A note on Copernicus' 'correction' of Ptolemy's mean synodic month

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Introduction.

In *A History of Ancient Mathematical Astronomy*[1], Neugebauer wrote that the equation

$$126007\ (=35,0,7)\ \text{days 1 hour} = 4267\ (=1,11,7)\ \text{synodic months} \qquad (1)$$

was supposedly the source of Ptolemy's value (*Almagest* IV.2)[2] for the length of the mean synodic month ($29;31,50,8,20^d$), and he added that Copernicus[3] was the first to check Ptolemy's arithmetic to find that equation (1) in fact leads to

[1] Neugebauer 1975, p. 310.

[2] Ptolemy's text is ambiguous: "For from the observations he [= Hipparchus] set out he shows that the smallest constant interval defining an ecliptic period in which the number of months and the amount of [lunar] motion is always the same, is 126007 days plus 1 equinoctial hour. In this interval he finds comprised 4267 months, 4573 complete returns in anomaly, and 4612 revolutions on the ecliptic less about 7 ½°, which is the amount by which the sun's motion falls short of 345 revolutions (here too the revolution of sun and moon is taken with respect to the fixed stars). (Hence, dividing the above number of days by the 4267 months, he finds the mean length of the [synodic] month as approximately 29;31,50,8,20 days). He shows, then, that the corresponding interval between two lunar eclipses is always precisely the same when they are taken over the above period [$126007^d\ 1^h$]" (Toomer 1984, pp. 175-6).

[3] *De revolutionibus* IV.4 (Nuremberg 1543, 101v): "Quapropter idem Hipparchus ulterior ista perquisiuit, nempe collatis adnotationibus, quas in eclipsibus lunaribus

1 mean synodic month = $29;31,50,8,9,20^d$

and not to $29;31,50,8,20^d$, a parameter that is now known to come from Babylonian system B.[4]

Later, in their *Mathematical Astronomy in Copernicus's De revolutionibus*, Swerdlow and Neugebauer pointed out that, according to Ptolemy, Hipparchus did not derive the mean motions of his lunar theory from equation (1), but merely accepted the Babylonian periods

1 synodic month = $29;31,50,8,20^d$,

251 synodic months = 269 anomalistic months,

5458 synodic months = 5923 draconitic months,

adding that Copernicus, however, following Regiomontanus's implicit misunderstanding in *Epitome* IV.3[5], took the cycle as a source of Hipparchus's mean synodic month, and computed that

diligentissime obseruauit, ad eas quas à Chaldaeis accepit: tempus in quo reuolutiones mensium et anomaliae simul reuerterentur, definiuit esse CCCLXV annos Aegyptios, LXXXII dies, & unam horam, & sub eo tempore menses IIII.CCLXVII, anomaliae uero IIII.DLXXIII circuitus compleri. Cum ergo per numerum mensium distributa fuerit proposita dierum multitudo, suntque centena vigintisex millia & vii dies atque una hora, inuenitur unus mensis aequalis dierum XXIX, scrup. primorum XXXI, secundorum L, tertiorum VIII, quartorum IX, quintorum XX. Qua ratione patuit etiam cuiuslibet temporis motus. Nam diuisis CCCLX unius menstruae reuolutionis gradibus per tempus menstruum, prodijt diarius Lunae cursus à Sole gradus [X]II, scrupula prima XI, secunda XXV, tertia XLI, quarta XX, quinta XVIII." The Basel edition of 1566 gives "vigintisex milia & xii dies"; Copernicus' autograph manuscript of *De revolutionibus* (Kraków, Biblioteka Jagiellońska, MS 10000, f. 110r) "vigintisex milia et vij dies".

[4] Cf. Kugler 1900, pp. 7, 24 and 111; Aaboe 1955; Neugebauer 1955, pp. 75-8.

