# Shape phase transitions and critical points

C.E. Alonso*, J.M. Arias*, L. Fortunato† and A. Vitturi†

* *Departamento de Física Atómica, Molecular y Nuclear, Facultad de Física Universidad de Sevilla, Apartado 1065, 41080 Sevilla, Spain*
†*Dipartimento di Fisica Galileo Galilei and INFN, Via Marzolo 8, 35131 Padova, Italy*

**Abstract.**
We investigate different aspects connected with shape phase transitions in nuclei and the possible occurrence of dynamical symmetries at the critical points. We discuss in particular the behaviour of the neighbour odd nuclei at the vicinity of the critical points in the even nuclei. We consider both the case of the transition from the vibrational behaviour to the gamma-unstable deformation (characterized within the collective Bohr hamiltonian by the E(5) critical point symmetry) and the case of the transition from the vibrational behaviour to the stable axial deformation (characterized by the X(5) symmetry). The odd particle is assumed to be moving in the three single particle orbitals j=1/2,3/2,5/2, a set of orbitals that is known to lead to possible supersymmetric cases. The coupling of the odd particle to the Bohr hamiltonian does lead in fact in the former case at the critical point to the E(5/12) boson-fermion dynamical symmetry. An alternative approach to the two shape transitions is based on the Interacting Boson Fermion Model. In this case suitably parametrized boson-fermion hamiltonians can describe the evolution of the odd system along the shape transitions. At the critical points both energy spectra and electromagnetic transitions were found to display characteristic patterns similar to those displayed by the even nuclei at the corresponding critical point. The behaviour of the odd nuclei can therefore be seen as necessary complementary signatures of the occurrence of the phase transitions.

**Keywords:** Shape phase transition. Interacting Boson Fermion Model. Critical point symmetries.
**PACS:** 21.60.-n, 21.60.Fw, 21.60.Ev

## Introduction

The study of shape phase transitions in finite nuclear quantal systems has recently been the subject of many investigations. Most of the work has been carried out for even-even nuclei, using either the Bohr Hamiltonian and the surface collective variables or algebraic approaches based on the use of interacting bosons (IBM). In the former case interesting analytic solutions have been advanced [1, 2] at the critical point in the case of the transition from sphericity to deformed gamma-instability (E(5) dynamical symmetry) and in the case of the transition from the spherical vibrational behaviour to stable axial deformation (X(5) dynamical symmetry). In this presentation we briefly discuss the occurrence of phase transitions and critical point symmetries in the case of the neighbour odd-even nuclei, where an odd particle is coupled to an even core undergoing a phase transition. The problem is again treated either in the collective Bohr hamiltonian (leading for example to the occurrence of the boson-fermion dynamical symmetries E(5/4) and E(5/12)) or in the algebraic interacting boson-fermion model (IBFM). Our results show that at the critical points both energy spectra and electromagnetic transitions display characteristic patterns similar to those displayed by the even nuclei at the correspond-

CP1120, *Perspectives in Nuclear Physics,* edited by S.-C. Jeong, H. Miyatake, Y. Utsuno, H. Ikezoe, T. Motobayashi, and A. Bracco

ing critical points. The behaviour of the odd nuclei can therefore be seen as necessary complementary signatures of the occurrence of the phase transitions.

## Phase transitions within the collective model

A characteristic feature of the structure of atomic nuclei is the large variety of behaviours along the nuclear chart. These different behaviours are often simply classified in terms of the degrees of freedom of the nuclear surface. Different situations can be described, for example, in terms of the collective Bohr hamiltonian with a proper choice of the intrinsic potential $V$ in the deformation variables $\beta$ and $\gamma$. This variation of structure (often referred to as shape transition) can occur, for example along isotope or isotone lines, and follows a corresponding variation of the properties of the potential $V(\beta,\gamma)$. The situation is becoming more interesting if the transition occurs rather rapidly along the mass chain. One can in this case profitably introduce the concept of phase transitions and study the nuclear behaviour at the corresponding critical points.

Figure 1 display examples of energy surfaces (and corresponding potentials) obtained as a function of the deformation parameter $\beta$ along a transitions from sphericity to deformed gamma-instability (left frame) and in the case of a transition from the spherical vibrational behaviour to stable axial deformation (right frame). In the former case all the energy surfaces are independent of the triaxiality parameter $\gamma$, while in the latter case all the curves have a minimum for $\gamma = 0$ (axial symmetry). The flatness of the energy surfaces at the critical points led in the two cases to the E(5) and X(5) analytical solutions obtained by Franco Iachello within the Bohr hamiltonian approximating the flat potentials with infinite square well shapes [1, 2].

