Towards a Practical Argumentative Reasoning with Qualitative Spatial Databases

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Abstract. Classical database management can be flawed if the Knowledge database is built within a complex Knowledge Domain. We must then deal with inconsistencies and, in general, with anomalies of several types. In this paper we study computational and cognitive problems in dealing qualitative spatial databases.

Keywords: Knowledge Management and Processing, Spatial Reasoning.

1 Introduction

Spatio-temporal representation and reasoning are topics that have attracted quite a lot of interest in AI. Since the spatial notions used by the humans are intrinsically qualitative, the reasoning about spatial entities, their properties and the relationship among them, are central aspects in several intelligent systems. But the problem is far to be solved in general. The spatial reasoning is more complex than the temporal one. The higher dimension of the things is not the unique problem. The topology is, in qualitative terms, hard to represent by formalisms with amenable calculus. The semantic of these representations offers incomplete support to our daily reasoning (the *poverty conjecture*: there is no purely qualitative, general purpose kinematics). Different ontologies have been proposed, but they are not of general purpose.

Among them, the theory called *Region Connection Calculus* (hereafter referred as RCC), developed by Randell, Cui and Cohn [4] have been extensively studied in AI [11], and in the field of Geographic Information Systems (GIS) [2]. A common deficiency of the theories representing topological knowledge, is that either the full theory is computationally unacceptable or they fail to meet basic desiderata for these logics [9]. For constraint satisfaction problems there are algorithms to work with the relational sublanguage, and tractable subsets of the calculus RCC-8 (a relational sublogic of RCC) have been found [11]. The intractability of the full theory is mainly due to the complexity of its models (topological spaces with separation properties [7]). We propose a practical approach (using an automated theorem prover) to investigate the verification

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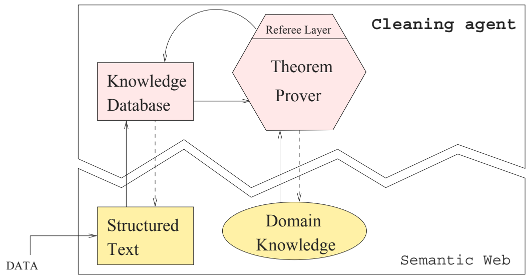

Fig. 1. Cleaning service process

problem of knowledge bases (KB) written in the language of RCC. In general, due to the complexity of the theories involved, the knowledge base may be inconsistent, although the environment was *well represented* by the relational part of the database. For a proper understanding of our framework, it is important to point out the following characteristics of the problem:

– The knowledge database is never completed (the user will write new facts in the future). Thus the difficulties begin with the future introduction of data.
– The *intensional theory* of the database is not clausal. Thus, is highly possible that Reiter's axiomatization of database theory [12] becomes inconsistent. Nevertheless, the self database represents a real spatial configuration.
– The knowledge base does not contain facts about all the relationships of the RCC language. It seems natural that only facts on primary relationships appear (we have selected for our experiment the relations `Connect`, `Overlaps` and `Part-of`, that one can consider as the *primary* relationships).

The preceding characteristics are important in order to classify the anomalies. The first one may produce inconsistencies that the user can repair, but the second one is a logical inconsistency and it is hard to solve. Thus, we have to reason with inconsistent knowledge. The last one implies that the (logic-based) deduction of new knowledge must replace to solving methods for CSP.

Our problem is only an interesting example of the more general problem of cleaning incomplete databases in the Semantic Web: the *cleaning agent* must detect anomalies in knowledge bases written by the user (in structured text), and associated to a complex ontology (see Fig. 1). A methodological approach to the cleaning problem was shown in [1] (where a cleaning cycle to the agent was proposed). It is necessary to point out that it is not our aim to find inconsistencies in the domain knowledge. In [6] it is shown an application of an automated theorem prover (the SNARK system) to provide a declarative semantics for languages for the Semantic Web, by translating first the forms from the semantic markup languages to first-order logic in order to apply the theorem prover to find inconsistencies. Our problem is not exactly that. We assume that the domain knowledge (the RCC theory and eventually the composition table for the relations of figure 3) is consistent, and that it is highly possible that RCC jointly with the database becomes inconsistent. However, in one of the experiments the theorem prover found an error in the composition table for the RCC-8 shown in [4]. Our problem has also another interesting aspect: the data inserted have not any spatial indexing.

