# Bad data identification when using ampere measurements

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# Bad Data Identification When Using Phasor Measurements

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***Abstract*—Synchronized phasor measurements obtained by phasor measurement units (PMU) are rapidly populating power systems. Among the expected benefits of having these measurements are the potential improvements in the accuracy and reliability of state estimation. On the other hand, state estimation formulation will have to be modified when phasor measurements are incorporated due to two issues. One is related to the reference bus selection which is no longer needed when phasor measurements are present. The other is the numerical problems encountered during initialization if current phasor measurements exist. This paper presents a reference-free rectangular state estimation method to address both of these problems. Furthermore, it is also shown that bad data in any of the phasor measurements can be detected and identified provided certain redundancy requirements are met. These requirements are illustrated using example systems and measurement configurations.**

***Index Terms* — PMU, power system state estimation, network observability, bad data processing, choice of reference bus, rectangular coordinates.**

## I. INTRODUCTION

PHASOR measurement units (PMU) are devices, which provide synchronized phasor measurements. This set of measurements enlarge the scope of the measurements used by the power system state estimator in addition to the traditional measurements such as power flows, net power injections and voltage and current magnitudes. Since phasor measurements are synchronized with respect to the time reference provided by the global positioning system (GPS) satellites, it eliminates the need to artificially define a phase angle of a bus voltage as the reference angle as done in conventional state estimators.

Devices that can measure synchronized phasors are developed and potential benefits of PMU measurements are recognized earlier by Phadke et al. [1-2]. One of the main objectives of installing these units is to improve the accuracy and reliability of the state estimator. Since the number of PMU devices installed in existing power systems is not sufficient to carry out state estimation exclusively based on PMU measurements, state estimation formulation and solution remains nonlinear and iterative respectively. A two-step procedure, which allows linear recursive estimation for the PMU measurements, is also possible [3].

In the absence of phase angle measurements, state estimation problem is formulated by choosing an arbitrary bus as the phase reference which is commonly assumed to be zero. This practice can be abadoned if phase angle measurements are made available. Furthermore, keeping an arbitrarily chosen angle reference will create inconsistencies. Treating the phase angle measurement of the reference bus may be one simple solution. In that case however, any errors associated with the measured phase angle will be unidentifiable since it will be assumed as the reference. Therefore, it is best to eliminate the reference bus [4] altogether and formulate the state estimation problem without any specified reference bus. This way, errors in any phasor measurement can be detected and identified provided there is enough measurement redundancy.

Phasor measurement units not only provide synchronized voltage phasors but also current phasors as well. While having these current phasors adds to the measurement redundancy and consequently improves estimation variance, they also create certain numerical problems. In particular, state estimation solution algorithm initialization becomes difficult when current phasor measurements are present. Entries of the measurement Jacobian corresponding to the current phasors will become undefined when they are evaluated at flat start. This issue can be addressed in a variety of ways. A simple alternative is to exclude current phasor measurements in the first iteration and incorporate them back after the second iteration. However, this approach will fail if one of the current phasor measurements is critical, i.e. excluding it will cause the system to become unobservable.

This paper presents a revised formulation using rectangular coordinates. This formulation allows the use of phasor voltage and current measurements without any numerical difficulties and allows detection and identification of errors in these phasor measurements. A convenient simplification is also presented by illustrating how the new Jacobian in rectangular coordinates can be decoupled, leading to a simpler observability test.

The paper is organized such that section II presents the proposed formulation with the discussion of issues related to network observability and bad data processing. Implementation details and the results of simulations are given in Section III. Section IV concludes the paper.

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## II. PROBLEM FORMULATION

### *Jacobian without a Reference Bus*

The power measurement model is:

$$z = h(x) + e \quad (1)$$

where:
$z$ is the measurement vector,
$h(x)$ is the nonlinear function relating the measurements to the system states,
$x$ is the system state vector including bus voltage magnitudes and phase angles,
$e$ is the vector of measurement errors.

