On a Păun's Conjecture in Membrane Systems

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Abstract. We study a Păun's conjecture concerning the unsolvability of **NP**–complete problems by polarizationless P systems with active membranes in the usual framework, without cooperation, without priorities, without changing labels, using evolution, communication, dissolution and division rules, and working in maximal parallel manner. We also analyse a version of this conjecture where we consider polarizationless P systems working in the minimally parallel manner.

1 Introduction

Every deterministic Turing machine working in polynomial time can be simulated in polynomial time by a family of recognizing P systems using only basic rules, that is, evolution, communication, and rules involving dissolution [14]. If a decision problem is solvable in polynomial time by a family of recognizing P systems (using only basic rules), then there exists a deterministic Turing machine solving it in polynomial time [20]. As a consequence of these results, the class of all decision problems solvable in polynomial time by this kind of P systems is equal to the standard complexity class **P** [5]. For that reason, recognizing P systems constructing in polynomial time an exponential workspace, expressed in the number of objects, cannot solve **NP**–complete problems in polynomial time (unless $\mathbf{P} = \mathbf{NP}$).

Hence, in order to efficiently solve **NP**–complete problem by P systems it seems necessary to be able to construct an exponential workspace (expressed by the number of membranes) in polynomial time. These models abstract the way of obtaining new membranes through the processes of *mitosis* (membrane division) and *autopoiesis* (membrane creation).

P systems with active membranes (using division rules) have been successfully used to efficiently solve **NP**–complete problems. The first solutions were given constructing a P system associated with each instance of the problem due to the systems lack of an input membrane. Actually, we say that this kind of solutions are *semi–uniform* if the following is true: (a) there exists a deterministic Turing

machine working in polynomial time which constructs the P system processing an instance of the problem (we say the family of P systems associated with all the instances is *polynomially uniform by Turing machines*); and (b) the instance of the problem has an affirmative answer if and only if every computation of the P system associated with it is an accepting computation (we say the P system is *confluent*).

The first semi–uniform polynomial–time solutions of computationally hard decision problems were given by Gh. Păun [11,12], C. Zandron et al. [20], S.N. Krishna et al. [7], and A. Obtulowicz [8]. In 2003, P. Sosik [19] gave a semi–uniform polynomial–time solution to **QSAT**, a well known **PSPACE**–complete problem.

There is another way to solve decision problems by P systems when we consider the possibility to have an input membrane in the systems in which we can introduce objects before the system starts to work. In this case, all instances of a decision problem having the same size (according to a prefixed polynomial time criterion) are processed by the same system.

P systems with active membranes have also been successfully used to design uniform polynomial–time solutions to some well-known **NP**–complete problems, such as **SAT** [17], *Subset Sum* [15], *Knapsack* [16], *Partition* [6], and the *Common Algorithmic Problem* [18].

All papers mentioned above deal with P systems with three polarizations using only division of elementary membranes (in [19] also division for non–elementary membranes are permitted), and working in the *maximal parallelism* in using the rules, that is, in each step, the assignment of objects to the rules to be applied is maximal, no further rule can be applied in any region. The number of polarizations can be decreased to two [1] without loss of efficiency.

It seems clear that the usual framework of P systems with active membranes to solve decision problems is too powerful from the complexity point of view. Then, it would be interesting to analyse which features allows to P systems with active membranes, but without polarizations, to still get polynomial–time solutions to computationally hard problems, and what features, once removed, only allows to obtain polynomial–time solutions to tractable problems, in the classical sense.

The present paper is a contribution to the problem of describing borderlines between tractability and intractability in terms of descriptional resources required in (recognizing) membrane systems using division rules.

The paper is organized as follows. In the next section we present the Păun's conjecture concerning polarizationless P systems with active membranes with three electrical charges and working in the maximally parallel mode. Also we provide some partial solutions to this conjecture by using the notion of dependency graph associated with a P system. Section 3 is devoted to formulate a new version of the Păun's conjecture, addressing P systems working in the minimally parallel mode. We give some partial solutions to this new version. Some conclusions and open problems are given in the last Section.