[5] "Hyparchus autem quantitatem huius interualli reperit 126007 dies et horam unam et in hoc interuallo fuerunt menses lunares 4267, quod facile per numerum nouiluniorum considerare potuit. Reditiones autem in circulo diuersitatis fuerunt 4573, quod etiam per motus lune conditionatos tardum medium uelocem et medium deprehendit. Reditiones uero in orbe signorum 4612 minus septem gradibus et medietate fere. Tantum enim sol minuit in 347 reuolutionibus huius temporis, eo quod in reditionibus istis processum est in relatione ad stellas fixas. Interuallum itaque dierum diuisum per numerum mensium ostendit quantitatem unius mensis lunaris" (*Epytoma Joannis De monte regio In almagestum ptolomei* IV.3, Venice, 1496; repr. 1972, p. 116). Note however that Regiomontanus does not give in the text the result of the division $126007^d\ 1^h / 4267$.

$$1 \text{ mean synodic month} = 126007^d\ 1^h / 4267 = 29;31,50,8,\mathit{9},\mathit{20}^d, \qquad (2)$$

> a mistake that has been repeated in modern literature, entirely independent of Copernicus. He then claims that dividing this number into 360° gives a mean daily elongation of 12;11,26,41,20,18°, although the division would correctly give 12;11,26,41,24,42°. In fact, and fortunately, Copernicus has merely rounded from the value in the Almagest, based upon the correct synodic month used by Hipparchus, that is,
>
> $\eta^d = 6,0° / 29;31,50,8,20^d = 12;11,26,41,20,17,59^{o/d}$.
>
> Thus he saves himself from the error of computing what he believes to be Hipparchus's mean motions from the wrong synodic month.[6]

The object of this note is (i) to show that the parameter $29;31,50,8,9,20^d$ was widely considered throughout the Latin Middle Ages as the 'correct' Ptolemaic value and simply taken from Gerard of Cremona's Latin translation of the *Almagest* (depending on this point on al-Ḥajjāj ibn Maṭar's Arabic version), and (ii) to suggest that very likely Copernicus never checked the computation, but merely took these inconsistent parameters for the synodic month and the daily mean elongation from the Latin version of the *Almagest*[7].

The Arabic, Hebrew, and Latin traditions of *Almagest* IV.2.

Only two Arabic translations of the *Almagest* are extant: one dated 827/8 by al-Ḥajjāj, the other completed ca. 879-90 by Isḥāq ibn Ḥunayn, later revised by Thābit ibn Qurra (d. 901). Al-Ḥajjāj's translation has $...8,9,20^d$

[6] Swerdlow and Neugebauer 1984, pp. 198-9. Pedersen (1974, pp. 162-3) also mentioned Copernicus's passage, asserting that an easy explanation for the discrepancy between Ptolemy's $29;31,50,8,20^d$ and the correct result in equation (2), $...8,9,20^d$, is to assume that the parameter $...8,20^d$ was not derived from equation (1). When discussing these issues with me, B.R. Goldstein suggested a reasonable solution to the puzzle: Ptolemy had no intention of changing the Babylonian parameter, and computed $29;31,50,8,20^d \times 4267 = 35,0,7;2,42,38,20^d = 126007^d\ 1;5,2,41,..^h$, rounding this result to $126007^d + 1^h$.

[7] Copernicus used Gerard of Cremona's translation (a copy of the Venice edition, 1515, annotated by him is preserved at Uppsala) and also Trebizond's version, first published at Venice in 1528. In 1539 Rheticus brought Copernicus the 1538 edition of the Greek text with Theon's commentary, but it is assumed that the use Copernicus could make of this at so late a date was limited (Swerdlow and Neugebauer 1984, p. 92).

