamma i}$. Also, $\tilde{\rho}(\tilde{\Gamma},\tau)$ is the transformed probability distribution,

$$\tilde{\rho}(\tilde{\Gamma},\tau) = (\omega_0 t)^{-d(N_1+N_2)} \rho(\Gamma,t)\,. \tag{29}$$

Then, the only change is that a new term appears in the streaming part of the equation. This new term can be understood as an accelerating homogeneous force proportional to the velocity of the particle.

The scaling of the HCS probability distribution in the phase space $\tilde{\Gamma}$ reads

$$\tilde{\rho}_{HCS}(\tilde{\Gamma},\tau) = [\tilde{v}_0(\tau) l]^{-d(N_1+N_2)}\, \rho^*\left(\left\{\frac{\boldsymbol{r}_{\gamma i,\lambda j}}{l}, \frac{\boldsymbol{w}_{\gamma i}}{\tilde{v}_0(\tau)}\right\}\right), \tag{30}$$

where

$$\tilde{v}_0(\tau) = \left[\frac{2\tilde{T}_{HCS}(\tau)}{\mu}\right]^{1/2} \tag{31}$$

with

$$\tilde{T}_{HCS}(\tau) = \sum_{\gamma=1,2} \frac{N_\gamma}{N_1+N_2} \tilde{T}_{\gamma,HCS}(\tau)\,, \tag{32}$$

and

$$\tilde{T}_{\gamma,HCS}(\tau) = \frac{m_\gamma}{d}\langle w_{\gamma 1}^2(\tau)\rangle = (\omega_0 t_0 e^{\omega_0\tau})^2 T_{\gamma,HCS}(t)\,. \tag{33}$$

Using Eq. (22) it is found that the time dependence of the partial temperatures in the τ scale is given by

$$\tilde{T}_{\gamma,HCS}(\tau) = \left(\frac{2\omega_0}{\overline{\zeta}_\gamma}\right)^2 \left[1 + \left(\frac{2\omega_0}{\overline{\zeta}_\gamma \tilde{T}_{\gamma,HCS}(0)^{1/2}} - 1\right) e^{-\omega_0\tau}\right]^{-2}. \tag{34}$$

Then, any initial value of the partial temperatures will equilibrate to a steady final value given by

$$\tilde{T}_{\gamma,st} = \left(\frac{2\omega_0}{\overline{\zeta}_\gamma}\right)^2. \tag{35}$$

Notice that this is consistent with the condition $\overline{\zeta}_1 \tilde{T}_{1,st} = \overline{\zeta}_2 \tilde{T}_{2,st}$, expressing the equalities of the cooling rates in the new variables. Also, Eq. (35) shows that the partial steady temperatures for a given value of ω_0 directly provide the reduced partial cooling rates $\overline{\zeta}_\gamma$.

Finally, the distribution function of the final steady state has the form

$$\tilde{\rho}_{HCS}^{st}(\tilde{\Gamma}) = \left[\tilde{v}_0^{st} l\right]^{-d(N_1+N_2)} \rho^*\left(\left\{\frac{\boldsymbol{r}_{\gamma i,\lambda j}}{l}, \frac{\boldsymbol{w}_{\gamma i}}{\tilde{v}_0^{st}}\right\}\right), \tag{36}$$

where $\tilde{v}_0^{st} = (2\tilde{T}_{HCS,st}/\mu)^{1/2}$, $\tilde{T}_{HCS,st} = \sum_{\gamma=1,2} N_\gamma \tilde{T}_{\gamma,st}/N$, and ρ^* obeys the equation

$$\omega_0 \sum_{\gamma=1,2} \sum_{i=1}^{N_\gamma} \frac{\partial}{\partial \boldsymbol{w}_{\gamma i}} \cdot \left[\boldsymbol{w}_{\gamma i} \rho^*(\tilde{\Gamma})\right] = \overline{\mathscr{L}}(\tilde{\Gamma}) \rho^*(\tilde{\Gamma})\,. \tag{37}$$

SIMULATION RESULTS

In order to check the practical usefulness of the steady representation, we have carried out particle simulations by using the Direct Monte Carlo method (DSMC) of a binary mixture of hard spheres ($d = 3$). This method is a N-particle simulation algorithm that mimics the dynamics of a low density gas [23]. This includes fluctuations and non-equilibrium correlations, although the precise relationship between the simulation algorithm and the theory describing the system from non-equilibrium statistical mechanics has not been established yet. One of the technical advantages of DSMC is that it allows to incorporate in the simulation the symmetries of the particular situation considered. For instance, here we want the system to stay in the HCS, so the information on the position of the particles is not necessary.