2 A Păun's Conjecture

Usual P systems with active membranes use three *electrical charges* for membranes, controlling the application of the rules which, basically, can be of the following types: *evolution rules*, by which single objects evolve to a multiset of objects, *communication rules*, by which an object is introduced in or expelled from a membrane, maybe modified during this operation into another object, *dissolution rules*, by which a membrane is dissolved, under the influence of an object, which may be modified into another object by this operation, and *membrane division rules* (both for elementary and non-elementary membranes, or only for elementary membranes).

Definition 1. *A P system with polarizationless active membranes of the initial degree $n \geq 1$ is a tuple of the form $\Pi = (\Gamma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_n, R, h_o)$, where:*

1. *Γ is the alphabet of objects;*
2. *H is a finite set of labels for membranes;*
3. *μ is a membrane structure, consisting of n membranes having initially neutral polarizations, injectively labeled with elements of H;*
4. *$\mathcal{M}_1, \ldots, \mathcal{M}_n$ are strings over Γ, describing the multisets of objects placed in the n initial regions of μ;*
5. *R is a finite set of developmental rules, of the following forms:*
   (a) *$[\, a \to v\,]_h$, for $h \in H, a \in \Gamma, v \in \Gamma^*$ (object evolution rules).*
   (b) *$a[\ ]_h \to [\, b\,]_h$, for $h \in H, a, b \in \Gamma$ (in communication rules).*
   (c) *$[\, a\ ]_h \to b[\ ]_h$, for $h \in H, a, b \in \Gamma$ (out communication rules).*
   (d) *$[\, a\ ]_h \to b$, for $h \in H, a, b \in \Gamma$ (dissolution rules)*
   (e) *$[\, a\ ]_h \to [\, b\ ]_h [\, c\ ]_h$, for $h \in H, a, b, c \in \Gamma$ (weak division rules for elementary or non-elementary membranes).*
   (a) *$h_o \in H$ or $h_o = env$ indicates the output region (in this case, usually h_o do not appear in the description of the system).*

Also, we can consider rules of the form $[\,[\,]_{h_1}[\,]_{h_2}\,]_{h_3} \to [\,[\,]_{h_1}\,]_{h_3}\,[\,[\,]_{h_2}\,]_{h_3}$, where h_1, h_2, h_3 are labels: if the membrane with label h_3 contains other membranes than those with labels h_1, h_2, these membranes and their contents are duplicated and placed in both new copies of the membrane h_3; all membranes and objects placed inside membranes h_1, h_2, as well as the objects from membrane h_3 placed outside membranes h_1 and h_2, are reproduced in the new copies of membrane h_3. These rules are called *strong division rules for non–elementary membranes*.

Using the *maximally parallel manner*, at each computation step (a global clock is assumed) in each region of the system we apply the rules in such a way that no further rule can be applied to the remaining objects or membranes. In each step, each object and each membrane can be involved in only one rule.

A halting computation provides a result given by the number of objects present in region h_o at the end of the computation; this is a region of the system if $h_o \in H$ (and in this case, for a computation to be successful, exactly one membrane with label h_o should be present in the halting configuration), or it is the environment if $h_o = env$.

We denote by $\mathcal{AM}^0$ the class of recognizing polarizationless P systems with active membranes, and we denote by $\mathcal{AM}^0(\alpha,\beta)$, where $\alpha \in \{-d,+d\}$ and $\beta \in \{-ne,+new,+nes\}$, the class of all recognizing P systems with polarizationless active membranes such that: (a) if $\alpha = +d$ (resp. $\alpha = -d$) then dissolution rules are permitted (resp. forbidden); and (b) if $\beta = +new$ or $+nes$ (resp. $\beta = -ne$) then division rules for elementary and non–elementary membranes, weak or strong (respectively only division rules for elementary) are permitted.