(MS Leiden, Or. 680, f. 50b: 6), whereas Isḥāq-Thābit's version has Ptolemy's figure ...8,20^d (MS Tunis, Bibliothèque Nationale, 07116, f. 53b:19-20; there is at this place in the manuscript a marginal note which reads "in the translation of al-Ḥajjāj nine fourths and twenty fifths", thus confirming both readings)[8]. It is likely that al-Ḥajjāj was embarrassed that equation (1) did not produce the expected result, and so he silently changed Ptolemy's text for the length of the mean synodic month, not appreciating the meaninglessness of the correction. In his *al-Qānūn al-Mas'ūdī* [9], al-Bīrūnī (973-1048) gives ...8,9,20,13^d (a value very close to the accurate result: 29;31,50,8,9,20,12,22...^{d}). Jābir ibn Aflaḥ, in his *Iṣlāḥ al-Majisṭī* (middle of the XIIth century), also translated by Gerard of Cremona and frequently cited in Western Europe, gives ...8,9,20^d[10]. Naṣīr al-Dīn al-Ṭūsī (d. 1274), in his *Taḥrīr al-majisṭī*, when discussing the mean synodic month, quotes the Babylonian parameter given in *Almagest* IV.2 and comments that instead of 29 days, 31 minutes, 50 seconds, 8 thirds, and 20 fourths, "Ḥajjāj's copy [of the *Almagest* had the value] 9 fourths, 20 fifths and 12 sixths, which was the correct [value]"[11]. A slightly different value is found in Ibn Yūnus' *al-Zīj al-Ḥākimī* (about 990)[12], where the length of Muḥarram is given as 29;31,50,8,9,24^d. The same value, ...8,9,24^d, was attributed to Ptolemy by al-Biṭrūjī (end of the XIIth c.) in his *Kitāb fī-l-hay'a*[13], a work translated into Latin by Michael Scot at Toledo in 1217 (*De motibus coelorum*) and often mentioned in thirteenth and fourteenth century Scholastic discussions of the Ptolemaic

[8] This information was kindly supplied by P. Kunitzsch.

[9] al-Bīrūnī 1954-6, book 7, ch. 2, vol. 2, p. 730. I wish to thank B. R. Goldstein for this information.

[10] "Cum ergo diuiserunt istos dies quos inuenerunt huic tempori reuolubili per numerum mensium qui sunt in eo, exiuit tempus mensis medij 29 dies, et 31 minutum [sic], et 50 secunda, et 8 tertia, et 9 quarta, et 20 quinta cum propinquitate..." (*Gebri Filii Affla Hispalensis Astronomi uetustissime pariter et peritissimi, libri IX de Astronomia...*, Nuremberg: Petreius, 1534, f. 49r).

[11] India Office MS No. Loth 741, f. 19v; quoted by Saliba 1987, p. 150.

[12] MS Leiden, Cod. Or. 143, p. 20 (I am grateful to B.R. Goldstein for checking this manuscript for me); see also Delambre 1819, p. 96, and Neugebauer 1979, p. 18.

[13] However, the values for the motion of the moon in longitude during a synodic month, the mean daily motion of the moon, and the daily increment in elongation given by al-Biṭrūjī agree with the corresponding parameters in the *Almagest*; cf. Goldstein 1971, 1: 145.

system. It was also used by Abū Shāker in his *Ḥasāba 'ālam* (or *Chronology*; ca. 1256)[14].

Al-Khwārizmī (d. ca. 850) informs us that the Jewish calendar used a mean synodic month of 29 days and 12 793/1080 hours (each of the 1080 equal parts of an hour was called in Hebrew a *ḥeleq*, a unit ultimately of Babylonian origin)[15], that is, in sexagesimal form, $29;31,50,8,20^d$, and an identical report can be found in al-Bīrūnī[16]. Abraham Bar Ḥiyya, in his *Sefer Ḥeshbon mahlekot ha-kokabim* (*Calculation of the celestial motions*; ca. 1136), gives 12 793/1080 hours[17]; Abraham Ibn Ezra (1089-1164), in his *Liber de rationibus tabularum*[18], writes that the lunar (synodic) month is 29 days and 12;44,3,20 hours, identical to $...8,20^d$, and, in the *Sanctification of the Moon*, Maimonides (1135-1204) also uses 29^d and 12 793/1080 hours[19]. However, in Jacob Anaṭoli's Hebrew version of the *Almagest* (ca. 1230-36) we find $...8,9,20^d$ (MS Turin, Biblioteca Nazionale Universitaria, A.II.10, f. 40v:27), the parameter from Ḥajjāj's Arabic version[20], and this agrees with the apparent dependence of Anaṭoli's work on the Latin translation by Gerard of Cremona[21]. In subsequent astronomical literature, this 'corrected' value is always given when Ptolemy's *Almagest* is quoted. Thus, for instance, Levi ben Gerson (1288-1344) in chapter 64 of his *Astronomy* (*Milḥamot Adonai* V.1), when introducing his own value for the synodic month ($29;31,50,7,54,25,3,32^d$ or 29^d, $12;44^h$ and nearly 1/1138 of 'an hour), attributes $...8,9,20^d$ to Ptolemy and ...12 793/1080 hours to "our ancient scholars" [22].