By using weighted least square state estimation, equation (1) yields the following linear model:

$$\Delta z = H \cdot \Delta x + e \quad (2)$$

where:
$\Delta x = x - x_m \quad \Delta z = z - h(x_m)$
$x_m$ is the system state vector at the m-th iteration.

$$H = \frac{\partial h(x)}{\partial x} \quad (3)$$

When using phasor measurements, the matrix H will have 2N columns where N is the number of buses. This will allow estimation of all phase angles corresponding to all bus voltages in the system. If there are no phasor measurements, then an artificial phase angle measurement of zero will be introduced in order to make the column rank of H full. Hence, this formulation will work with or without phasor measurements.

### *Application of Rectangular Coordinates*

As mentioned in the introduction, the use of current phasor measurements leads to numerical difficulties during initialization of iterative state estimation solution. The expressions corresponding to the derivatives of current phasor measurements in the Jacobian will be as follows:

$$\frac{\partial I}{\partial V_n} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial V_n} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial V_n}\right) - \frac{S}{V_n^2} \quad (4)$$

$$\frac{\partial I}{\partial V_m} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial V_m} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial V_m}\right) \quad (5)$$

$$\frac{\partial I}{\partial \theta_n} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial \theta_n} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial \theta_n}\right) \quad (6)$$

$$\frac{\partial I}{\partial \theta_m} = \frac{1}{V_n}\left(\frac{P}{S}\cdot\frac{\partial P}{\partial \theta_m} + \frac{Q}{S}\cdot\frac{\partial Q}{\partial \theta_m}\right) \quad (7)$$

$$\frac{\partial \delta}{\partial V_n} = -\frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial V_n} - Q\cdot\frac{\partial P}{\partial V_n}\right) \quad (8)$$

$$\frac{\partial \delta}{\partial V_m} = -\frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial V_m} - Q\cdot\frac{\partial P}{\partial V_m}\right) \quad (9)$$

$$\frac{\partial \delta}{\partial \theta_n} = 1 - \frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial \theta_n} - Q\cdot\frac{\partial P}{\partial \theta_n}\right) \quad (10)$$

$$\frac{\partial \delta}{\partial \theta_m} = -\frac{1}{S^2}\left(P\cdot\frac{\partial Q}{\partial \theta_m} - Q\cdot\frac{\partial P}{\partial \theta_m}\right) \quad (11)$$

where:
$S$ is the magnitude of complex power from bus n to bus m,
$P$ and $Q$ are the real and reactive power flow from bus n to bus m,
$V$ and $\theta$ are bus voltage magnitude and phase angle,
$I$ and $\delta$ are current magnitude and phase angle.

At flat start, which assumes that all bus voltage magnitudes are equal to 1.0 and all the phase angles are equal to 0, the complex power $S$ which appears in the denominator of the above equations will be zero. Hence, several Jacobian entries will be undefined at flat start. Admittedly, by excluding the current measurements in the first iteration and introducing them afterwards, this problem can be avoided. This may or may not be possible depending on the criticality of current phasor measurements. A better solution which will work independent of criticality is the use of rectangular coordinates in the formulation. The resulting new Jacobian H will have the form shown in Table 1 and will be labeled as $H_n$ to differentiate it from the polar version $H$.

Table 1 Structure of $H_n$

| | $E$ | $F$ |
|---|---|---|
| $P_i$ | $\frac{\partial P_i}{\partial E}$ | $\frac{\partial P_i}{\partial F}$ |
| $Q_i$ | $\frac{\partial Q_i}{\partial E}$ | $\frac{\partial Q_i}{\partial F}$ |
| $P_f$ | $\frac{\partial P_f}{\partial E}$ | $\frac{\partial P_f}{\partial F}$ |
| $Q_f$ | $\frac{\partial Q_f}{\partial E}$ | $\frac{\partial Q_f}{\partial F}$ |
| $E$ | 1 | --- |
| $F$ | --- | 1 |
| $C$ | $\frac{\partial C}{\partial E}$ | $\frac{\partial C}{\partial F}$ |
| $D$ | $\frac{\partial D}{\partial E}$ | $\frac{\partial D}{\partial F}$ |

where:
$P_i$, $Q_i$, $P_f$, $Q_f$ are real and reactive power injections and flows, respectively,
$E$ and $F$ are real and imaginary parts of bus voltages,
$C$ and $D$ are real and imaginary parts of currents.