The class of all decision problems solvable in uniform (resp. semi–uniform) way, and in polynomial time by a family of recognizing membrane systems is denoted by $\mathbf{PMC}_{\mathcal{R}}$ (resp. $\mathbf{PMC}^*_{\mathcal{R}}$)

Proposition 1. *For each $\alpha \in \{-d,+d\}$, $\beta \in \{-ne,+new,+nes\}$, and $\epsilon = *, \lambda$, we have:*

1. $\mathbf{PMC}_{\mathcal{AM}^0(\alpha,\beta)} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0(\alpha,\beta)}$
2. $\mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,-ne)} \subseteq \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,+new)}$
3. $\mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,-ne)} \subseteq \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,+nes)}$
4. $\mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(-d,\beta)} \subseteq \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(+d,\beta)}$

where $\epsilon = *$ (respectively $\epsilon =$ empty string) means that the complexity classes are associated with semi–uniform (respectively, uniform) solutions.

At the beginning of 2005, Gh. Păun (problem **F** from [13]) wrote: *My favorite question (related to complexity aspects in P systems with active membranes and with electrical charges) is that about the number of polarizations. Can the polarizations be completely avoided? The feeling is that this is not possible – and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non–efficiency to efficiency.*
That is, formally we can formulate the called conjecture of Păun as follows:

> *The class of all decision problems solvable in polynomial time by polarizationless P systems with active membranes using evolution, communication, dissolution and division rules for elementary membranes (working in the maximally parallel mode) is equal to the class* **P**

This conjecture can be expressed in terms of complexity classes in P systems as follows: $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0(+d,-ne)} = \mathbf{PMC}^*_{\mathcal{AM}^0(+d,-ne)}$
Next, we study possible answers to the conjecture of Păun.

2.1 A Partial Affirmative Answer

Let us recall that using the concept of *dependency graph* associated with a P system, a partial affirmative answer to the Păun's conjecture can be given.

Let Π be a P system whose working alphabet is Γ and the set of labels is H, and we denote by *env* the label of the environment. The *dependency graph associated with* the system Π is the directed graph G_Π whose nodes are the pairs $(a,h) \in \Gamma \times (H \cup \{env\})$ such that the object a in membrane (maybe the environment) labelled by h either triggers a rule or it is produced by a rule, and $((a,h),(a',h'))$ is an arc in the graph if there exists a rule r of Π such that the

object a in membrane labelled by h produces the object a' in membrane (maybe the environment) labelled by h' by the application of rule r.

It can be proved that there exists a deterministic Turing machine that constructs the dependency graph, G_{Π}, associated with Π, in polynomial time, that is, in a time bounded by a polynomial function depending on the total number of rules and the maximum length of the rules (see [4]).

Let Δ_{Π} be the set whose elements are the pairs $(a, h) \in \Gamma \times (H \cup \{env\})$ such that there exists a path (within the dependency graph) from (a, h) to $(\mathtt{yes}, env)$. Having in mind that the *reachability problem* (see chapter 1 from [10]) can be solved by a search algorithm running in polynomial (quadratic) time, there exists a deterministic Turing machine that constructs the set Δ_{Π} in polynomial time, that is, in a time bounded by a polynomial function depending on the total number of rules and the maximum length of the rules (see [4]).

Let $\mathbf{\Pi} = \{\Pi(n) : \ n \in \mathbf{N}\}$ be a family of recognizing P systems with input membranes (not using dissolution rules) solving a decision problem X, in a uniform way. Let (cod, s) be a polynomial encoding associated with that solution. An instance u of the problem will be accepted by the system $\Pi(s(u))$ with input $cod(u)$ if and only if there is an object a in a membrane h of the initial configuration of the system such that there exists a path in the associated dependency graph from (a, h) to $(\mathtt{yes}, env)$. As a consequence of the previous results we have the following:

Theorem 1. *For each* $\beta \in \{-ne, +new, +nes\}$*, we have* $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0(-d,\beta)}$.