2 The Theory of RCC

The Region Connection Calculus is a topological approach to qualitative spatial representation and reasoning where the *spatial entities* are non-empty regular sets[1] (a good introduction to the theory is [4]). The primary relation between such regions is the connection relation $\mathtt{C}(\mathtt{x},\mathtt{y})$, which is interpreted as *"the closures of* x *and* y *intersect"*. The axioms of RCC are two basic axioms on $\mathtt{C}$, $\mathtt{A}_1 := \forall \mathtt{x}[\mathtt{C}(\mathtt{x},\mathtt{x})]$ and $\mathtt{A}_2 := \forall \mathtt{x},\mathtt{y}[\mathtt{C}(\mathtt{x},\mathtt{y}) \to \mathtt{C}(\mathtt{y},\mathtt{x})]$, plus several axioms/definitions on the main spatial relationships (see Fig. 2).

$\mathtt{A}_{\mathtt{DC}}$:	$\mathtt{DC}(\mathtt{x},\mathtt{y}) \leftrightarrow \neg\mathtt{C}(\mathtt{x},\mathtt{y})$	(x is disconnect from y)
$\mathtt{A}_{\mathtt{P}}$:	$\mathtt{P}(\mathtt{x},\mathtt{y}) \leftrightarrow \forall \mathtt{z}[\mathtt{C}(\mathtt{z},\mathtt{x}) \to \mathtt{C}(\mathtt{z},\mathtt{y})]$	(x is part of y)
$\mathtt{A}_{\mathtt{PP}}$:	$\mathtt{PP}(\mathtt{x},\mathtt{y}) \leftrightarrow \mathtt{P}(\mathtt{x},\mathtt{y}) \wedge \neg\mathtt{P}(\mathtt{y},\mathtt{x})$	(x is proper part of y)
$\mathtt{A}_{\mathtt{EQ}}$:	$\mathtt{EQ}(\mathtt{x},\mathtt{y}) \leftrightarrow \mathtt{P}(\mathtt{x},\mathtt{y}) \wedge \mathtt{P}(\mathtt{y},\mathtt{x})$	(x is identical with y)
$\mathtt{A}_{\mathtt{O}}$:	$\mathtt{O}(\mathtt{x},\mathtt{y}) \leftrightarrow \exists \mathtt{z}[\mathtt{P}(\mathtt{z},\mathtt{x}) \wedge \mathtt{P}(\mathtt{z},\mathtt{y})]$	(x overlaps y)
$\mathtt{A}_{\mathtt{DR}}$:	$\mathtt{DR}(\mathtt{x},\mathtt{y}) \leftrightarrow \neg\mathtt{O}(\mathtt{x},\mathtt{y})$	(x is discrete from y)
$\mathtt{A}_{\mathtt{PO}}$:	$\mathtt{PO}(\mathtt{x},\mathtt{y}) \leftrightarrow \mathtt{O}(\mathtt{x},\mathtt{y}) \wedge \neg\mathtt{P}(\mathtt{x},\mathtt{y}) \wedge \neg\mathtt{P}(\mathtt{y},\mathtt{x})$	(x partially overlaps y)
$\mathtt{A}_{\mathtt{EC}}$:	$\mathtt{EC}(\mathtt{x},\mathtt{y}) \leftrightarrow \mathtt{C}(\mathtt{x},\mathtt{y}) \wedge \neg\mathtt{O}(\mathtt{x},\mathtt{y})$	(x externally connected to y)
$\mathtt{A}_{\mathtt{TPP}}$:	$\mathtt{TPP}(\mathtt{x},\mathtt{y}) \leftrightarrow \mathtt{PP}(\mathtt{x},\mathtt{y}) \wedge \exists \mathtt{z}[\mathtt{EC}(\mathtt{z},\mathtt{x}) \wedge \mathtt{EC}(\mathtt{z},\mathtt{y})]$	(x tangential prop. part of y)
$\mathtt{A}_{\mathtt{NTPP}}$:	$\mathtt{NTPP}(\mathtt{x},\mathtt{y}) \leftrightarrow \mathtt{PP}(\mathtt{x},\mathtt{y}) \wedge \neg\exists \mathtt{z}[\mathtt{EC}(\mathtt{z},\mathtt{x}) \wedge \mathtt{EC}(\mathtt{z},\mathtt{y})]$	(x non-tang. prop. part of y)

Fig. 2. Axioms of RCC

The eight jointly exhaustive and pairwise disjoint relations shown in Fig. 3 form the relational calculus RCC-8, that has been deeply studied by J. Renz and B. Nebel [11]. In that work CSP problems on RCC-8 are classified in terms of (un)tractability. These problems are, in some cases, tractable, but the relational language can be too weak for some applications. The consistency/entailment problems in the full theory RCC have a complex behaviour. If we consider topological models, the problem is computationally unacceptable. The restriction to *nice* regions of $\mathbb{R}^2$ is also hard to compute [8].