The partial derivatives of $P_f$, $Q_f$, $C$ and $D$ can be derived as below. Consider the real and reactive power flows from bus n to bus m:

$$P_{nm} = \frac{\left(E_n^2 + F_n^2\right)r - \left(E_n E_m + F_n F_m\right)r + \left(F_n E_m - E_n F_m\right)x}{r^2 + x^2} \quad (12)$$

$$Q_{nm} = \frac{\left(E_n^2 + F_n^2\right)x - \left(F_n E_m - E_n F_m\right)r - \left(E_n E_m + F_n F_m\right)x}{r^2 + x^2} \quad (13)$$
$$-\left(E_n^2 + F_n^2\right)b$$

where $r$, $x$ and $b$ are resistance, reactance and susceptance of line n-m respectively.
Their partial derivatives are:

$$\frac{\partial P}{\partial E_n} = \frac{2rE_n - rE_m - xF_m}{r^2 + x^2} \quad (14)$$

$$\frac{\partial P}{\partial E_m} = \frac{xF_n - rE_n}{r^2 + x^2} \quad (15)$$

$$\frac{\partial P}{\partial F_n} = \frac{2rF_n - rF_m + xE_m}{r^2 + x^2} \quad (16)$$

$$\frac{\partial P}{\partial F_m} = \frac{-rF_n - xE_n}{r^2 + x^2} \quad (17)$$

$$\frac{\partial Q}{\partial E_n} = \frac{2xE_n + rF_m - xE_m}{r^2 + x^2} - 2bE_n \quad (18)$$

$$\frac{\partial Q}{\partial E_m} = \frac{-rF_n - xE_n}{r^2 + x^2} \quad (19)$$

$$\frac{\partial Q}{\partial F_n} = \frac{2xF_n - rE_m - xF_m}{r^2 + x^2} - 2bF_n \quad (20)$$

$$\frac{\partial Q}{\partial F_m} = \frac{rE_n - xF_n}{r^2 + x^2} \quad (21)$$

Expressing the current from bus n to bus m:

$$I = \frac{V_n - V_m}{Z} + V_n b \quad (22)$$

where $Z = r + jx$ is the impedance of line n-m.
Real and imaginary parts of the current n-m will be:

$$C = \frac{\left(E_n - E_m\right)r + \left(F_n - F_m\right)x}{r^2 + x^2} - F_n b \quad (23)$$

$$D = \frac{\left(F_n - F_m\right)r - \left(E_n - E_m\right)x}{r^2 + x^2} + E_n b \quad (24)$$

and their partial derivatives can be derived as:

$$\frac{\partial C}{\partial E_n} = \frac{r}{r^2 + x^2} \quad (25)$$

$$\frac{\partial C}{\partial E_m} = \frac{-r}{r^2 + x^2} \quad (26)$$

$$\frac{\partial C}{\partial F_n} = \frac{x}{r^2 + x^2} - b \quad (27)$$

$$\frac{\partial C}{\partial F_m} = \frac{-x}{r^2 + x^2} \quad (28)$$

$$\frac{\partial D}{\partial E_n} = b - \frac{x}{r^2 + x^2} \quad (29)$$

$$\frac{\partial D}{\partial E_m} = \frac{x}{r^2 + x^2} \quad (30)$$

$$\frac{\partial D}{\partial F_n} = \frac{r}{r^2 + x^2} \quad (31)$$

$$\frac{\partial D}{\partial F_m} = \frac{-r}{r^2 + x^2} \quad (32)$$

Note that these terms are well defined even at flat start, allowing calculation of the Jacobian and successful initialization of state estimation solution.

