Boson expansion methods applied to a two-level model in the study of multiple giant resonances

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Abstract

We apply boson expansion methods to an extended Lipkin-Meshkov-Glick model including anharmonicities in analogy with previous microscopic calculations. We study the effects of different approximations present in these calculations, among which the truncation of the hamiltonian and of the space, in connection with the study of the properties of two-phonon and three-phonon states. By comparing the approximate results on the spectrum with the exact ones we conclude that the approximations made in the microscopic calculations on two-phonon states are well justified. We find also that a good agreement with the exact results for the three-phonon state is obtained by using a bosonic hamiltonian truncated at the fourth order. This result makes us confident that such approximation can be used in realistic calculations, thus allowing a theoretical study of triple excitations of giant resonances.

1 Introduction

Collective excitations have been known for many years in nuclear physics [1], both in the low-lying spectra and in the Giant Resonance (GR) region. A basic microscopic theory for such collective modes is the Random Phase Approximation (RPA) [2, 3] which can be seen as the lowest order in a boson expansion such that the hamiltonian can be put in the form of a sum of hamiltonians of harmonic oscillators, each one corresponding to a collective mode (phonon). Therefore, RPA predicts the existence of one-phonon, two-phonon,...etc states with a harmonic spectrum. In addition to the well known low-lying two-phonon states, recently heavy ion inelastic scattering experiments at intermediate and relativistic energies and double charge-exchange reactions have shown the existence of states in the high excitation energy region which can be described as a GR built on top of another GR [4, 5]. The study of the properties of these states allows to test our comprehension of the GR's as small amplitude vibrations and therefore the harmonic picture. The systematics on the energies and widths is in qualitative agreement with the harmonic approximation, namely the energy of a doubly excited GR is nearly twice that of the single GR and its width is between $\sqrt{2}$ and 2 times larger. However, the inelastic cross sections, when calculated within the harmonic picture, are almost always smaller than the measured ones. In particular, the data concerning Coulomb excitation show a discrepancy ranging from 30% up to a factor 4, according to the nucleus studied. In order to understand the origin of this discrepancy, corrections to the harmonic approximation have been proposed [6] by including anharmonicities in the internal hamiltonian and non-linearities in the external field. As shown in [6], small anharmonicities in the excitation spectrum of the target nucleus can lead to a large enhancement of the Coulomb excitation cross section. The model used there was an oversimplified one, namely the target was described as an anharmonic linear oscillator. The parameters of the cubic and the quartic terms in its hamiltonian were fixed so that the energy of the second excited state was shifted down by $\approx 2\ MeV$ with respect to twice the energy of the first excited state. In [7] a 3-dimensional extension of this model was considered and similar effects coming from anharmonicities were found. We remark that, in the case of Coulomb excitation of ^{136}Xe in the reaction ^{136}Xe + ^{208}Pb at E/A = 700 MeV, the experimentally observed peak, interpreted as the Double Giant Dipole Resonance (DGDR), is shifted by ≈ -2 MeV from that expected in the harmonic case [8]. A more refined study was performed in [9], where anharmonicities and non-linearities were included by starting from RPA and extending it by means of boson mapping techniques [3, 10]. Such microscopic approach was applied in [9] to realistic cases. In the case of ^{208}Pb + ^{208}Pb at E/A = 641 MeV, for example, it was found that the inclusion of anharmonicities and non-linearities gives rise to an enhancement of 30% of the cross section in the region of the DGDR, bringing the theoretical results closer to the experimental ones. Important contributions coming from other two-phonon states in the same energy region were also found.

The studies reported in the above quoted papers have of course some limitations. The model used in [6] is clearly very schematic and has, among others, the drawback that it is based on a purely bosonic description so that Pauli blocking effects are not

