Agents with other-regarding preferences in the commons

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Abstract

We present a unified approach to study the problem of the commons for agents with other-regarding preferences. This situation is modeled as a game with vector-valued utilities. Several types of agents are characterized depending on the importance assigned to the components of their utility functions. We obtain the set of equilibria of the game with two types of agents, pro-social and pro-self, and some refinements of this set for conservative agents. The most relevant result is that only a pro-social agent is required to avoid the tragedy of the commons, regardless of the behavior of the rest of the agents.

Keywords: Other-regarding preferences, the game of the commons, vector-valued utilities, equilibria.

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1 Introduction

The sustainability of common-pool resources has become one of the central issues in economic models and political agendas. Traditional theoretical models are pessimistic because they predict the collapse of the common resources. Since these are non-excludable and rival, the rational behavior of the individuals leads inexorably to an overuse of the resource. This is what has been called the *tragedy of the commons*, expression coined by Hardin (1968). The tragedy seems to be unavoidable in traditional models where the self-interest appears to be the only motivation of the agents.

However, human behavior is very complex and the assumption of the strict self-interested behavior is increasingly questioned. The evidence provided by psychologists (Tabibnia et al. (2008)) and biologists (Nowack (2006)) indicates that people often care for the well-being of others. In addition, the experimental economic literature shows that the behavior exhibited by agents in situations like gift exchange, bargaining or cooperation experiments cannot be explained by the self-interest hypothesis (see Cooper and Kagel (2013), and the references therein). In this sense, Cárdenas (2000) and Casari and Plott (2003) provide evidence of cooperative agents in the laboratory that voluntarily contribute to avoid depletion of common resources. Rustagi et al. (2010), Fehr and Leibbrandt (2011), Polania-Reyes and Echeverry (2015) find evidence of other-regarding preferences (ORP) in field experiments with local users that exploit a common resource. This could be suggesting that, at least, some individuals have ORP. Therefore, standard microeconomic models which represent strategic decisions in the commons could be useless if they do not take into account the possibility that the preferences of the agents are other than self-interest.

For this reason, we analyze the problem of the commons when the agents show ORP. To this end, we consider that the agents not only care about their own interest, but also they care about the other agents' welfare. In other words, they are concerned about the utility functions of all the agents. Thus, the incorporation of theses ORP into the problem leads us to consider our model as a game with vector-valued utilities. One of the most important consequences of this new approach is that we can study the situation under different degrees of concern of each agent with respect to the utilities of the other agents. Since the agents act independently, we do not consider the possibility of institutional arrangements or collective actions for managing the common resource as proposed in Ostrom (1990).

We obtain the set of equilibria in the vector-valued game of the commons. Since this is a wide set and, depending on the situation, not all the strategies in the reaction set are likely to be adopted by the agents, it remains unclear which of these possible equilibria will eventually be attained in the game. Therefore, it is worth investigating which equilibria will be reached when the agents choose their best responses according to different attitudes with respect to the others. We consider that the preferences of each agent are represented by a weighted additive value function, where the weights are interpreted as the relative importance that this agent assigns to the components of his vector-utility function. This allows us to characterize several types of agent as equanimous, altruistic or impartial, depending on the importance they give to their own utility with respect the utility of the other agents. We focus on the latter, defining an agent as impartial if he considers all the others equally. Within this type of agent, we distinguish between pro-social agents, who are those for which their own utility is not more important than the utility of the others, and pro-self agents, for which their own utility is at least as important as the utility of the others.

We study the case in which the agents are all of the same type and also the case in which both types of agent are involved. In the first case, when all agents are pro-social, the tragedy of the commons is avoided since the total quantity at equilibrium ranges between the social maximum, obtained when the aggregated utility is maximized, and the absolute underuse of the resource, known as the tragedy of the anti-commons. When all agents are pro-self, the tragedy of the commons cannot be excluded since it constitutes one of the equilibria. However, it is not the sole equilibrium. In this situation, the total quantity at equilibrium varies from the social maximum to the quantity in which the resources are exhausted. In the second case, when the agents are of different type, the strong conclusion is that only a pro-social agent is required to avoid the tragedy of the commons, and it is not conditional on other agents' pro-social behavior, which differs from the conclusion of Ostrom (1998).

A natural approach in order to more realistically predict the final results of interaction consists of using conservative modeling techniques. Some experimental evidence supports the hypothesis that social responsibility explains more conservative social decisions when the choice of each agent influences the well-being of others as well as his own well-being (Charness and Jackson (2009), Bolton et al. (2015)). Thus, in the last part of the paper, an additional decision rule based on a conservative attitude of the agents is considered and the corresponding subsets of equilibria are identified. A remarkable conclusion is that when all the agents are of the same type, either pro-social or pro-self, in a conservative equilibrium all the agents make use of the resource at the same level.

Our results have interesting implications for economic and environmental policies. If they are inspired in the self-interest assumption and the consequent inexorability of the tragedy of the commons, then the efficient solutions to avoid the depletion of the resources are either giving the control of the most natural resource system to the central government or the assignment of ownership right. However, if the agents show ORP, sustainability is not only a possible result of the model but also the most likely result. Therefore, environmental policies should go beyond the usual economic recommendations. In short, the design of more effective policies requires integration of ORP into the environmental policy theory.

The rest of the paper is organized as follows. Section 2 contains notation and a summary of the theoretical results that will be used in our analysis. In Section 3, we formalize the types of agent with other-regarding preferences which will be involved in our model. Section 4 analyzes the game of the commons with other-regarding agents and presents the results on the equilibria for different types of agent. In Section 5, in order to reduce the sets of equilibria, we consider a conservative attitude of each agent with respect to the utility values that can be obtained. Section 6 is devoted to the concluding remarks. In order to ease the presentation, proofs are included in an Appendix.

2 Preliminaries

In this section we introduce notations and definitions and, in order to make the paper self-contained, we summarize some results that will be applied in the following sections. The results in Theorems 1 and 2 have been established in Mármol et al. (2016) for general games in which the agents have different vector-valued utilities. The games arising when the agents show other-regarding preferences are special cases in which all the agents have the same vector-valued utility. The mentioned results are applied in Section 4 in order to obtain the equilibria for these games.

The following notation will be used. Let $\mathbb{R}(\mathbb{R}_+)$ denote the set of all (non-negative) real numbers and let $\mathbb{R}^k(\mathbb{R}^k_+)$ be the k-fold Cartesian product of $\mathbb{R}(\mathbb{R}_+)$. The origin of $\mathbb{R}^k$ is 0^k and 1^k is an k-dimensional vector with components equal to one. We use the conventional notation for comparison of vectors: $x \geq y$ indicates that $x \geq y$ and $x \neq y$, and $x > y$ indicates that $x_i > y_i$ for all $i = 1, \dots, k$.

A vector-valued normal-form game is represented by $G = \{(A^i, u^i)_{i\in N}\}$, where $N = \{1, \ldots, n\}$ is the set of agents, A^i is the set of strategies that agent $i \in N$ can adopt and the mapping $u^i : \times_{i\in N} A^i \to \mathbb{R}^{s^i}$, is the vector-valued utility function of agent i, $u^i := (u^i_1, \ldots, u^i_{s^i})$, where s^i indicates the number of components of the utility function of agent i. Denote by $J^i = \{1, ..., s^i\}$ the set of indices of such components. A profile of strategies, $a = (a^1, \ldots, a^n)$, with $a^i \in A^i$, for a game G can be written as $a = (a^i, a^{-i})$, where a^i is a strategy of agent i, and $a^{-i} = (a^1, \ldots, a^{i-1}, a^{i+1}, \ldots, a^n)$ stands for the strategy combination of all players except player i.

Definition 1. An action profile $a^* = (a^{*1}, \ldots, a^{*n})$ is an equilibrium for the vector-valued game $G = \{(A^i, u^i)_{i\in N}\}$ if $\nexists\, i \in N$ with $a^i \in A^i$ such that $u^i(a^i, a^{*-i}) \geq u^i(a^{*i}, a^{*-i})$.

The set of all equilibria of game G is denoted by $E(G)$.

Definition 2. An action profile $a^* = (a^{*1}, \ldots, a^{*n})$ is a weak equilibrium for the vector-valued game $G = \{(A^i, u^i)_{i\in N}\}$ if $\nexists\, i \in N$ with $a^i \in A^i$ such that $u^i(a^i, a^{*-i}) > u^i(a^*)$.

The set of all weak equilibria of game G is denoted by $E^w(G)$.

When for all $i \in N$ the sets of strategies A^i are nonempty convex subsets of a finite dimensional space and the functions u^i_j are strictly concave in a^i for all $j \in J^i$ then the set of weak equilibria and the set of equilibria coincide.