[14] Neugebauer 1987, p. 280.

[15] Kennedy 1964, p. 55. On the Babylonian origin of the division of the hour into 1080 parts, see Neugebauer 1956, p. 117.

[16] al-Bīrūnī 1879, p. 143.

[17] Millás Vallicrosa 1959, p. 55.

[18] Millás Vallicrosa 1947, p. 99: "Et hoc potest probari nam in mense lunari qui est ab adunatione solis et lune cum cursu medio suo donec iterum coniungantur sunt 29 dies et 12 hore et 44 puncta [*sic*; read: minuta] hore et medietas none minuti."

[19] Neugebauer 1949, p. 326; Neugebauer 1956, p. 114.

[20] I am grateful to M. Zonta for checking this manuscript.

[21] Zonta 1993, p. 332.

[22] "Et dico quod Ptolomeus declarauit experientijs antiquorum et suis, et Abarcas [= Hipparchus] ante eum declarauit hoc idem, scilicet, quod tempus medij mensis lunaris est $29;31,50,8,9,20^d$. Et nos inuenimus istum computum ita ueritati propinquum quod in

It is well known that Gerard of Cremona's translation of the *Almagest* was made in 1175 using the Arabic al-Ḥajjāj's version for Books I-IX, and Isḥāq-Thābit's version for Books X-XIII[23]. Consequently, Gerard of Cremona gives the 'corrected' parameter ...8,9,20^d, which we can find in the manuscript tradition as well as in the printed edition[24]. The same value is given in other widely used Latin texts, as the well known *Almagestum parvum* (or *Almagesti minoris libri VI*)[25]. George of Trebizond's

toto de cursu temporis a Ptolomeo usque ad presens non inuenitur defectus nisi 0;12°, in quibus inuenimus distantiam lune a sole in tempore nostro maiorem quam esse deberet secundum computum Ptolomei. Et iste computus quasi consentit computui cui consenserunt sapientes nostri antiqui, qui ponebant tempus mensis lunaris 29 dies, 12 horas, 793 puncta, atribuendo 1080 puncta hore cuilibet, que sunt 29;31,50,8,20^d; qui computus excedit computum Ptolomei in 0;0,0,0,10,40^d [...] Set nos in hoc considerantes subtiliter per experientias antiquorum et nostras, ut declarabitur in futuro, ista inquisitione completa inuenimus tempus medij mensis lunaris 29;31,50,7,54,25,3,32^d [...] Et secundum computum nostrum esset mensis lunaris 29 dies, 12;44^h et circa unum punctum atribuendo hore 1138 puncta" (Vat. Lat. 3098, f. 57rb). See also Mancha 1998, pp. 307-9. According to B.R. Goldstein, Levi (along with medieval Jews in general) believed that the length of the month in the Jewish calendar was already used by the rabbis of the Talmudic period, if not earlier, as attested, for example, in Judah Halevi's *Kuzari* (12th c., Spain): "The calendar, based on the rules of the revolutions of the moon, as handed down by the House of David, is truly wonderful. Though [the medieval Hebrew translation adds: "thousands and"] hundreds of years have passed, no mistake has been found in it, whilst the observations of Greek and other astronomers are not faultless. They were obliged to insert corrections and supplements every century, whilst our calendar is always free from error, as it rest on a prophetic tradition." (Hirschfeld 1905, p. 123).

[23] See, e.g., Kunitzsch 1974, pp. 99-102. Two Latin versions of the *Almagest* made from the Greek are extant (cf. Haskins 1924, pp. 103-10), which I have not checked. A fourth version, probably from the Arabic (Haskins 1924, p. 108) and of Spanish origin prior to the early thirteenth century, is preserved only in fragments.

