Core Excitation Effects in the Breakup of Halo Nuclei

A. M. Moro*, R. de Diego*, J. A. Lay*, R. Crespo†,**, R. C. Johnson‡, J. M. Arias* and J. Gómez-Camacho*,§

*Departamento de FAMN, Universidad de Sevilla, Apartado 1065, E-41080 Sevilla, Spain
†Centro de Física Nuclear, Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal
**Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Prof. Cavaco Silva, Taguspark, 2780-990, Oeiras, Portugal
‡Physics Department, University of Surrey, Guildford Surrey, GU2 7XH, U.K.
§Centro Nacional de Aceleradores, Universidad de Sevilla/Junta de Andalucía, E-41092 Sevilla, Spain

Abstract. The role of core excitation in the structure and dynamics of two-body halo nuclei is investigated. We present calculations for the resonant breakup of ^{11}Be on protons at an incident energy of 63.7 MeV/nucleon, where core excitation effects were shown to be important. To describe the reaction, we use a recently developed extension of the DWBA formalism which incorporates these core excitation effects within the no-recoil approximation. The validity of the no-recoil approximation is also examined by comparing with DWBA calculations which take into account core recoil. In addition, calculations with two different continuum representations are presented and compared.

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INTRODUCTION

Reactions induced by halo nuclei have been described during the past 25 years using a variety of reaction formalisms, such as the Continuum–Discretized Coupled–Channels (CDCC) method [1], the adiabatic (*frozen-halo*) approximation [2, 3, 4], several semi-classical approaches [5, 6, 7] and, more recently, the Faddeev equations [8, 9].

In their standard formulations, these methods are typically based on a simple mean-field description of the halo nucleus in which the excitation (or breakup) of this nucleus is modeled as a single-particle excitation of the valence particle, with the core remaining in its ground state. This simplified picture ignores the presence of core admixtures in the states, as well as possible transitions between these core states during the collision. These core excitation (CEX hereafter) effects are however expected to be important in halo nuclei with a well deformed core, such as ^{11}Be or ^{19}C.

At intermediate energies, the effect of CEX in inelastic and breakup reactions can be studied in the DWBA formalism. In this scheme, the non-central part of the core-target interaction, which is responsible for the dynamic CEX mechanism, gives rise to an additional term in the transition amplitude [10]. Furthermore, in situations in which the core is much heavier than the valence particle, this additional term acquires a very simple

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and transparent form [11]. The model was applied to the scattering of ^{11}Be on protons and revealed that CEX plays a very significant role in the dynamics of the reaction, affecting the magnitude and shape of the breakup angular distributions.

In this contribution, we revisit the CEX model and explore in more detail its limits of validity. For this purpose, we compare two different discretization methods to describe the continuum of the halo nucleus, namely, a *binning* method and a pseudostate method. Also, we present additional DWBA calculations performed without making use of the no-recoil approximation, which show that this approximation is very well fulfilled in the kinematical conditions considered in the calculations of Refs. [10, 11].

STRUCTURE OF THE HALO NUCLEUS IN THE PARTICLE-CORE MODEL

The halo structure is treated in the particle-rotor model, in terms of the Hamiltonian

$$H_{\text{proj}} = T_r + V_{vc}(\vec{r},\vec{\xi}) + h_{\text{core}}(\vec{\xi}), \tag{1}$$

where $\vec{r}$ is the relative coordinate between the valence and the core, $\vec{\xi}$ denote the internal degrees of freedom of the core, T_r the core-valence kinetic energy operator, V_{vc} the valence-core interaction and $h_{\text{core}}(\vec{\xi})$ the intrinsic Hamiltonian of the core.

