Equivalent Circuit Model to Explain Extraordinary Transmission

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***Abstract*— This work proposes a circuit model based explanation for the extraordinary transmission (ET) of light phenomenon studied in recent scientific literature [1], [2]. ET mainly stands for unexpected transmission of light through periodic arrays of subwavelength holes in a metal screen. The study of this phenomenon has attracted the attention of many scientists working in the fields of Optics and Condensed Matter Physics, giving place to some controversial explanations. The existence of surface plasmons supported by the metal/air interface at optical frequencies has been considered the underlying reason behind ET. Our contribution tries to offer a relatively simple explanation of ET based on conventional waveguide/transmission-line theory. It will be shown how this simplified microwave-engineering standpoint offers satisfactory explanation for most ET findings. Indeed, ET should be expected not only at optical frequencies but also at lower frequencies, when surface plasmons are not possible.**

***Index Terms*— Extraordinary transmission, periodic structures, circuit modeling, surface plasmons.**

I. INTRODUCTION

Since Ebbesen and coworkers [3] reported on extraordinary transmission of light through perforated metal screens, hundreds of scientific papers have been published giving explanations and details on this (or related) phenomenon. The reported surprising fact was the obtaining, at a certain frequency, of a significant peak of transmission through an opaque screen perforated with holes whose diameters were significantly smaller than the corresponding wavelength (Bethe's theory for small holes predicted much less transmitted power than observed). The distinctive feature of the experimental device was that holes were arranged into a 2D periodic square lattice whose unit cell has dimensions close to the wavelength of the transmitted light. This strongly suggests that periodicity can play a crucial role. However, the first theoretical explanations put a lot of emphasis on the behavior of metals at optical frequencies. At these high frequencies metals are described by a complex permittivity having a large negative real part. Metals are then penetrable materials that can support a special kind of surface waves, the socalled surface plasmons. The excitation of such waves scattered by the periodic structure were then assumed to be the physical fact behind extraordinary transmission. However, ET has also been found in metal structures at millimeter wave frequencies [4], where metals are described by a real conductivity and penetration of electromagnetic fields is marginal (skin effect). In this situation surface plasmons are not possible in the metal/air interface, although a periodically perforated metal screen (even if metal is considered a perfect conductor) can still support surface waves, as it has been well known for decades in the microwaves community [5] and has been recently rediscovered [6]. Full-wave models accounting for both propagating and evanescent fields around the scattering surface (dynamical diffraction models) correctly account for ET phenomena and predict ET through periodically perforated perfect conductor plates and perfect dielectric slabs [7], [8], [9], [10]. Nevertheless, in recent review papers [1], [2] the role of surface plasmons is still considered essential.

Our purpose here is to offer a different point of view of the problem based on the obvious similarity of ET structures with frequency selective surfaces (FSS). Thus, we propose to apply the same methodology commonly used to deal with FSS structures [11] also to study ET structures. This new perspective allows us to use common and simple ideas coming from waveguide theory to explain the observed ET phenomena. The surprising predictive power of the proposed model will be shown in the following sections.

II. TRANSMISSION LINE MODELING

Let us consider the periodically perforated perfect conducting screen shown in Fig. 1(a). For normal incidence of a y-polarized planar TEM wave, the scattering problem for the whole structure can be reduced to the waveguide discontinuity problem in Figs. 1(c) and (d). The equivalent circuit model for this discontinuity is shown in Fig. 1(e). It should be noted that due to symmetry of both the structure and the excitation, AA' in Fig. 1(c) is an electric wall, while BB' is a magnetic one, which means that only even order modes are allowed in the structure under consideration (TE$_{2n,2m}$, TM$_{2n,2m}$ in the regions outside the hole, apart from the TEM mode). The π-circuit made of

