

**A CMOS IMPLEMENTATION OF
FITZHUGH-NAGUMO NEURON MODEL**

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Abstract— A CMOS circuit is proposed that emulates FitzHugh–Nagumo’s differential equations using OTAs, diode connected MOSFETs and capacitors. These equations model the fundamental behavior of biological neuron cells. Fitz–Hugh–Nagumo’s model is characterized by two threshold values. If the input to the neuron is between the two thresholds the output yields a sequence of firing pulses, if the input is outside this range, no output is observed. The resulting circuit due to the (voltage) programmability of the OTA allows one to easily vary parameters. Thus a large family of solutions can be obtained including the Van der Pol’s equation. Experimental results from a CMOS prototype are given that show the suitability of the technique used, and their potential for biological CMOS system emulation.

I. INTRODUCTION. Several types of artificial neuron models have been used until now which are usually inspired in the biological neural cells. Most of these artificial neuron models are extremely simplified versions of the real ones, such as those used by Hopfield, Anderson, Rumelhart, Kohonen, and in most of the papers in the expanding neural literature. These conventional neuron models [1] can be classified as non-oscillatory neurons. There has also been attempts to model the oscillatory nature of the biological neurons using hysteretic devices. These models are supposed to have a similar behavior to the more detailed modeling of FitzHugh–Nagumo [3] which is also a simplified version of one of the most exact models due to Hodgkin and Huxley [4]. In this paper, we propose a CMOS implementation of the FitzHugh–Nagumo’s equations. The devices used are CMOS operational transconductance amplifiers (OTAs), capacitors and MOS diodes. In the next section we propose a circuit that solves FitzHugh–Nagumo’s [3] equations. Finally, experimental results of a CMOS prototype circuit fabricated in a $2\mu m$ p-well, double-metal, double-poly process are discussed.

II. CIRCUIT IMPLEMENTATION. FitzHugh–Nagumo’s equations represent a second order system as

$$I_{o1} + g_{m1}V_2 - g_{m3}V_1 - C_{11}\dot{V}_1 = 0 \quad (1a)$$

$$I_{o2} - g_{m2}V_1 - f(V_2) - C_{22}\dot{V}_2 = 0 \quad (1b)$$

where the dot over V_1 and V_2 implies time derivatives, $f(\cdot)$ is an N-shaped nonlinear function that will be shown later, and g_{m_i} , C_{ij} are positive parameters. The exact form of the function $f(\cdot)$ does not seem to be very critical. Originally [3] a cubic polynomial was suggested, but a piece-wise linear dependance can give the same basic properties to the system. We will consider this latter approach, $f(\cdot)$ being as shown in Fig. 1. This element is built as a nonlinear resistor, using the technique proposed in [5]. This technique implents nonlinear I-V transfer characteristics with OTAs and MOS diodes. The resulting circuit for this element is shown in Fig. 2 where $g_l = g_b - g_a = g_c - g_a$. The complete circuit implementation of eq.(1) using OTAs and MOS-diodes is shown in Fig. 3.

III. EXPERIMENTAL RESULTS. A CMOS prototype has been fabricated in a $2\mu m$ double metal, double polysilicon process. When the two external inputs I_{01} and I_{02} (see Fig. 3) are set to zero the circuit behaves as a free running oscillator. The waveforms at the two nodes of the circuit are shown in Fig. 4. They compare very well to the computed solution of the differential equation (1). The system described by (1) and implemented in Fig. 3 has two threshold values for both I_{01} or I_{02} . Beyond each one of them oscillation will cease. This is illustrated by the measurement shown in Fig. 5, where the lower trace is the input to the neuron and the upper trace is its output. Output oscillations are observed only if the input is between the two thresholds. Frequency is proportional to the difference between the input and the threshold. Interesting results are obtained when interconnecting several such neuron cells. This opens possibilities of small biological system emulations. We have interconnected just two of them as shown in Fig. 6 using two neurons of Fig. 3 and two additional integrators. When $A=0$ the response is stable and is shown in Fig. 7. But for certain values of A the pattern observed at the output changes randomly between different oscillation patterns. One of these temporary patterns can be seen in Fig. 8. This is a clear phenomenon of out-of-synchronization which has been shown to have a chaotic nature [6].

