A Consistent Decomposition of the Redistributive Effect. An application to Taxes and Welfare Expenditures.

Abstract

The aim of this work is to solve the problem of non-additivity revealed by the works that calculate the redistributive effect of the budget or public policies made up of different instruments of income or public spending. We use Shapley's value to do this. This technique allows us to decompose the redistributive effect and the vertical and horizontal effects consistently and not arbitrarily. The main result obtained for the case of taxes and social transfers in the US is that previous calculations undervalued the redistributive effects and their vertical and horizontal components for taxes and transfers. Undervaluation is more important for taxes.

1. Introduction

Within public economy studies there is an important tradition in the analysis of the redistributive effects of taxes. One of the most used indicators to calculate redistributive effects is the difference in Gini indexes before and after taxes (Reynolds y Smolensky, 1977). Its versatility and simplicity has meant the extending of the measure to all kinds of public intervention instruments[1] and even to the analysis of redistributive effect of the whole budget[2]. An additional refinement is in decomposing the Reynolds-Smolensky index to assess the effects on vertical equity and reranking, according to the method of Kakwani (1984), or also on horizontal inequality, according to the proposal of Aronson, Johnson and Lambert (1994).

The problem of works that calculate the redistributive impacts of policies made up of various instruments or the whole budget is that they tend to present inconsistent results. Additive inconsistence is produced when the sum of the redistributive effects of each instrument does not coincide with the redistributive effect of the measures taken as a whole. This non-additivity is due to calculating the redistributive effects of each instrument only taking as a reference the original income and excluding the other instruments. That is, the interaction between instruments having an effect on the redistributive effect is not taken into account.

The most elemental solution to take this interaction into account is to sequentially aggregate the different instruments to calculate their redistributive impacts. This was done by: Ferrarini and Nelson (2003) in a compared study of the financing and spending of the social insurance in ten countries, Keselman and Cheung (2004) to analyze the budget in Canada, Mahler and Jesuit (2006) for the budget in developed countries via the *Luxembourg Income Study (LIS) data set*, or Wolff and Zacharias (2007) for the budget in the US. The problem that this solution produces is that the redistributive effects calculated for the same policy are different according to the sequence chosen. That is, a problem of arbitrariness and judgment values arises.

The aim that we set out for this work is to solve the problem of inconsistency of the decomposition of the redistributive effects without incurring in arbitrariness. To achieve this we propose additively decomposing the Reynolds-Smolensky index, and its disaggregation into vertical and horizontal effects in the manner of Kakwani (1984) or Aronson, Johnson and Lambert (1994), applying the Shapley´s value (1953). In broad terms, this technique considers the marginal effect on budget of eliminating each of the contributory factors in sequence, and then assigns to each factor the average of its marginal contributions in all possible elimination sequences. Shorrocks (1999), instigator of its use for the decomposing of inequality indicators,[3] shows that the sum of these effects is consistent and symmetrical.

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[1] For spending policies in the US compared with other countries we can see Mahler and Jesuit (2006), Marical, et. al (2008), Tanzi (2008), Warren (2008) or Prasad (2008).

[2] Among the most recent we find those of Atkitson (2004), Smeeding (2005), Förster and Mira d'Ercole (2005) or Garfinkel, Rainwater and Smeeding (2006) in which the redistributive effects of the US Administration budget are compared with the results obtained in other countries. For other countries see Forteza and Rossi (2009) for Uruguay, Bargain and Callan (2010) for the EU, Clark and Leicester (2004) for the UK or Fuchs and Lietz (2007) for Austria.

[3] Previous Works belong to Rongve (1995) and Chantreuil and Trannoy (1997). This methodology has been employed more profusely in the decomposing of poverty indicators, see as examples Kolenikov and Shorrocks (2005), Sami and Duclos (2008) or Deustsch and Silber (2008).

To present analytical results we are going to take as a reference the recent work of Kim and Lambert (2009) in which the redistributive impact is analyzed for taxes and social spending in the USA for the years 1994, 1999 and 2004.[4] Our calculations will allow the consistent and symmetrical decomposing of the total redistributive effect, the vertical effect, the horizontal inequality and the reranking that taxes and social benefits produce in the US and compare the results with those of the article cited. From this comparison we will be able to draw conclusions about the consequences of assessing redistributive effects that do not produce consistent results.

This paper is organized as follows. The next section discuss the problems of traditional methodologies of redistributive decomposition and the relative merits of the Shapley decomposition. In section 3 we apply our proposal. The last section summarizes the results obtained and offers some concluding remarks.