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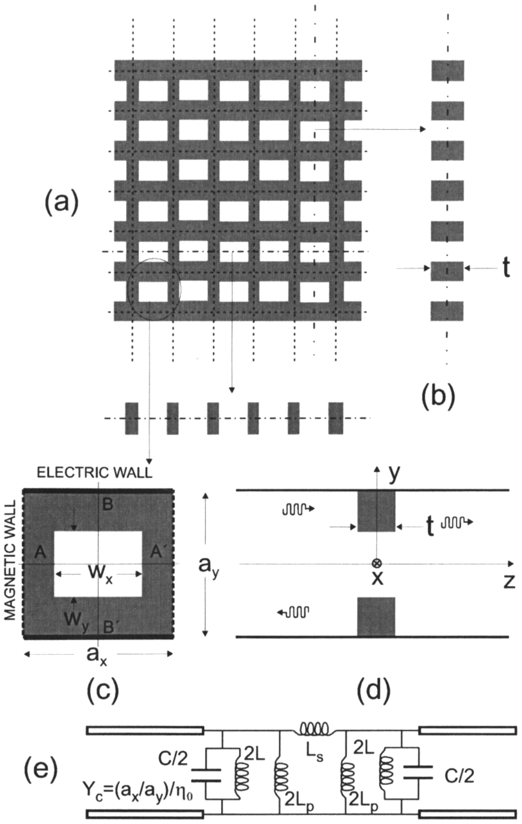

Fig. 1. Perfect conductor screen perforated with rectangular holes: front view (a) and two lateral cuts (b). Front (c) and lateral (d) views of the structure unit cell. Equivalent circuit for the fundamental TEM propagating and transmitted mode and discontinuity effects (e).

inductances in Fig. 1(e) accounts for the magnetic energy in excess stored by the below cutoff TE modes excited inside the rectangular hole. Obviously, this π-this circuit will not be present in the case of zero-thickness screens. In this last situation, the circuit model predicts total transmission of the impinging TEM wave at the resonance frequency of the remaining LC-tank circuit.

In common microwave engineering practice, the above C and L parameters are associated with the capacitance and inductance corresponding to the rectangular diaphragm. The size of this diaphragm is typically chosen so as to produce resonant transmission at a frequency well below the onset frequency of the first grating lobe (here denoted as $f_W = c/a_y$, and that is well known to be connected with a transmission zero known as *Wood's anomaly*). Total transmission ($|S_{21}|$=1) frequency can then be obtained as the first resonance frequency of a slot waveguide short-circuited at both ends. (The resonance condition would be $\beta_{\text{slot}} w_x = \pi$, where β_{slot} is the propagation constant of the slot mode, very close to the free space wavenumber $k_0 = \omega/c$.)

Alternatively, C can also be interpreted as the lumped circuit element that accounts for the electric energy (in excess) associated with the excitation of the TM modes below cutoff around the diaphragm, while L accounts for the magnetic energy (in excess) associated with the below cutoff TE modes. If C and L are large enough to produce zero shunt admittance at certain frequency below f_W, the circuit model in Fig. 1(e) will predict total transmission at such frequency. The bandwidth of this transmission peak would depend on the specific proportion of L and C yielding resonance. If losses were present, transmission would be limited.

Although some researchers have considered the above situation as "extraordinary transmission" (for instance, in [12] this kind of extraordinary transmission is called "localized waveguide resonance"), it is clear that this is the usual operation of a frequency selective surface and nothing "extraordinary" actually happens. Obviously, the resonance frequency and the details of this transmission peak will closely depend on the shape and size of the diaphragm.

The situation is slightly different when the diaphragm width (w_x) is well below half a wavelength of the impinging radiation. In such cases a very narrow peak of power transmission can be observed very close to but slightly below f_W [2]. This peak is particularly narrow for electrically very thin screens, such as those commonly used in practice. Moreover, operation near f_W has scarce practical interest because of the onset of the first grating lobes. The above two facts could explain why this peak has not been reported previously in the microwave area. However the circuit model in Fig. 1(e) accounts for the existence of that peak. The explanation is very simple: the capacitance C in Fig. 1(e) is almost constant for frequencies sufficiently far from (and below) $f_W = c/a_y$. However, for frequencies close to f_W, it should be taken into account that the imaginary admittance associated with the TM_{02} mode in the input and output transmission lines quickly goes to infinity at the cutoff frequency of the mode (which also is f_W). This means that C reaches very high values near f_W. More precisely, the expected frequency dependence of C can be written as

$$C(f) = C_0 + C_{\text{TM}_{02}}(f)\ , \tag{1}$$

where C_0 is the contribution of the higher order modes to the total capacitance and $C_{\text{TM}_{02}}$ is the particular contribution of the mode TM_{02}, given by

$$C_{\text{TM}_{02}}(f) = \frac{A_{\text{TM}}}{2\pi f \eta_0 \sqrt{\left(\frac{f_W}{f}\right)^2 - 1}}\ , \tag{2}$$

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with η_0 being the free-space impedance (the TM_{02} mode is assumed to be close to its cutoff frequency, f_W).