included. The approach presented in [9] is certainly much more realistic and the effects of the Pauli exclusion principle are taken into account to some extent. However, for computational reasons, only one- and two-phonon states were considered there.
The purpose of this paper is to present an analysis based on an extension of the Lipkin-Meshkov-Glick (LMG) model [11] including a residual interaction between the phonons, in analogy with the microscopic approach used in ref.[9]. This model has several advantages with respect to the others. Because of its group structure, this model is exactly solvable. Besides, it directly takes into account the Pauli principle. In this context, we discuss several approximations, corresponding to the cases considered in [6, 9] and, by direct comparison with the exact results, we test how severe are the limitations of these approximations. We focus on one hand on two-phonons states, studied in ref.[9] and on the other hand on the properties of three-phonon states. In fact, in order to have a more stringent test on the validity of the harmonic picture, experimental and theoretical studies of triple excitation of GR's should be envisaged. From the theoretical point of view, the same approach as the one in [9] can be used, but this would imply huge calculations. In order to make them feasible in the context of boson expansion methods, one may consider an approximation in which the same fourth order hamiltonian used in [9] is diagonalized in a space containing up to 3-phonon states. We present a test on how well this approximation works, in the context of the extended LMG hamiltonian. We will see that the exact results are very close to the approximate ones in all the cases studied for both the second and the third excited states.

The paper is organized as follows. In section 2 the LMG model is shortly reviewed and our extension introduced. In section 3 the boson mapping of this hamiltonian is presented. In section 4 the approximate results on the energy of the first, second and third excited states, obtained by truncating at different levels the mapping, are compared among themselves and with the exact calculations.

2 The model

2.1 The Lipkin-Meshkov-Glick model

In the original Lipkin-Meshkov-Glick (LMG) model [11] a finite number Ω of particles can be arranged in two levels separated by ε in energy. Ω different quantum states are available in each level. Each particle is therefore identified by two quantum numbers. The first σ indicates if the particle is in the upper ($\sigma = +$) or in the lower ($\sigma = -$) energy level. The second quantum number s is related to the Ω different available quantum states in each level. We will consider systems whose hamiltonian can be written as function of the operators K_+ , K_- , K_0 :

$$K_+ \;=\; \sum_{s=1}^{\Omega} a^\dagger_{+,s} a_{-,s} \tag{1}$$

$$K_- = (K_+)^\dagger = \sum_{s=1}^{\Omega} a^\dagger_{-,s} a_{+,s}$$
$$K_0 = \frac{1}{2}\sum_{s=1}^{\Omega} (a^\dagger_{+,s} a_{+,s} - a^\dagger_{-,s} a_{-,s})$$

where $a^\dagger_{\sigma,s}$ (and the hermitian conjugate $a_{\sigma,s}$) creates (annihilates) a particle in the quantum state (σ, s). Since the operators K_+ , K_- , K_0 satisfy the SU(2) commutation relations :

$$[K_+, K_-] = 2K_0 \qquad [K_0, K_\pm] = \pm K_\pm \tag{2}$$

they are often called the quasi-spin operators. According to (1), there can not be two particles in the same quantum state, that is the Pauli exclusion principle is included in the model. A system of Ω particles has 2^Ω states. However, since the hamiltonian is a function of the generators of SU(2) and K^2 is a Casimir operator of such algebra, the space can be separated into subspaces, each corresponding to an eigenvalue K. The state with all particles in the lower level, $\sigma = -$, is an eigenvector of K^2 and K_0, belonging to their maximum and minimum eigenvalue, respectively. This state, with the ones generated by applying K_+ to it, are the elements of a subspace of dimension $\Omega + 1$, that we will denote by $|\Omega/2, m\rangle$ with $m \in [-\Omega/2, \Omega/2]$.
Following (1) the original hamiltonian of LMG [11] can be written as :

$$H_{LMG} = \varepsilon \bar{K}_0 + V_1 K_+ K_- + V_2 (K_+ K_+ + K_- K_-) \tag{3}$$