The bound and unbound states of the system are represented by the eigenstates of the Hamiltonian (1) for negative ($\varepsilon < 0$) and positive ($\varepsilon > 0$) eigenvalues. In both cases, the dependence on $\vec{\xi}$ in V_{vc} gives rise to core admixtures in the projectile states. For example, for a bound state with total angular momentum J and projection M, the projectile wave function can be expressed as

$$\Psi_{JM}(\vec{r},\vec{\xi}) = \sum_{\alpha} \left[\varphi^J_\alpha(\vec{r}) \otimes \Phi_I(\vec{\xi}) \right]_{JM}, \tag{2}$$

where the label α denotes the set of quantum numbers $\{\ell, s, j, I\}$, with I and s the core and valence intrinsic spins, ℓ the orbital angular momentum, and $\vec{j} = \vec{\ell} + \vec{s}$. The functions $\Phi_I(\vec{\xi})$ and $\varphi^J_\alpha(\vec{r})$ describe, respectively, the core states and the valence–core relative motion.

The radial parts of the functions $\varphi^J_\alpha(\vec{r})$, denoted hereafter $R^J_\alpha(r)$, can be obtained in different ways. In the calculations presented in Refs. [10, 11], these functions were obtained by direct integration of the Schrödinger equation subject to the appropriate boundary conditions, for bound or unbound states. Continuum states were grouped into energy intervals, using the standard average binning method [1]. Alternatively, the projectile Hamiltonian can be represented by the eigenstates of the Hamiltonian in a truncated basis of square-integrable functions (generically referred to as *pseudostates*). For positive-energy states ($\varepsilon > 0$), the eigenvalues obtained in the pseudostate method, and their corresponding eigenstates, should be regarded as a discrete representation of the continuum spectrum.

In this contribution, we compare the calculated resonant breakup observables using the binning and the pseudostates methods. A related work was done in Ref. [12] but in that case the projectile consisted on a three-body Borromean nucleus. As pseudostates we use the Transformed Harmonic Oscillator (THO) basis used in Ref. [13]. This basis is obtained by applying an analytic local scale transformation (LST) [14] to the conventional harmonic oscillator basis such that it transforms the asymptotic Gaussian behaviour of the HO functions into an exponential.

In this manuscript our working example is the ^{11}Be nucleus, described in terms of the hamiltonian of Ref. [15] which assumes that the ^{10}Be core has a permanent axial deformation with $\beta_2 = 0.67$. This model reproduces the separation energy for the ground state ($1/2^+$; $\varepsilon = -500$ keV) and the $1/2^-$ excited state ($\varepsilon = -180$ keV). In addition, it produces narrow resonances at $\varepsilon = 1.2$ ($5/2^+$), 2.7 ($3/2^-$) and 3.2 MeV ($3/2^+$), which can be identified with the states observed by Fukuda *et al.* [16] in the ^{11}Be+^{12}C reaction at 70 MeV/nucleon.

In Fig. 1 we show the wavefunctions for the ground state, and the $5/2^+$ and $3/2^+$ resonances, which are the most relevant for the reaction calculations presented below. The solid lines are the *exact* calculations, obtained by direct integration of the Schrödinger equation, whereas the dashed lines correspond to a pseudostate calculation, using the THO basis. Resonant states are identified with stabilized energies with respect to the basis size (N), as explained in [13]. The dominant configuration in the ground state and the $5/2^+$ resonance correspond to the core in the ground state, whereas the $3/2^+$ resonance has a dominant $^{10}\mathrm{Be}(2^+) \otimes \nu s_{1/2}$ parentage. It is apparent from Fig. 1 that, for the ground-state, the pseudostate representation is almost identical to the *exact* solution. For the resonances, the radial wavefunctions are very similar in the interior part, but they necessarily differ at large distances, due to the exponential asymptotic behaviour of the THO functions. In the next section, we explore the effect of the differences between the radial wavefunctions on the calculated breakup observables.