*Observability of Network*

Assuming that $|F| << |E|$, and $r << x$, which are typically true in most power systems, it can be shown that the Jacobian can be decoupled. Substituting $E = 1$, $F = 0$, $r = 0$ and $x = 1$, in equations (14)-(21) and (25)-(32):

$$\frac{\partial P}{\partial E_n} = 0 \quad \frac{\partial P}{\partial E_m} = 0 \quad \frac{\partial P}{\partial F_n} = 1 \quad \frac{\partial P}{\partial F_m} = -1 \quad (33)$$

$$\frac{\partial Q}{\partial E_n} = 1 \quad \frac{\partial Q}{\partial E_m} = -1 \quad \frac{\partial Q}{\partial F_n} = 0 \quad \frac{\partial Q}{\partial F_m} = 0 \quad (34)$$

$$\frac{\partial C}{\partial E_n} = 0 \quad \frac{\partial C}{\partial E_m} = 0 \quad \frac{\partial C}{\partial F_n} = 1 \quad \frac{\partial C}{\partial F_m} = -1 \quad (35)$$

$$\frac{\partial D}{\partial E_n} = -1 \quad \frac{\partial D}{\partial E_m} = 1 \quad \frac{\partial D}{\partial F_n} = 0 \quad \frac{\partial D}{\partial F_m} = 0 \quad (36)$$

Note that P and C are strongly correlated with F while Q and D are strongly correlated with E. So the Jacobian $H_n$ can be decoupled as shown below:

$$H_n \doteq \left[\begin{array}{c|c} H_{PF} & \\ H_{FF} & \\ H_{CF} & \\ \hline & H_{QE} \\ & H_{EE} \\ & H_{DE} \end{array}\right] \quad (37)$$

where:

$H_{PF} = \partial P / \partial F$, $H_{FF} = \partial F / \partial F$, $H_{CF} = \partial C / \partial F$
$H_{QE} = \partial Q / \partial E$, $H_{EE} = \partial E / \partial E$, $H_{DE} = \partial D / \partial E$

The gain matrix can then be written as:

$$G_n = \begin{bmatrix} G_F & 0 \\ 0 & G_E \end{bmatrix} \quad (38)$$

where:

$G_F = H_F^T W_F H_F$ and $G_E = H_E^T W_E H_E$

$$H_F = \begin{bmatrix} H_{PF} \\ H_{FF} \\ H_{CF} \end{bmatrix} \text{ and } H_E = \begin{bmatrix} H_{QE} \\ H_{EE} \\ H_{DE} \end{bmatrix}.$$

$W_F$ and $W_E$ are the diagonal weight matrices of the corresponding measurements.

Using the well documented numerical observability method, network observability can be checked by identifying zero pivots during the factorization of the gain matrix [5]. As done with the conventional measurements, it is sufficient to form the decoupled gain matrix that uses only the real power measurements as defined below:

$$G_{AA} = H_{AA}^T \cdot R_A^{-1} \cdot H_{AA} \qquad (16)$$

where

$H_{AA} = \frac{\partial h_A}{\partial \theta}$ is the decoupled Jacobian for the real power measurements.

$R_A$ is the diagonal matrix of real power measurement variances.

In forming the measurement Jacobian $H_{AA}$ the columns corresponding to all bus voltage phase angles are used. Hence, when using the conventional formulation with a single reference bus, the system will be declared observable if only one zero pivot is encountered during the Cholesky factorization of $G_{AA}$. The observability analysis with phasor measurements will be carried out the same way except that the matrix $G_{AA}$ will be substituted by $G_F$. Since no reference bus exists, the system will be declared as observable if no zero pivots are encountered while factorizing $G_F$.

*Bad Data Processing*

Detecting and identifying bad data not only in conventional measurements but also in phasor measurements is very important since these errors will have a significant impact on the estimated state for the entire system.