The set of equilibria of these games can be described in terms of the reaction function of the components of the utility function under certain conditions. Let r^i_j denote the correspondence of best response of agent i in relation to the j-th utility component.

Theorem 1. *(Mármol et al. (2016)) If for all $i \in N$, A^i is a nonempty convex compact subset $A^i \subseteq \mathbb{R}$, and u^i_j is strictly concave in its own action for each $j \in J^i$, then the set of equilibria of the game with vector-valued utilities $G = \{(A^i, u^i)_{i\in N}\}$ is*

$$E(G) = \{(a^1, \ldots, a^n) \in \times_{i\in N} A^i : \underline{r}^i(a^{-i}) \leq a^i \leq \bar{r}^i(a^{-i}),\ i \in N\},$$

where $\underline{r}^i(a^{-i}) = min_{j\in J^i}\, r^i_j(a^{-i})$, and $\bar{r}^i(a^{-i}) = max_{j\in J^i}\, r^i_j(a^{-i})$.

A similar result characterizes the set of weak equilibria when the assumption of strict concavity of the components of the multidimensional utility is relaxed to concavity.

We assume that the preferences of each agent are represented by a weighted additive value function. Let $\Delta^{s^i} = \{\lambda^i \in \mathbb{R}^{s^i} : \sum_{j=1}^{s^i} \lambda^i_j = 1, \lambda^i_j \geq 0\}$, and $\Delta = \times_{i=1}^{n} \Delta^{s^i}$. For $\lambda \in \Delta$, weighted scalar game $G_\lambda = \{(A^i, v^i_\lambda)_{i \in N}\}$ is defined, with

$$v^i_\lambda(a) = \sum_{j=1}^{s^i} \lambda^i_j u^i_j(a).$$

Definition 3. For $\lambda \in \Delta$, an action profile $a^* = (a^{*1}, \ldots, a^{*n})$, is a Nash equilibrium for the game G_λ if $\nexists\, i \in N$ with $a^i \in A^i$ such that $v_\lambda(a^i, a^{*-i}) > v_\lambda(a^*)$.

$E(G_\lambda)$ denotes the set of Nash equilibria of game $G_\lambda = \{(A^i, v^i_\lambda)_{i \in N}\}$.

Bade (2005) establishes the relationship between the set of equilibria of a vector-valued game and the sets of equilibria of weighted games with positive weights and with non-negative weights. Moreover, in Mármol et al. (2016) it is proven that the equilibria of weighted games with non-negative weights are weak equilibria of the corresponding game with vector-valued utilities.

When information about the preferences of the agents is available, it can be applied in order to reduce the set of equilibria. The information is formalized by means of information sets, which in general are different for each agent. Consider a subset of weights for each agent, $\Lambda^i \subseteq \Delta^{s^i}$, representing partial information on the relative importance that the agent assigns to the components of his vector-valued utility function. Denote $\Lambda = \times_{i \in N} \Lambda^i$ the set containing all the preference information.

Definition 4. An action profile $a^* = (a^{*1}, \ldots, a^{*n})$ is an equilibrium for the game with preference information Λ if for each $i \in N, \exists \lambda^i \in \Lambda^i$ such that $v^i_\lambda(a^*) \geq v^i_\lambda(a^i, a^{*-i})$ for all $a^i \in A^i$.

$E_\Lambda(G)$ denotes the set of equilibria of the game with preference information.

When the information sets are polyhedra they can be characterized by their extreme points. For $i = 1, \ldots, n$, let Λ^i be a polyhedron with p_i extreme points $\{\bar{\lambda}(1), \ldots \bar{\lambda}(p_i)\}$, and let B^i be the $p_i \times m_i$ matrix whose rows are the extreme points of Λ^i. For each $i \in N$, define a function, v^i_Λ, with values in $\mathbb{R}^{p_i}$, given by $v^i_\Lambda = B^i \cdot u^i$.

Theorem 2. *(Mármol et al. (2016)) Let $G = \{(A^i, u^i)_{i \in N}\}$ be a game with vector-valued utilities such that each A^i is a nonempty convex subset of a finite dimensional space and for each i, u^i is concave in a^i. Then the set of equilibria of the game with preference information Λ coincides with the set of weak equilibria of the game $\{(A^i, v^i_\Lambda)_{i \in N}\}$.*

If for each $i \in N$, the components of u^i are strictly concave, then the set of equilibria and the set of weak equilibria of $\{(A^i, v^i_\Lambda)_{i\in N}\}$ coincide. Note that, for the case in which no preference information exists (that is, for $\Lambda = \times_{i\in N}\Delta^{s^i}$), this result establishes that under concavity assumptions, the set of weak equilibria of the vector-valued game coincides with the set containing all the equilibria of the weighted games with non-negative weights.

3 Agents with other-regarding preferences

In this section we consider different attitudes of the agents with respect to their self-interest and the well-being of the others. As mentioned, the literature shows that the agents' behavior is not always selfish. Rather, in some situations they care about the other agents' welfare.

We introduce these ORP of the agents into the model by considering decision problems in which a set of agents $N = \{1, ..., n\}$, each one with an individual real-valued utility function, u_j, $j \in N$, takes into account the utilities of all of the group. In this case, we denote the vector-valued utility function of each agent as $u := (u_j)_{j\in N}$. Note that in this initial setting, the vector-valued utility functions of all the agents coincide. However, the preferences which the agents exhibit with respect to the values of these utilities are generally different. That is, they assign different importance to their own individual utility than to the utilities of others. We analyze the behavior of the agents under different social attitudes which are represented by the preferences they show with respect to all the agents' utility functions.

We assume that the preferences of agent i on the utilities of the set of agents is represented by a value function $\nu^i : \mathbb{R}^n \to \mathbb{R}$. This function ν^i provides the evaluation that agent i gives to each vector of utilities of all the group. We define different types of agents which depend on the attitudes that they exhibit with respect to these utilities.

A permutation π in the set of agents N is a bijection $\pi : N \to N$. Let Π_N denote the set of permutations in N. Consider $\pi \in \Pi_N$, for a vector $u \in \mathbb{R}^n$, denote $u_\pi := (u_{\pi(j)})_{j\in N}$. Let π_{-j} denote a permutation of the set of agents $N \setminus j$.

Definition 5. Let N be a set of agents in which for all $i \in N$ the vector-valued utility function of agent i is $u := (u_j)_{j\in N}$, and let the preferences of agent i be represented by the value function ν^i. An agent $i \in N$ is

a) *equanimous* if for each $u \in \mathbb{R}^n$ and each $\pi \in \Pi_N$, $\nu^i(u) = \nu^i(u_\pi)$.

b) *impartial* if for each $u \in \mathbb{R}^n$, and each $\pi \in \Pi_N$, $\nu^i(u_i, u_{-i}) = \nu^i(u_i, u_{\pi_{-i}})$.

c) *altruistic* if there exist $u, \bar{u} \in \mathbb{R}^n$, with $u_i < \bar{u}_i$, such that $\nu^i(\bar{u}) \leq \nu^i(u)$.

d) *egoistic* if for all $u, \bar{u} \in \mathbb{R}^n$, with $u_i < \bar{u}_i$, $\nu^i(u) < \nu^i(\bar{u})$.

The property of equanimity is a property of symmetry stating that the names of the agents do not matter. That is, the evaluation of agent i of the vector of utilities of all the group does not change if the agents permute their results. Impartiality means that the agent considers all the others equally. Altruism and egoism are opposites. Altruism is defined here in its widest sense. It means that it is possible that, at least for one utility vector, the agent puts the benefit of others before to his own.

Since we assume that the preferences of the agents are additive, then for each $i \in N$ the value function is $\nu^i(u) = \sum_{j=1}^n \lambda_j^i u_j$, where $\lambda^i \in \Delta^n$, and each component of λ^i, λ_j^i, can be interpreted as the relative importance that agent i assigns to the individual utility of agent j. The different attitudes of the agents are characterized in terms of the weights in the following Lemma.

Lemma 1. *If the value function of agent $i \in N$ is $\nu^i(u) = \sum_{j=1}^n \lambda_j^i u_j$, with $\lambda^i \in \Delta^n$, then agent i is*

a) equanimous if and only if $\lambda_j^i = \lambda_k^i$ for all $j, k \in N$.

b) impartial if and only if $\lambda_j^i = \lambda_k^i$ for all $j, k \neq i$.

c) altruistic if and only if $\lambda_j^i > 0$ for some $j \neq i$.

d) egoistic if and only if $\lambda_j^i = 0$ for all $j \neq i$.

