ON SIMPLIFICATION TECHNIQUES FOR SYMBOLIC ANALYSIS OF ANALOG INTEGRATED CIRCUITS

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Abstract: This paper addresses the topic of formula simplification for symbolic analyzers. Previously reported criteria for flat analysis approaches are briefly reviewed and their limitations illustrated via examples of practical circuits. An adaptive simplification method using pole/zero monitoring for error control is given to alleviate some of these problems. Then a new simplification strategy is presented which takes into account potential ranges of variation of the different symbolic parameters and which overcomes the drawbacks of previous criteria. An algorithm for the approximation of symbolic formula obtained via hierarchical symbolic analysis approaches is also outlined.

Introduction

During the last few years symbolic analyzers have received considerable attention as very valuable tools for different topics related to analogue integrated circuits, as for *teaching* [1], *design space exploration* [2], *optimum topology selection* [3], *design automation* [4], *behavioral modeling* [4], *fault diagnosis* [5], etc. Symbolic analyzers work on networks defined as the interconnection of linear models and calculate related network functions where all or part of the model circuit parameters are kept as symbols. The efficient calculation of symbolic network functions defines the elementary basic functionality of symbolic analyzers. However this basic functionality is not enough for most applications and large amount of formula post-processing is needed in order the results to be appropriately exploited.

Among the different types of formula postprocessing usually required, this paper focuses on that of *formula approximation*. By formula approximation (equivalently formula simplification) we mean the reduction of the *complexity* (measured in number of terms) of symbolic expressions via the elimination of *insignificant terms* according to the numerical estimation on the symbolic terms using *typical values* of the parameters. Reducing the formula complexity is mandatory in analog integrated circuits described at the device level due to the huge amount of terms appearing in even elementary building blocks. Consider to the purpose of illustration the calculation of the output impedance of the cascode current mirror of Fig.1a, using the MOS transistor model of Fig.1b. The formula delivered by the symbolic analyzer ASAP [2] contains 1384 different terms. This number is not definitively astonishing by itself, but assume we are trying to use this formula for educational purposes, to gain insight into the operation of the circuit. It is clear that a compact formula, containing just a few dominant terms, is much more appropriate to this purpose. In fact the expressions that can be found in advanced analog circuit textbooks for this characteristics contain less than 10 terms [6,7,8]. Applications involving repetitive formula evaluations also impose the need of reducing formula complexity. Consider for instance the task of model compilation for automated circuit dimensioning using statistical optimization [9]. Assume a not very complex CMOS building block as it is the nine transistor folded cascode OTA shown in Fig.1c. The formula for the voltage gain of this opamp using the model of Fig.1b contains 97953!! different terms [2]. Similar number of terms will be obtained for other characteristics which are worth considering for analog design like CMRR, PSRR, noise, input capacitance, etc. Trying to create a model including all the characteristics needed for the AC design of this device and using exact symbolic expressions will be probably beyond the capabilities of many compilers. Even if the compilation were possible the required computation time for the iterative design procedure would be extremely high. The necessity of simplifications can be hence seen also from this application point of view.

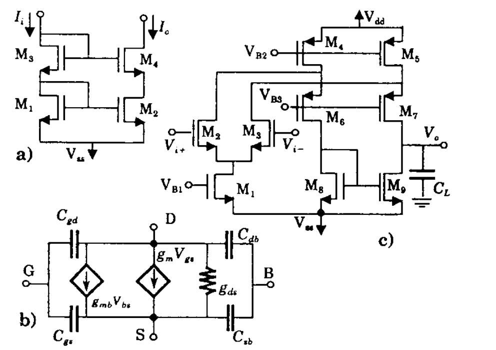

Figure 1: Current mirror, AC MOS model and cascode OTA

Previous approaches to symbolic formula approximation have focused only on flat symbolic analysis techniques [2,4,10,11]. Besides, these approaches consider just a single point of the complete parameter space to estimate relative sizes of symbolic terms. In this paper we first briefly review previously reported simplification criteria, illustrating via examples some of their drawbacks. An adaptive criterion is then introduced which uses a variable error margin controlled via pole/zero monitorization during the simplification process. Also a completely different simplification approach is presented where numerical estimation is not made at a single point of the parameter space but inside a region defined by taking into account typical variation ranges of the different symbolic parameters. Finally, an algorithm is outlined for the simplification of formulas resulting from hierarchical symbolic analysis [12].