When there is only one phase angle measurement in the system, then this case can be reduced to the conventional formulation with an assigned reference bus. Since the value of its phase angle is irrelevant, errors in this measurement will not affect the estimation results. When there are two or more phasor measurements in the system, detection of phasor measurement errors requires higher redundancy as discussed below.

Excluding the phasor measurements, conventional network observability analysis [5] will yield the number of observable islands in a given system. Having at least one phasor measurement in every observable island will ensure observability for the entire network.

In order to be able to detect and identify errors in the phasor measurements, higher levels of redundancy will be required in their respective observable islands. It can be shown that two phasor measurements will ensure detectability and three will be necessary for identification of bad data associated with any phasor measurement in a given observable island.

The following section will present several simulated cases which will be used to experimentally verify the properties of the proposed state estimation formulation. Specifically, analysis of network observability and bad data processing in the presence of phasor measurements will be illustrated.

## III. Simulation Results

The proposed rectangular state estimator is implemented and tested using the IEEE 57-bus system as an example.

### *A. State estimation with and without a reference*

This case simply illustrates the effect of eliminating phase reference in state estimation when using phasor measurements. A single error in phase angle measurement is introduced in the 57-bus system which is fully observable. Three PMUs are assumed to exist at buses 5, 18 and 37. Each PMU is assumed to measure one voltage and two pairs of current phasors. Two cases are simulated:

Test A: Rectangular state estimation using bus 5 as reference bus. Note that there are three PMUs in the system and the one at bus 5 is assumed to be the reference PMU. This reference phase angle measurement is considered to have an error.

Test B: Rectangular state estimation formulated without any specific reference bus. Again, the phase angle measured by the PMU at bus 5 is assumed to be in error.

Table 2 shows the sorted normalized residuals $r^N$ obtained for test B. Statistical threshold of 3.0 is used for $r^N$ in all tests. Note that the state estimation of test A fails to converge, while test B correctly identifies the erroneous phase measurement at bus 5.

Table 2 Results of Error Identification

| Test A | Test B | |
|---|---|---|
| | **Measurement** | **Normalized residual** |
| Did not converge | $f_5$ | 53.99 |
| | $f_{18}$ | 44.59 |
| | $c_{18-4}$ | 34.24 |
| | $p_{15-45}$ | 16.19 |
| | $f_{37}$ | 15.85 |

As evident from the above, keeping the reference bus will lead to divergence of the state estimation process when using rectangular coordinates. This undesirable situation can be avoided by a simple revision in the problem formulation where no explicit phase angle is used as the reference. The following cases will elaborate on this further by studying observability and bad data processing issues for different phasor measurement configurations.

### B. Merging Observable Islands Using Phasor Measurements

This case considers the situation where phasor measurements are used to merge several observable islands. Only few phasor measurements are required to be introduced into such system to merge the islands into one.

The test system is shown in Figure 1. Note that phasor measurements are not considered yet in this figure. The system is composed of 4 observable islands that are indicated by the lines in the figure.

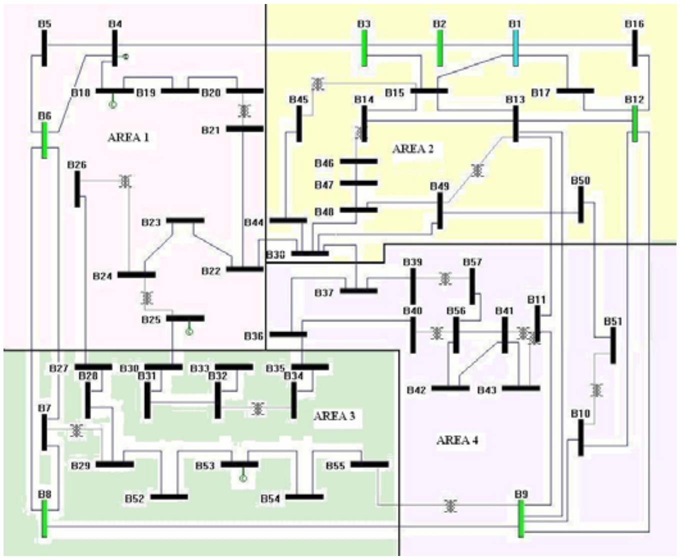


Fig 1. 57-bus system with 4 observable islands

Following tests are carried out:

Test A: Three voltage phasor measurements ($e_1$, $f_1$), ($e_{29}$, $f_{29}$), ($e_{42}$, $f_{42}$).

