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Analysis of the frictional slip between a layer and a half-space

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ABSTRACT

The numerical analysis of a boundless elastic layer on an elastic half-space with different material properties under the effects of an uniform surface pressure and a cyclic tangential surface force is presented. Frictional contact conditions are assumed. The study is focussed on the evaluation of the maximum amplitude of the tangential load which produces localized slip between the two regions during the first load cycle but not the subsequent ones. The more simple limit for which no slip exist even for the first cycle is also established

INTRODUCTION

The study of the slip and separation that may take place between two surfaces in contact when they are under tangential cyclic loading conditions is very important to prevent the failure known as "fretting". This failure mechanism is initiated by localized slip between two surfaces from which some particles are detached. These particles act as an abrasive in subsequent load cycles and deteriorate the material rapidly. The damage mechanism may be combined with corrosion in the case of metals[1].

The slip between the two surfaces can be avoided by introducing a normal pressure between them. If this pressure is enough the prevent slip during the first load cycle then the system behaves linearly and slip will not take place during subsequent cycles of the same amplitude. This welded contact problem can be solved easily. The limit for which the first slip takes place can be obtained from the welded contact model. It corresponds to the tangential load for which the shear traction at a point becomes equal to the normal pressure times the friction coefficient. There are still higher values of the tangential load for which there is partial slip during the first load cycle but not in the subsequent ones. Therefore no fretting will take place for this

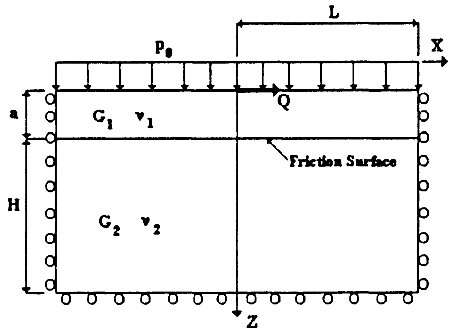

Figure 1. Elastic layer on elastic half-space under tangential load Q and normal load p_0

load. The slip of the first cycle leaves residual stresses which prevent the two surfaces to slip during the next cycles. The evaluation of the higher value of the load for which the system has this kind of behavior is important since it is the actual limit of the loads which do not produce fretting. The study requires a more complicated model and a numerical solution approach as shown below.

The problem analyzed in this work refers to a semi-infinite domain consisting of a boundless horizontal elastic layer with depth "a" on an elastic half-space. The surface of the layer is subject to a uniform constant in time pressure p_o and to a concentrated tangential load of amplitude Q which has a cyclic time dependence. The variation of the load with time is assumed to be sufficiently slow so that inertial effects are negligible. Therefore, a quasi-static analysis is carried out. The finite region with the boundary conditions shown in Figure 1 is used for the study.

The problem at hand remains linear for small values of the load Q. The expressions for the tractions along the interface in such case can be found for instance in Ref.[2]. The study of a similar problem with two materials of identical properties and a single loading process which produces first, slip at a point then, one or two slip zones and finally, separation, was done analytically be Schmueser, Comminou and Dundurs [3]. Comminou and Barber [4] extended this analysis to the case of cyclic loading and assuming a friction coefficient $\mu = 0.5$. These authors identified a value λ_1 of the parameter $\lambda = Q/(p_0 a)$ such that for loads $\lambda < \lambda_1$ the problem remains linear, and a load range $\lambda_1 < \lambda < \lambda_2$ for which there is slip but only in the first load cycle. Loads corresponding to $\lambda > \lambda_2$ produce multiple slip and/or separation which may become general along the interface during the subsequent load cycles.

The purpose of this paper is to carry out a study of these ranges of the load for the more general case when the layer and the half-space have different material properties. The study is done numerically by means of the Boundary Element Method (BEM) which is very well suited for this kind of problems. A parametric study is done using as a parameter to define the relative stiffness of the two materials, the square root of the ratio between the shear modula RCs = $(G_1/G_2)^{1/2}$, the Poisson's ration being the same for both materials.

NUMERICAL APPROACH

The analysis of the problem in Figure 1 starts by discretizing the boundaries into constant elements and computing the usual BEM system of equations for the boundary nodes of each one of the two subregions

$$\boldsymbol{H}^i \boldsymbol{u}^i = \boldsymbol{G}^i \boldsymbol{p}^i \tag{1}$$

where $\boldsymbol{u}^i$ and $\boldsymbol{p}^i$ are the displacement and traction vectors respectively, along the boundary of the "i" subregion. The coupling between the two regions is done by means of the compatibility and equilibrium conditions along the interface corresponding to one of the following situations: bonding, separation or sliding with friction. A Coulomb type friction is assumed.