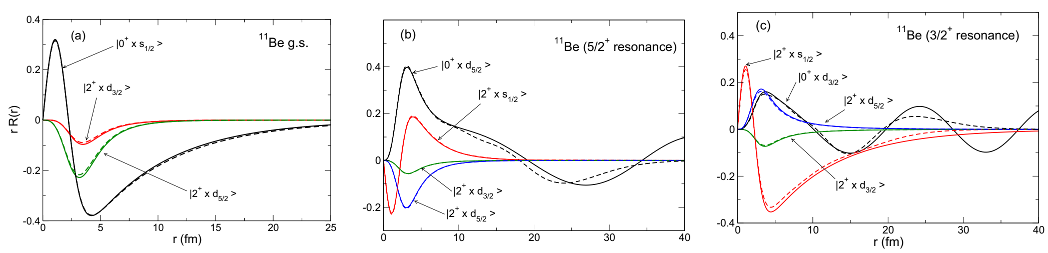

FIGURE 1. (Colour online) Radial parts of the ground state wave function for the ^{11}Be nucleus and the $5/2_1^+$ and $3/2_1^+$ resonances obtained by direct integration of the Schrödinger equation (solid lines) and by diagonalization in a THO basis (dashed lines).

A DWBA MODEL FOR CORE EXCITATION

We consider the excitation (or breakup) of the halo nucleus resulting from its collision with a target nucleus that, for simplicity, will be considered as inert. Core excitation affects the reaction dynamics in two ways. Firstly, the presence of core admixtures in the states of the projectile means that these states cannot be simply treated as single-particle states calculated in some mean field potential. Secondly, the interaction of the core with the target may give rise to transitions between these core states, leading also to the breakup of the projectile. As shown below, the standard DWBA method can be naturally extended to accommodate these two effects.

This breakup process is described in terms of the three-body Hamiltonian

$$H = T_R + H_{\text{proj}} + V_{ct}(\vec{R}_{ct}, \vec{\xi}) + V_{vt}(\vec{R}_{vt}), \tag{3}$$

where T_R represents the kinetic energy operator for the projectile–target relative motion, V_{ct} and V_{vt} are in the core–target and the valence–target interactions and H_{proj} is the projectile Hamiltonian introduced in the previous section.

Using the Hamiltonian (3), the DWBA transition amplitude for the transition between the states Ψ^i and Ψ^f is given by

$$\mathscr{T}_{pt}^{JM,J'M'} = \langle \chi^{(-)}_{\vec{K}'}(\vec{R})\Psi^f_{J'M'}(\vec{r},\vec{\xi})|V_{vt}(\vec{R}_{vt}) + V_{ct}(\vec{R}_{ct},\vec{\xi})|\chi^{(+)}_{\vec{K}}(\vec{R})\Psi^i_{JM}(\vec{r},\vec{\xi})\rangle, \tag{4}$$

where $\chi^{(+)}_{\vec{K}}(\vec{R})$ and $\chi^{(-)}_{\vec{K}'}(\vec{R})$ are distorted waves describing the projectile-target relative motion in the initial and final channels, respectively.

Although the expression (4) could be evaluated directly, without further approximations, in [10, 11] we have proposed a simple way of evaluating this expression which makes use of a multipole expansion of the core-target interaction:

$$V_{ct}(\vec{R}_{ct}, \vec{\xi}) = \sum_{\lambda,\mu} V^{(\lambda)}_{ct}(R_{ct}, \xi) Y^*_{\lambda\mu}(\hat{R}) Y_{\lambda\mu}(\hat{\xi}). \tag{5}$$

Replacing this expansion in the DWBA amplitude, Eq. (4), and separating the central ($\lambda = 0$) from the non-central ($\lambda > 0$) parts gives rise to two terms,

$$\mathscr{T}_{pt}^{JM,J'M'}(\vec{K}', \vec{K}) = \mathscr{T}_{val}^{JM,J'M'} + \mathscr{T}_{\text{corex}}^{JM,J'M'}. \tag{6}$$

The first term, that we denote *valence amplitude* for shortness, is explicitly given by

$$\mathscr{T}_{val}^{JM,J'M'}(\vec{K}', \vec{K}) = \sum_{\alpha,\alpha'} \langle \chi^{(-)}_{\vec{K}'}(\vec{R})\psi^{J'}_{\alpha'}(\vec{r})|V_{vt}(R_{vt}) + V^{(0)}_{ct}(R_{ct})|\chi^{(+)}_{\vec{K}}(\vec{R})\psi^{J}_{\alpha}(\vec{r})\rangle \delta_{I,I'}, \tag{7}$$

where we have written the initial and final states in the form of Eq. (2). This amplitude contains only the central part of V_{ct} and, therefore, it cannot induce transitions involving excitations of the core.