We particularly analyze two types of behavior depending of the agents' attitude toward their own utility and the utilities of the others. First, we consider *pro-social* agents, who are those impartial agents for which their own utility is not more important than that of the others. Secondly, *pro-self* agents, who are those impartial agents for which their own utility is at least as important as that of the others.

Definition 6. Let $\nu^i(u) = \sum_{j=1}^n \lambda_j^i u_j$ be the value function of an impartial agent $i \in N$, with $\lambda^i \in \Delta^n$. Agent i is *pro-social* if $\lambda_i^i \leq \lambda_j^i$ for all $j \in N$. Agent i is *pro-self* if $\lambda_i^i \geq \lambda_j^i$ for all $j \in N$.

Note that pro-self agents can be either altruistic agents or egoistic agents. However, a pro-social agent is always altruistic. In addition, the union of the set of pro-social agents and the set of pro-self agents constitutes the set of impartial agents. As a particular case, agents who are both pro-social and pro-self are equanimous.

4 The behavior of the agents in the commons

In this section we analyze the well-known game of the commons in the extended setting where the strategic behavior of the agents is determined by a certain social attitude. Previously, we state the usual formulation of the game of the commons.

4.1 The standard game of the commons

Let $N = \{1, \ldots, n\}$ be a set of individuals that have access to a finite common-pool resource. Let m^i be the number of units used by agent $i, i \in N$, and $M = \sum_{i=1}^{n} m^i$ the total units used. Let $V(M)$ be the unit value when M units have been used. We assume that this function verifies some conditions. There is a maximum number of units that can be used, M_{max}, such that from this value, function $V(M)$ is equal to zero. In addition, $V(M)$ is strictly decreasing, twice differentiable and strictly concave in $(0, M_{max})$.

In this game the strategies refer to quantities, thus $A^i \subseteq \mathbb{R}_+$. Moreover, the total units the agents are able to use is bounded by M_{max}, that is $A^i = [0, M_{max}]$ for $i \in N$. The utility function for agent j is $u_j : \times_{i=1}^{n} A^i \to \mathbb{R}$, with $u_j(m^1, ..., m^n) = m^j V(\sum_{i=1}^{n} m^i)$ which, under the assumptions on V, is strictly concave with respect to his own action. For each agent $i \in N$, $r^i(m^{-i})$ denotes the reaction function to the actions of the other agents, $m^{-i} = (m^1, \ldots, m^{i-1}, m^{i+1}, \ldots, m^n)$. Under these assumptions, there is a unique Nash equilibrium $m^* = (m^{*1}, ..., m^{*n})$ in which each agent maximizes his utility given the actions of the other agents. This equilibrium is obtained as the intersection of the graphs of the reaction functions of all the agents, and is symmetric, that is, $m^* = (\frac{M^*}{n}, ..., \frac{M^*}{n})$, where M^* is the total quantity at equilibrium. This quantity yields the overuse of the resource since each agent only considers their own situation and not the effect of his decisions over the others.

In this standard model, the alternative to the agents acting individually is the possibility of maximizing the aggregated utility, $u_S(m) = \sum_{j=1}^{n} u_j(m)$, that is, $u_S(m^1, ..., m^n) = (\sum_{i=1}^{n} m^i)V(\sum_{i=1}^{n} m^i)$. For each agent i, the reaction function with respect to the aggregated utility is denoted by $r_S^i(m^{-i})$. The maximizers of this aggregated utility are $\{(m^1, ..., m^n) \in \mathbb{R}^n_+ : \sum_{i=1}^n m^i = S^*\}$, where the quantity $S^* = \arg\max_{\times_{i=1}^n A^i} MV(M)$ is called the social maximum. By comparing the total amount at equilibrium, M^*, and the social maximum S^*, it follows that $M^* > S^*$. That is, when all the agents act as one, a more rational use of the common is made, since the number of units used would be lower, but all the agents can obtain a higher utility when dividing the maximum aggregated utility.

4.2 The commons with other-regarding agents

In order to analyze other-regarding preferences of the agents in the commons, we suppose that each agent takes into account all the agents' utility functions. Therefore, this new situation can be analyzed as a n-person game in which each agent considers the same vector-valued utility function $u : \times_{i=1}^n A^i \to \mathbb{R}^n$, with $u := (u_j(m))_{j\in N}$, defined for $j \in N$ as $u_j(m) = m^j V(\sum_{i=1}^n m^i)$. The *game of the commons with other-regarding agents* is denoted by $G = \{(A^i, u)_{i\in N}\}$.

Note that this game is a special case of a vector-valued game, as described in Section 2, in which all the agents have the same vector-valued utility. Therefore, the definition of equilibrium is the following.

Definition 7. An action profile (m^{*i}, m^{*-i}) is an equilibrium for the vector-valued game $G = \{(A^i, u)_{i\in N}\}$ if $\nexists\, i \in N$ with $m^i \in A^i$ such that $u(m^i, m^{*-i}) \geq u(m^{*i}, m^{*-i})$.

For $i \in N$, denote by R^i the correspondence which represents the best response of agent i to the actions of the other agents. In the case of vector-valued utilities, the best response of one agent given the actions of the other agents is not in general a singleton, but a subset of its set of strategies, $R^i(m^{-i}) \subseteq A^i$: those strategies of agent i, such that he does not improve his vector-valued utility by deviating from them. Thus, an action profile (m^{*i}, m^{*-i}) is an equilibrium for the game $G = \{(A^i, u)_{i\in N}\}$ if and only if $m^{*i} \in R^i(m^{*-i})$ for $i \in N$. For $i, j \in N$, let $r^i_j(m^{-i})$ be the reaction function of agent i corresponding to the j-component function u_j of his vector-valued utility u, that is to say, with respect to the utility of agent j.

The following result identifies the equilibria for the game $G = \{(A^i, u)_{i\in N}\}$.

Proposition 1. *The set of equilibria of game $G = \{(A^i, u)_{i\in N}\}$ is*

$$E(G) = \{(m^1, ..., m^n) : 0 \leq m^i \leq r^i(m^{-i}), i \in N\}.$$

Example 1. Suppose that n herdsmen take their sheep to a pasture open to all. Let $m^i, i = 1, \ldots, n$, be the number of sheep of herdsman i in the pasture, and

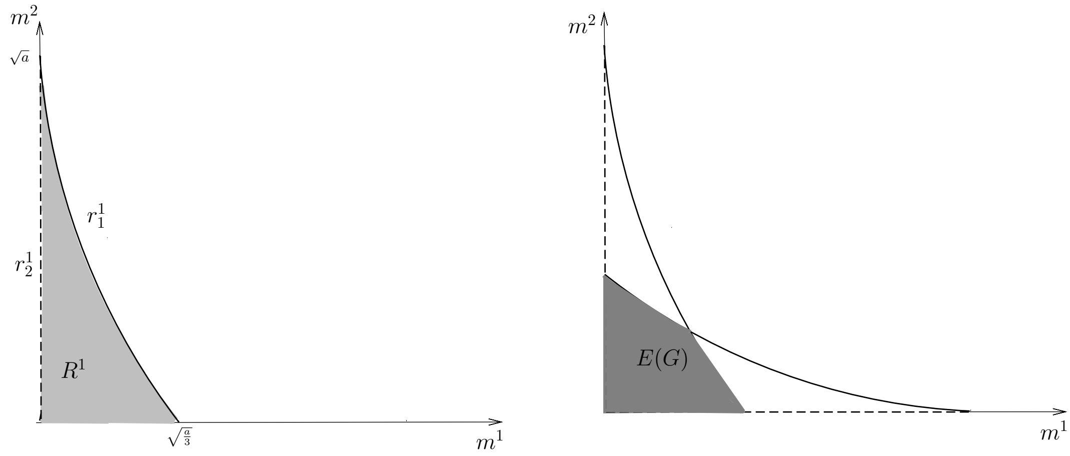

Figure 1: The reaction set of agent 1 and the set of equilibria

$V(M) = a - M^2$ the value of one sheep when there are M sheep in the pasture. The individual benefit for herdsman j is

$$u_j(m) = m^j(a - (\textstyle\sum_{i=1}^n m^i)^2).$$

Under the assumption that the preferences of the herdsmen are other-regarding, the situation can be modeled as a vector-valued game. In this game all the herdsmen consider the same vector-valued utility function. The components of this function are the individual benefits of each herdsman, that is,

$$u(m) = (u_i(m))_{i \in N}, \quad u_i(m) = m^i(a - (\textstyle\sum_{j=1}^n m^j)^2), i \in N.$$

For a two-agent situation, the corresponding best response function of agent i to the actions of agent j with respect to the first and second component of his utility function are

$$r_1^i(m^j) = \tfrac{-2m^j + \sqrt{(m^j)^2 + 3a}}{3}, \quad r_2^i(m^j) = 0, \; i, j = 1, 2, i \neq j.$$

By applying Proposition 1 the set of equilibria is

$$E(G) = \left\{ m \in \mathbb{R}_+^2 : m^1 \leq \tfrac{-2m^2 + \sqrt{(m^2)^2 + 3a}}{3}, \, m^2 \leq \tfrac{-2m^1 + \sqrt{(m^1)^2 + 3a}}{3} \right\}.$$

In Figure 1, the reaction set for agent 1 (left-hand side) and the set of equilibria $E(G)$ for the two-person, two-component game (right-hand side) are represented.