A Glimpse on Flat Simplification Techniques

Assume a generic symbolic network function $H(s,\mathbf{x})$, where s represents the complex frequency and $\mathbf{x}^T=\{x_1, x_2, \ldots x_N\}$ is the vector of circuit symbolic parameters. The process of simplification consists in the pruning of insignificant terms in the

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numerator and the denominator of $H(.)$ to yield a reduced function $H_r(s,\mathbf{x})$ which approximates the original one under a predefined error and inside a given interval of both s and $\mathbf{x}$.

In this paper no restriction will be assumed on the frequency range for simplification. This is the case covered typically in literature [2,4,10,11]. Also, this assumption does not mean any loss of generality since it includes as subcases those situations where either a limited frequency range or a single frequency is specified.

Network functions resulting from a flat symbolic analysis may either be presented in a *nested* format, like for instance,

$$\frac{[\{(a_2+a_3)a_4-a_5(a_6+a_7)\}+(a_8+a_9)]a_1}{[\{b_1b_2b_3-b_4\}+b_5b_6b_7(b_8+b_9)]} \quad (1)$$

where b_i and a_i represents subexpressions in the complex frequency, or in an *expanded* format, like

$$\frac{s^4a_4+s^3a_3+s^2a_2+sa_1+a_0}{s^4b_4+s^3b_3+s^2b_2+sb_1+b_0} \quad (2)$$

where b_i and a_i do not contain the complex frequency. In what follows previously reported simplification techniques for each one of these formats are briefly reviewed.

Expanded Format

In an expanded format both the numerator and the denominator of the symbolic network functions are given in the form of ordered polynomial in s,

$$H(s,\underline{x}) = \frac{\sum_{i=0,N} s^i f_i(\underline{x})}{\sum_{j=0,M} s^j g_j(\underline{x})} \quad (3)$$

the coefficients of the different powers being in their turn polynomials in the symbolic parameters,

$$h_k(\underline{x}) = \sum_{l=1,L} h_{kl}(\underline{x}) \quad (4)$$

where $h_k(\mathbf{x})$ represents either $f_i(\mathbf{x})$ or $g_j(\mathbf{x})$ in (3) and the terms $h_{kl}(\mathbf{x})$ are product of symbols.

Simplifications are performed by deleting the least significant terms in each coefficient polynomial. To this purpose numerical information concerning the relative size of the parameters must be provided. Four basic simplification criteria have been reported [2,4,11]. For all of them a single typical numerical value for each parameter is used (It is to say, for simplifications the symbolic expressions are evaluated at a single point $\mathbf{x}_0$ of the parameter space).

The simplest criteria can be found in [11]. There the largest magnitude term is identified and multiplied by an error figure specified by the user. Then all the terms whose magnitude is below the resulting value are eliminated. It is to say, a term $h_{kl}(\mathbf{x})$ will be eliminated in case it fulfills the following,

$$|h_{kl}(\underline{x})| < \begin{matrix}\varepsilon\{max(|h_{kl}(\underline{x})|)\}\\ for, l = 1, 2, \ldots L\end{matrix} \quad (5)$$

Since no assessment is made on the relative value of the maximum term as compared to the result of the summation of deleted terms, large errors are to be expected as a consequence of using this criteria. In order to increase the accuracy of the simplification process the eliminated terms should be compared not just to the most significant term but to all the terms in $h_k(\mathbf{x})$. It can be made by using one of the three alternatives shown in Table 1, where expressions are given for the deletion of the P least significant terms of $h_k(\mathbf{x})$. Notice in all these cases it is required that the numbers $h_{kl}(\mathbf{x}_0)$ are ordered according to their magnitude. Hence these criteria are more demanding from the computational point of view that the one in (5).