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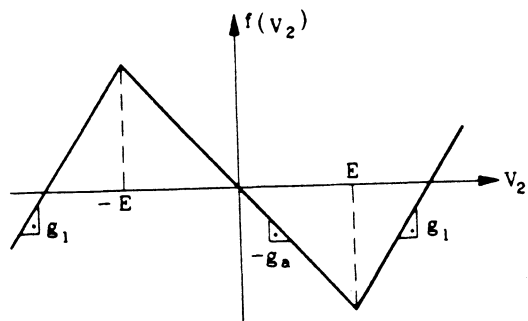


Fig. 1. N-Shaped Transfer Function $f(V_2)$.

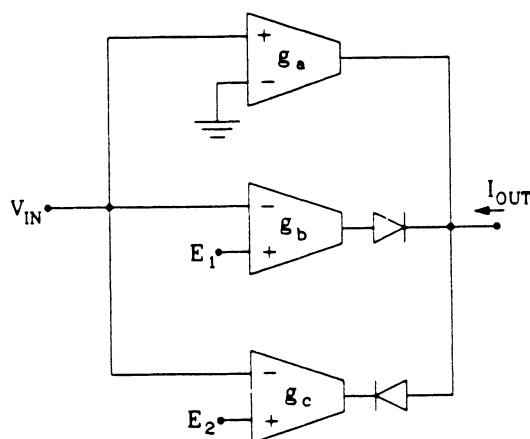


Fig. 2 Piece Wise Linear Implementation for the Circuit of Fig. 1.

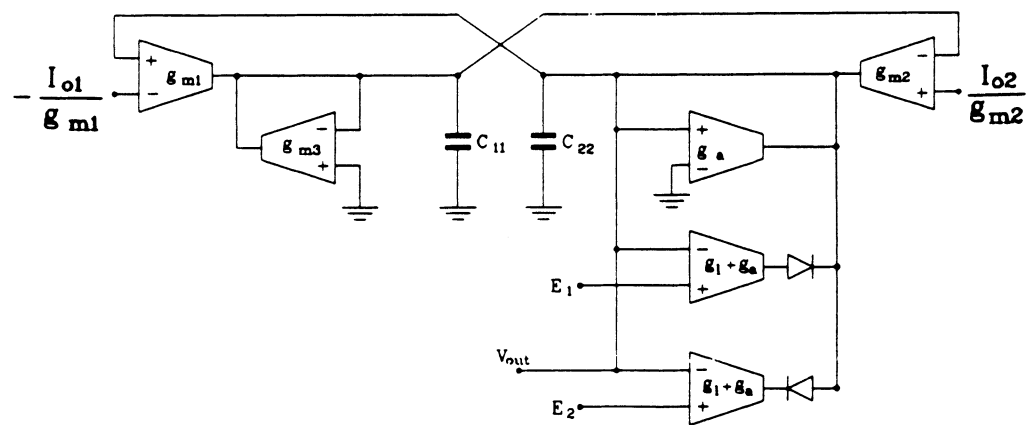


Fig. 3 Complete FitzHugh-Nagumo Neuron Architecture.

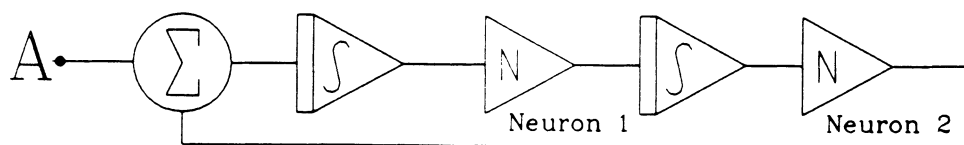


Fig. 6 Connection of two oscillatory neurons in a loop.