Note that in this setting, the set of Nash equilibria for the game $G = \{(A^i, u)_{i \in N}\}$ is a wide set. Since, depending on the situation, not all the strategies in the reaction set are likely to be adopted by the agents, it remains unclear which of these possible equilibria will eventuate in the game. For this reason, it is worth investigating which equilibria will be reached when the agents choose their best responses according to different attitudes with respect to the results of others.

4.3 Pro-social agents in the commons

In this section we focus on the game of the commons with agents for which their own benefits are not more important than the benefits of others, and consider the benefits of the remaining agents equally. Recall that in Section 3 an agent of this type is named a pro-social agent. The set of weights for a pro-social agent i is

$$\Lambda^i_{soc} = \{\lambda^i \in \Lambda^i : \lambda^i_i \leq \lambda^i_j, \lambda^i_j = \lambda^i_k,\ j, k \neq i\}.$$

Denote $\Lambda_{soc} = \times_{i \in N} \Lambda^i_{soc}$.

It follows from Theorem 2, that when information about the preferences of the agents is incorporated into the original game of the commons, $G = \{(A^i, u)_{i \in N}\}$, the equilibria coincide with those of a transformed vector-valued utility game. The game of the commons with preference information Λ_{soc} is established in the following result.

Proposition 2. *The set of pro-social equilibria of game $G = \{(A^i, u)_{i \in N}\}$ coincides with the set of equilibria of game $\{(A^i, v^i_{\Lambda_{soc}})_{i \in N}\}$, where*

$$v^i_{\Lambda_{soc}}(m^1, ..., m^n) = \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m^j}{n-1} V(M), \sum_{j=1}^{n} \frac{m^j}{n} V(M) \right).$$

In this situation, the game is transformed into a vector-valued game whose utility function has two components. The first component of the transformed vector-valued utility of agent i is the average of the benefits of all the agents excluding agent i. This evaluation represents an extreme concern for the other agents, given that agent i does not take into account his own profit. The second component is the average of the benefits of all agents, which represents a responsible social behavior. The set of equilibria is described in the following result.

Proposition 3. *The set of pro-social equilibria of game $G = \{(A^i, u)_{i \in N}\}$ is*

$$E_{\Lambda_{soc}}(G) = \{(m^1, \ldots, m^n) \in \mathbb{R}^n_+ : \sum_{i \in N} m^i \leq S^*\}.$$

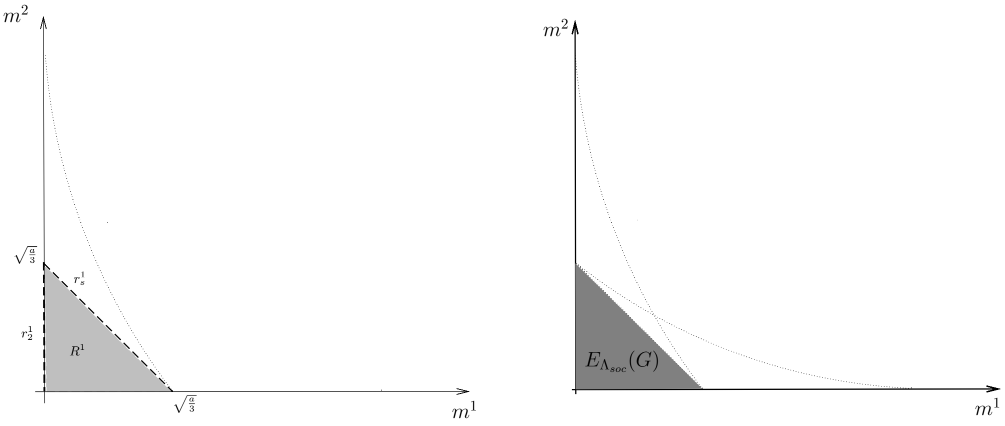

Figure 2: Reaction set of a pro-social agent and pro-social equilibria

Example 2. (Example 1 continued) For the commons game when the two agents are pro-social the preference information for each agent is

$$\Lambda^i_{soc} = \{\lambda^i \in \Lambda^i : \lambda^i_i \leq \lambda^i_j\}.$$

The corresponding best response function of agent i to the actions of agent j for each component of his new vector-valued utility function are $\hat{r}^i_1(m^j) = 0$ and $r^i_S(m^j) = \sqrt{\frac{a}{3}} - m^j$. The set of pro-social equilibria is

$$E_{\Lambda_{soc}(G)} = \left\{(m^1, m^2) \in \mathbb{R}^2_+ : m^1 + m^2 \leq \sqrt{\tfrac{a}{3}}\right\}.$$

Figure 2 represents the reaction set of a pro-social agent 1 (left-hand side) and the set of pro-social equilibria $E_{\Lambda_{soc}}(G)$ (right-hand side).

Note that when all the agents are pro-social, the Nash equilibrium of the standard game is never attained. Instead, only those points with total quantity below the social maximum, $S^* = \sqrt{\frac{a}{3}}$, are equilibria.

4.4 Pro-self agents in the commons

We now analyze the game of the commons $G = \{(A^i, u)_{i \in N}\}$ with agents for which their own benefits are at least as important as the benefits of others, and consider the benefits of the remaining agents equally. In Section 3 an agent of this type is named a pro-self agent. The set of weights for a pro-self agent i is

$$\Lambda^i_{self} = \{\lambda^i \in \Lambda^i : \lambda^i_i \geq \lambda^i_j,\ \lambda^i_j = \lambda^i_k, j, k \neq i\}.$$

Denote $\Lambda_{self} = \times_{i\in N}\Lambda^i_{self}$.

The incorporation of the information about the preference of each agent i into the original game $G = \{(A^i, u)_{i\in N}\}$, permits the identification of the set of equilibria for the game with preference information Λ_{self}, as well as a corresponding two-component utility function for each agent in this game, as the following result states.

Proposition 4. *The set of pro-self equilibria of game $G = \{(A^i, u)_{i\in N}\}$ coincides with the set of weak equilibria of game $\{(A^i, v^i_{\Lambda_{self}})_{i\in N}\}$, where*

$$v^i_{\Lambda_{self}}(m^1, ..., m^n) = \left(m^i V(M), \sum_{j=1}^{n} \frac{m^j}{n} V(M)\right).$$

Note that the first component of the transformed utility of agent i is his own benefit, i.e. his own utility function u_i, which represents a rational behavior, and the second component is the average of the benefits of all agents, which represents a responsible social behavior. The following result identifies the equilibria for the game of the commons with pro-self agents.

Proposition 5. *The set of pro-self equilibria of game $G = \{(A^i, u)_{i\in N}\}$ is*

$$E_{\Lambda_{self}}(G) = \{(m^1, \ldots, m^n) \in \mathbb{R}^n_+ : \sum_{i\in N} m^i \geq S^*, \, m^i \leq r^i(m^{-i}), \forall i \in N\}.$$

Example 3. For the commons game with preference information Λ_{self} in Example 1, the best response function of agent i to the actions of agent j with respect to each component of his new vector-valued function are

$$r^i(m^j) = \frac{-2m^j + \sqrt{(m^j)^2 + 3a}}{3} \quad \text{and} \quad r^i_S(m^j) = \sqrt{\frac{a}{3}} - m^j.$$

The set of pro-self equilibria is

$$E_{\Lambda_{self}}(G) = \left\{ m \in \mathbb{R}^2_+ : m^1 + m^2 \geq \sqrt{\frac{a}{3}}, \, m^1 \leq \frac{-2m^2 + \sqrt{(m^2)^2 + 3a}}{3}, m^2 \leq \frac{-2m^1 + \sqrt{(m^1)^2 + 3a}}{3} \right\}.$$

Figure 3 represents the reaction set of agent 1 (left-hand side) and the set of pro-self equilibria $E_{\Lambda_{self}}(G)$.

Note that this set includes the Nash equilibrium of the standard game of the commons, and that no profile of strategies which yields a total quantity below the social maximum, $S^* = \sqrt{\frac{a}{3}}$, is an equilibrium.