Sliding:

$$\begin{array}{ll} | p_s^{A,B} | = | \mu \, p_n^{A,B} | & p_n^A = p_n^B \leq 0 \\ p_s^A = p_s^B & u_n^A + u_n^B = 0 \end{array} \tag{2}$$

Bonding:

$$\begin{array}{lll} | p_s^{A,B} | < | \mu \, p_n^{A,B} | & & u_n^A + u_n^B = 0 \\ & p_n^A = p_n^B \leq 0 & \\ p_s^A = p_s^B & & u_s^A + u_s^B = 0 \end{array} \tag{3}$$

Separation:

$$p_n^{A,B} = 0 \qquad p_s^{A,B} = 0 \qquad \delta > 0 \tag{4}$$

where the superindex indicates the region to which the variable refers and the subindexes s and n stand for tangential and normal direction to the interface, respectively; μ is the friction coefficient and δ the distance along the normal direction between two points which could be in contact.

The sign of the shear traction during the sliding is determined by the condition of negative work (dissipation of energy) during the process. In the three cases above four equations can be written for each couple of points on the contact surfaces. Those equations plus the two BEM equations for each domain determine the values of the two displacement components and two traction components for each one of the two points of a couple which are or may be, in contact.

The solution of the system of equations requires of an iterative process within each load step. This process for a load step N starts by writing the B.E. equations (1) for each boundary node as

$$\boldsymbol{A}\,\boldsymbol{x}_N = \boldsymbol{f}_N \tag{5}$$

where $\boldsymbol{x}_N$ and $\boldsymbol{f}_N$ are the boundary unknown vector and the known vector, respectively. The latter is obtained from the product of the known boundary values and the corresponding columns of the $\boldsymbol{H}$ and $\boldsymbol{G}$ matrix. The system matrix $\boldsymbol{A}$ contains more columns than rows as both tractions and displacements are unknown along the interface.

The contact equations for the nodes on the interface are written as

$$\boldsymbol{C}_N\,\boldsymbol{x}_N = \boldsymbol{0} \tag{6}$$

The above system of equations (5) plus (6) allows for the solution of the problem for the load step N provided that the contact conditions for all the points on the interface (represented by $\boldsymbol{C}_N$) remain constant during the load step. Since these conditions will, in general, change during the load step, each load step must include several iterations with different contact

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conditions C_N^K. For each load step one works with increments of the variables with respect to the previous step

$$\begin{bmatrix} A \\ C_N \end{bmatrix} (x_N - x_{N-1}) = \begin{bmatrix} f_N - f_{N-1} \\ 0 \end{bmatrix} \qquad (7)$$

$$\begin{bmatrix} A \\ C_N \end{bmatrix} \Delta x_N = \Delta f_N$$

The increments of the variables are subdivided into M possible sub-increments and written as

$$\Delta f_N = \sum_{k=1}^{M} \Delta f_N^k \qquad \Delta x_N = \sum_{k=1}^{M} \Delta x_N^k \qquad (8)$$

where each sub-increment corresponds to different contact conditions C_N^k.

The iteration process for load step N begins by solving the system (7) assuming the same contact conditions of the end of the previous step and applying the complete load increment Δf_N. A scale factor β_N^1 is computed from the solution of this system such that it represents the part of the load which can be applied without change in the contact conditions. This load can be written as

$$\Delta f_N^1 = \beta_N^1 \, \Delta f_N \qquad \textit{and} \qquad \Delta x_N^1 = \beta_N^1 \, \Delta x_N^1 \qquad (9)$$

where Δx_N is the solution of (7) with Δf_N. Next the contact conditions are modified and the load $(1 - \beta_N^1)\, \Delta f_N$ applied. The process continues up to the point when $\beta_N^k \geq 1$. Then the computation for load step N is finished. A similar iterative process was applied by the authors for dynamic problems in Refs. [6] and [7].

LAYER ON A HALF-SPACE

The following values of the parameters have been used for the model in Figure 1: a = 1 m; L = 10 m ; H = 10 m; Poisson's ratio $\upsilon_1 = \upsilon_2 =$

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1/3 ; $p_0 = 1$ Nw/m^2 ; friction coefficient $\mu = 0.5$ and a shear modulus of the half-space $G_2 = 1$ Nw/m^2. The parametric study is done by changing the shear modulus of the layer material and the load amplitude Q. The load is assumed to be applied in 100 steps following the variation shown in Figure 2.