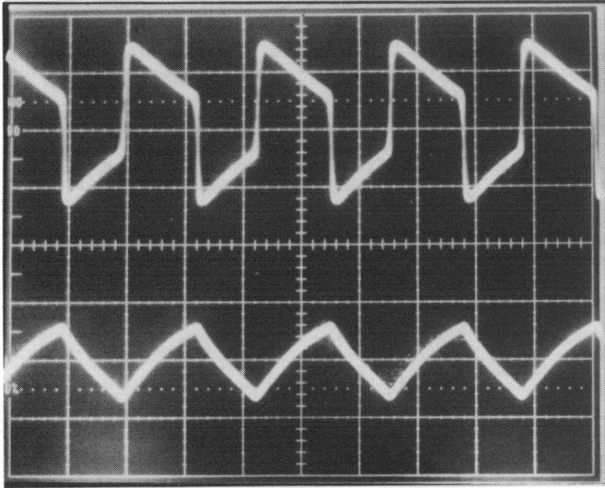


Fig. 4 Observed Waveforms at the Nodes of the FitzHugh-Nagumo Oscillator Circuit Implementation.

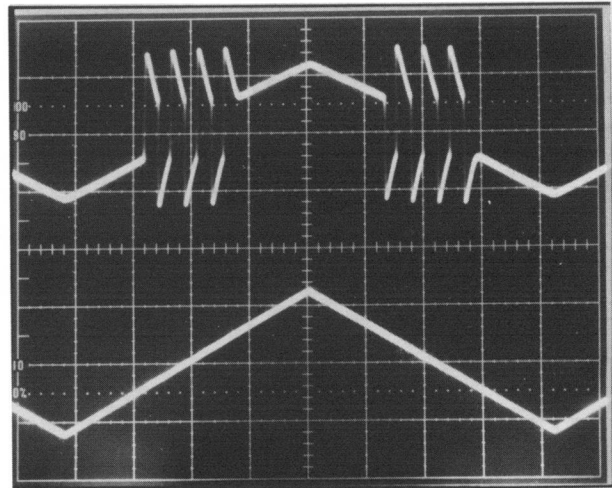


Fig. 5. Lower trace is input to neuron and upper trace is the output. Oscillations are observed only if the input is between the two threshold values.

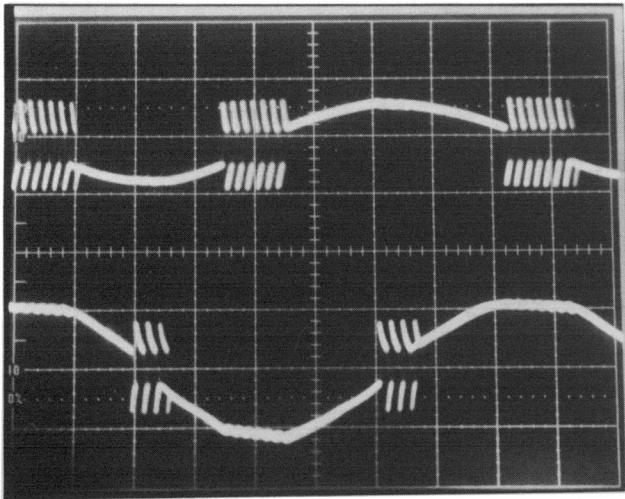


Fig. 7 Steady response of a two neuron loop oscillator when $A=0$.

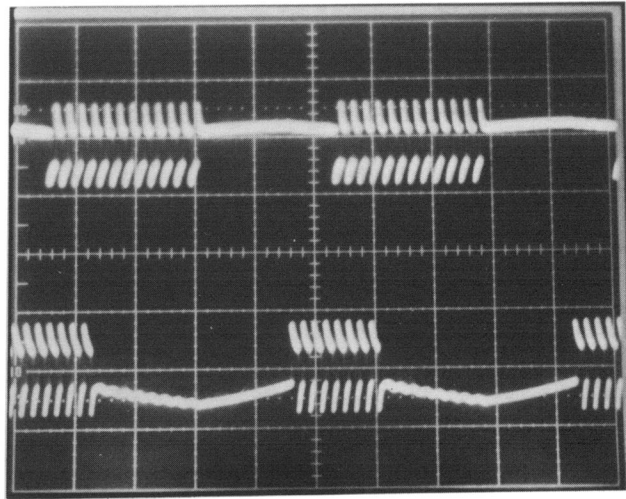


Fig. 8 Temporarily stable pattern observed when A is such that the two neuron loop is chaotic.