where

$$\bar{K}_0 = K_0 + \Omega/2 \tag{4}$$

The $\varepsilon \bar{K}_0$ term can be viewed as corresponding to the Hartree-Fock part. The lowest eigenstate of K_0 corresponds then to the uncorrelated HF ground state $|\Omega/2, -\Omega/2\rangle$. In order to stress the analogy with the microscopic calculations in [9], we will often denote by h (hole) a $(-, s)$ state, i.e. a single particle state which is occupied in $|\Omega/2, -\Omega/2\rangle$, and by p (particle) a $(+, s)$ single particle state. Therefore, by looking at eq.(1), we can say that the residual interaction in (3) only contains particle-hole terms $a^\dagger_p a^\dagger_{p'} a_h a_{h'}$ and $a^\dagger_p a^\dagger_h a_{p'} a_{h'}$ terms, that is those usually included in RPA.
If $V_1 = V_2 = 0$ then $H_{LMG} = \varepsilon \bar{K}_0$. The corresponding eigenvalues are $n\varepsilon$ with $n \in [0, \Omega]$. From now on we will denote these states by $|n>$. The energy eigenvalues are equidistant and form a harmonic spectrum truncated at $\Omega + 1$ levels. If V_1 or V_2 are different from zero, then the energy spectrum is not harmonic any more and only if $V_2 \neq 0$ the energy eigenvectors correspond to a mixing of $|n\rangle$ states. In fact, $V_1 K_+ K_-$ is diagonal in the $|n\rangle$ space. This term of $ph - p'h'$ type does not mix states with different particle-hole numbers. On the contrary, the $pp' - hh'$ type term, $V_2 \left(K_+^2 + K_-^2\right)$ mixes states with n and $n \pm 2$ particle-holes. To conclude this part, we would like to recall that the LMG model is exactly solvable using group techniques and that it includes the Pauli exclusion principle.

2.2 An extension of the LMG model

The LMG model in its original form already includes some anharmonicities, essentially those related to the fact that the Pauli principle is treated exactly. However, parts of the residual interaction are neglected, namely the $pp' - p''p'''$, $pp' - p''h$, $hh' - h''p$, $hh' - h''h'''$ ones, which were considered in the microscopic calculations [9] where the anharmonicities arising from them were also studied. In order to simulate them, we propose an extension of the LMG hamiltonian which is still quadratic in the K_+ , K_- and K_0 operators :

$$H = H_{LMG} + \Delta V = H_{LMG} + V_3(K_+\bar{K}_0 + \bar{K}_0 K_-) + V_4(\bar{K}_0 - 1)\bar{K}_0 \tag{5}$$

The K_+ $\bar{K}_0$ term and its hermitian conjugate introduce a coupling between $|n\rangle$ and $|n \pm 1\rangle$ whereas the last term shifts the energies of the $|n\rangle$ states, except those with $n = 0$ and 1. Therefore, the eigenstates $|\phi_\alpha >$ of the hamiltonian (5) are superpositions of the $|n >$ states

$$|\phi_\alpha >= \sum_n X_n^\alpha |n > \tag{6}$$

In section 4 we will compare the exact eigenvalues of the hamiltonian (5) with those corresponding to the bosonic hamiltonian obtained by mapping the fermionic one up to the fourth order.

3 The boson hamiltonian

We apply boson expansion methods to the fermionic hamiltonian (5) and truncated the so obtained boson hamiltonian to second and fourth order. In ref.s[12, 13] a similar study was done for the original LMG hamiltonian (3). In the present case, the quadratic hamiltonian corresponds to RPA while the quartic can be directly compared with what was done in [9], where some further approximations were performed, as discussed below.

Let's take the normal ordered Holstein-Primakoff boson expansion of the SU(2) generators [12, 14, 15] :

$$\begin{array}{l} (K_+)_b = \sum_{i=0}^{\infty} a_i (b^\dagger)^{i+1} b^i \\ (K_-)_b = \sum_{i=0}^{\infty} a_i (b^\dagger)^i b^{i+1} \\ (\bar{K}_0)_b = b^\dagger b \end{array} \tag{7}$$

where

$$a_i = \sum_{m=0}^{i} \frac{(-1)^{i-m}}{m!(i-m)!} (\Omega - m)^{1/2} \tag{8}$$

In the case of SU(2), the Hage-Hassan-Lambert mapping used in ref.[9] is equivalent to the Holstein-Primakoff mapping (7,8) [16] with the shifted operator $(\bar{K}_0)_b$. By this shift one eliminates terms linear in $b^\dagger$ and b in the hamiltonian which corresponds to a redefinition of the mean field.