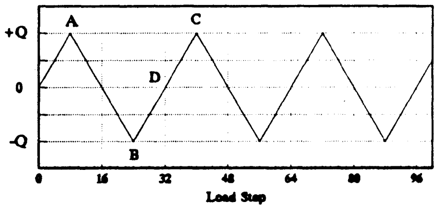

Figure 2. Tangential load variation

Constant Boundary Elements are used for the study. The discretization consists of 50 equal elements on each side of the contact interface and on the layer free surface, 3 equal elements on each lateral boundary of the layer, 5 on each lateral boundary of the half-space and 10 equal elements for the bottom of the model.

In order to test this model which extends to a finite region around the load to represent an infinite domain and includes a constant Boundary Element approximation, a problem with known analytical solution is solved. Such solution was obtained by Comninou and Barber [5] for the case of two identical material properties for the layer and the half-space. Figure 3 shows a comparison between the analytical and the present numerical results for the maximum load of the second load cycle (point C of Figure 2) with a load amplitude $\lambda = 2.353$.

Slip between the layer and the half-space takes place for this value of λ during the first cycle but not in the subsequent ones.

The normal and shear tractions along the interface versus the horizontal distance x to the point load are shown in Figure 3. The normal tractions have a smooth variation and are almost the same as in the linear case. On the other hand, the shear tractions show a clear difference with those of the linear case because of the residual tractions existing in the zones where slip

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took place in the previous load cycle. These residual tractions avoid the sliding in the second and subsequent load cycles. The results presented in Figure 3 show a very good agreement between the analytical and the present numerical solution.

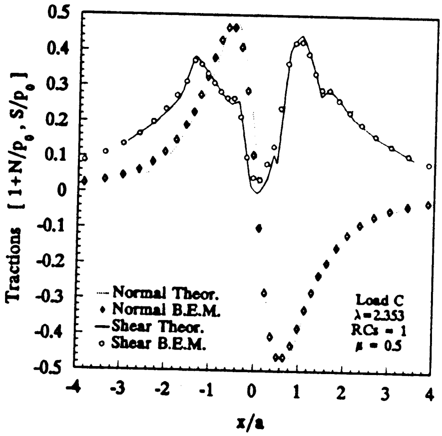

Figure 3. Results for two identical materials

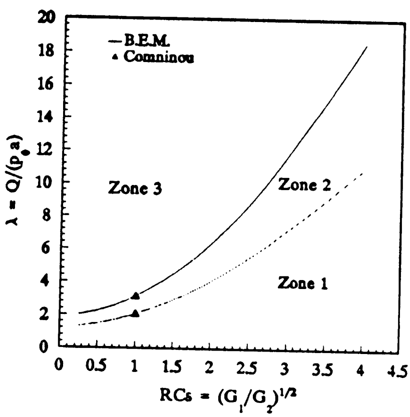

Figure 4. Fretting zones

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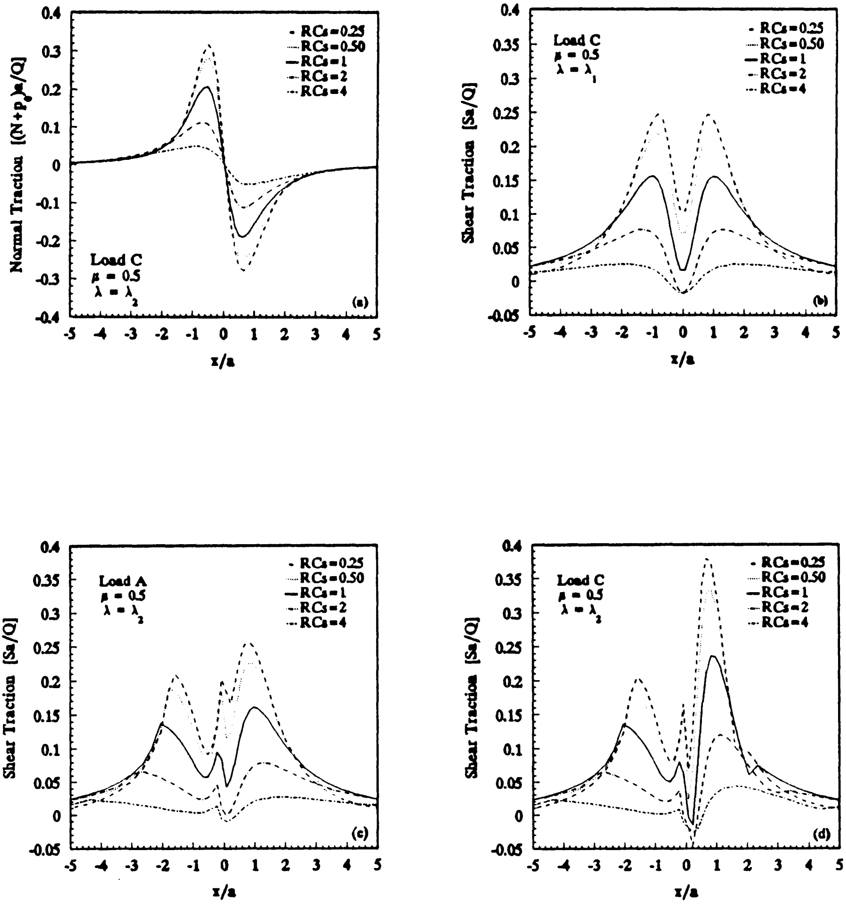