The second term (*core excitation amplitude*) acquires a particularly simple form when evaluated in the no-recoil approximation [11],

$$\mathscr{T}_{\text{corex}}^{JM,J'M'} = \sum_{\lambda>0,\mu} \langle J'M'|JM\lambda\mu\rangle \sum_{\alpha,\alpha'} \langle R^{J'}_{\alpha'}|R^{J}_{\alpha}\rangle G^{(\lambda)}_{\alpha J,\alpha' J'} \widetilde{\mathscr{T}}^{(\lambda\mu)}_{\text{ct}}(I \to I'), \tag{8}$$

where $G^{(\lambda)}_{\alpha J,\alpha' J'}$ is a geometric factor [10, 11] and $\widetilde{\mathscr{T}}^{(\lambda\mu)}_{ct}$ is related to the core-target two-body transition amplitude for a core transition $IM_c \to I'M'_c$ of multipolarity λ as $\mathscr{T}_{ct}^{IM_c,I'M'_c} = \langle IM_c\lambda\mu|I'M'_c\rangle \widetilde{\mathscr{T}}^{(\lambda\mu)}_{ct}$. This approximation is expected to be valid when the mass of the core is much larger than that of the valence particle.

APPLICATION TO ^{11}BE BREAKUP

Now we apply the DWBA model, extended to include CEX, to the breakup of ^{11}Be on protons at 63.7 MeV/nucleon. This reaction has been already analysed in [10, 11] using this framework and a binning description of the ^{11}Be continuum. The calculated angular distributions, integrated in the relative energy intervals ε=0–2.5 MeV and ε=2.5–5.0 MeV, were found to describe reasonably well the data of Ref. [17]. Furthermore, it was shown that the core excitation mechanism gives a sizable contribution to the breakup cross section, particularly in the second energy interval, which contains the $3/2^+$ resonance.

In this contribution we present further calculations for this reaction. We adopt the same structure model and fragment-target potentials used in [11] so we refer the reader to this reference for further details. Here, we focus on the excitation of the 1.78 MeV ($5/2^+$) and 3.41 MeV ($3/2^+$) resonances.

In Fig. 2 we display the calculated breakup angular distributions as a function of the center-of-mass scattering angle. The left and right panels correspond, respectively, to the $5/2^+$ and $3/2^+$ resonances. The dashed and solid lines use the no-recoil approximation for the CEX amplitude, Eq. (8). For the dashed lines, the ^{11}Be wavefunctions were obtained by direct integration of the Schrödinger equation and the resonance region was represented by a continuum bin spanning the intervals $\varepsilon = 0.8 - 1.4$ MeV and $\varepsilon = 2.5 - 3.5$ MeV for the $5/2^+$ and $3/2^+$ resonances, respectively. On the other hand, in the calculations represented by the solid lines, the ^{11}Be states were represented by eigenstates of the ^{11}Be Hamiltonian in the THO pseudostate basis. It is seen that the two basis representations provide almost identical results. This result indicates that this observable is mostly sensitive to the interior part of the projectile wavefunctions. For the CEX amplitude, this independence of the asymptotic behaviour can be readily inferred from Eq. (8) since the overlap between the radial parts of the continuum wavefunctions and the ground-state wavefunction will suppress the contribution of the asymptotic part of the former.

For each basis representation, we show in Fig. 2 the separate contributions arising from the valence [Eq. (7)] and the CEX amplitude [Eq. (8)], as well as their coherent sum [Eq. (6)]. It is seen that the core excitation mechanism is very important in both cases, particularly for the $3/2^+$ state, due to the dominance of the $^{10}\mathrm{Be}(2^+) \otimes \nu s_{1/2}$ configuration in this state.