$$\frac{\left|\sum_{l=1,P} h_{kl}(\underline{x}_o)\right|}{\left|\sum_{l=1,L} h_{kl}(\underline{x}_o)\right|} < \varepsilon, \quad \frac{\sum_{l=1,P} |h_{kl}(\underline{x}_o)|}{\left|\sum_{l=1,L} h_{kl}(\underline{x}_o)\right|} < \varepsilon, \quad \frac{\sum_{l=1,P} |h_{kl}(\underline{x}_o)|}{\sum_{l=1,L} |h_{kl}(\underline{x}_o)|} < \varepsilon$$

Table 1: Relative value simplification criteria

Consider first the so called signed object signed reference technique given in the first column of Table 1.Notice signed terms are added in both the numerator and the denominator. Thus, contributions from opposite signed terms mutually cancel themselves. As a consequence this criterion gives the most accurate results at the nominal point $\mathbf{x}_0$. However the eventual cancellation of large magnitude opposite terms may result in important errors when the simplified formula is evaluated at points of the parameter space different from the nominal one. Unfortunately, this is not an uncommon situation in analogue integrated circuits, due to the mismatch among nominally matched devices. Consider for the sake of illustration the calculation of the DC voltage gain of the circuit in Fig.2 where a positive feedback OTA for high-Q SC filters is shown [13]. This spec will be given as a ratio of two symbolic coefficients a/b. Application of the criterion in column 1 of Table 1 for a 25% margin error (parameter ε) yields an actual error at the nominal point of 24.9% for both a and b. However when a mismatch of .1% is considered among the nominally matched transconductances of the amplifiers the error in b increases to 3,170%!! while does not significantly change for a.

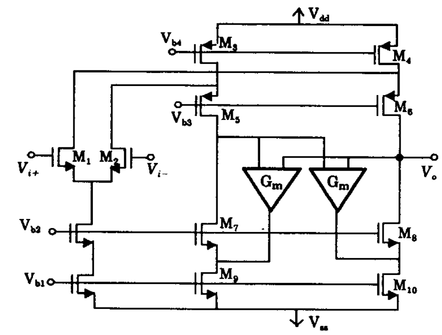

Figure 2: Positive feedback OTA [13]

The problems mentioned above can be overcome by the use of the unsigned object signed reference criterion given in the second column of Table 1. With this criterion opposite sign terms are eliminated only in case they are really nonsignificant. Notice however that, since terms in the reference used for simplification (the denominator) remain signed, the actual error margin may be much smaller for those coefficients containing opposite sign terms than for those others containing just negative or positive terms. As a consequence, large disparities will appear among the coefficients of the different powers of s in the numerator andr the denominator of the network functions, thereby yielding large pole/zero displacements. To avoid this the unsigned object unsigned reference criterion shown in third column of Table 1 can be used.

This criterion may also produce problematic situations in case the value of the error margin is not chosen small enough. As for the first criterion the cause for these problems will be again the existence of coefficients containing terms with large magnitude value and opposite sign. Nevertheless the problems arising in both criteria are not of the same type. In the first criterion problems arise when the simplified formula is evaluated at points different from the nominal one. However,in the latter criterion large inaccuracies may be observed at the nominal point while no important additional changes are to be expected when a displacement around this nominal point is made.

To illustrate problems in the unsigned object unsigned reference criterion consider again the circuit of Fig.2 and an error margin of 25%. Fig.3 gives the magnitude and phase plots for both the exact and the simplified expression of the voltage gain of that circuit. Large errors can be observed. They are due to the fact that the independent coefficient as well as the coefficient for the s power and the s^2 power of the transfer function denominator contain large terms of opposite sign. As a consequence the reference used for simplification is large and many terms are eliminated which are comparable in magnitude to the modulus of the coefficient value