Let us then consider the bosonic mapping of our extended hamiltonian (5) in the particular case $V_2 = V_1/2$ which is the value used in the numerical applications

we present in the next section. By including all the terms of the expansion (7) which contribute up to the second order in the $b^\dagger, b$ operators, we get the quadratic hamiltonian

$$H_b^{(2)} = \varepsilon(1+\delta_1)b^\dagger b + \frac{\delta_1\varepsilon}{2}\left(1-\frac{1}{\Omega}\right)^{1/2}\left(b^{\dagger 2}+b^2\right) \tag{9}$$

where

$$\delta_1 = \Omega V_1/\varepsilon \tag{10}$$

is related to the strength of the particle-hole residual interaction. The hamiltonian $H_b^{(2)}$ can be written in diagonal form by introducing the Bogoliubov transformation [3] :

$$\begin{array}{l} b^\dagger = XQ^\dagger + YQ \\ b = XQ + YQ^\dagger \end{array} \tag{11}$$

and imposing

$$H_{20} = 0 \tag{12}$$

with the condition

$$X^2 - Y^2 = 1 \tag{13}$$

which guarantees the correct commutation relation

$$\left[Q, Q^\dagger\right] = 1 \tag{14}$$

Thus one gets the RPA hamiltonian

$$H_b^{(2)} = H_{11}Q^\dagger Q \tag{15}$$

where

$$H_{11} = \varepsilon(1+\delta_1)(X^2+Y^2) + 2XY\delta_1\varepsilon\left(1-\frac{1}{\Omega}\right)^{1/2} \tag{16}$$

The conditions (12,13) determine the X and Y's amplitudes as solutions of the set of equations :

$$\left\{\begin{array}{rcl} X^2 - Y^2 & = & 1 \\ \varepsilon(1+\delta_1)XY + \frac{\delta_1\varepsilon}{2}\left(1-\frac{1}{\Omega}\right)^{1/2}(X^2+Y^2) & = & 0 \end{array}\right. \tag{17}$$

Of course the terms in V_3 and V_4 do not appear at the quadratic order. In eq.(15), the H_{00} which corresponds to a shift of the eigenenergies was omitted.
The violations to the Pauli exclusion principle introduced by truncating the boson expansion at the lowest order as well as a contribution coming from the residual interaction of the $pp' - p''p'''$, $pp' - p''h$, $hh' - h''p$, $hh' - h''h'''$ type can be partly introduced by going one step further, i.e. including all terms up to the fourth order, as it was done in ref.[9]. At this level, the terms in V_3 and V_4 will therefore enter. Their presence, together with the corrections for the Pauli principle, will give rise to an anharmonic bosonic hamiltonian.
By mapping the hamiltonian of eq.(5) up to fourth order we get:

$$
\begin{aligned}
H_b^{(4)} &= \varepsilon(1+\delta_1)b^\dagger b + \tfrac{\delta_1\varepsilon}{2}\left(1-\tfrac{1}{\Omega}\right)^{1/2}\left(b^{\dagger 2}+b^2\right) \\
&+V_3\sqrt{\Omega}\left(1-\tfrac{1}{\Omega}\right)^{1/2}\left(b^{\dagger 2}b+b^\dagger b^2\right)+(V_4-V_1)\,b^{\dagger 2}b^2 \\
&+\tfrac{\delta_1\varepsilon}{2}\left(1-\tfrac{1}{\Omega}\right)^{1/2}\left[\left(1-\tfrac{2}{\Omega}\right)^{1/2}-1\right]\left(b^{\dagger 3}b+b^\dagger b^3\right)
\end{aligned}
\tag{18}
$$

which we rewrite in terms of the $Q^\dagger$ and Q operators as :

$$
\begin{aligned}
H_b^{(4)} = H_{11}Q^\dagger Q + H_{30}(Q^{\dagger 3}+Q^3) + H_{21}(Q^{\dagger 2}Q+Q^\dagger Q^2)+ \\
H_{31}(Q^{\dagger 3}Q+Q^\dagger Q^3) + H_{22}Q^{\dagger 2}Q^2 + H_{40}(Q^{\dagger 4}+Q^4)
\end{aligned}
\tag{19}
$$

whose coefficients are given in the appendix (eqs.21-27), together with the new equations for X and Y (20) coming again from the conditions (12,13). In eq.(19) the H_{00} was omitted as well as a term, linear in the $Q^\dagger$ and Q operators, which would introduce a redefinition of the mean field, in analogy with what was done in [9].
The quartic hamiltonian (19) corresponds to that used in [9] where, however, some approximations were introduced in order to make feasible the calculations in the realistic cases considered there. First, only one- and two-phonon states were considered. Therefore, the terms in H_{30}, H_{31} and H_{40} were not effective. Second, the X and Y amplitudes appearing in the fourth order hamiltonian were not recalculated but taken equal to those obtained at the second order, i.e. the RPA ones. In order to get an indication on how good these approximations on the space, on the hamiltonian and on the X and Y amplitudes are, in the next section we will study them within the present schematic model.