Similar characterizations of **P** can be obtained when we deal with semi–uniform solutions in the framework of recognizing polarizationless P systems with active membranes, and where dissolution rules are forbidden. The proofs are similar, it is enough to consider the system $\Pi(u)$, for each instance u of the decision problem, instead of the system $\Pi(s(u))$ with input the multiset $cod(u)$.

Theorem 2. *For each* $\beta \in \{-ne, +new, +nes\}$*, we have* $\mathbf{P} = \mathbf{PMC}^*_{\mathcal{AM}^0(-d,\beta)}$.

2.2 A Partial Negative Answer

It has been shown ([4]) that the class of decision problems solvable in polynomial time in a semi–uniform way by families of recognizing polarizationless P systems with active membranes where dissolution rules are permitted, and using division rules for elementary and non–elementary membranes, contains the standard complexity class **NP**.

Theorem 3. *We have the following:*

1. $\mathbf{SAT} \in \mathbf{PMC}^*_{\mathcal{AM}^0(+d,+nes)}$
2. $\mathbf{NP} \cup \mathbf{co–NP} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0(+d,+nes)}$

Moreover, it has been obtained an efficient *uniform* solution for the **QSAT**–problem ([2]) in this framework.

Theorem 4. *We have the following:*

1. $\mathbf{QSAT} \in \mathbf{PMC}_{\mathcal{AM}^0(+d,+nes)}$.
2. $\mathbf{PSPACE} \subseteq \mathbf{PMC}_{\mathcal{AM}^0(+d,+nes)}$.

Hence, in the framework of polarizationless P systems with active membranes and working in the maximally parallel mode, dissolution rules play a crucial role from the computational efficiency point of view. Specifically, if dissolution rules are forbidden then it is not possible to solve **NP**–complete problems in polynomial time (unless $\mathbf{P} = \mathbf{NP}$). Nevertheless, if dissolution rules are permitted then it is possible to efficiently solve computationally hard problems.

That is, the Păun's conjecture has a (partial) negative answer (assuming that $\mathbf{P} \neq \mathbf{NP}$). Nevertheless, the answer will be (partially) affirmative if dissolution rules are forbidden.

3 A New Version of a Păun's Conjecture

Recently, in [3] a more relaxed strategy of using the rules was introduced, the so-called *minimal parallelism*: in each region where at least a rule can be applied, at least one rule must be applied (if there is no conflict with the objects), without any other restriction. This introduces an additional degree of non-determinism in the system evolution.

P systems with active membranes working in the *minimally parallel mode* means the following:

- All the rules of any type involving a membrane h form the set R_h, this means all the rules of the form $[\, a \to v\,]_h$, all the rules of the form $a[\ ]_h \to [\, b\,]_h$, and all the rules of the form $[\, a\,]_h \to z$ and $[\, a\,]_h \to [\, b\,]_h [\, c\,]_h$, with the same h, constitute the set R_h.
- If a membrane h appears several times in a given configuration of the system, then for each occurrence of the membrane we consider a different set R_h.
- Then, in each step, from each set R_h associated with each membrane labelled by $h \in H$, from which at least a rule *can* be used, at least one rule *must* be used (if there is no conflict with the objects; for example, if we have only an object a in membrane h and we have an evolution rule $[\, a \to b\,]_h$ and a send–in rule $a[\ ]_{h'} \to [\, c\,]_{h'}$, being h' the label of a membrane immediately inside membrane h, then we can apply at least a rule from R_h and from $R_{h'}$, but we will apply only one between these two rules).

Of course, as usual for P systems with active membranes, each membrane and each object can be involved in only one rule, and the choice of rules to use and of objects and membranes to evolve is done in a non-deterministic way. In each step, the use of rules is done in the bottom-up manner (first the inner objects and membranes evolve, and the result is duplicated if any surrounding membrane is divided).

In this kind of P systems still universality and semi–uniform polynomial–time solutions to **SAT** were obtained in the new framework by using P systems with active membranes, with three polarizations [3].