Test B: Four voltage phasor measurements ($e_1$, $f_1$), ($e_{29}$, $f_{29}$), ($e_{42}$, $f_{42}$), ($e_5$, $f_5$).

The simulation results are shown in Table 3.

Table 3 Results of Network Observability Analysis

| Test A | Test B |
|---|---|
| Two observable islands: Island 1: Buses 4, 5, 6, 18, 19, 20, 21, 22, 23, 24, 25, 26 Island 2: Rest of the buses. | Entire system is a single observable island. |

Assigning one phasor measurement per observable island is sufficient to merge the islands into an observable network.

### C. Bad Data Identification in Phasor Measurements

Consider the measurement system shown in Figure 1. This 57-bus system is divided into 4 areas. Among them, area 1 and rest of the system form two observable islands when there are no phasor measurements. Four voltage phasor measurements at buses 1, 5, 29, and 42 are introduced.

Following cases are simulated:

Test A: There is only one voltage phasor measurement in area 1 at bus 5. The phase angle measurement at bus 5 is assumed to be in error.

Test B: Two more phasor measurements ($e_{23}$, $f_{23}$), ($e_{20}$, $f_{20}$) are assumed to exist in the system. Again, the phase angle measurement of bus 5 is considered to be in error.

Note that in test A there is a single phasor measurement in the first observable island while in test B there are a redundant set of three. No current phasor measurements are considered in this case. The results are shown in Table 4.

Table 4 Identification of Phasor Measurement Errors

| Test A | | Test B | |
|---|---|---|---|
| **Measurement** | **Normalized residual** | **Measurement** | **Normalized residual** |
| $p_5$ | 0.044 | $f_5$ | 14.06 |
| $p_{4-5}$ | 0.036 | $f_{20}$ | 12.30 |
| $p_{5-6}$ | 0.035 | $p_{18-19}$ | 10.12 |
| $p_{11}$ | 0.015 | $p_{4-18}$ | 8.64 |
| $p_{41-43}$ | 0.015 | $e_5$ | 6.20 |

In case A, phasor measurement at bus 5 is a critical measurement. Hence, any error in this measurement will go undetected, as evident from the normalized residuals shown in Table 4, column 2. When additional phasor measurements are considered in case B, bad data in voltage phasor at bus 5 is detected and identified by using the largest normalized residual test.

## IV. Conclusions

This paper studies the use of phasor measurements in state estimation and addresses two issues related to their utilization. One of them is the choice of a reference phasor. It is shown that no such reference is needed when phasor measurements are present. Elimination of the reference phasor also facilitates bad data processing for conventional as well as phasor measurements in the system. The paper also proposes a rectangular coordinate formulation by which numerical problems encountered during flat start when using current phasors, will be avoided.

Simulation results are used to illustrate how phasor measurements can be used to merge observable islands. Bad data processing for phasor measurements is also studied and cases of undetectable phasor measurement errors are illustrated. These results may be useful for the placement of new phasor measurement units in a power system.

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## VI. BIOGRAPHIES

**Jun Zhu** was born in Shanghai, China, on February 15, 1978. He received his B. S. degree from Department of Electrical Engineering, Shanghai Jiaotong University, Shanghai, China, in 2000. In the following two years, he worked as a system engineer dealing with the control system of power plant for Shanghai Automation Instrumentation Co. Ltd. In 2004, he received his M. S. degree from Department of Electrical Engineering, Texas A&M University, College Station, TX. He is currently pursuing his Ph.D. degree in Department of Electrical Engineering at Northeastern University. His research field is power system monitoring and control.

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