The parameter A_{TM} depends on the excitation degree of the TM_{02} mode, which increases around the ET frequency. The key point here is that for any value of L in Fig. 1(e), C in (1) always reaches the appropriate value to satisfy the resonance condition (thus producing extraordinary transmission). The presence of losses in the metal screen would probably affect the level of transmission but would not modify the essential physics of the phenomenon. The transmission peak near f_W in zero-thickness perfect conducting screens with small circular holes has been reported, for instance, in [8], where a much more complex analysis is reported. It is worth mentioning at this point that the proposed circuit model is also consistent with the existence of Wood's anomaly (namely, total reflection). Since $C \to \infty$ as $f \to f_W$, a virtual short circuit arises in such case and total reflection is then expected.

An additional interesting experimental observation is that extraordinary transmission also happens for electrically thick screens [2]. This observation is clearly accounted for byrigorous and cumbersome dynamical diffraction theory models, which predict (in agreement with experiments) the existence of *two* transmission peaks near f_W. It is thus interesting to see what our simplified circuit model predicts for this situation. If the screen has a significant thickness, the hole itself should be rather considered as a small section of rectangular waveguide operating well below its first cutoff frequency (corresponding to the TE_{10} mode of the small waveguide in Figs. 1(c) and (d)). From waveguide theory a reasonable circuit model for the TE_{10} mode below cutoff is the π-circuit made of inductances in Fig. 1(e). Circuit analysis for that situation predicts the existence of *two* transmission peaks near f_W; namely, the same result given by dynamical diffraction theory models and experiments.

Since our modeling of the situation reduces the original problem to the scattering of an impinging TEM mode by a thick diaphragm (see Figs. 1(c) and (d)), the well-known mode matching method can be applied to find reflection and transmission coefficients with relatively low computational effort. In next section we will provide numerical results obtained with this method and how these results match previously reported data (using different numerical models) and are consistent with the predictions of the equivalent circuit model in Fig. 1(e). Also it is important to mention that our model predicts that the existence of extraordinary transmission peaks is basically linked to periodicity along y-direction. eriodicity along x-direction does not play any relevant role in the appearance of those transmission peaks. Indeed, the model could be extrapolated to structures having no periodicity along the x-direction. This is consistent with previously reported results based on sophisticated full-wave simulations: a single row of holes (along the y-direction) also exhibits extraordinary transmission [13]. Finally it should be noticed that the sort of extraordinary transmission here analyzed can also happens in standard metallic waveguides with small diaphragms. From a physical point of view the situation would be almost the same, the only changes will come from the presence of electric walls instead of magnetic walls in the vertical boundary conditions in Fig. 1(c). Thus, a peak of transmission will occur at certain frequency close to the cutoff frequency of the first excited TM mode (below cutoff, of course), with this latter frequency also being a zero-transmission frequency. As a matter of fact, it is worth mentioning that our simplified model would provide a complementary physical explanation of the tunneling of electromagnetic energy through sub-wavelength channels and bends reported in [14].

III. RESULTS

In order to illustrate the previous theory, a couple of examples will be next considered. In Fig. 2 we show some details of the transmission spectrum of a perforated zero-thickness perfect metal screen (t is taken very small in the simulation) with small rectangular slits having two different orientations. From unit cell dimensions, f_W=59.959 GHz. From the diaphragms dimensions (inset in Fig. 2), using the short circuited slot-waveguide model, the intrinsic resonance frequency would be around 100 GHz, in such a way that no *regular* frequency selective surface operation is expected. Thus, transmission through the perforated screen is expected to be marginal below f_W. But the results of our mode-matching simulation shows that the situation dramatically changes near and below f_W (our mode-matching results have been validated after an excellent comparison with data from other methods, as the FDTD results in [12]). This frequency region is explored in Fig. 2 for the same rectangular slit with two different orientations with respect to the polarization