Theorem 5. *The* **SAT** *problem can be solved in a semi–uniform way and in a linear time by polarization P systems with active membranes, without dissolution rules and using (weak) division for non–elementary membranes, and working in the minimally parallel mode.*

Next, we define new classes of P systems related to $\mathcal{AM}^0$. Let $\alpha \in \{-d, +d\}$, $\beta \in \{-ne, +new, +nes\}$ and $\gamma \in \{m, M, md, Md\}$. Then we denote by $\mathcal{AM}^0(\alpha, \beta, \gamma)$ the class of recognizing P systems with polarizationless active membranes such that:

- $\alpha = +d$: dissolution rules are permitted.
- $\alpha = -d$: dissolution rules are forbidden.
- $\beta = +new$: division rules for elementary and (weak) non–elementary membranes are permitted.
- $\beta = +nes$: division rules for elementary and (strong) non–elementary membranes are permitted.
- $\beta = -ne$: only division rules for elementary membranes are permitted.
- $\gamma = m$: working in the minimally parallel mode.
- $\gamma = md$: working in the deterministic minimally parallel mode.
- $\gamma = M$: working in the maximally parallel mode.
- $\gamma = Md$: working in the deterministic maximally parallel mode.

Proposition 2. *We have the following:*

1. $\mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,\beta,md)} \subseteq \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,\beta,m)}$
2. $\mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,\beta,m)} \subseteq \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,\beta,M)}$
3. $\mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,\beta,Md)} \subseteq \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(\alpha,\beta,M)}$

where $\epsilon = *$ or $\epsilon =$ empty string.

We can formulate the Păun's conjecture in P systems working in the minimally parallel mode.

The class of all decision problems solvable in polynomial time by polarizationless P systems with active membranes using evolution, communication, dissolution and division rules for elementary membranes (working in the minimally parallel mode) is equal to the class **P**

This conjecture can be expresses in terms of complexity classes in P systems as follows: $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0(+d,-ne,m)} = \mathbf{PMC}^{*}_{\mathcal{AM}^0(+d,-ne,m)}$.
Next, we study possible answers to the new version of Păun's conjecture.

3.1 A Partial Affirmative Answer

Let us recall that through the concept of *dependency graph* associated with a P system, we have given a partial affirmative answer to the Păun's conjecture related with P systems working in the maximally parallel mode, and without using dissolution rules.

Let us also recall that, in order to give that answer, the main property that the dependency graph associated with a P system must satisfy is the following:

every computation of the system is an accepting computation if and only if there exists an object a in an initial membrane (labelled by h) of the system such that there exists a path (within the dependency graph) from (a, h) to $(\texttt{yes}, env)$.

When a polarizationless P system with active membranes works in the minimally parallel mode, in each transition step we can think that objects are assigned to rules, non–deterministically choosing the rules and the objects assigned to each rule, according to the semantic of the minimally parallel mode. The objects which remain unassigned are left where they are, and they are passed unchanged to the next configuration (and belonging to the same membrane because dissolution rules are not permitted). So, the above property is satisfied by the computations of this kind of P systems working in the minimally parallel mode (without using dissolution rules) because we can pass from a node (a, h) to another node (a', h') in the dependency graph if and only if there exists a transition step producing (a', h') from (a, h).

Hence, we have a negative answer to the to the Păun's conjecture related with P systems working in the minimally parallel mode (deterministically or not), and without using dissolution rules.

Theorem 6. $\mathbf{P} = \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(-d,\beta,md)} = \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(-d,\beta,m)}$, *where* $\epsilon = *$ *or* $\epsilon =$ *empty string.*

3.2 A Partial Negative Answer

Next, we give a semi–uniform linear–time solution to **SAT** by using polarizationless P systems with active membranes working in the minimally parallel mode, and now using dissolution rules.