2. Methodology

One of the most usual ways of measuring the redistributive effect is the comparison of the Gini index of the initial income (market income) with the Gini index of the final income (post-tax income) (Reynolds and Smolensky 1977):

$$RE_{X-T} = \mathrm{G}_X - G_{X-T} \qquad (1)$$

At the same time, the most used ways of calculating the vertical and horizontal redistributive effects of the taxes are those devised by Kakwani (1984) and Aronson, Johnson and Lambert (1994).[5] The first distinguishes between the vertical effect and reranking:

$$RE_{X-T} = \mathrm{V}_{X-T} - R_{X-T} \qquad (2)$$

And the second also identifies the horizontal inequality without an order change:[6]

$$RE_{X-T} = \mathrm{V}_{X-T} - H_{X-T} - R_{X-T} \qquad (3)$$

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[4] Other recent works that are concerned with the redistributive effect of this type of policies are those of Verbist (2007) for taxes on work, pensions and unemployment benefits in the EU-15, Ferrario and Zanardi (2009) for the National Health of Italy or Biggs et al (2009) for the National Health in the USA. There is also a broad literature of health spending from the work of Wagstaff and van Doorslaer (1997)for the health spending and financing in Holland, which later Van Doorslaer et. al (1998) extended to an important group of countries.

[5] There is another decomposing alternative put forward by Lerman and Yitzaki (1995), in which the order change is assessed first, and then V is resolved as the income change, but with the final order.

[6] In spite of the terminology being undistinguishable, it must be pointed out that the original values of V and R are not coincident in both formulas. The work of Urban and Lambert (2008) recently put forward a reformulating of the values given by Aronson, Johnson and Lambert that identifies the relationship between both equations.

The vertical effect is defined by the difference between pre-tax distribution inequality index and the inequality index of a fictitious income distribution obtained with the aid of taxation not generating either horizontal effects or reranking effects. This fictitious taxation leads to the pre-tax income of a household of rank in the distribution of pre-tax income being equal to its expected net income before taxation. Furthermore, the effect (*V)* measures the redistributive effect generated by the taxation system. This difference is positive if the taxation system is progressive and negative if it is regressive. The *horizontal inequality (H),* on the other hand, relates to unequal treatment of equals. A further concept of *reranking (R)* corresponds to the income scale rank-switching induced by the fiscal system. [7]

In theory, for each of the *m* instruments a_j*,(j=1,...,m)* that make up a policy *M* we can calculate its respective redistributive effects RE_{X+a_j} and the problem of inconsistency or asymmetry is given because $RE_{X+M} \neq \sum_{J=1}^{M} RE_{X+a_J}$. The same takes place for *V*, *H* and *R.*

As was cited above, the most elemental solution to the problem is to establish a sequence. To see the problem that it produces, we take the example of the work of Wolff and Zacharias (2007), in which the effects of taxes and spending for the years 1989 and 2000 in U.S. are analyzed. The authors first calculate the redistributive effects for the taxes (*T*), then for the transfers (*Tr*) and finally for public consumption (*E*) and the results fulfil the additive property [8]. Now, why choose this sequence and not another (*T+E+Tr*, *E+T+Tr*, *E+Tr+T*, *Tr+E+T* o *Tr+T+E*)? If we did this, we would have obtained different redistributive impacts for each instrument according to the sequence used. That is, the order influences the result. To sum up, if we change the order, the redistributive effect values change, producing asymmetrical results.

If this is so, what value do we take? Why must taxes precede public spending? It would not seem reasonable to us to consider that the administration first receives payments and then spends. Then, this sequence does not have the guarantee of certainty. Spending can be financed from the public deficit and in this case the spending is first and the taxes come afterwards. What happens when taxes fall back on transfers received? The reality is that modern economies are complex and it is not possible to find unquestionable sequences in the budget procedure. This is why to choose a sequence implies arbitrariness and the establishing of a value judgment that conditions the results.

We can find the problem´s solution calculating the decomposition via the value of Shapley (1953). This methodology comes from game theory and has been applied in the study of inequality since the work of Shorrocks (1999).[9] The applying of Shapley´s value allows an additive, symmetrical and exact decomposition for any index.

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[7] See decompostion of taxes in Creedy and Van Ven (2001), for Australia, Urban (2008) for Croacia, Lambert and Thoresen, T (2009) for Norway.

[8] This can be checked in their Table 6 (Wolff and Zacharias, 2007, p.710).