The problem of a good representation of a model by a knowledge base arises. Concretely, we must consider three classes of models: the class of all models (according to the classical definition from first order logic), the class of the topological models, and $\mathbb{R}^n$ where the constants are interpreted as the regular sets under study (*the intended model*). Formally,

[1] A set x of a topological space is regular if it agrees with the interior of its closure.

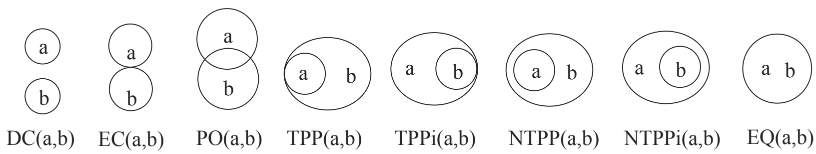

Fig. 3. The eight basic relations of RCC-8

Definition 1. *Let Ω be a topological space, and* X *be a finite set of constants. A structure Θ is called a* topological model *on Ω if it has the form*

$$\langle \mathcal{R}(\Omega)_{/\sim}, \mathtt{C}_{\Theta}, \{\mathtt{a}_{\Theta} : \mathtt{a} \in \mathtt{X}\}\rangle$$

where $\mathcal{R}(\Omega)$ is the class of nonempty regular sets, $\sim$ is the equivalence relation "the closures agree"[2], $\mathtt{C}_\Theta$ *is the intended interpretation of* C *and whenever* $\mathtt{a} \in \mathtt{X}$, $\mathtt{a}_\Theta \in \mathcal{R}(\Omega)_{/\sim}$.

Every structure is expanded to one in the full language of RCC, by the natural interpretation of the other relationships [7]: If Ω is a nontrivial connected T_3-space, the natural expansion of any topological structure on Ω to the full language is a model of RCC.

3 Towards an Automated Argumentative Reasoning

The logic-based argument theory is a formalism to reason with inconsistent knowledge [5]. An *argument* in T is a pair $\langle \Pi, \phi\rangle$ where $\Pi \subseteq T$ and $\Pi \vdash \phi$. The argumentative structure of K is an hierarchy of arguments which offers a method to obtain useful knowledge from T with certain properties. Also, it provides a method to estimate the robustness of an argument via *argument trees* [3]. However, this approach can not be directly applied on huge databases (it needs, for example, to find all maximally consistent subsets of the database). The problem can be solved in practice by adapting the notion of *argument* to an automated theorem prover. For this work we choose OTTER [10], a resolution-based automated theorem prover. Since it is not our aim to describe the methodology we need to work with the theorem prover, we simply assume that the system works in autonomous mode, a powerful feature of OTTER.

Definition 2.

1. *An* O*-argument (an argument for OTTER) is a pair $\langle \Pi, \phi\rangle$ such that Π is the set of axioms from the OTTER's refutation of $\{\neg\phi\}$ (that we write $\Pi \vdash^{\mathtt{O}} \phi$).*
2. *If $\langle \Pi, \phi\rangle$ is an* O*-argument, the length of $\langle \Pi, \phi\rangle$, denoted as $len(\langle \Pi, \phi\rangle)$, is the length of the refutation of $\Pi \cup \{\neg\phi\}$ by OTTER.*

[2] The relation is necessary because of the extensionality of P given in the axiom $\mathtt{A}_{\mathtt{P}}$. With this relation, the mereological relation EQ agree with the equality.

The argumentative structure can not be directly translated, because of the consistency notion[3]. By example, the argument class $\mathtt{A}\exists(\mathtt{K})$ may be adapted: $\mathtt{A}^{\mathtt{0}}\exists(\mathtt{K}) = \{\langle \Pi, \phi\rangle \ : \ \Pi$ is consistent and $\Pi \vdash^{\mathtt{0}} \phi\}$.