Theorem 7. SAT *can be solved in a semi–uniform way and in a linear time by polarizationless P systems with active membranes, using evolution, communication, dissolution and (weak) division for non–elementary membrane rules, and working in the deterministic minimally parallel mode.*

Proof. Let $\varphi = C_1 \wedge \cdots \wedge C_m$ be a propositional formula in conjunctive normal form, such that each clause C_j, $1 \leq j \leq m$, is of the form $C_j = y_{j,1} \vee \cdots \vee y_{j,k_j}$, $k_j \geq 1$, for $y_{j,r} \in \{x_i, \neg x_i \mid 1 \leq i \leq n\}$, and being $\{x_1, \ldots, x_n\}$ the set of variables of φ. For each $i = 1, 2, \ldots, n$, let us denote

$$t(x_i) = \{C_j \mid \text{there is } r, 1 \leq r \leq k_j, \text{ such that } y_{j,r} = x_i\},$$
$$f(x_i) = \{C_j \mid \text{there is } r, 1 \leq r \leq k_j, \text{ such that } y_{j,r} = \neg x_i\}.$$

That is, $t(x_i)$ (respectively, $f(x_i)$) is the set of clauses which assume the value *true* when x_i is *true* (resp. when x_i is false). Obviously, these sets have at most m elements.

We construct a recognizing P system

$$\Pi(\varphi) = (\Gamma, H, \mu, \mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M}_m, \mathcal{M}_p, \mathcal{M}_q, \mathcal{M}_r, \mathcal{M}_s, \mathcal{M}_{s'}, \mathcal{R})$$

associated with the formula φ as follows:

$$\begin{aligned}
\Gamma &= \{a_i, f_i, t_i \mid 1 \le i \le n\} \cup \{c_j, d_j \mid 1 \le j \le m\} \cup \{p_i \mid 1 \le i \le 2n+7\} \\
&\quad \cup \{q_i \mid 1 \le i \le 2n+1\} \cup \{r_i \mid 1 \le i \le 2n+5\} \cup \{b_1, b_2, y, \texttt{yes}, \texttt{no}\}, \\
H &= \{0, 1, 2, \ldots, m, p, q, r, s, s'\}, \\
\mu &= [_s[_{s'}[_p\,]_p[_0[_q\,]_q[_r]_r[_1\,]_1[_2\,]_2 \cdots [_m\,]_m]_0]_{s'}]_s, \\
\mathcal{M}_p &= p_1,\ \mathcal{M}_q = q_1,\ \mathcal{M}_r = r_1,\ \mathcal{M}_0 = a_1, \mathcal{M}_s = \mathcal{M}_{s'} = \mathcal{M}_j = \lambda,\ (1 \le j \le m)
\end{aligned}$$

The set of evolution rules, $\mathcal{R}$, consists of the following rules:

(a) $[p_i \to p_{i+1}]_p$, for all $1 \le i \le 2n+6$
$[q_i \to q_{i+1}]_q$, for all $1 \le i \le 2n$
$[r_i \to r_{i+1}]_r$, for all $1 \le i \le 2n+4$

(b) $[a_i]_0 \to [f_i]_0[t_i]_0$, for all $1 \le i \le n$
$[f_i \to f(x_i)a_{i+1}]_0$ and $[t_i \to t(x_i)a_{i+1}]_0$, for all $1 \le i \le n-1$,
$[f_n \to f(x_n)]_0$; $[t_n \to t(x_n)]_0$

(c) $c_j[\,]_j \to [c_j]_j$ and $[c_j]_j \to d_j$, for all $1 \le j \le m$.

(d) $[q_{2n+1}\,]_q \to q_{2n+1}[\,]_q$; $[q_{2n+1} \to b_1^m]_0$

(e) $b_1[\,]_j \to [b_1]_j$; and $[b_1]_j \to b_2$, for all $1 \le j \le m$,
$[b_2\,]_0 \to b_2$; $[p_{2n+7}\,]_p \to p_{2n+7}[\,]_p$; $[p_{2n+7}\,]_{s'} \to \texttt{no}[\,]_{s'}$; $[\texttt{no}\,]_s \to \texttt{no}[\,]_s$
$[r_{2n+5}\,]_r \to r_{2n+5}$; $[r_{2n+5}\,]_0 \to y[\,]_0$; $[y\,]_{s'} \to \texttt{yes}$; $[\texttt{yes}\,]_s \to \texttt{yes}[\,]_s$

An overview of the computation of $\Pi(\varphi)$

The rules of type (a) are used for evolving general counters p_i, q_i and r_i in membranes labelled by p, q and r, respectively, making possible the correct synchronization.