Remark. It is interesting to note that the whole set of equilibria of the game of the commons coincides with the union of the set of pro-social equilibria and the set of pro-self equilibria of the game of the commons even though we are not considering all the possible weights, only those associated to impartial agents, i.e., $\Lambda_{soc} \cup \Lambda_{self} \subset \Delta^n$.

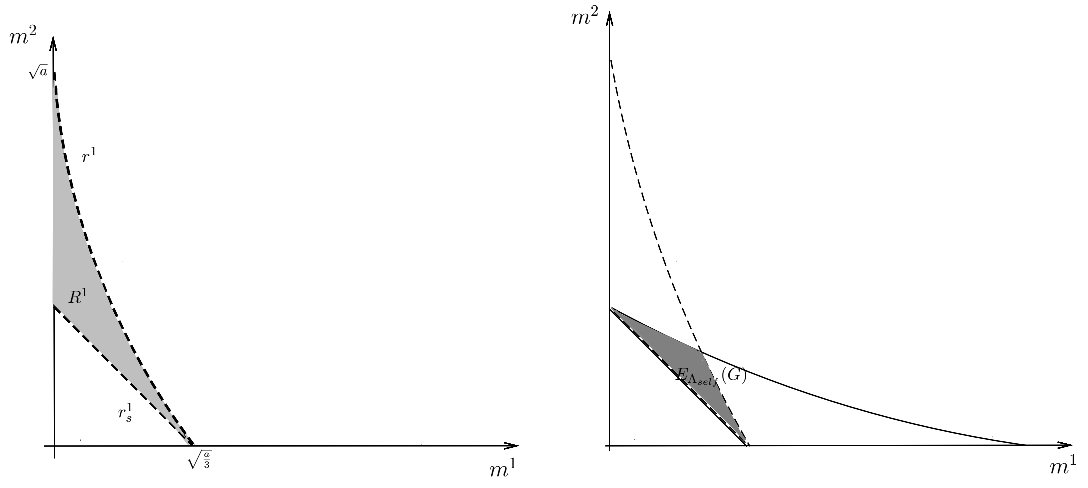

Figure 3: Reaction set of a pro-self agent and pro-self equilibria

4.5 Pro-social and pro-self agents in the commons

We now analyze the game of the commons with both types of agent. That is, the set of agents is divided into two groups: the set of pro-social agents, $N_{soc} = \{1, \dots, k\}$, and the set of pro-self agents, $N_{self} = \{k+1, \dots, n\}$.

Let $\Lambda_{cro} = \Lambda_{soc}^1 \times \dots \times \Lambda_{soc}^k \times \Lambda_{self}^{k+1} \times \dots \times \Lambda_{self}^n$ be the set of information weights for the game of the commons $G = \{(A^i, u)_{i \in N}\}$. We name the equilibria of the game with this information crossed equilibria. The set of crossed equilibria is stated in the following results.

Proposition 6. *The set of crossed equilibria of game $G = \{(A^i, u)_{i \in N}\}$ coincides with the set of weak equilibria of game $\{(A^i, v^i_{\Lambda_{cro}})_{i \in N}\}$, where*

$$v^i_{\Lambda_{cro}}(m^1, ..., m^n) = \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m^j}{n-1} V(M), \sum_{j=1}^{n} \frac{m^j}{n} V(M) \right) \ \textit{for } i \in N_{soc}.$$

$$v^i_{\Lambda_{cro}}(m^1, ..., m^n) = \left(m^i V(M), \sum_{j=1}^{n} \frac{m^j}{n} V(M) \right) \ \textit{for } i \in N_{self}.$$

Note that the average of the benefits of all agents, which represents a responsible social behavior is always a component of the transformed utility of all the agents.

Recall that for a pro-social agent i the reaction functions with respect to the components of the vector-valued utility function are $r_1^i(m^{-i}) = 0$ and $r_S^i(m^{-i})$ respectively, and for a pro-self agent are $r_1^i(m^{-i})$ (the reaction function of agent i with respect to his own benefit) and $r_S^i(m^{-i})$. The following proposition identifies the equilibria for the game of the commons with crossed agents.

Proposition 7. *The set of crossed equilibria of game $G = \{(A^i, u)_{i\in N}\}$ is*

$$E_{\Lambda_{cro}}(G) = \{(m^1, \ldots, m^n) : \sum_{i\in N} m^i = S^*\}.$$

Example 4. For the commons game with preference information Λ_{cro} in Example 1, the best response functions of agent 1, who is pro-social, are $r_1^1(m^2) = 0$ and $r_S^1(m^2) = \sqrt{\frac{a}{3}} - m^2$. For agent 2, who is pro-self, the best response functions are $r_1^2(m^1) = \frac{-2m^1+\sqrt{(m^1)^2+3a}}{3}$, and $r_S^2(m^1) = \sqrt{\frac{a}{3}} - m^1$. The set of crossed equilibria is

$$E_{\Lambda_{cro}}(G) = \left\{(m^1, m^2) \in \mathbb{R}_+^2 : \ m^1 + m^2 = \sqrt{\tfrac{a}{3}}\right\}.$$

Remark. It is interesting to note that the set of crossed equilibria of the game of the commons coincides with the intersection of the set of pro-social equilibria and the set of pro-self equilibria.

5 Conservative equilibria in the commons

The set of equilibria of game $G = \{(A^i, u)_{i\in N}\}$ has been reduced by incorporating information into the model about the preferences of the agents with respect to the results of all the group. However, we still have a set with an infinite number of equilibria. Therefore, further refinements based on additional decision rules are needed in order to identify a more realistic set of equilibria. In this section, we consider a conservative rule to identify a reduced set of pro-social equilibria, a reduced set of pro-self equilibria, and a reduced set of crossed equilibria of the game of the commons with other-regarding agents.

We assume that the agents show a conservative attitude with respect to the different utility values that can be obtained. That is, we consider that the agents are averse to the uncertainty underlying the weights assigned to the different utilities. In a conservative equilibrium, the agents select the strategies that maximize the worst outcomes that can be obtained with the possible weights. Thus, agent i evaluates an action profile $m \in \times_{i=1}^n A^i$ as the minimum weighted value among all the feasible weights in his preference information set. Formally, the value function of agent i is:

$$v^c_{\Lambda^i}(m) = \min_{\lambda^i \in \Lambda^i} \sum_{j=1}^{n} \lambda^i_j u_j(m).$$

5.1 Conservative equilibria with pro-social agents

If an agent i is pro-social and conservative then his value function is

$$v^c_{\Lambda^i_{soc}}(m) = \min_{\lambda^i \in \Lambda^i_{soc}} \sum_{j=1}^{n} \lambda^i_j u_j(m).$$

The minimum of a linear function on a polyhedron is reached at one of its extreme points. Since for each m, $\sum_{j=1}^{n} \lambda^i_j u_j(m)$ is linear with respect to λ, and the extreme points of Λ^i_{soc} are $(\frac{1}{n-1}, \ldots, 0_{i)}, \ldots, \frac{1}{n-1})$, $(\frac{1}{n}, \ldots, \frac{1}{n})$, then it follows that

$$v^c_{\Lambda^i_{soc}}(m) = \min \left\{ \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m^j}{n-1} V(M), \sum_{j=1}^{n} \frac{m^j}{n} V(M) \right\} =$$

$$= \begin{cases} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m^j}{n-1} V(M) & \text{if} \quad m^i \geq \sum_{j=1}^{n} \frac{m^j}{n} \\ \sum_{j=1}^{n} \frac{m^j}{n} V(M) & \text{if} \quad m^i \leq \sum_{j=1}^{n} \frac{m^j}{n} \end{cases}$$

When a pro-social agent applies this conservative rule, the valuation of a profile of strategies depends on the relationship between the number of units used by himself and the average of the total units used. That is, when $m^i \geq \sum_{j=1}^{n} \frac{m^j}{n}$ then the preferences of agent i leads to consider the average of the benefits of all agents except for his own. However, when $m^i \leq \sum_{j=1}^{n} \frac{m^j}{n}$, agent i considers the average benefit of all agents including his own.

The following result characterizes the set of conservative pro-social equilibria of the game of the commons $G = \{(A^i, u)_{i \in N}\}$.