Along a similar approach one can treat the neighbour odd nuclei. If the odd particle can only move in the j=3/2 single particle orbital and the even core is described by the E(5) hamiltonian one can obtain [3] again an analytical solution to the Bohr hamiltonian (E(5/4)). A similar situation occurs when the allowed single-particle orbitals are j=1/2,3/2,5/2, where one can use the separation of the single-particle angular momentum into a pseudo-spin and pseudo-orbital angular momentum (0 and 2) and assume no coupling due to the pseudo-spin. In this case at the critical point one can introduce a hamiltonian where a suitable term couples the odd particle to the E(5) collective hamiltonian in the form

$$\begin{aligned} H = & -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}-\frac{1}{4\beta^4}\sum_\kappa\frac{Q_\kappa^2}{\sin^2\left(\gamma-\frac{2}{3}\pi\kappa\right)}\right] \\ & + u(\beta)+k\,g(\beta)[2\hat{\mathscr{L}}_B\circ\hat{\mathscr{L}}_F]+k'\,g(\beta)\hat{\mathscr{L}}_F^2 \ , \end{aligned} \tag{1}$$

with

$$\begin{aligned} u(\beta) &= 0, \quad \beta < \beta_w, \qquad u(\beta)=\infty, \quad \beta \geq \beta_w \ , \\ g(\beta) &= \frac{\hbar^2}{2B\beta^2} \ . \end{aligned} \tag{2}$$

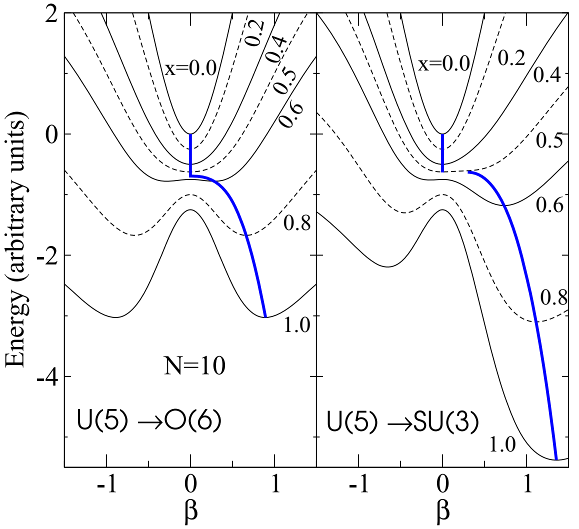


**FIGURE 1.** Evolution of the energy surfaces along a transition from sphericity to deformed gamma-instability (left frame) and in the case of a transition from the spherical vibrational behaviour to stable axial deformation (right frame). Different curves correspond to different values of the control parameter $x$ in the hamiltonian 3. The bold lines give the values of the order parameter $\beta$ at the minimum of the energy surface. This order parameter varies continuously for the U(5)→O(6) transition (second order), while has a discontinuity at the critical point for the U(5)→SU(3) transition (first order).

Note that $\hat{\mathscr{L}}_B$ and $\hat{\mathscr{L}}_F$ are the five-dimensional boson and fermion angular momenta, and that the dot $\circ$ indicates the five-dimensional scalar product. This hamiltonian displays a E(5/12) symmetry, which means that the core transition is characterized by the Euclidean group in five dimensions (E(5)) while the fermion space is described by the U(12) algebra. In this case, the associated wave functions and energies can be determined analytically for each choice of the $k$ and $k'$ coupling constants [4]. The corresponding spectrum is shown in Figure 2 in the lower frame, compared with the corresponding E(5) spectrum in the upper frame. As can be seen from the figure the spectra in the even and odd nuclei display similar characteristic patterns, aside from the occurrence of "new" bands in the odd-even nucleus that arise from the proper coupling of the odd particle to the even-even core nucleus. More details on the E(5/12) model can be found in ref.[5].

## Phase transition within the Interacting Boson and Interacting Boson Fermion Models

The Interacting Boson Model (IBM) has been widely used to describe the collective properties of even-even nuclei. By proper choice of the boson hamiltonian one is able to cover different behaviours present in the nuclear chart. In particular, analytic solutions can be obtained for the case of spherical vibrational nuclei (U(5) dynamical symmetry),