$$\left|\sum_{l=1,L} h_{kl}(x_o)\right| \equiv S_D \tag{6}$$

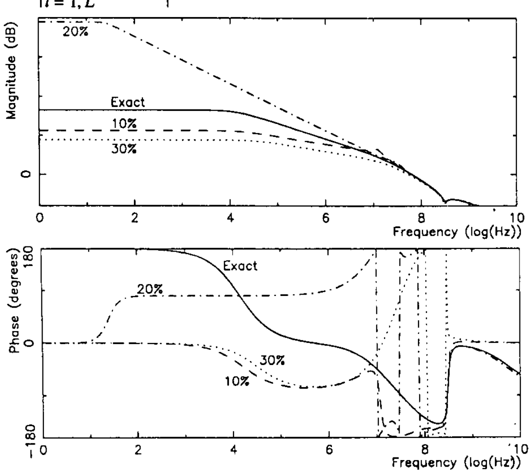

Figure 3: Magnitude and phase plots for Fig.2

One solution to this problem is to monitor the magnitude of each term in order to avoid the elimination of any terms whose magnitude is greater than S_D [4]. Another approach providing better control on the accuracy of the simplified expression consists in the implementation of an adaptive ε in such a way that simplifications are stopped when pole/zero displacements are beyond a predefined margin [2]. Performance of this approach is illustrated for the current gain of the active current mirror of Fig.4a [14]. Fig.4b shows the pole loci for this gain as functions of ε in case simplifications are made by using the unsigned object unsigned reference criterion. As it can be seen there are a pair of complex conjugate poles passing from the left to the right of the complex frequency plane for ε=0.17. This movement, which induces qualitative errors in evaluation of the circuit operation, is neither observed using exact formula nor using the adaptive error formula. However it is not corrected by using the approach of avoiding the elimination of terms whose magnitude is greater than S_D.

Nested Format

Few has been done concerning simplification for symbolic expressions given in this format [10,15]. The technique in [10] is based on pruning insignificant subexpressions at the nested levels. This is made without assessing the sensitivities of the

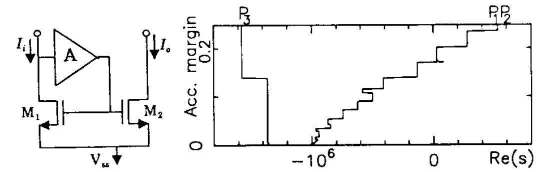

Figure 4:Active current mirror [14] and associated pole loci.

coefficients of the different powers of s in the symbolic network function to changes in the deleted terms. Hence, large errors may be expected as long as a pruned subexpression can only slightly influence some coefficients but very strongly influence others. To avoid these errors a partial fraction expansion is needed, at least, to separate the different powers of the complex frequency. Then, the advantages of the nested symbolic format disappear. But, even taking this expansion, some problems may still appear:

a) Pruning significant terms while keeping insignificant ones. Consider for instance the nested expression

$$(A+B)\ (C+D) + (E+F)\ (G+H) \tag{7}$$

If $A \gg B$, $C \cong D$, $E \cong F$, $G \cong H$, in the expression tree B will be pruned although perhaps $B(C+D) > E(G+H)$.

b) Uncontrolled large errors because of cancellations in the expression tree. This problem has been partially solved with the "lazy expansion technique" given in [15], but only cancellations in consecutive levels of the expression tree are detected.

Simplifications with Ranges of Variation

Problems with previous simplification criteria can be overcome if the numerical evaluations to estimate relative sizes of the symbolic parameters are not made at a single point $\mathbf{x_0}$ of the parameter space, but inside a region of this space. It can be so done by considering a typical range of variation for each parameter, instead of considering a typical value. This approach provides a natural simple mechanism to cope with mismatches. Also, simplifications are more realistic since in symbolic analysis the exact numerical value of the parameters is not known a priori.

Concept and Basic Operators.

We assume for the simplification procedure to be carried out that each symbol may take any value inside a given range of variation,

$$x \in \{x_{iS}, x_{iH}\} \tag{8}$$

In order simplifications to be made several operators among ranges have to be defined [16]:

Product of Ranges: Since each term $h_{kl}(\mathbf{x})$ is made up of a product of symbols, the product of ranges must be defined, and the range extrema calculated. In case one of the factors is a numerical coefficient the range of the other is scaled according to the value of this coefficient. In other case all the possible combinations among the limits of the individual parameters have to be considered. The required computations become largely simplified in case the symbolic parameters do not change their sign inside the corresponding definition intervals [16].