[24] MS Memmingen F.33, f. 39v: "...per 4267 menses prouenit enim numerus dierum mensis lunaris 29 dies et 31 minuta et 50 secunda et 8 tertia et 9 quarta et 20 quinta fere..."; printed edition, Venice: Lichtenstein, 1515, f. 36r: "Et ex hoc inuenit Abrachis tempus medium mensurnum lunare, vbi diuisit numerum horum dierum per quatuor milia ducentos et sexagintaseptem menses. Prouenit enim numerus dierum mensis lunaris 29 dies et 31 minuta et 50 secunda et 8 tertia et 9 quarta et 20 quinta fere".

[25] "...et est hic numerus prefinito tempore 4573 reuersiones diuersitatis. Hijs itaque cognitis, numerus dierum et unius hore inter duas eclipses per numerum mensium diuidendus et exibit tempus equalis lunacionis, et est sicut ex premissis deprehenditur 29;31,50,8,9,20^d..." (*Almagestum parvum*, IV.3, MSS British Library, Harley 625, f. 101v, and Prague, Univ. V.A.11, f. 24r). However, the copy of this work in MS Memmingen F.33, f. 169v, has 29;31,50,8,9,25^d, very close to the value given by Ibn Yūnus, al-Biṭrūjī, and Abū Shāker (see above).

translation has also ...8,9,20^{d}, despite claiming to have been made directly from the Greek[26]. Once Ptolemy's Greek text was printed in 1538[27], the two values were clearly distinguished and their sources identified, as is attested in the marginal notes in some copies of printed editions of *De revolutionibus*, now attributed to Paul Wittich (ca. 1550-87) and some time ago wrongly to Tycho Brahe[28]. So, in the copy of *De revolutionibus* preserved at the University Library at Prague[29], in the margin of f. 101v, next to the passage where Copernicus asserts that Ptolemy's mean synodic month is 29;31,50,8,9,20^{d}, there is the annotation: *Sic habet translatio Arabica, sed graeca sic: 29;31,50,8,20^{d}, qua et usus Ptolemaeus hinc colligit diurnum motum distantiae lunae a solis 12;11,26,41,20,18^{o}* [30], and in the text, the value for the elongation of the Moon, 12;11, 26,41,20,18$^{o/d}$, rounded from Ptolemy's result of the subtraction of the mean daily motion of the Sun from the mean daily motion of the moon in longitude, namely

$$13;10,34,58,33,30,30^{o/d} - 0;59,8,17,13,12,31^{o/d} = 12;11,26,41,20,17,59^{o/d}$$

is corrected to 12;11,26,41,24,42$^{o/d}$, which results from the consistent calculation

$$\frac{360^{o}+0;59,8,17,13,12,31^{o}\times 29;31,50,8,9,20^{d}}{29;31,50,8,9,20^{d}} = 13;10,34,58,37,54,41^{o/d}$$

and, therefore,

$$13;10,34,58,37,54,41^{o/d} - 0;59,8,17,13,12,31^{o/d} = 12;11,26,41,24,42,10^{o/d}.$$

[26] Venice: Junta, 1528, f. 33r.

[27] *Claudii Ptolemaei Magnae Constructionis, id est Perfectae coelestium motuum pertractationis, Libri XIII*, Basel: Hervagius, 1538.

[28] Gingerich and Westman 1981.

[29] Basel: Petrina, 1566, shelf-mark No. 14 B 16.Tres M 11; a facsimile of it was published in Horský 1971.

[30] At the bottom of the folio Wittich adds: *Mensis Synodicus Iuxta Hipparchum 29;31,50,8,9,20,12^{d}.*

Regiomontanus, who owned a Greek copy of the *Almagest* and intended to publish a Latin translation, was surely aware of the discrepancy between the original text and Gerard of Cremona's version from the Arabic, and perhaps his perplexity in confronting the dilemma ('wrong' Ptolemy *versus* 'correct' translation) may explain his refusal to give us the exact result deriving from equation (1).