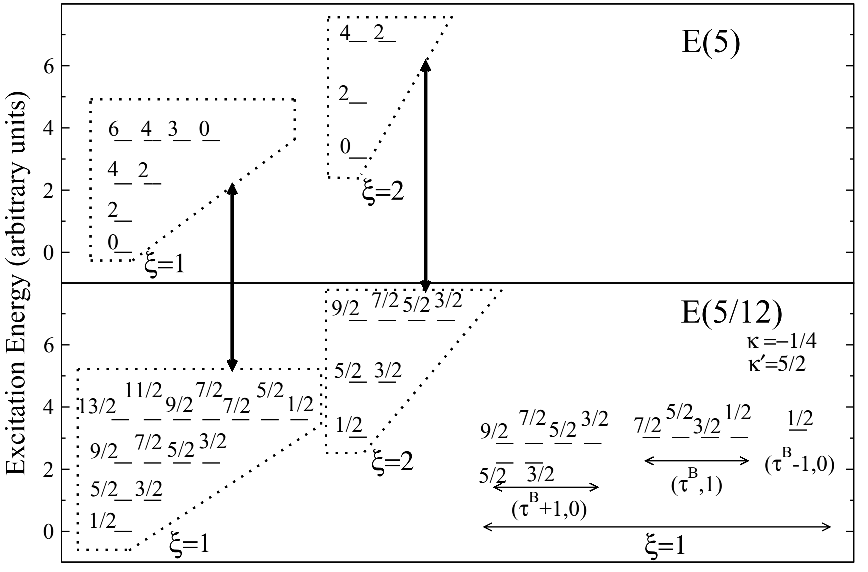


**FIGURE 2.** E(5) energy spectrum (upper frame) and E(5/12) energy spectrum (lower frame) for parameters $k = -1/4$ and $k' = 5/2$. The two group of bands at the left in the E(5/12) spectrum are in clear correspondence to the displayed bands in the E(5) one (just including the coupling of the pseudo-orbital $\ell$ with the pseudo-spin $s = 1/2$ angular momenta). However, new bands arise in the odd-even case due to the coupling of the odd particle to the even-even core.

axially symmetric rotors (SU(3)) and deformed gamma-unstable nuclei (O(6)). Along this line simple parametrized hamiltonians have been introduced to describe the possible transition between these dynamical symmetries. One such hamiltonian is of the simple form

$$H_B = (1-x)\hat{n}_d - \frac{x}{4N}\hat{Q}_B \cdot \hat{Q}_B \; , \tag{3}$$

where the operators in the Hamiltonian are given by

$$\hat{n}_d = \sum_\mu d^\dagger_\mu d_\mu \;\; , \tag{4}$$

$$\hat{Q}_B = (s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s} + \chi \; d^\dagger \times \tilde{d})^{(2)} \;\; , \tag{5}$$

and $N$ is the total number of bosons. With the choice $\chi = 0$ this hamiltonian produces a transition between spherical ($x = 0$) and $\gamma$-unstable ($x = 1$) phases, the critical point being obtained for $x_c = N/(2N-2)$. In a similar way the transition from sphericity ($x = 0$) to stable axial deformation ($x = 1$) is obtained with the choice $\chi = -\sqrt{7}/2$, with the critical point obtained for $x_c = 16N/(34N-27)$. The spectra at the critical points are qualitatively similar to those obtained with the E(5) and X(5) models within the Bohr hamiltonian. Detailed comparisons among the different approaches can be found in refs. [6, 7].

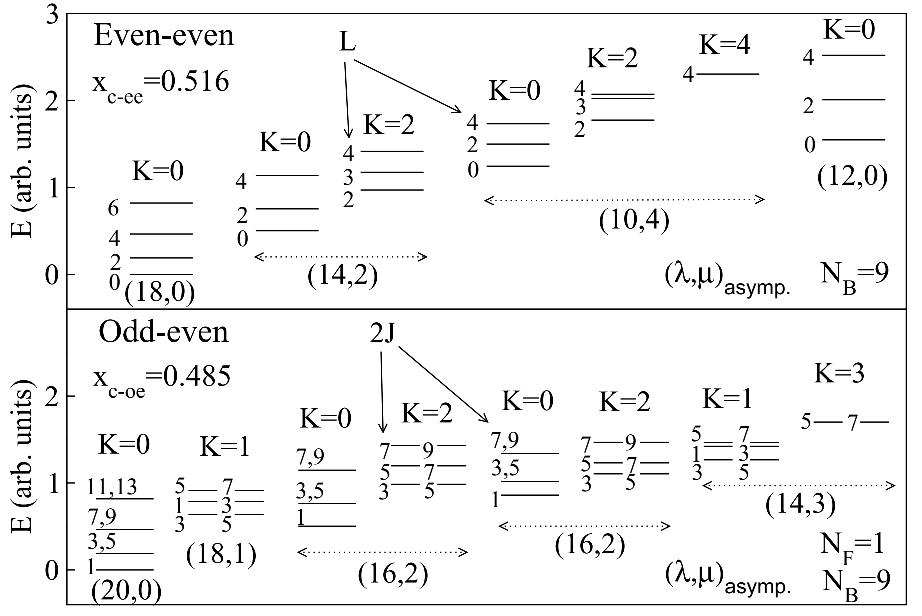


**FIGURE 3.** Spectrum obtained at the critical point for the even core (upper frame) and the the odd case (lower frame) along the spherical to axial symmetric phase transition within the IBM-IBFM framework. Spectra are obtained for $N = 9$ bosons and bands are identified by the asymptotic $(\lambda, \mu)$ quantum numbers (see Ref. [9]).