We have also assessed the validity of the no-recoil approximation used to derive Eq. (8). For this purpose, we have performed additional calculations evaluating directly Eq. (4), without further approximations. The pseudostate representation was chosen to represent the ^{11}Be states. The results, shown in Fig. (2) by open circles, are found to be very close to the no-recoil calculations. This justifies the use of this approximation for this reaction, and supports the calculations presented in Ref. [11].

SUMMARY AND CONCLUSIONS

In summary, we have investigated the effect of core excitation (CEX) in the breakup of halo nuclei, within the framework of the DWBA method. Within this formalism, the non-

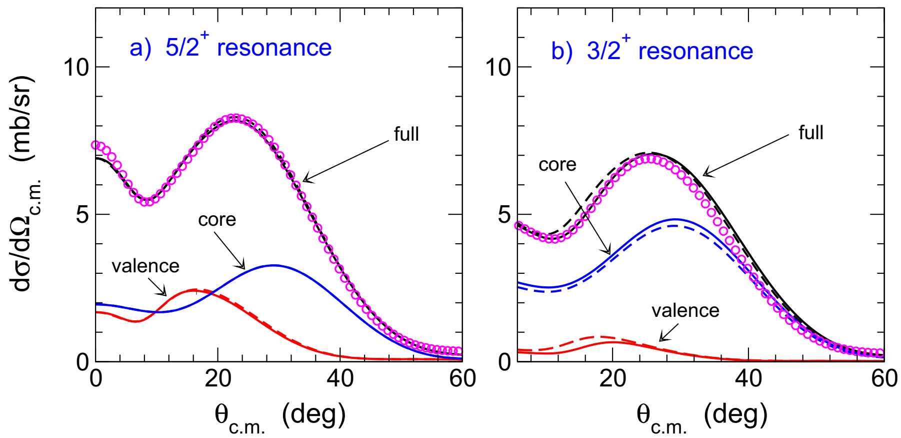

FIGURE 2. (Color online) Angular distribution for the breakup of ^{11}Be on protons at 63.7 MeV/nucleon. The left and right panels correspond, respectively, to the $5/2^+$ ($E_x = 1.78$ MeV) and the $3/2^+$ ($E_x = 3.41$ MeV) resonances. Solid and dashed lines correspond to the DWBA calculations using the no-recoil model, representing the resonances by a continuum bin (dashed) or by a pseudostate (solid). The separate contributions coming from the valence and core amplitudes are also shown. The open circles correspond to the DWBA calculation taking into account recoil effects.

central part of the core-target potential gives rise to an additional term in the transition amplitude, which accounts for the dynamic CEX mechanism. This extra term acquires a particularly simple form when evaluated in the no-recoil approximation.

The model has been applied to the resonant breakup of ^{11}Be on protons at an incident energy of 63.7 MeV/nucleon. We find that the CEX mechanism is very important in both cases, affecting the magnitude and shape of the breakup angular distributions.

We have compared two different representations for the ^{11}Be continuum, a binning method and a pseudostate method. Both representations are found to give very similar results for the breakup cross sections.

We have also tested the validity of the no-recoil approximation used to evaluate the CEX amplitude. For this purpose, we have performed calculations evaluating directly the DWBA amplitude, Eq. (4). The results are very close to those obtained in the no-recoil approximation, thus justifying the use of this approximation in the present case.

We note that all calculations presented in this work rely on the Born approximation, and hence higher order couplings (such as continuum-continuum couplings) are ignored. These higher-order couplings could be included in the coupled-channels framework, as described in [18]. These kind of calculations are in progress and will be presented elsewhere.

From the calculations presented in this work, we may conclude that these core-excitation effects might be also important in other reactions induced by weakly-bound projectiles with deformed constituents. The DWBA method used here can provide a useful and simple estimate of these effects in those situations in which the assumptions of the model (i.e., the validity of the Born approximation and the possibility of neglecting core-recoil) are justified.

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