Proposition 8. *The set of conservative pro-social equilibria of the game* $G = \{(A^i, u)_{i \in N}\}$ *is*

$$E^c_{\Lambda_{soc}}(G) = \left\{ (m^1, ..., m^n) \,:\, m^i = m^j, \forall i, j \in N,\, 0 \leq m^i \leq \frac{S^*}{n} \right\}.$$

Example 5. In Example 2 the value function for agent i when a conservative attitude is considered in the set of feasible weights Λ_{soc} is

$$v^c_{\Lambda^i_{soc}}(m^1, m^2) = \begin{cases} m^j V(M) & \text{if} \quad m^i \geq m^j \\ \frac{m^i+m^j}{2} V(M) & \text{if} \quad m^i \leq m^j \end{cases}$$

The corresponding best response function of agent i to the actions of agent j is

$$r^i(m^j) = \begin{cases} m^j & \text{if} \quad 0 \leq m^j \leq \sqrt{\frac{a}{12}} \\ \sqrt{\frac{a}{3}} - m^j & \text{if} \quad \sqrt{\frac{a}{12}} \leq m^j \leq \sqrt{\frac{a}{8}} \end{cases}$$

and the set of conservative pro-social equilibria is

$$E^c_{\Lambda_{soc}}(G) = \left\{ (m^1, m^2) = \left(\sqrt{\tfrac{a}{12}}, \sqrt{\tfrac{a}{12}} \right) - \gamma \left(\sqrt{\tfrac{a}{12}}, \sqrt{\tfrac{a}{12}} \right), 0 \leq \gamma \leq 1 \right\}.$$

Figure 4 represents the reaction function of agent 1 (left-hand side) and the set of conservative pro-social equilibria (right-hand side).

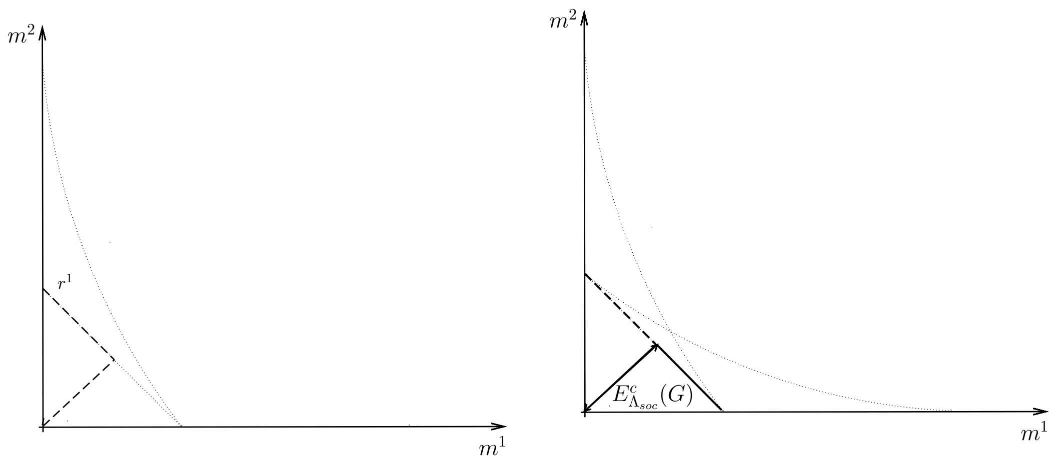

Figure 4: The reaction set of agent 1 and conservative pro-social equilibria

5.2 Conservative equilibria with pro-self agents

If an agent i is pro-self and conservative then his value function is

$$v^c_{\Lambda^i_{self}}(m) = \min_{\lambda^i \in \Lambda^i_{self}} \sum_{j=1}^{n} \lambda^i_j u_j(m).$$

Taking into account that the extreme points of Λ^i_{self} are $(0, \ldots, 1_{i)}, \ldots, 0)$ and $(\frac{1}{n}, \ldots, \frac{1}{n})$, and that this function is linear, it follows that

$$v^c_{\Lambda^i_{self}}(m) = \min \left\{ m^i V(M), \sum_{j=1}^{n} \frac{m^j}{n} V(M) \right\} =$$

$$= \begin{cases} m^i V(M) & \text{if} \quad m^i \leq \sum_{j=1}^n \frac{m^j}{n} \\ \sum_{j=1}^n \frac{m^j}{n} V(M) & \text{if} \quad m^i \geq \sum_{j=1}^n \frac{m^j}{n} \end{cases}$$

Under this conservative rule, the valuation of the preferences of a pro-self agent depends also on the relationship between the number of units used by himself and the average of the total units used. When $m^i \leq \sum_{j=1}^n \frac{m^j}{n}$, this agent considers his own benefit. However, when $m^i \geq \sum_{j=1}^n \frac{m^j}{n}$, agent i considers the average of the benefits of all agents including his own benefit.

The following result characterizes the set of conservative pro-self equilibria of the game of the commons $G = \{(A^i, u)_{i \in N}\}$.

Proposition 9. *The set of conservative pro-self equilibria of* $G = \{(A^i, u)_{i \in N}\}$ *is*

$$E^c_{\Lambda_{self}}(G) = \{(m^1, ..., m^n) \, : \, m^i = m^j, \forall i, j = 1, \ldots, n, \, \frac{S^*}{n} \leq m^i \leq \frac{M^*}{n}\}.$$

Example 6. In Example 3 the value function of agent i when a conservative attitude is considered in the set of feasible weights Λ_{self} is

$$v^c_{\Lambda^i_{self}}(m^1, m^2) = \begin{cases} m^i V(M) & \text{if} \quad m^i \leq m^j \\ \frac{m^i + m^j}{2} V(M) & \text{if} \quad m^i \geq m^j \end{cases}$$

The corresponding best response function of agent i to the action of agent j is

$$r^i(m^j) = \begin{cases} \sqrt{\frac{a}{3}} - m^j & \text{if} \quad 0 \leq m^j \leq \sqrt{\frac{a}{12}} \\ m^j & \text{if} \quad \sqrt{\frac{a}{12}} \leq m^j \leq \sqrt{\frac{a}{8}} \\ \frac{-2m^j + \sqrt{(m^j)^2 + 3a}}{3} & \text{if} \quad \sqrt{\frac{a}{8}} \leq m^j \leq \sqrt{a} \end{cases}$$

and the set of conservative pro-self equilibria is

$$E^c_{\Lambda_{self}}(G) = \left\{(m^1, m^2) = \gamma \left(\sqrt{\tfrac{a}{12}}, \sqrt{\tfrac{a}{12}}\right) + (1 - \gamma) \left(\sqrt{\tfrac{a}{8}}, \sqrt{\tfrac{a}{8}}\right), 0 \leq \gamma \leq 1\right\}.$$

Figure 5 shows the reaction function of agent 1 (left-hand side) and the set of conservative pro-self equilibria (right-hand side).

Note that when agents show a conservative attitude both the conservative pro-social equilibria and the conservative pro-self equilibria are symmetric equilibria.

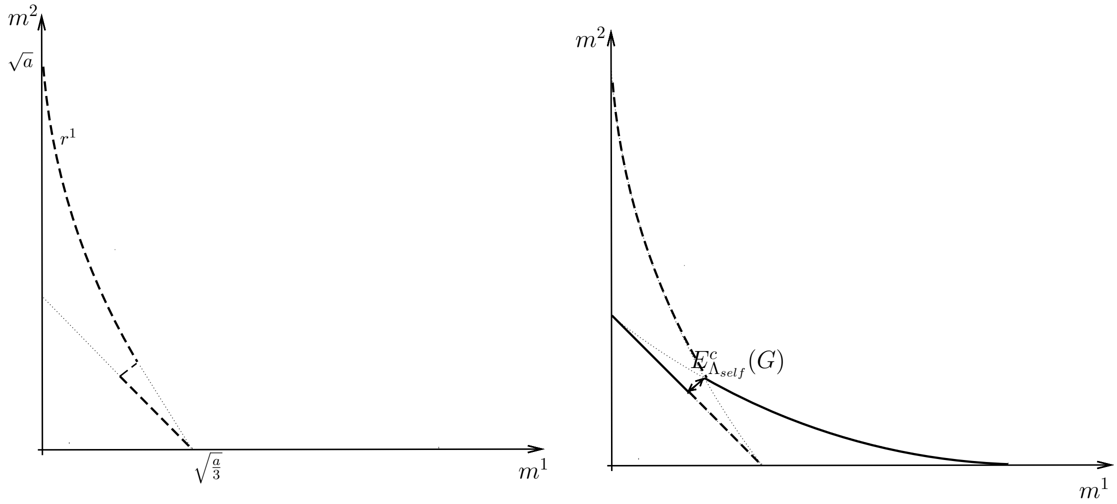

Figure 5: The reaction set of agent 1 and conservative pro-self equilibria

5.3 Conservative equilibria with pro-social and pro-self agents

Assume that the set of agents is divided into two groups: the set of pro-social agents, N_{soc}, and the set of pro-self agents, N_{self}. The following result characterizes the set of conservative crossed equilibria of the game of the commons $G = \{(A^i, u)_{i \in N}\}$.