Along the same line one can describe the corresponding phase transitions in the neighbour odd nuclei. This is formally obtained in the Interacting Boson Fermion Model (IBFM) by a boson-fermion hamiltonian

$$H = H_B + H_F + V_{BF} , \tag{6}$$

where

$$H_F + V_{BF} = \sum_j \varepsilon_j a_j^\dagger \cdot a_j - \frac{x}{2N} \hat{Q}_B \cdot \hat{q}_F . \tag{7}$$

With proper choices of the single particle energies $\varepsilon_j$ and of the form of the fermion quadrupole operator $\hat{q}_F$, with the choice $\chi = 0$ for the boson quadrupole operator one can describe, by varying the mixing parameter $x$, the transition from sphericity to deformed gamma-instability. Assuming the fermion to be in the single j=3/2 orbital, at the critical point the spectrum and the properties of the odd mass states produced by the IBFM hamiltonian are rather similar [8] to those obtained in the E(5/4) model [3].

With the same approach we can describe odd nuclei along another leg of the Casten shape triangle, namely the transition from the spherical vibrational behaviour to stable axial deformation. We will assume again that the odd particle can sit in a triplet of orbitals, namely $j = 1/2, 3/2$ and $5/2$. The parameters of the boson, boson-fermion and pure fermion parts of the boson-fermion hamiltonian are chosen in such a way that varying the control parameter $x$ one obtains a transition from the spherical boson-fermion symmetry $U^{BF}(5)$ to the deformed $SU^{BF}(3)$. More details can be found in ref. [9].

As in the case of the even-even nuclei, the position of the critical point can be determined by resorting to the energy surfaces. In this case we have a number of different energy surfaces associated with the intrinsic states obtained by coupling the core intrinsic state to the different single particle states with different component $k$ along the core symmetry axis. When the core is well deformed all energy surfaces in the odd system also display clear deformed minima. Around the critical point, however, the coupling of the different orbitals modify in different ways the core energy surface. As a consequence, in the odd case, the different intrinsic states leads to critical points that differ from the even one and among themselves, with differences of the order of 1/N (N being the number of bosons of the core system).

The spectrum obtained at the critical point associated with the lowest state is shown in the lower frame of Figure 3. For comparison, in the upper frame we display the corresponding critical point spectrum for the even core. One can see that both energy spectra display similar characteristic patterns. The same arguments apply to the electromagnetic transition rates. We can conclude, therefore, that the behaviour of the odd nuclei provides a necessary complementary signatures of the occurrence of the phase transition.


## Acknowledgements

This work has been partially supported by the Spanish Ministerio de Ciencia e Innovación and by the European regional development fund (FEDER) under project number FIS2008-04189, and by an italian-spanish INFN-MICINN agreement.


## REFERENCES


1. F. Iachello, Phys. Rev. Lett. **85**, 3580 (2000).
2. F. Iachello, *Phys. Rev. Lett.* **87**, 052502 (2001).
3. F. Iachello, *Phys. Rev. Lett.* **95**, 052503 (2005).
4. C. E. Alonso, J. M. Arias, and A. Vitturi, *Phys. Rev. Lett.* **98**, 052501 (2007).
5. C.E. Alonso, J.M. Arias and A. Vitturi *Phys. Rev. C* **75**, 064316 (2007).
6. D.J. Rowe, Nucl. Phys. A **745**, 47 (2004).
   P.S. Turner and D.J. Rowe, Nucl. Phys. A **756**, 333 (2005).
   G. Rosensteel and D.J. Rowe, Nucl. Phys. A **759**, 92 (2005).
   D.J. Rowe and G. Thiamova, Nucl. Phys. A **760**, 59 (2005).
7. J. M. Arias, C. E. Alonso, A. Vitturi, J. E. García-Ramos, J. Dukelsky, and A. Frank, *Phys. Rev. C* **68**, 041302 (2003).
   J. E. García-Ramos, J. Dukelsky, and J. M. Arias, *Phys. Rev. C* **72**, 037301 (2005).
8. C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, *Phys. Rev. C* **72**, 061302(R) (2005).
   C.E. Alonso, J.M. Arias, and A. Vitturi, Phys. Rev. C **74**, 027301 (2006).
9. C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C (2009) (in press).