4 Anomalies in Complex Knowledge Bases

From now on, we will consider fix a topological model Θ, the spatial model with we will work, and $\mathtt{K}$ a database representation of Θ (that is, $\mathtt{K}$ is a set of ground atomic formulae such that $\Theta \models \mathtt{K}$). To simplify we assume that the model satisfies the unique names axiom. Three theories describe the model: the formalization of Reiter's database theory $\mathtt{T_{DB}}(\mathtt{K})$, the theory $\mathtt{RCC}(\mathtt{K})$, whose axioms are those of $\mathtt{K}$ plus RCC, and $\mathtt{RCC}(\mathtt{T_{DB}}(\mathtt{K}))$. The three theories has a common language, $\mathtt{L_K}$. The following is an intuitive ontology of the anomalies in RCC-databases suggested by the experiments:

- A1: The contradictions of the base due to the bad implementation of the data (e.g. absence of some knowledge)
- A2: The anomalies due to the inconsistency of the model: the theorem prover derives from the database the existence of regions which do not have a name (possibly because they have not been introduced by the user yet). This anomaly may also be due to the *Skolem's noise*, produced when we work with the domain closure axioms but the domain knowledge is not clausal.
- A3: Disjunctive answers (a logical deficiency).
- A4: Inconsistency in the Knowledge Domain.

As we remarked, the anomalies come from several sources: the set may be inconsistent with the Domain Knowledge due to formal inconsistencies produced by wrong data, the database is not complete with respect to a basic predicate (the user will continue introducing data), etc. In Fig. 4 the most simple problem is shown. The system shows arguments with the Skolem function of the clausal form of $\mathtt{A_O}$ to questions as *"give us a region which is part of* $\mathtt{a}$*"*.

If the spatial regions used in a GIS are semialgebraic sets, Skolem functions can be semi-algebraically defined [13]. In the practice, the spatial interpretation may be thought as a partial function. In the case of RCC, Skolem functions come from axioms $\mathtt{A_P}, \mathtt{A_O}, \mathtt{A_{TPP}}$ and $\mathtt{A_{NTPP}}$. It is possible to give a spatial interpretation of such functions. For example, the Skolem function for $\mathtt{A_O}$, $\mathtt{f_O}(\mathtt{x}, \mathtt{y})$ gives the intersection region of $\mathtt{x}$ and $\mathtt{y}$, if $\mathtt{O}(\mathtt{x}, \mathtt{y})$. This idea allows one to eliminate useless results by a partial axiomatization of the intersection (see Fig.6).

5 Consistent Databases and Arguments

Definition 3. *Let Θ be a topological model. The* graph *of Θ, denoted by Θ_G, is the substructure of Θ whose elements are the interpretation of the constants.*

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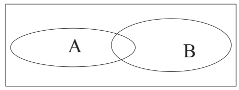
[3] An associated automated model finder, MACE, may be considered for a complete description.