Proposition 10. *The set of conservative crossed equilibria of* $G = \{(A^i, u)_{i \in N}\}$ *is*

$$E^c_{\Lambda_{cro}}(G) = \{m \in I\!R^n_+ : \sum_{i \in N} m^i = S^*,\ m^i \leq \frac{S^*}{n},\ \textit{for } i \in N_{soc},\ \frac{S^*}{n} \leq m^i \leq \frac{M^*}{n},\ \textit{for } i \in N_{self}\}.$$

Example 7. For the commons game with preference information Λ_{cro} in Example 4, when agent 1 is pro-social, agent 2 is pro-self and a conservative attitude is considered, then the set of conservative crossed equilibria is

$$E^c_{\Lambda_{cro}}(G) = \left\{(m^1, m^2) = \gamma\left(\sqrt{\tfrac{a}{12}}, \sqrt{\tfrac{a}{12}}\right) + (1-\gamma)\left(0, \sqrt{\tfrac{a}{3}}\right), 0 \leq \gamma \leq 1\right\}.$$

Figure 6 shows the set of conservative crossed equilibria.

6 Concluding remarks

The role of the agents with other-regarding preferences when they use a common-pool resource have been analized in this paper. Agents care about the relationship between their results and the results of the other agents using the commons. In this context we specifically focus on those agents that consider the outcomes of the others

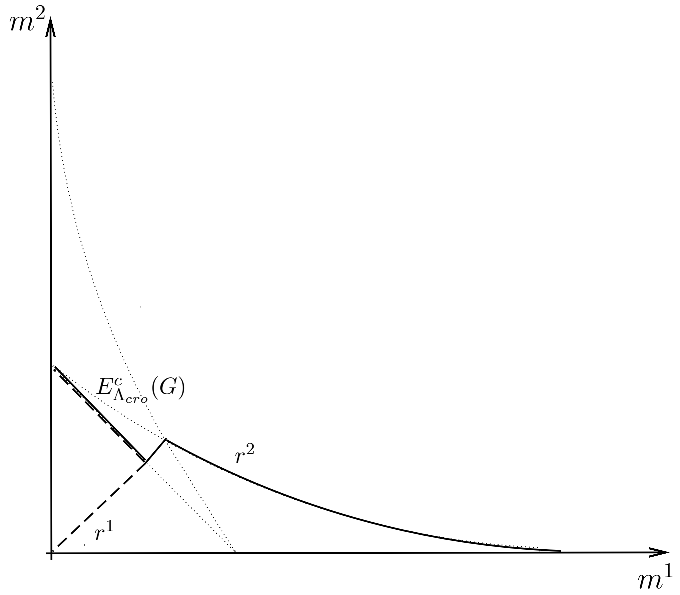

Figure 6: The set of conservative crossed equilibria

equally, and also on those agents which are capable of diminishing their personal well-being to increase that of the others.

We have modeled the value functions of the agents as depending on the results of the other agents. The preferences of the agents could more generally be modeled as depending on the value judgements of the others. This requires more complex functional forms which under certain assumptions can be reduced to the representation adopted here (see, for instance, Hori (2009)). However, in our competitive setting, since in order to choose their strategies, agents act individually, a representation of the preferences based only upon the observed results seems to be a more accurate description of the judgements of the agents.

Our analysis helps explain how the different behavior of the agents can mitigate the overuse of the commons, since the incorporation of other-regarding preferences into the model suggests situations where the overuse of the resources is reduced. The results herein show, on the one hand, that if only an agent exhibits a pro-social behavior, regardless of the behavior of the rest of the agents, the tragedy of the commons can be avoided. On the other hand, pro-self agents can mitigate depletion of the common resources, because the tragedy of the commons is only one of the possible equilibria to where the agents can eventually arrive.

Furthermore, it is worth remarking that when all the agents show the same attitude, either in the case of pro-social agents, or in the case of pro-self agents, a conservative rule leads them to adopt strategies in which all of them make use of the common at the same level. In other words, at equilibria, they make an equitable use of the resources regardless of the total quantity they are using. In contrast, when agents with different attitudes are using the resources, a conservative rule forces that the quantity at equilibria be the social maximum, although in these cases, not surprisingly, at equilibria pro-social agents never use the resources at a higher level than pro-self agents.

These results imply that recommendations on the choice of economic policies can go beyond the traditional measures based on self-interested agents. In models with this kind of agents, the literature shows that the tragedy of the commons might be avoided with cooperative agreements. The fulfillment of these agreements requires from the policy maker the design of an effective mechanism of punishment. However, our results point out that policy makers should stress the incentives that lead agents to adopt a pro-social behavior, instead of focusing of punishment. In addition, the incentive to pro-social attitude could be more effective in preventing the depletion of the resources. This appreciation is specially relevant because the punishment applies once the agreement has been breached and therefore the damage is already done, while the role of the incentive is to prevent the misuse of the resource. In this sense, our findings provide information that might be useful in the formulation of management strategies for common-pool resources and may contribute to design more effective environmental policies.

7 Appendix: proofs

Proof of Lemma 1.

a) If agent i is equanimous, then for all $u \in \mathbb{R}^n$, $\nu^i(u) = \nu^i(u_\pi)$. Thus, for $\bar{u}$ such that $\bar{u}_j = 1$ and $\bar{u}_k = 0$ for $k \neq j$, $\nu^i(\bar{u}) = \lambda^i_j$. Since for all k, a permutation exists such that $\nu^i(\bar{u}_\pi) = \lambda^i_k$, it follows that $\lambda^i_j = \lambda^i_k$.
The reverse is straightforward.
b) The result follows by applying the same reasoning as in a) to the set $N \setminus i$.
c) Suppose on the contrary that $\lambda^i_j = 0$ for all $j \neq i$, thus $\lambda^i_i = 1$, then $\nu^i(u) = \lambda^i_i u_i < \lambda^i_i \bar{u}_i = \nu^i(\bar{u})$, and this contradicts $\nu^i(\bar{u}) \leq \nu^i(u)$.
Reciprocally, consider $\lambda^i_j > 0$ for some $j \neq i$. Take u such that $u_k = 0$ for $k \neq j$, and

$u_j = a$, with $a > \dfrac{\lambda^i_i + \lambda^i_j}{\lambda^i_j}$, and $\bar{u}$ such that $\bar{u}_i = 1$, $\bar{u}_j = 1$, and $\bar{u}_k = 0$ for $k \neq i, j$. Then $\nu^i(\bar{u}) < \nu^i(u)$.

d) Since an egoistic agent is the opposite of an altruistic agent, then from c) it follows that i is egoistic if and only if $\lambda^i_j = 0$ for all $j \neq i$.

Proof of Proposition 1.

It follows from the strict concavity of each u_j with respect to the action of agent i and from Theorem 1, that the set of equilibria is $\{(m^1, \dots, m^n) \in \times_{i \in N} A^i : \underline{r}^i(m^{-i}) \leq m^i \leq \bar{r}^i(m^{-i}),\ i \in N\}$, where $\underline{r}^i(m^{-i}) = min_{j \in N}\, r^i_j(m^{-i})$, and $\bar{r}^i(m^{-i}) = max_{j \in N}\, r^i_j(m^{-i})$.

For each agent i, the i-th component of u, $u_i(m) = m^i V(\sum_{i \in N} m^i)$, provides the same reaction function as the scalar game with pay-off function $m^i V(\sum_{i \in N} m^i)$, that is, $r^i_i(m^{-i}) = r^i(m^{-i})$. When agent i considers the j-component of u, $j \neq i$, $u_j(m) = m^j V(\sum_{i \in N} m^i)$, since $V(M)$ is strictly decreasing, the maximum of $u_j(m^i, m^{-i})$ with respect to m^i is attained at $m^i = 0$. That is to say, $r^i_j(m^{-i}) = 0$, for $j \neq i$. The result follows.

Proof of Proposition 2.

The set Λ^i_{soc} is a polyhedron whose extreme points are $(\frac{1}{n-1}, \dots, 0_{i)}, \dots, \frac{1}{n-1})$ and $(\frac{1}{n}, \dots, \frac{1}{n})$. Since the components of the utility functions of the original game are strictly concave, and the transformed utility of agent i is $v^i_{\Lambda_{soc}} = B^i \cdot u_i$, where B^i is a matrix whose rows are the extreme points of Λ^i_{soc}, then from Theorem 2 the result follows.

Proof of Proposition 3.

It follows from the strict concavity of each u_j with respect to the action of agent i and from Theorem 1, that the set of equilibria is $\{(m^1, \dots, m^n) \in \times_{i \in N} A^i : \underline{r}^i(m^{-i}) \leq m^i \leq \bar{r}^i(m^{-i}),\ i \in N\}$, where $\underline{r}^i(m^{-i}) = min_{j \in N}\, r^i_j(m^{-i})$, and $\bar{r}^i(m^{-i}) = max_{j \in N}\, r^i_j(m^{-i})$.