In parallel with these rules, the non–elementary membrane 0 evolves by means of the rules of the type (b). In step $2i-1$ ($1 \le i \le 2n$), object a_i produces the division of the membrane 0 (with f_i, t_i corresponding to the truth values *false, true*, respectively, for variable x_i). In step $2i$ we introduce inside membrane 0 the clauses satisfied by x_i or $\neg x_i$, respectively. Let us recall that when we divide membrane 0, all inner objects and membranes are replicated. At the end of this phase, all 2^n truth assignments for the n variables are generated and they are encoded in membranes labeled by 0.

In parallel with the process of membrane division, in the odd steps (until step $2n+1$), if a clause C_j is satisfied by the previously expanded variable, then the corresponding object c_j enters membrane j, by means of the first rule of the type (c), producing their dissolution in the next step by means of the second rule of that type and sending objects d_j to membrane 0.

In step $2n+2$, in each membrane 0, the counters q_i and r_i follow evolving and the second rule of the type (d) produces m copies of the object b_1.

Thus, the configuration $\mathcal{C}_{2n+2}$ of the system obtained after $2n+2$ steps, consists of 2^n copies of membrane 0, each of them encoding a truth assignment of the variables associated with φ, and containing the membrane q empty, the

membrane r with the object r_{2n+3}, possible objects c_j and d_j, $1 \leq j \leq m$, as well as copies of only membranes with labels $1, 2, \ldots, m$ corresponding to clauses which were not satisfied by the truth assignment generated in that copy of membrane 0. Also, in that configuration the membrane p contains the object p_{2n+3} and membranes s' and s are empties.

Hence, formula φ is satisfied if and only if there is a membrane 0 where all membranes $1, 2, \ldots, m$ have been dissolved. In order to check this last condition, we proceed as follows.

In step $2n+3$ we use the first rule of the type (e) which introduces objects b_1 in each membrane j which has not been dissolved. In parallel, the counters p and r follow evolving. In step $2n+4$ objects b_1 in membrane j (in each such membrane appearing in the configuration $\mathcal{C}_{2n+2}$) dissolve these membranes producing object b_2 in membrane 0. Therefore, the presence of objects b_2 in membrane 0 of the configuration $\mathcal{C}_{2n+4}$ means that the truth assignment encoded by that membrane makes the formula false.

In step $2n+5$ the counter r_{2n+5} exits from membrane r and, simultaneously, each membrane 0 containing an object b_2 is dissolved by the third rule of the type (e). Hence, formula φ is satisfied if and only if in the configuration $\mathcal{C}_{2n+5}$ there exists a membrane 0 that has not been dissolved.

In step $2n+6$, the counter p_i evolves to p_{2n+7} in membrane p, and if there is a membrane 0 that has not been dissolved, the object r_{2n+5} sends to membrane s' an object y. On the contrary, only the counter p_i evolves.

In step $2n+7$ the counter p_{2n+7} exits from membrane p to membrane s', by applying the first rule of the type (f). Moreover, in that step, if the formula φ is satisfiable then an object y dissolves the membrane s' by applying the sixth rule of the type (f) producing an object `yes` in the skin, that in step $2n+8$ is sent to the environment; and the system halts. On the contrary, if membrane s' has not been dissolved, in step $2n+8$ by applying the second rule of the type (f) the object p_{2n+7} exits from membrane s' producing an object `no` in the skin, and in step $2n+9$ sends to the environment an object `no`; then, the system halts.