4 Results and discussion

First of all we have to fix the parameters entering in the hamiltonian (5). For the single particle energy we use the parametrization $\varepsilon = 41/A^{1/3}$ MeV. We take $V_2 = 0.5V_1$ and the strength V_1 is fixed so that the first excited state lies at an energy around $80/A^{1/3}$ MeV corresponding to the systematics of GDR in nuclei. This criterion gives $V_1 = 1.2$ MeV. We have studied the behaviour of the energies of the three lowest states, E_1, E_2 and E_3, as a function of V_4 for two values of V_3, namely $V_3 = 0$ MeV (Fig.1) and $V_3 = 0.25$ MeV (Fig.2). The latter value for V_3 gives $< 2|\Delta V|1 > \approx 1$ MeV in analogy with the microscopic calculations [9]. Note that the sign of V_3 is irrelevant. As far as the sign of V_4 is concerned, we show results only for negative values, which give a downward shift of E_2 and E_3 with respect to $2E_1$ and $3E_1$, respectively, i.e. the harmonic (RPA) values. In the figures we show the results obtained for $\Omega = 8$.

To compare the spectrum of the exact hamiltonian and its different boson expansions we study the energy differences $(E_n - E_{n-1})$ between the lower excited states. In RPA they have the common value 15.11 MeV for the considered parameters, independent of n, V_3 and V_4. In the figures this value is represented by a dashed line. This should be compared with the exact results for the fermionic hamiltonian eq.(5), shown as solid lines. We see that, even for $V_3 = V_4 = 0$, the RPA results deviate from the exact

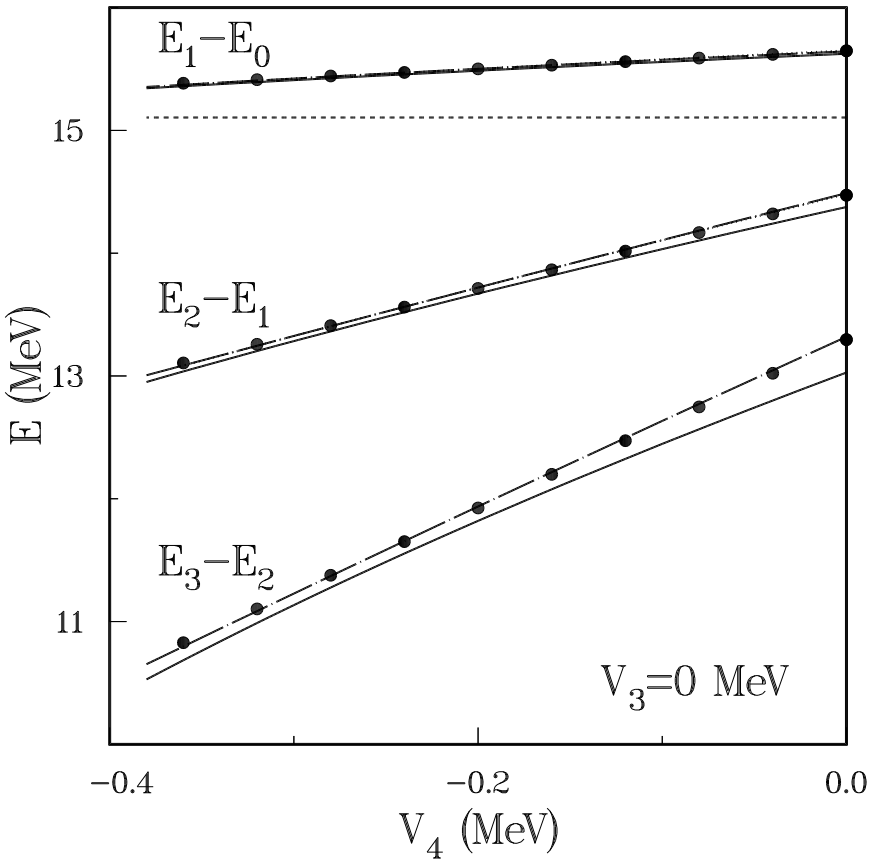

Figure 1: Energy differences as functions of V_4 for a fixed value of $V_3 = 0$ MeV. The ones corresponding to the fermionic hamiltonian (5) are plotted as solid lines. The others correspond to different approximations on the bosonic expansion of the hamiltonian (see text).