Note that since $V(M)$ is decreasing, then the maximum of $\sum_{\substack{j=1 \\ j \neq i}}^n \frac{m^j}{n-1} V(M)$ with respect to m^i is attained at $m^i = 0$. Therefore, for the first component of the transformed game the reaction function is $\hat{r}^i_1(m^{-i}) = 0$ for $j \neq i$. For the second component, the reaction function $\hat{r}^i_2(m^{-i})$ is the reaction function with respect to the aggregated utility, $r^i_S(m^{-i}) = S^* - \sum_{\substack{j=1 \\ j \neq i}}^n m^j$. The result follows.

Proof of Proposition 4.

Since the set Λ^i_{self} is a polyhedron whose extreme points are $(0,\ldots,1_{i)},\ldots,0)$ and $(\frac{1}{n},\ldots,\frac{1}{n})$, the components of the utility functions of the original game are concave and $v^i_{\Lambda_{self}} = B^i \cdot u_i$, where B^i is a matrix whose rows are the extreme points of Λ^i_{self}, by applying Theorem 2, the result follows.

Proof of Proposition 5.

In order to determine the set of pro-self equilibria for the game, we consider the reaction function of agent i for each component of his value function. Since the first component is his own benefit then the reaction function of agent i with respect to the first component is $\hat{r}^i_1(m^{-i}) = r^i(m^{-i})$. For the second component, the reaction function $\hat{r}^i_2(m^{-i})$ coincides with the reaction function of the aggregated utility, $r^i_S(m^{-i}) = S^* - \sum_{\substack{j=1\\ j\neq i}}^n m^j$.

To prove that $r^i_S(m^{-i}) \leq r^i(m^{-i})$ for all $i \in N$, first note that $r^i_S(m^{-i})$ is the best action of agent i when he maximizes $MV(M)$, hence, $V(r^i_S(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j) + (r^i_S(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j)V'(r^i_S(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j) = 0$. Moreover, $r_i(m^{-i})$ is the best action of agent i when he maximizes $m^iV(M)$, and therefore, $V(r^i(m^{-i})+\sum_{\substack{j=1\\ j\neq i}}^n m^j)+ r^i(m^{-i})V'(r^i(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j) = 0$.

Suppose that for some $i \in N, r^i_S(m^{-i}) > r^i(m^{-i})$. Since V is strictly decreasing and strictly concave, that is, V' is also strictly decreasing, then $V(r^i_S(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j) < V(r^i(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j)$ and $V'(r^i_S(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j) < V'(r^i(m^{-i}) + \sum_{\substack{j=1\\ j\neq i}}^n m^j) < 0$. Therefore, $V(r^i_S(m^{-i})+\sum_{\substack{j=1\\ j\neq i}}^n m^j)+(r^i_S(m^{-i})+\sum_{\substack{j=1\\ j\neq i}}^n m^j)V'(r^i_S(m^{-i})+ \sum_{\substack{j=1\\ j\neq i}}^n m^j) < V(r^i(m^{-i})+\sum_{\substack{j=1\\ j\neq i}}^n m^j)+r^i(m^{-i})V'(r^i(m^{-i})+\sum_{\substack{j=1\\ j\neq i}}^n m^j)$. This contradicts the fact that the two expressions above are equal to zero.

Therefore, $r^i_S(m^{-i}) \leq r^i(m^{-i})$ for all $i \in N$. The result now follows from Theorem 1.

Proof of Proposition 6.

This result follows analogously as in Propositions 2 and 4.

Proof of Proposition 7.

It follows from the proofs of Propositions 3 and 5, that for $i \in N_{soc}$, $\hat{r}^i_1(m^{-i}) = 0$ for $j \neq i$, Hence, from Theorem 1, if $m \in E_{\Lambda_{cro}}(G)$, then $m^i \leq S^* - \sum_{\substack{j=1\\ j\neq i}}^n m^j$, that is, $S^* \geq \sum_{j=1}^n m^j$.

For $i \in N_{self}$, $r^i_S(m^{-i}) < r^i(m^{-i})$, thus, if $m \in E_{\Lambda_{cro}}(G)$, them $m^i \geq r^i_S(m^{-i}) = S^* - \sum_{\substack{j=1\\ j\neq i}}^n m^j$, and $m^i \geq S^* - \sum_{\substack{j=1\\ j\neq i}}^n m^j$, that is, $S^* \leq \sum_{j=1}^n m^j$. As a consequence,

$S^* = \sum_{j=1}^n m^j$.

Proof of Proposition 8.

We first prove that the conservative pro-social equilibria, as defined above, are pro-social equilibria. That is, $E^c_{\Lambda_{soc}}(G) \subseteq E_{\Lambda_{soc}}(G) = \{(m^1, \ldots, m^n) \in \mathbb{R}^n_+ : \sum_{i\in N} m^i \leq S^*\}$. Let $m \in \times_{i=1}^n A^i$ such that $\sum_i m^i > S^*$, then any agent $i \in N$ can improve his conservative utility by moving to $m^i - \varepsilon$, since both $\sum_{\substack{j=1\\j\neq i}}^n \frac{m^j}{n-1} V(M)$, and $\sum_{j=1}^n \frac{m^j}{n} V(M)$ are decreasing with respect to m^i.

Now, consider a profile of strategies $m \in E_{\Lambda_{soc}}(G)$ such that $m^i < \sum_{j=1}^n \frac{m^j}{n}$ and $\sum_{j\in N} m^j < S^*$, then $v^c_{\Lambda^i_{soc}}(m) = \sum_{j=1}^n \frac{m^j}{n} V(M)$. Since $v^c_{\Lambda^i_{soc}}$ is strictly concave with respect to the actions of agent i, and its maximum is attained at S^*, then if agent i moves to $m^i + \varepsilon$, the function $v^c_{\Lambda^i_{soc}}$ increases. Therefore, $m \notin E^c_{\Lambda_{soc}}(G)$.

For $m \in E_{\Lambda_{soc}}(G)$ such that $m^i < \sum_{j=1}^n \frac{m^j}{n}$ and $\sum_{j\in N} m^j = S^*$, an agent $k \neq i$ exists, such that $m^k > \sum_{j=1}^n \frac{m^j}{n}$. For this agent $v^c_{\Lambda^k_{soc}}(m) = \sum_{\substack{j=1\\j\neq i}}^n \frac{m^j}{n-1} V(M)$. Since the best response is now $m^k = 0$, then, if agent k moves to $m^k - \varepsilon$, the function $v^c_{\Lambda^k_{soc}}$ increases. Therefore, $m \notin E^c_{\Lambda_{soc}}(G)$.

Analogously, for any $m \in E_{\Lambda_{soc}}(G)$ with $m^i > \sum_{j=1}^n \frac{m^j}{n}$, if agent i moves to $m^i - \varepsilon$, then function $v^c_{\Lambda^i_{soc}}(m) = \sum_{\substack{j=1\\j\neq i}}^n \frac{m^j}{n-1} V(M)$ increases and $m \notin E^c_{\Lambda_{soc}}(G)$.

Finally, consider $m \in E_{\Lambda_{soc}}(G)$, such that for all $i \in N$, $m^i = \sum_{j=1}^n \frac{m^j}{n}$. It follows that $m^i = m^j$ for all $i, j \in N$. Then, if agent i moves to $m^i + \varepsilon$, $v^c_{\Lambda^i_{soc}}(m) = \sum_{\substack{j=1\\j\neq i}}^n \frac{m^j}{n-1} V(M)$ decreases, and if he moves to $m^i - \varepsilon$, $v^c_{\Lambda^i_{soc}}(m) = \sum_{j=1}^n \frac{m^j}{n} V(M)$ decreases. Therefore, $m \in E^c_{\Lambda_{soc}}(G)$.

Since $\sum_{i\in N} m^i \leq S^*$ and $m^i = m^j$ for all $i, j \in N$, it also follows that $0 \leq m^i \leq \frac{S^*}{n}$.

Proof of Proposition 9. Following an analogous reasoning to that of the proof of Proposition 8, it can be shown that if point $m \in E_{\Lambda_{self}}(G)$ with $\frac{S^*}{n} \leq m^i \leq \frac{M^*}{n}$, $m^i = m^j$, for all $i, j \in N$, then $m \in E^c_{\Lambda_{self}}(G)$, and that if $i, j \in N$ exist such that $m^i \neq m^j$ then $m \notin E^c_{\Lambda_{self}}(G)$.

Proof of Proposition 10. Analogously to the proof of Proposition 8 and Proposition 9.

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