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Database                                          OTTER's proof
--------------------------------------------------------------------------
all x (x=A|x=B).                               1[] x=x.
A!=B.                                          5[] x!=A|y!=B|O(x,y).
all x y (x=A&y=B|x=B&y=A|x=A&y=A|              14[] P($f1(x,y),x)| -O(x,y).
         x=B&y=B -> O(x,y)).                   16[] -P(x,A)|$Ans(x).
all x y (x=A& y=B|x=B& y=A|x=A&y=A|            27[hyper,5,1,1]O(A,B).
         x=B&y=B -> C(x,y)).                   65[hyper,14,27]P($f1(A,B),A).
all x y ((exists z (P(z,x)&P(z,y)))<->O(x,y)).66[binary,65.1,16.1]
all x (P(x,A)->$Ans(x)).                           $Ans($f1(A,B)).
```

Fig. 4. A simple anomaly and an O-argument

Definition 4. *Let* K *be a set of formulas.*

– *The* world *of* K*, denoted by* W(K)*, is the set of the interpretations in* Θ *of the constants in the language of* $\mathrm{L}_{\mathtt{K}}$*.*
– *Consider an interpretation of the Skolem functions of the clausal form of* RCC*. The* cognitive neighborhood *of* K*,* $\Gamma(\mathtt{K})$*, is the least substructure of the expansion of* Θ *to the clausal language of* RCC*, containing* W(K)*.*

It seems that the consistency of an argument depends only on its cognitive neighborhood. It is true for arguments with enough *credibility.*

Definition 5. *An* undercut *of* $\langle \Pi_1, \phi\rangle$ *is an argument* $\langle \Pi, \neg(\phi_1 \wedge \cdots \wedge \phi_n)\rangle$ *where* $\{\phi_1, \ldots, \phi_n\} \subseteq \Pi_1$*. The undercut is called* local *if* $\Gamma(\Pi) \subseteq \Gamma(\Pi_1)$*.*

Definition 6. $\langle \Pi, \alpha\rangle$ *is more conservative than* $\langle \Pi', \beta\rangle$ *if* $\Pi \subseteq \Pi'$ *and* $\beta \vdash^{\mathtt{O}} \alpha$*.*

Definition 7. *Let* T *be a theory, and* ϕ *a formula of the clausal language of* T*.*

– *A clause has* Skolem's noise *if it has occurrences of Skolem function symbols.*
– *The* degree of credibility *of an argument* $\langle \Pi, \phi\rangle$ *is*

$$gr(\langle \Pi, \phi\rangle) = \frac{len(\langle \Pi, \phi\rangle) - |\{\eta \in Proof^{\mathtt{O}}(\Pi, \phi) \ : \ \eta \textit{ has Skolem's noise}\}|}{len(\langle \Pi, \phi\rangle)}$$

The degree of credibility estimates the robustness of the argument according to the use of Skolem functions in the proof, functions which become *ghost* regions (the credibility degree of the argument shown in Fig. 4 is 4/7).

Theorem 1. *Let* $\langle \Pi, \phi\rangle$ *be an* O*-argument of* RCC(K)*. If* $gr(\langle \Pi, \phi\rangle) = 1$ *then* $\langle \Pi, \phi\rangle \in \mathtt{A}^{\mathtt{O}}\exists(\mathtt{RCC}(\mathtt{K}))$ *and* $\Gamma(\Pi) \models \Pi + \phi$*.*

Corollary 1. *If* $gr(\langle \Pi, \phi \rangle) = gr(\langle \Pi', \phi' \rangle) = 1$ *and the first argument is an undercutting argument of the second one, then*

- $\Gamma(\Pi) \not\subseteq \Gamma(\Pi')$ *(thus there is not local undercutting argument with degree of credibility 1).*
- *If* $\langle \Pi, \phi \rangle$ *is a canonical undercut, that is* $\phi \equiv \neg(\phi_1 \wedge \cdots \wedge \phi_n)$ *with* $\phi_i \in \Pi'$*, then* $\Gamma(\Pi') \subsetneq \Gamma(\Pi)$*.*

The preceding corollary relates undercutting arguments and spatial configurations, and it may be useful to estimate the size of *argument trees* [3].

Definition 8. *Let* $\mathtt{K}$ *be a knowledge database (a set of ground atomic formulae) for* Θ*. The base* $\mathtt{K}$

- *is* $\mathtt{C}$-complete *if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L}_\mathtt{K}$*, if* $\Theta \models \mathtt{C}(\mathtt{a}, \mathtt{b})$*, then* $\mathtt{C}(\mathtt{a}, \mathtt{b}) \in \mathtt{K}$*.*
- *is* extensional *for* $\mathtt{P}$ *if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L}_\mathtt{K}$