ones. Namely, the first three differences obtained in the exact calculations are equal to 15.62 MeV, 14.38 MeV and 13.03 MeV, respectively.
The agreement with the exact energies can be improved by using the quartic hamiltonian, eq.(19). When the bosonic hamiltonian is diagonalized in the space containing up to two-phonon states we get the results presented in Fig. 1 and 2 by dotted lines. We remark that now the first excited state is very close to the exact one, while for the second one there is a discrepancy of not more than 250 KeV. In the more realistic microscopic calculations presented in [9] a further approximation was introduced. Instead of reobtaining the X and Y amplitudes by solving the equations analogous to eqs (20), the RPA amplitudes were used. In the present model, this corresponds to use in eq. (19) the solutions of eqs (17) rather than those of (20). The results obtained are indistinguishable from the dotted lines in Fig. 1 and 2. This supports the validity of the procedure used in [9].

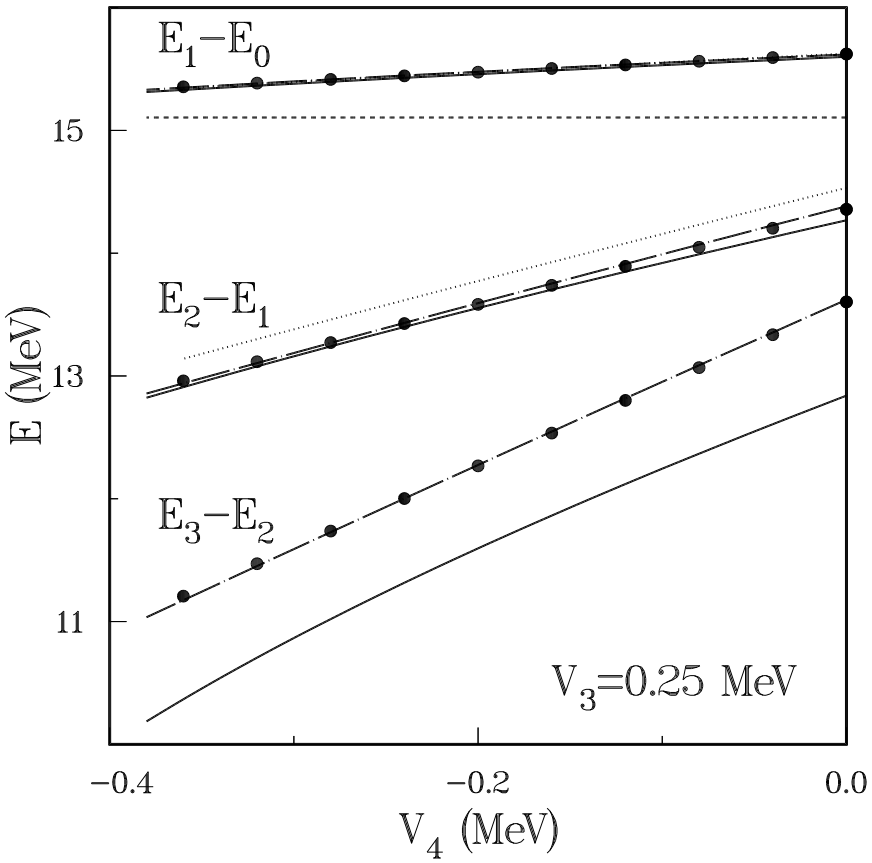

Figure 2: As in fig. 1, but for $V_3 = 0.25$ MeV.