[9] See Chantreuil and Trannoy (1999); Sastre and Trannoy (2002), Deustch and Silber (2008).

The system consists of quantifying the impact of a factor calculating the average of the marginal effects, determined by the elimination of the said factor in all the possible sequences.

Indeed, let M be the set of instruments $a_j, j \in M = \{1,2,....m\}$, that make up a public policy. Let G be a subset of M made up of g instruments. Let RE_{X+G} be the value of RE when the instruments a_j, $j \notin G \subseteq M$ have been eliminated. If all the sizes are equally probable, a determined size g will occur with a probability of $1/g$. On the other hand, if we have a determined instrument a_j that belongs to a subgroup G, the $(g-1)$ instruments remaining of this subgroup can be chosen between the $(m-1)$ instruments remaining in a number of sequences equal to:

$$\frac{(m-1)!}{[(m-1)-(g-1)]!(g-1)!} = \frac{(m-1)!}{(m-g)!(g-1)!} \qquad (4)$$

The probability of one of these combinations being chosen is the reciprocal. Therefore, the probability of a group G containing the element a_j is equal to the said reciprocal multiplied by $1/m$.

If we take into account the marginal contribution of $a_j \in G$ to the inequality of the subgroup G it is:

$$RE^{j}_{X+G} = RE_{X+G} - RE_{X+(G-a_j)} \qquad (5)$$

The contribution of the instrument a_j to the total value of the index RE_{X+M} is equal to the summation of all the groupings that can contain it, pondered by the probability of them occurring:

$$\overline{RE}^{j}_{X+M} = \sum_{\substack{G\subseteq M \\ j\in G}} \frac{(m-g)!(g-1)!}{m!}\left[RE_{X+G} - RE_{X+(G-a_j)}\right] \qquad (6)$$

Such that:

$$RE_{X+M} = \sum_{j=1}^{m} \overline{RE}^{j}_{X+M} \qquad (7)$$

This expression is generalizable to determine the contribution of each a_j to the indices V_{X+M}, H_{X+M} and R_{X+M} such that:

$$V_{X+M} = \sum_{j=1}^{m} \overline{V}^{j}_{X+M}$$
$$H_{X+M} = \sum_{j=1}^{m} \overline{H}^{j}_{X+M} \qquad (8)$$
$$R_{X+M} = \sum_{j=1}^{m} \overline{V}^{j}_{X+M}$$

Therefore:

$$RE_{X+M} = \sum_{j=1}^{m} RE_{X+M}^{j} = V_{X+M} - H_{X+M} - R_{X+M} = \sum_{j=1}^{m} \overline{V}_{X+M}^{j} - \sum_{j=1}^{m} \overline{H}_{X+M}^{j} - \sum_{j=1}^{m} \overline{R}_{X+M}^{j} \qquad (9)$$

With this methodology the sequence ceases to be determinant. All the possible combinations are considered and the interaction of the instruments that make up the policy is taken into account.

To appreciate the consequences of its application, we are going to use the case of a policy with two instruments, for example taxes (T) and transfers (B). Such that for equation 7 must be met:

$$RE_{X-T+B} = \overline{RE}_{X-T+B}^{T} + \overline{RE}_{X-T+B}^{B} \qquad (10)$$

Being:

$$\overline{RE}_{X-T+B}^{T} = \frac{1}{2}.(RE_{X-T+B} - RE_{X+B)}) + \frac{1}{2} RE_{X-T}$$
$$\overline{RE}_{X-T+B}^{B} = \frac{1}{2}(RE_{X-T+B} - RE_{X-T}) + \frac{1}{2} RE_{X+B} \qquad (11)$$

Taking these results, we can construct Table 1, in which are included the solution obtained via applying Shapley´s value and the two possible sequences. These latter produce consistent results but they are not symmetrical.

Table 1. Decomposition of the redistributive effect of a public policy made up of two instruments, taxes and transfers.

	Sequence 1 *(-T+B)*	*Sequence 2* *(B-T)*	*Shapley Solution*
Taxes	RE_{X-T} *(I)*	$RE_{X-T+B} - RE_{X+B}$ *(III)*	$\overline{RE}_{X-T+B}^{T} = \frac{1}{2}(RE_{X-T+B} - RE_{X+B)}) + \frac{1}{2} RE_{X-T}$ *(V)*
Transfers	$RE_{X-T+B} - RE_{X-T}$ *(II)*	RE_{X+B} *(IV)*	$\overline{RE}_{X-T+B}^{B} = \frac{1}{2}(RE_{X-T+B} - RE_{X-T}) + \frac{1}{2} RE_{X+B}$ *(VI)*
Total	RE_{X-T+B} *(VII)*	RE_{X-T+B} *(VII)*	RE_{X-T+B} *(VII)*

Source: own elaboration.