$$\mathtt{P}(\mathtt{a}, \mathtt{b}) \notin \mathtt{K} \quad \Longrightarrow \quad \exists \mathtt{c} \in \mathtt{L}_\mathtt{K} \, [\mathtt{C}(\mathtt{c}, \mathtt{a}) \in \mathtt{K} \wedge \mathtt{C}(\mathtt{c}, \mathtt{b}) \notin \mathtt{K}]$$

- *is* refined *if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L}_\mathtt{K}$

$$\Theta \models \mathtt{O}(\mathtt{a}, \mathtt{b}) \quad \Longrightarrow \quad \exists \mathtt{c} \in \mathtt{L}_\mathtt{K} \, [\{\mathtt{P}(\mathtt{c}, \mathtt{a}), \mathtt{P}(\mathtt{c}, \mathtt{b})\} \subseteq \mathtt{K}] \quad \Longrightarrow \quad \mathtt{O}(\mathtt{a}, \mathtt{b}) \in \mathtt{K}$$

- recognizes frontiers *if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L}_\mathtt{K}$ *such that* $\Theta \models \mathtt{P}(\mathtt{a}, \mathtt{b})$

$$\Theta \models \mathtt{TPP}(\mathtt{a}, \mathtt{b}) \Longrightarrow \exists \mathtt{c} \in \mathtt{L}_\mathtt{K}[\{\mathtt{C}(\mathtt{c}, \mathtt{a}), \mathtt{C}(\mathtt{c}, \mathtt{b})\} \subseteq \mathtt{K} \wedge \{\mathtt{O}(\mathtt{c}, \mathtt{a}), \mathtt{O}(\mathtt{c}, \mathtt{b})\} \cap \mathtt{K} = \emptyset]$$

The preceding definition shows a practical interpretation in some KB of the RCC-relationships. In fact, we have the following theorem.

Theorem 2. *If* $\mathtt{K}$ *has the above four properties, then* $\Theta_G \upharpoonright_{\mathtt{W(K)}} \models \mathtt{RCC(K)}$*.*

An useful parameter on Θ_G is the *compactness level.*

Definition 9. *The* compactness level *of* Θ *is the least* $n > 0$ *such that the intersection of any set of regions of* Θ_G *is equal to the intersection of* n *regions of the set.*

In general, a database is not refined. Notice that when a database $\mathtt{K}$ is refined, the Skolem function $\mathtt{f_O}$ is interpretable within $\mathtt{W(K)}$. Thus, we can add to the database a set of axioms with basic properties of such function, and $\mathtt{f_O}$ can be *syntactically defined* in $\mathtt{K}$, and simplified by compactness level, if possible. If it is not refined, the partial definition is also useful (see table 1).

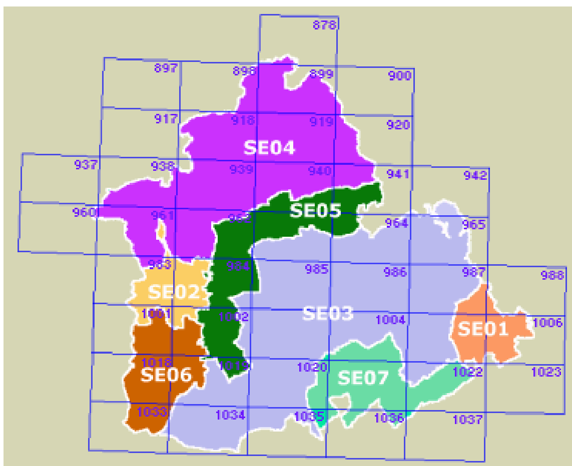
6 Experiments

We now report experiments with a spatial database on the relationships among three types of regions: counties, districts, and available maps on Andalucía, a Spanish autonomous region. The system works on a database built with the relationships of connection (`Connect`), nonempty-intersection (`Overlaps`), and

Table 1. Experiment without and with ($^+$) axiomatization of the compactness level

`P(x,Jaen) -> $Ans(x)`

Exp.	CPU time (sec.)	generated clauses	results	(A1)	(A2)	(A3)
(R1)	54.21	175	1	0	0	0
(R1)$^+$	55,20	180	1	0	0	0
(R2)	59	671	25	102	1	0
(R2)$^+$	60,26	677	25	0	2	0
(R3)	316	19,812	232	0	5	1
(R3)$^+$	320	31,855	287	0	5	1
(R4)	54.79	570	1	0	1	0
(R4)$^+$	55.6	575	1	0	1	0



K = {⟨Connect : SE04, SE05⟩, ⟨Connect : SE04, Map − 941⟩, ⟨Overlaps : Map − 920, SE04⟩, ⟨Part − of : SE04, SEVILLA⟩ · · · }

Fig. 5. Partial view of the autonomous region and some facts from the database

part-of (`Part-of`). Thus there exists hidden information, knowledge with respect to other topological relations among regions, not explicit in the database, that the theorem prover might derive (and, eventually, add to the database). The graph of Θ is formed by 260 regions, approximately, for which we have a database with 34000 facts (included the first-order formalization of databases, but the number can be reduced using some features of the theorem prover). This database has been made by hand, and it might have mistakes. It produces 40242 clauses (the processing takes 6.5 seconds). It has been used OTTER 3.2 on a computer with two Pentium III (800 Mhz) processors and 256 Mb RAM, running with Red Hat Linux operating system 7.0.

The database is `C`-complete but it is not refined. Thus, it is highly possible that the theorem prover detects anomalies of type (A2). It recognize accidentally the frontiers, its compactness level is 2, and it can be axiomatized and incorpo-