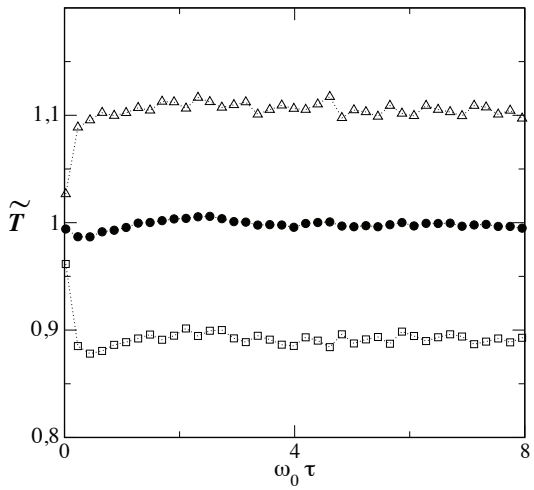

FIGURE 1. Evolution of the scaled temperatures for a system with $\alpha_{11} = 0.92$, $\alpha = 0.98$, $\Delta = 8$. The filled circles are the total temperature of the system, the empty squares the temperature of species 1, and the empty triangles that of species 2.

Since the technical details of the method have been extensively discussed in the literature [23, 24], they will not be reproduced here.

The simulations we will present are for a binary mixture with $N_1 = N_2 = N/2$, N being the total number of particles. In order to reduce the number of parameters still characterizing the problem, we have considered the diameters of the particles of the two species to be the same, i.e. $\sigma_1 = \sigma_2 = \sigma$, and also in all the cases it is $\alpha_{12} = (\alpha_{11} + \alpha_{22})/2$. Then, the properties of the system will depend on α_{11}, α_{22} and the mass ratio, $\Delta = m_2/m_1$. The number of particles in the system is $N = 2 \cdot 10^4$, but it must be noticed that this number has only an statistical meaning, since the dynamics corresponds always to that of an homogeneous, low density gas. Also, for each set of values of the parameters characterizing the system, twenty trajectories have been studied.

The simulations have been performed in the τ scale, i.e., the dynamics given by Eqs. (27), that is expected to yield a steady state after a transient period. The value of ω_0 was chosen in all the cases as $\omega_0 = \overline{\zeta}/2$, with $\overline{\zeta}$ approximated by its gaussian value, that has been given several times in the literature [3, 16]. If the gaussian value was exact, notice that the steady value of the total temperature would be one. Once in the steady state, properties can be time-averaged, as well as averaged over different trajectories. This improves the statistics of the measured properties, and it must be remembered that the steady state properties are directly related with those of the HCS. In Fig. 1 the evolution of the total temperature of the system (filled circles) and the partial temperatures of the two components (empty symbols) are shown for given a value of the inelasticity and mass ratio. The observed behavior is consistent with the theoretical prediction, and after a transient, all the three temperatures reach a steady value. It is important to remark that this implies also that the partial temperatures are proportional to the total one, as it follows from the scaling properties of the HCS. Also notice that the partial temperatures of the two components are different, showing that equipartition of energy is violated in the system.

From the steady values of the temperatures, the cooling rates in the HCS as well as the temperature ratio can be obtained. The results for two sets of values of the inelasticities are shown in Fig. 2 as functions of the mass asymmetry. In the case of the cooling rate, it has been scaled with $\lambda^{-1} v_0$, with λ the mean free path and v_0 the thermal velocity in the steady situation. The symbols are the results from the simulation, while the lines are the theoretical prediction using a gaussian approximation. The agreement is very good. Notice that, as expected, for a given value of the mass ratio the scaled cooling rate is larger the more inelastic the system. Also, the temperature difference between the two components is larger the more inelastic the system. For given inelasticity, a non-monotonic behavior of the scaled cooling rate with the mass ratio is observed, again clearer the more inelastic the system. For these values of the inelasticity, the scaled cooling rate is larger if the mass of the particles is similar. We must mention that results similar to these had been found in previous studies of the HCS of mixtures [8, 25], although not using the steady representation