The problem of inconsistency that tends to appear in works of this kind, such as happens in Kim and Lambert (2009), originates in calculating the total effect, VII and the effects I and IV. The three effects are estimated as the marginal effect on the initial income, but in VII the interaction of both instruments is taken into account, while in I and IV it is not. This inconsistency is also produced in the case of our calculating the partial effects III and II (calculating the redistributive effect of each policy, eliminating it in the final income instead of aggregating it to the initial income), as can be observed in Inmervoll et. al (2006).

For their part, in the works that adopt the sequential solution, as happens in the work of Wolff and Zacharias (2007), sequence 1 is opted for (values I and II), which fulfils the additive property, but incurs in arbitrariness. Sequence 2 could be taken and the result would also be consistent, but with different values (values III and IV).[10]

Shapley´s solution consists of calculating the average of the marginal effects of each instrument in each sequence. It fulfills the properties of additivity and symmetry (*V+VI=VII*) and takes into account any interaction between instruments.

In the same way in which we proceed with RE, the vertical, horizontal and reordering effects *V*, *H* and *R* can be decomposed.

3. Data and Results

As was pointed out in the introduction, to contrast the efficiency of the methodology proposed and its consequences, we employ the data of the work of Kim and Lambert (2009), which is the most recent study for the United States, and we will analyze the consequences that derive from the new estimation. In this work, data of personal taxes and transfers for social benefits are taken in the United States for the years 1994, 1999 and 2004 proceeding from the U.S. Current Population Survey (CPS). The different indexes that we use are computed with the relation specified in equation (3).

In Table 2 the data offered by the authors is reproduced and the new results with value´s Shapley for the three years considered is included.[11] As can be observed, taxes and transfers reduce the income inequality of the market around 30%. The greatest inequality after taxes and transfers is in 2004, as a consequence of the market income being divided more unequally. Taxes have their greatest redistributive effect and their greatest relative weight, close to 30%, in 1999. Later they lose importance in favour of transfers which are responsible for 76.29% of the redistribution in 2004. Likewise, taxes and transfers lose their capacity to correct the vertical inequality of the 1999 market income. Nevertheless, the increase of the vertical equality of the transfers means that in 2004 taxes and transfers together generate the greatest horizontal inequality correction.

In comparative terms we can see that the redistributive effects of taxes and transfers separately are greater than those previously calculated and explain the whole redistributive effect. To explain this, we take taxes as an example and with Shapley value we calculate the average of the two redistributive effects on the original income and on the income after social transfers (*I* and *III* of Table 1). When taxes are applied after the social transfers, these have already reduced the inequality and consequently with the same quantity of resources a greater equality can be attained. The most relevant effect for the taxes is that their redistributive effect is broader: they explain between

[10] As was pointed out above, in the work of Wolff and Zacharias the marginal effect on the initial income of the transfers is also calculated, which is why it can be seen that II and IV do not coincide.

[11] As we did not have at our disposal the intermediate of the Gini indices of the different arrangement sequences, we will limit ourselves to disaggregating the redistributive effect of transfers and taxes, without continuing the disaggregation of them into the different subelements that they make up.

29.7% (1999) and 23.7% (2004) of the respective total effects. The new calculations also show a greater weight in the vertical effects of the taxes. This situation is coherent with its greater total redistributive effect.

It is also observed that, as Kim and Lambert pointed out, there is a certain stability of equality loss due to reranking, between 30 and 35% of the redistributive effect. Now, with the new calculations we also moreover identify the contribution of each instrument to the reranking. Thus, in 1994, 76% of the reranking was produced by transfers and 24% by taxes. In 2004 the taxes hardly contributed 17.5% of the reranking, which means that the transfers have a more unequal distribution in terms of horizontal equality.

Finally we have to highlight, also in the tax ambit, the sign change of the *H* values. In the original calculations the taxes produced inequality as a consequence of unequal treatment of equals. The new calculations point out that taxes and transfers follow a similar pattern. Inequality due to horizontal inequality is not produced, but rather this operates in favour of redistribution.