Bibliography

Aaboe, A. 1955. "On the Babylonian origin of some Hipparchian parameters", *Centaurus*, 4:122-5.

al-Bīrūnī, 1954-56. *al-Qānūn al-Mas'ūdī*. Hyderabad. 3 vols.

al-Bīrūnī, 1879. *The Chronology of Ancient Nations. An English Version of the Arabic Text of the Athâr ul-Bâkiya of Albīrūnī or the "Vestiges of the Past"*. Transl. C. E. Sachau, London.

Delambre, J.B.J. 1819. *Histoire de l'Astronomie du Moyen Age*. Paris.

Gingerich, O. and Westman, R.S. 1981. "A Reattribution of the Tychonic Annotations in Copies of Copernicus's *De Revolutionibus*", *Journal for the History of Astronomy*, 12:53-54.

Goldstein, B.R. 1971. *Al-Biṭrūjī: On the Principles of Astronomy*. New Haven-London. 2 vols.

Haskins, C.H. 1924. *Studies in the History of Mediaeval Science*. Cambridge, Mass.

Hirschfeld, H. 1905. *Judah Hallevi's 'Kitab al Khazari'*, London.

Horský, Z. 1971. *Nicolai Copernici De revolutionibus orbium coelestium libri sex (editio Basileensis) cum commentariis manu scriptis Tychonis Brahe*. Praga.

Kennedy, E.S. 1964. "Al-Khwārizmī on the Jewish Calendar", *Scripta Mathematica*, 27:55-59.

Kugler, F.X. 1900. *Babylonischen Mondrechnung*. Freiburg.

Kunitzsch, P. 1974. *Der Almagest. Die Syntaxis Mathematica des Claudius Ptolemäus in arabisch-lateinischer Überlieferung*. Wiesbaden.

Mancha, J.L. 1998. "The Provençal Version of Levi ben Gerson's Tables for Eclipses", *Archives Internationales d'Histoire des Sciences*, 48:269-352.

Millás Vallicrosa, J.M. 1947. *Abraham Ibn Ezra. El libro de los fundamentos de las tablas astronómicas*. Madrid.

Millás Vallicrosa, J.M. 1959. *La obra Séfer Ḥešbón mahlekot ha-kokabim (Libro del cálculo de los movimientos de los astros) de R. Abraham Bar Ḥiyya ha-Bargeloni*. Madrid.

Neugebauer, O. 1949. "The Astronomy of Maimonides and Its Sources", *Hebrew Union College Annual*, 22:321-363.

Neugebauer, O. 1955. *Astronomical Cuneiform Texts*. 3 vols. London.

Neugebauer, O. 1956. *The Code of Maimonides. Book Three, Treatise Eight. Sanctification of the New Moon*. Trans. by S. Gandz, with supp. and intr. by J. Obermann, and astronomical commentary by O. Neugebauer. New Haven.

Neugebauer, O. 1975. *A History of Ancient Mathematical Astronomy*. Berlin-New York.

Neugebauer, O. 1979. *Ethiopic Astronomy and Computus*. Österreische Akademie der Wissenschaften, Philosophisch-Historische Klasse, Sitzungberichte, 347. Wien.

Neugebauer, O. 1987. "The Chronological System of Abū Shāker (A.H. 654)", in King, D.A. and Saliba, G. (eds.), *From Deferent to Equant: A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E.S. Kennedy*. Annals of the New York Academy of Sciences, 500. New York.

Pedersen, O. 1974. *A Survey of the Almagest*. Odense.

Saliba, G. 1987. "The Role of the *Almagest* Commentaries in Medieval Arabic Astronomy: A Preliminary Survey of Ṭūsī's Redaction of Ptolemy's *Almagest*", *Archives Internationales d'Histoire des Sciences*, 37:3-20 (reprinted in *A History of Arabic Astronomy. Planetary Theories during the Golden Age of Islam*, New York-London, 1994, pp. 143-60).

Swerdlow, N.M. and Neugebauer, O. 1984. *Mathematical Astronomy in Copernicus's De Revolutionibus*. Berlin-New York.

Toomer, G.J. 1984. *Ptolemy's Almagest*. London.

Zonta, M. 1993. "La tradizione ebraica dell'*Almagesto* di Tolomeo", *Henoch*, 15:325-350.