In order to test how much the truncation of the space affects the results, let us now enlarge the space up to three-phonon states. Of course, in this space, the quartic term H_{40} in (19) does not play any role. In the figures, we show the results corresponding to the complete quartic hamiltonian of eq.(19), with the X and Y amplitudes solutions of eq.s (20) (dot-dashed lines). Comparing the energies of the first two states we see that the agreement with the exact ones is now almost perfect. In the enlarged space we can also study the third excited state which, in the harmonic limit, would correspond to a three-phonon state. In this case the approximate results differ from the exact ones by 1 MeV, which can be considered as a good approximation since the excitation energy of this state is about 40 MeV. We would like to point out that the results obtained in the same model space but corresponding to the quartic hamiltonian of eq (19) without the H_{31} and H_{30} term and with the use of the RPA X and Y amplitudes, solution of eqs (17), (represented in figure 1 and 2 like solid circles) are practically coincident with the dot-dashed line. The use of an analogous approximation in realistic microscopic calculations would make them much simpler.
In summary, we have seen that quite good results can be obtained for both the second

and the third excited state by diagonalizing the fourth order bosonic hamiltonian with coefficients calculated with the RPA X and Y amplitudes, in the space containing up to three-phonon states. Nowadays there is interest in searching for triple giant resonance excitations [17]. The present results indicate that a realistic theoretical study of such states will be feasible in the near future.

5 Conclusions

We have applied boson expansion methods to an extended Lipkin-Meshkov-Glick model including a residual interaction between the excited states in analogy with the anharmonicities introduced in the microscopic calculations of ref.[9]. We have studied the effect of truncations of the space and of the hamiltonian as well as other approximations present in those calculations by comparing the approximate results on the energies of the first three excites states among them and with the exact ones. From the analysis presented we can conclude that the approximations made in [9] are well justified if one wants to study one- and two-phonon excitations. What about three-phonon states? Of course, in the context of boson expansion methods, the study of these states would need a boson expansion up to sixth order, but this would be a formidable task. The results shown make us confident that a quartic hamiltonian as that used in [9], diagonalized in an enlarged space including up to three-phonon states, may give reasonable results also in realistic cases.

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6 Appendix

The X and Y coefficients of the Bogoliubov transformation obtained by imposing conditions (12,16) for the quartic expansion $H_b^{(4)}$ of the hamiltonian (5) are solutions of the equations

$$\left\{\begin{array}{rcl} X^2 - Y^2 & = & 1 \\ \varepsilon(1+\delta_1)XY + \frac{\delta_1 \varepsilon}{2}\left(1-\frac{1}{\Omega}\right)^{1/2}(X^2+Y^2) + & & \\ (V_4 - V_1)\,XY\,(X^2+5Y^2) + 3\lambda\,(3X^2Y^2+Y^4) & = & 0 \end{array}\right. \tag{20}$$

The coefficients of the boson hamiltonian $H_b^{(4)}$ are given by the following expressions

$$H_{11} = \varepsilon(1+\delta_1)(X^2+Y^2) + 2XY\delta_1\varepsilon\left(1-\tfrac{1}{\Omega}\right)^{1/2} + 4\left(2X^2Y^2+Y^4\right)(V_4-V_1) + 6XY\lambda\left(X^2+3Y^2\right) \tag{21}$$

$$H_{30} = V_3XY\left(X+Y\right)\sqrt{\Omega}\left(1-\frac{1}{\Omega}\right)^{1/2} \tag{22}$$

$$H_{21} = V_3\sqrt{\Omega}\left(1-\frac{1}{\Omega}\right)^{1/2}\left(X^3+2XY^2+2X^2Y+Y^3\right) \tag{23}$$

$$H_{40} = \left(-V_1+V_4\right)X^2Y^2 + \lambda XY(X^2+Y^2) \tag{24}$$

$$H_{31} = \left(-V_1+V_4\right)2XY\left(X^2+Y^2\right) + \lambda\left(X^4+6X^2Y^2+Y^4\right) \tag{25}$$

$$H_{22} = \left(-V_1+V_4\right)\left(X^4+4X^2Y^2+Y^4\right) + 6XY\lambda\left(X^2+Y^2\right) \tag{26}$$

and

$$\lambda = \frac{\delta_1\varepsilon}{2}\left(1-\frac{1}{\Omega}\right)^{1/2}\left[\left(1-\frac{2}{\Omega}\right)^{1/2}-1\right] \tag{27}$$

In particular, if $V_1 = V_3 = V_4 = 0$, the relations (21)-(27) coincide with those of reference [12].

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