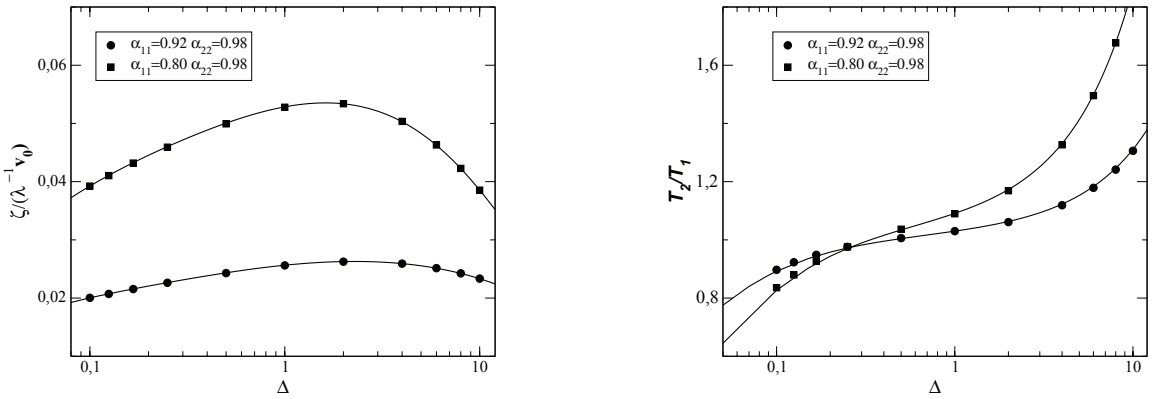

FIGURE 2. Scaled cooling rate (left) and partial temperatures ratio (right) as functions of the mass ratio for two set of values of the inelasticity, as indicated in the inset. The symbols are from the DSMC simulations, while the lines are the theoretical prediction using a gaussian approximation.

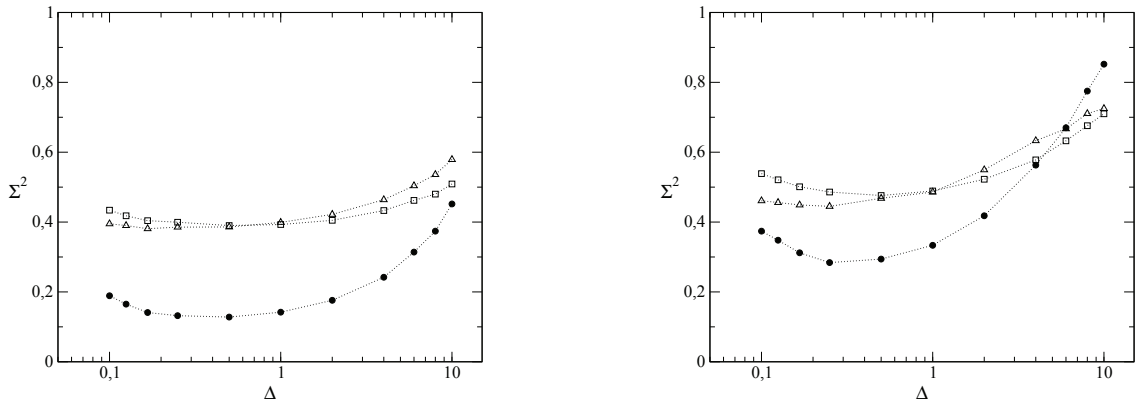

FIGURE 3. Second moment of the energy distribution multiplied by the number of particles for $\alpha_{11} = 0.92$, $\alpha_{22} = 0.98$ (left) and $\alpha_{11} = 0.80$, $\alpha_{22} = 0.98$ (right). The filled circles are for the energy of the total system,while the empty squares and triangles are for the energy of species 1 and 2, respectively (in the single component cases the moment is multiplied by $N/2$, the number of particles of the component).

of the HCS.

Finally, we have studied the total energy fluctuation distribution in the steady state for different inelasticities and mass ratios. In all the cases, it was found to be gaussian. The same applies for the distribution of energies of each of the components. In Fig. 3 we have plotted $\Sigma^2 = N\sigma_E^2$, where σ_E^2 is the second moment of the energy distribution, once in the steady state, i.e. $\sigma_E^2 = (\langle E^2\rangle - \langle E\rangle^2)/\langle E\rangle^2$. Notice that, as $\sigma_E \propto N^{-1/2}$, Σ^2 is expected to be independent of the number of particles. In the Figure, two different set of values of the inelasticities have been considered, and the results are shown as functions of the mass ratio. As it also happens in the case of a one-component system in the HCS [26], the fluctuations of the energy are larger the more inelastic the system. A non-monotonic behavior of the energy fluctuations on the mass ratio is observed, more clearly in the case of the total energy, and also stronger the

more inelastic the system. It must be noticed that, for not too different masses of the components, the fluctuations of the total energy are smaller than those of each component. This is not an statistical fact due to the different number of particles used to evaluate the total and partial energies, as this has been scaled out in Σ^2. The reason for this behavior are the correlations between the fluctuations of the two components, which are found to change in sign as the mass anisotropy increases.

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