$$\mathtt{Int}(\mathtt{x},\mathtt{x}) = \mathtt{x} \qquad \mathtt{P}(\mathtt{x},\mathtt{y}) \rightarrow \mathtt{Int}(\mathtt{x},\mathtt{y}) = \mathtt{x}$$

$$\mathtt{O}(\mathtt{x},\mathtt{y}) \rightarrow \mathtt{Int}(\mathtt{x},\mathtt{y}) = \mathtt{Int}(\mathtt{y},\mathtt{x})$$

$$\mathtt{O}(\mathtt{y},\mathtt{z}) \wedge \mathtt{O}(\mathtt{x},\mathtt{Int}(\mathtt{y},\mathtt{z})) \rightarrow \mathtt{Int}(\mathtt{x},\mathtt{Int}(\mathtt{y},\mathtt{z})) = \mathtt{Int}(\mathtt{Int}(\mathtt{x},\mathtt{y}),\mathtt{z})$$

$$\mathtt{O}(\mathtt{y},\mathtt{z}) \wedge \mathtt{O}(\mathtt{x},\mathtt{Int}(\mathtt{y},\mathtt{z})) \rightarrow \left\{ \begin{array}{l} \mathtt{Int}(\mathtt{x},\mathtt{Int}(\mathtt{y},\mathtt{z})) = \mathtt{Int}(\mathtt{y},\mathtt{z}) \vee \\ \mathtt{Int}(\mathtt{x},\mathtt{Int}(\mathtt{y},\mathtt{z})) = \mathtt{Int}(\mathtt{x},\mathtt{y}) \vee \\ \mathtt{Int}(\mathtt{x},\mathtt{Int}(\mathtt{y},\mathtt{z})) = \mathtt{Int}(\mathtt{x},\mathtt{z}) \end{array} \right\}$$

Fig. 6. An axiomatization of $\mathtt{f}_0$ (as `Int`) when the compactness level is 2

Table 2. Statistics for a complex question

`PP(x, Huelva) -> $Ans(x)`

Exp.	CPU time (sec.)	generated clauses	results	(A1)	(A2)	(A3)	(A4)
(R1)	2395.31	195,222	1	113	0	0	0
(R2)	2400	201,797	8	113	0	0	0
(R3)	2514.46	287,088	14	117	0	1	0
(R4)	54.15	286	0	1	0	0	0

Table 3. Statistics of an experiment when the composition table of [4] produces errors

`EC(x, Sevilla) -> $Ans(x):`

CPU time (sec.)	generated clauses	results	(A1)	(A2)	(A3)	(A4)
3845	11,673,078	25	113	0	6	72

rated to the theory if we use the (partial) spatial interpretation of the Skolem function as *partial intersection* (see Fig. 6). Likewise, the higher compactness levels can be axiomatized.

We selected the predicates `Part-of, Proper-part, Externally-connect` as targets of the experiments. Several results are in tables 1, 2 and 3. `(R1)` shows the statistics for the first correct answer to the question, `(R2)` for 5 seconds later, `(R3)` for the first useless result and `(R4)` for the first error found.

It is not our aim to use the theorem prover as a simple database programming language. The idea is to ask complex questions which are unsolvable by constraint satisfaction algorithms or simple SQL commands. The questions are driven to obtain knowledge on spatial relationships not explicit in the database (as `Proper-part` or boolean combination of complex spatial relations). Some of the questions require an excessive CPU time. Surprisingly, the time cost is justified: the theorem prover *thought* all the time on the database and it found many errors of the type (A1), errors which may to be unacceptable. The degree of credibility allows to temporally accept some arguments. The number of useless argument can be significatively reduced by the spatial interpretation of $\mathtt{f}_0$ (see table 1). As we remarked earlier, OTTER found an error in the composition table of RCC of [4] (type (A4)) working on a complex question (see table 3).

7 Conclusions and Future Work

We have focused on practical paraconsistent reasoning with qualitative spatial databases using logic-based argumentative reasoning. The problem is an example of *cleaning* databases within complex domain knowledge, which is a promising field of applications in the Semantic Web. This analysis supports —in a case study— a methodology for the computer-aided cleaning of complex databases [1]. A spatial meaning of some relationships between arguments has been shown. The next challenge is to model the robustness of an argument estimating the *number of arguments for or against* a particular argument by topological parameters on the graph of the model that it will be useful when we work with a vast amount of spatial information.

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