Addition of Ranges: The interval extrema of terms resulting as the sum of two can be obtained by adding the corresponding extrema of the addends.

Range Comparison: A range is defined to be smaller than other in case the following is fulfilled

$$\{R_{oS}, R_{oH}\} \leq \{R_{1S}, R_{1H}\} \qquad iff$$
$$max\,[|R_{oS}|, |R_{oH}|] \leq max\,[|R_{1S}|, |R_{1H}|] \tag{9}$$

Simplification Algorithm.

The simplification algorithm can be summarized in the following steps [16]:

1) Calculate the range for each term inside each coefficient polynomial using the product of ranges operator.

2) For each coefficient polynomial, defined as a sum of terms, calculate the range using the results of **1)** and the addition of ranges operator.

3) Look for pairs of terms with opposite sign and similar magnitude. For each pair of this type, if the range of the corresponding sum is smaller than any of the ranges of the single terms, a grouping is made and the range of the sum is associated to it.

4) Arrange the terms in an ordered array using the range comparison operator.

5) Beginning by the smallest term, the range of each new term is added to an accumulated sum range. Then, the maximum (in modulus) of the limits of this range is compared to the minimum of the limits of the range of the sum of all terms multiplied by a given threshold.

$$\forall k \qquad \sum_{j=1,k} [(T_{jS}), (T_{jH})] = [(A_{cS}), (A_{cH})]$$
$$\frac{max\{(A_{cS}), (A_{cH})\}}{min\{(S_S), (S_H)\}} < \varepsilon \tag{10}$$

Notice this is a very conservative criterion as there may be terms summed with their maximum value in the accumulated sum and with the minimum one in the total sum. For that reason, a slightly modified criterion has been developed in which terms that are summed with their maximum magnitude in the numerator are also summed in this way in the denominator. This little extra computation allows to eliminate negligible terms which were kept with previous criterion.

The new criterion has been tested for a large number of examples, in particular for the circuits discussed in previous Section. Results are free of the inaccuracies observed with the other criteria.

Simplifications in Hierarchical Analysis

One of the very points in symbolic analysis is the ability to handle nested expressions in order to face more complex circuits [12]. When dealing with this type of expressions some difficulties arise due to potential cancellations at intermediate levels of the hierarchy. In general a transfer function can be expressed as:

$$F = \frac{g_1 g_2 \cdots + g_i g_{i+1} \cdots + \cdots}{g_j g_{j+1} \cdots + g_l g_{l+1} \cdots + \cdots} \tag{11}$$

where the $g's$ may in their turn be either nested expressions or simple functions of the form

$$g_k = \frac{G_{Nk}(s)}{G_{Dk}(s)} \tag{12}$$

A simple transformation allows to rewrite F as

$$F = \frac{G_{Nk} C_{NNk}(s) + G_{Dk}(s) C_{DNk}(s) + R_N(s)}{G_{Nk} C_{NDk}(s) + G_{Dk}(s) C_{DDk}(s) + R_D(s)} \tag{13}$$

where the polynomials C_{NNk}, C_{DNk}, C_{NDk}, C_{DNk} express the contribution factor of G_{Nk} and G_{Dk} to the total transfer function.

The proposed algorithm computes the values for these factors prior to perform simplifications in g_k.In this way we know beforehand how much will those simplifications affect the total.

In the first phase the expression tree is traversed from the leaves to the root computing the contribution to parent nodes. During the second phase the tree is traversed from the root to the leaves assigning to each node its contribution to the root. The factors are computed numerically using ranges of values for the low-end symbols. It is not safe to assign typical values as it is possible for some expressions to be highly sensitive to variations of values and that fact goes unnoticed until we reach higher levels in the hierarchy.

Discussion of Results

Summarizing this paper on the one hand reviews previously reported simplification criteria and, on the other, outlines new simplification criteria for both symbolic expressions calculated via flat analysis approaches and for those calculated via hierarchical analysis. Information about the proposed new algorithms and their performance will be reported in greater details elsewhere.

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