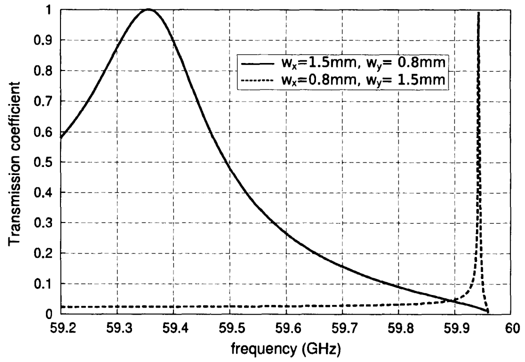

Fig. 2. Magnitude of the transmission coefficient. Dimension of the unit cell: $a_x = a_y = 5\,\mathrm{mm}$, $t = 0.1\,\mathrm{mm}$.

of the impinging field (y-directed). Note that a single narrow-width total transmission peak occurs near f_W. The transmission frequency for horizontal orientation of the slit is lower than the one for vertical orientation, and bandwidths are quite different. This result qualitatively agrees with the prediction of our equivalent-circuit model. The level of excitation of the below-cutoff TM_{02} mode is significantly higher for the horizontal orientation, which makes that C reaches higher values for frequencies lower than those expected for the vertical orientation. Obviously higher values of C leads to lower values of resonance frequency, and then of total transmission.

Our model also predicts the existence of two transmission peaks near f_W for finite-thickness screens, as can be clearly seen in Fig. 3. Note that, once again, the bandwidth of the first transmission peak is larger than the bandwidth of the second one, as corresponds to a smaller value of capacitance. It can also be seen that the two peaks are closer when the thickness is larger. This is consistent with the expected result of having larger L_s and smaller L_p as the thickness increases. In Fig. 3 we have compared our full-wave mode-matching results with data obtained from a circuit simulation using properly estimated values of L, L_s and L_p (see Fig. 1(e)), and C_0 and A_{TM} (see (1) and (2)). Perfect matching between the two sets of results clearly suggests that our circuit model is appropriate to account for this ET phenomenon.

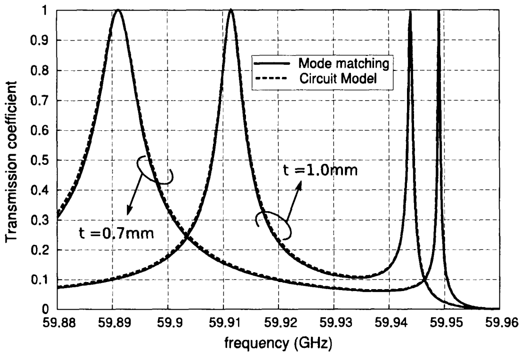

Fig. 3. Magnitude of the transmission coefficient. Dimension of the unit cell: $a_x = a_y = 5\,\mathrm{mm}$, $w_x = 1.5\,\mathrm{mm}$, $w_y = 0.5\,\mathrm{mm}$.

IV. CONCLUSION

The phenomenon of extraordinary transmission through metal screens periodically perforated with small holes can be interpreted in terms of the scattering of a TEM mode traveling along a parallel plate transmission line and scattered by a diaphragm discontinuity. This is a classical problem in the microwaves area which can be numerically solved using a mode matching scheme with high accuracy and low computational effort. Moreover the problem is amenable to a simple equivalent circuit that easily captures the physics of the problem. Although the results can be interpreted in terms of surface waves supported by the structured metal surface, the proposed model is simpler and opens the possibility of designing practical devices based on this phenomenon. Many more details of the experimentally observed and numerically computed transmission spectra can be also accounted for by our proposed equivalent-circuit model.

ACKNOWLEDGMENT

This work has been supported by the Spanish Ministry of Science and Education and European Union FEDER funds (projects TEC2007-65376 and TEC2007-68013-C02-01) and by Junta de Andalucía (project TIC-253).

The authors are indebted to Prof. M. Sorolla (Universidad Pública de Navarra, Spain) for his stimulus to work on this topic and clarifying discussions and to Profs. J.M. Rebollar and J.A. Ruiz-Cruz (Universidad Politécnica de Madrid and Universidad Autónoma de Madrid, Spain) for their invaluable help with mode matching method.

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