Figure 5. Traction distribution along the interface

Once the B.E. model and the iterative approach have been validated the parametric study for a range of the stiffness ratio RCs = $(G_1/G_2)^{1/2}$ going from 0 to 4.5 is carried out. For each stiffness ratio two limiting values of the load parameters $\lambda = Q/p_0 a$ are determined. The first one, λ_1 is the limit of the loads which do not produce non-linear effects on the contact interface. This limit define the linear zone (zone 1) in Figure 4. The second limit λ_2 defines the load zone (zone 2) for which there is slip along the interface during the first load cycle but the residual tractions avoid slip in the subsequent load cycles. Values of $\lambda > \lambda_2$ (zone 3) correspond to loads which produce slip in all the load cycles and therefore the fretting process can not be controlled. The values of λ_1 and λ_2 obtained analytically by Comninou and Barber [5] for the case of RCs = 1 are shown as dots in the figure.

Figure 5 shows some of the traction distributions which appear along the interface during the loading process. Only in one case are the normal tractions represented (Figure 5a) since those tractions show very little difference with the linear ones for zone 2.

The shear tractions represented in Figure 5b correspond to the second cycle (Point C) and $\lambda = \lambda_1$. Therefore, no slip has taken place and the curves in this figure represent the shear traction distribution for a linear problem. Figures 5c and 5d show the shear traction distribution for $\lambda = \lambda_2$ during the first and second load cycles (Points A and C, respectively). In the former case, there have been slip only in the negative part of the x-axis and residual tractions have appeared. It can be seen in Figure 5d that the residual tractions corresponding to point A remain and some more appear along the positive part of the x-axis which have been produced during the negative part of the first load cycle (point B).

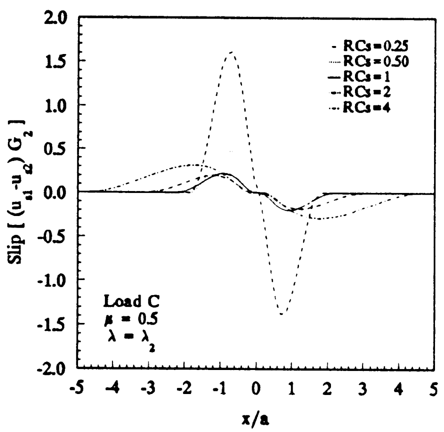

Figure 6. Relative tangential displacements along the interface

Figure 6 shows the relative displacement along the tangential direction for points on the interface when $\lambda = \lambda_2$ RCs varies between 0.25 and 4, and the load has follows a complete cycle up to the point C. The length of the sliding zone due to the first load ramp (Point A, x/a < 0) increases with RCs whereas the amplitude of the relative displacement decreases. When the load goes from A to B points along the positive part of the x axis (x/a > 0) undergo sliding displacements very similar to those which appeared along x/a < 0 for the first load ramp. The latter relative displacements are a little smaller than the former ones because of the residual stresses of the first part of the load cycle. After a complete cycle the relative displacement for points along the interface do not change. These points remain bonded and the behavior of the system linear.

CONCLUSIONS

A numerical formulation for frictional contact problems based on the Boundary Element Method has been presented. Sliding, bonding or separation may take place at the interface points.

The formulation has been applied to carry out a parametric study of the limit values of the amplitude of the cyclic tangential loads which can be applied on the surface of an elastic layer on an elastic half-space satisfying: first, the condition of having no slip between the two surfaces in contact and second, the condition of having slip only during the first load cycle.

The numerical results have been validated by comparison with the existing analytical solution for the simple case of two regions with the same material properties.

The approach is general and can be applied to the solution of different contact problems.

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