Table 2. Redistributive Effect of taxes and transfers. Non -additive decomposition vs Shapley decomposition

1994

	*Non additive decomposition**		*Shapley decomposition*		
	Absolute value	*In percentage of RE*	*Absolute value*	*In percentage of RE*	*In percentage of effect of taxes and public transfers*
Taxes (T)					
RE	0,03312		0,040595		26,28%
V^K	0,03961	119,60%	0,053275	131,24%	25,65%
V	0,03962	119,64%	0,05317	130,96%	25,73%
H	0,00001	0,02%	-0,000115	-0,28%	10,65%
R^K	0,0065	19,62%	0,012685	31,24%	23,83%
Public Transfers (B)					
RE	0,10638		0,1138505		73,72%
V^K	0,14075		0,154415		74,35%
V	0,1399	131,51%	0,15345	134,78%	74,27%
H	-0,00085	-0,80%	-0,000965	-0,85%	89,35%
R^K	0,03437	32,31%	0,040555	35,62%	76,17%
Total (N=T-B)					
RE	0,15445		0,1544455		100,00%
V^L	0,20769		0,20769		100,00%
T by V^L decomposition	0,03126		0,053275		
B by V^L decomposition	0,17643		0,154415		
V	0,20662	134%	0,20662	133,78%	100,00%
H	-0,00107	-0,69%	-0,00108	-0,70%	100,00%
R^K	0,05324	34,47%	0,05324	34,47%	100,00%

1999

	*Non additive decomposition**		*Shapley decomposition*		
	Absolute value	*In percentage of RE*	*Absolute value*	*In percentage of RE*	*In percentage of effect of taxes and public transfers*
Taxes (T)					
RE	0,03492		0,04296		29,70%
V^K	0,03847	110,17%	0,051685	120,31%	27,38%
V	0,03847	110,17%	0,05163	120,18%	27,43%
H	0,00001	0,02%	-0,00005	-0,12%	9,26%
R^K	0,00354	10,15%	0,008725	20,31%	19,79%
Public Transfers (B)					
RE	0,09367		0,10171		70,30%
V^K	0,12385		0,137065		72,62%
V	0,12342	131,76%	0,13658	134,28%	72,57%
H	-0,00043	-0,46%	-0,00049	-0,48%	90,74%
R^K	0,03018	32,22%	0,035365	34,77%	80,21%
Total (N=T-B)					
RE	0,14467		0,14467		100,00%
V^L	0,18875		0,18875		100,00%
T by V^L *decomposition*	0,03144		0,051685		
B by V^L *decomposition*	0,15731		0,137065		
V	0,18821	130%	0,18821	130,10%	100,00%
H	-0,00054	-0,38%	-0,00054	-0,37%	100,00%
R^K	0,04409	30,48%	0,04409	30,48%	100,00%

2004

	*Non additive decomposition**		*Shapley decomposition*		
	Absolute value	*In percentage of RE*	*Absolute value*	*In percentage of RE*	*In percentage of effect of taxes and public transfers*
Taxes (T)					
RE	0,02954		0,03766		23,71%
V^K	0,03323	112,49%	0,04754	126,23%	22,08%
V	0,03326	112,57%	0,04747	126,05%	22,14%
H	0,00002	0,08%	-0,000075	-0,20%	8,24%
R^K	0,00369	12,49%	0,00988	26,23%	17,49%
Public Transfers (B)					
RE	0,11307		0,12119		76,29%
V^K	0,1535		0,16781		77,92%
V	0,15276	135,10%	0,16697	137,78%	77,86%
H	-0,00074	-0,66%	-0,000835	-0,69%	91,76%
R^K	0,04043	35,76%	0,04662	38,47%	82,51%
Total (N=T-B)					
RE	0,15885		0,15885		100,00%
V^L	0,21535		0,21535		100,00%
T by V^L *decomposition*	0,02613		0,04754		
B by V^L *decomposition*	0,1892		0,16781		
V	0,21444	135%	0,21444	135,00%	100,00%
H	-0,00091	-0,57%	-0,00091	-0,57%	100,00%
R^K	0,0565	35,57%	0,0565	35,57%	100,00%

** Data obtained from Table 3 from the work of Kim, Lambert (2009). Source:own elaboration.*

The previous information can be completed with that which appears in Table 3. This includes the relative variation in the calculations for 2004. We appreciate that the data of Kim and Lambert (2009) undervalued all the indices, but in comparative terms the indicators of the redistributive effect of the taxes were more affected than those of the transfers. The undervaluing of the redistributive effect of the taxes triples that of the transfers. What is more, we can also see that the previous calculations undervalued more the inequality associated with the horizontal inequality and the reranking than the vertical redistributive effect.

