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ON COMPLETELY CONTINUOUS MAPS FROM LOCALLY CONVEX SPACES TO ℓ_1

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A Banach space E is said to be Grothendieck if weak* and weak sequential convergence in E' coincide. Grothendieck spaces are close to the heart of every vector measure theorist (see [2]). Here is a list of reformulations of the definition of Grothendieck spaces which can be also seen in [2].

Theorem 1 [2]. *Any one of the following statements about a Banach space E implies all the others:*

(1) *E is a Grothendieck space.*
(2) *Every bounded linear operator from E to c_0 is weakly compact.*
(3) *For all Banach spaces F such that F' has a weak* sequentially compact unit ball, every bounded linear operator from E to F is weakly compact.*
(4) *For all weakly compactly generated Banach spaces F, every bounded linear operator from E to F is weakly compact.*

In the framework of locally convex spaces, we are going to show that if we change "sequential convergence" for "unconditional convergence of series" in the definition of Grothendieck spaces, we can obtain a similar result to Theorem 1 with the Banach space ℓ_1 and some other classes of Banach spaces. Moreover, we also give some other characterizations which involve perfect sequence spaces, copies of c_0 and Orlicz–Pettis topologies.

In what follows, for space we mean Hausdorff locally convex space. We refer the reader to Jarchow's monograph [4] for terminology and notation.

Now, we list some facts and definitions that we will use in the theorem.

(1) A space E is said to be Σ–complete if every unconditionally Cauchy series in E is convergent (see [1]). It's clear that every sequentially complete space is a Σ–complete space.
(2) There exists a finest Hausdorff locally convex topology on every space $[E,\tau]$ which has the same summable sequences as the original one (see [6]). This topology is denoted by $\mathcal{OP}(\tau)$.
(3) Let E and F be Banach spaces. In [3], Edgar introduced the following order relation:
$$E < F \quad \equiv \quad E = \bigcap_{T\in L(E,F)} (T^{**})^{-1}(F),$$
where $L(E,F)$ denotes as usual the set of linear continuous maps from E to F.
(4) A sequence space λ is said to be perfect if $\lambda = \lambda^{\times\times}$ (see [5]).

Our theorem is the following.

Theorem 2. *Assume that $[E', \sigma(E', E)]$ is a Σ-complete space, then the following are equivalent:*

(1) *$[E', \sigma(E', E'')]$ is Σ-complete.*
(2) *$\mathcal{OP}(\sigma(E', E))$ is finer than $\beta(E', E)$.*
(3) *$[E', \beta(E', E)]$ has no copy of c_0.*
(4) *Weak* and weak unconditional convergence of series in E' coincide.*
(5) *Every linear weakly continuous map from E to ℓ_1 carry bounded sets into relatively compact sets.*
(6) *For all Banach spaces F such that $F < \ell_1$ in Edgar's ordering, every linear weakly continuous map from E to F carry bounded sets into weakly relatively compact sets.*
(7) *For all separable weakly sequentially complete Banach lattices F, every linear weakly continuous map from E to F carry bounded sets into weakly relatively compact sets.*
(8) *For all perfect sequence spaces λ, every linear $\sigma(E, E')$-$\sigma(\lambda, \lambda^{\times})$ continuous map from E to λ maps bounded sets into weakly relatively compact sets.*

We notice that if E is a Banach space, the maps which carry bounded sets into weakly relatively compact sets are exactly the weakly compact maps.

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