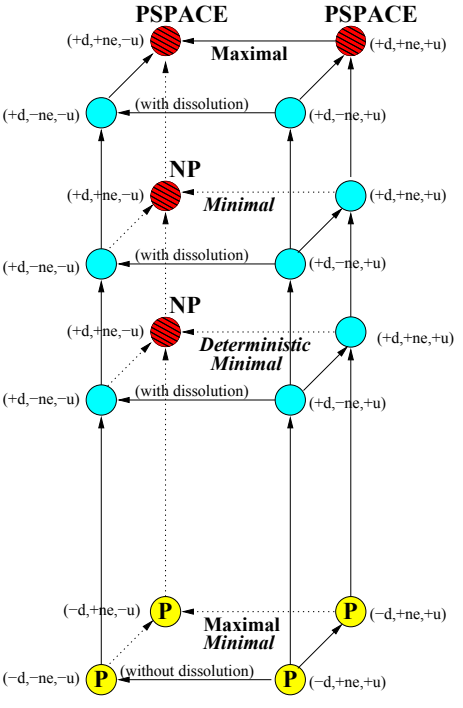
The system $\Pi(\varphi)$ uses $9n+2m+18$ objects, $m+6$ initial membranes, containing in total 4 objects, and $8n+4m+21$ rules. The length of any rule is bounded by $m+3$. Clearly, all computations stop (after at most $2n+9$ steps) and all give the same answer, `yes` or `no`, to the question whether formula φ is satisfiable.

Corollary 1. $\mathbf{NP} \cup \mathbf{co\text{–}NP} \subseteq \mathbf{PMC}^{*}_{\mathcal{AM}^{0}(+d,+new,md)}$.

As a consequence of this result, we have polarizationless P systems with active membranes, using dissolution rules and (weak) division for non–elementary membranes, and working in the deterministic minimally parallel mode are able to give semi–uniform solutions of **NP**–complete problems. That is, we have a (partial) negative answer to the new version of the Păun's conjecture.

The following picture describes the results obtained until now related to Păun's conjecture in both modes, where $-u$ (resp. $+u$) means semi–uniform (resp. uniform) solutions, $-ne$ (resp. $+ne$) means using division only for elementary membranes (resp. division for elementary and non–elementary membrane,

strong in the maximal parallelism and *weak* in the minimal parallelism). Through this graph, we try to specify whether or not it is possible to solve computationally hard problems by recognizing P systems of the class associated with each node. The direction of each arrow shows a relation of inclusion, and each blue node provides an open question.



4 Conclusions and Open Problems

A conjecture of Păun, related to the impossibility to solve NP–complete problems in polynomial–time by means of polarizationless P systems with active membranes, is studied in this paper. Partial solutions are given within the usual framework of P systems working in the maximally parallel mode. As a consequence of the results obtained, the crucial role played by dissolution rules when we try to solve computationally hard problems, is highlighted.

Besides, a new version of that conjecture is formulated, this time associated to polarizationless P systems with active membranes working in the minimally parallel mode. Other partial solutions also arise from this new version and once again, dissolution rules are shown to be a singular ingredient which gives a borderline between efficiency (the possibility of solving computationally hard problems using feasible membrane computing resources) and non–efficiency.

Finally, we propose some open problems.

1. $\mathbf{NP} \cup \mathbf{co–NP} \subseteq \mathbf{PMC}^{\epsilon}_{\mathcal{AM}^0(+d,-ne,m)}$?
2. $\mathbf{NP} \cup \mathbf{co–NP} \subseteq \mathbf{PMC}_{\mathcal{AM}^0(+d,+new,md)}$?
3. $\mathbf{PMC}_{\mathcal{AM}^0(\alpha,\beta,m)} \subseteq \mathbf{PMC}_{\mathcal{AM}^0(\alpha,\beta,Md)}$?
4. Consider other ingredients in the framework $\mathcal{AM}^0(-d,\beta,\gamma)$ that permit to solve **NP**–complete problems.
5. Study the computational efficiency of the class $\mathcal{AM}^0$ **with** evolution rules with length 2 (or **with** communication rules without evolution of objects).
6. Study the computational efficiency of the class $\mathcal{AM}^0$ **without** evolution rules (or **without** communication rules).

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