To sum up, for the years analyzed the result we obtain is that the redistributive effects of taxes and social transfers are greater than those calculated on the initial income. At the same time, the calculations allow us to conclude that the undervaluing of the redistributive effect is greater in the case of the taxes. According to our estimation, the relative weight of the taxes would represent around 25% of the total redistribution, which contrasts with 12% attributed by Kim and Lambert for 2004. That is, the weight of the taxes in the redistribution in the US for the years considered is notably greater when it is assessed taking into account the rest of the policy it is part of or with which it is associated.

Table 3. Non additive decomposition vs Shapley decomposition. % differences. 2004

	Taxes			*Public Transfers (B)*		
	Non additive decomposition	*Shapley methodology*	*Percentage change*	*Non additive decomposition*	*Shapley methodology*	*Percentage change*
	(I)	*(II)*	*((I-II)/II)*	*(I)*	*(II)*	*((I-II)/II)*
RE	0,02954	0,03766	-21,56%	0,11307	0,12119	-6,70%
V^K	0,03323	0,04754	-30,10%	0,1535	0,16781	-8,53%
V	0,03326	0,04747	-29,93%	0,15276	0,16697	-8,51%
H	0,00002	-0,00008	-126,67%	-0,00074	-0,00084	-11,38%
R^K	0,00369	0,00988	-62,65%	0,04043	0,04662	-13,28%
	Taxes and Public Transfers (C)			*A+B-C*		
	Non additive decomposition	*Shapley methodology*	*Percentage change*	*Non additive decomposition*	*Shapley methodology*	*Percentage change*
	(I)	*(II)*	*((I-II)/II)*	*(I)*	*(II)*	*((I-II)/II)*
RE	0,15885	0,15885	0,00%	0,11307	0,12119	-6,70%
V^K	0,21535	0,21535	0,00%	0,1535	0,16781	-8,53%
V	0,21444	0,21444	0,00%	0,15276	0,16697	-8,51%
H	-0,00091	-0,00091	0,00%	-0,00074	-0,00084	-11,38%
R^K	0,0565	0,0565	0,00%	0,04043	0,04662	-13,28%

Source: own elaboration whit table 2.

4. Repercussions for redistributive impact studies

As we have been able to check, applying Shapley value to the Reynolds-Smolensky index, and its decomposition according to Aronson, Johnson and Lambert (1994), we have managed to determine the contribution relative to the redistributive effect and its decomposition into vertical and horizontal impacts of the instruments considered in Kim and Lambert (2009). We have consistently calculated the contribution of taxes and transfers to the total redistributive effect of the social well-being policies of the United

States for the years 1994, 1999 and 2004 proceeding from the U.S. Current Population Survey (CPS). That is to say, we have achieved symmetrical results without arbitrarily introducing criteria, which has at least two repercussions for the redistributive assessment of public policies.

Firstly, that to value a policy we must take into account the problem of judgment values and their repercussion on the results. We are going to use Table 4 to show our argument. In this table, we have reproduced table 1, but with the values calculated in Table 2 for the redistributive effect of 2004. In it one can clearly appreciate the possibility of arbitrary influence in the results. Thus, if taxes and transfers are evaluated on initial rent (alternative of the boxes *I* and *IV)*, exactly as we have seen in Table 3, the effects of taxes and transfers are undervalued. On the contrary, if taxes and transfers are evaluated on income post-transfers or income post-taxes, respectively, (take the opposite alternative, that defined by boxes *II* and *III)* then what we do is to overvalue the effect of both. On the other hand, if we choose sequential *sequence 1* (initial income-taxes+transfers) the redistributive effect of the taxes is undervalued and that of the transfers is overvalued. Finally, if we choose *sequence 2* (initial income+transfers-taxes), the opposite occurs: the overvaluing is produced in the value of the taxes. That is to say, the choice of one alternative or the other gives rise to a specific bias. That is, selecting a methodology we can arbitrarily influence the results and these results have a greater or lesser redistributive impact on one instrument or another or both. Faced with this situation, Shapley´s methodology also produces consistent results and reduces the technical possibility of arbitrary manipulation.[12]

Table 4. Additive decompositions. Redistributive Effect of Taxes and Transfers 2004

	Sequence 1 *(-T+B)*	*Sequence 2* *(B-T)*	*Shapley Solution*
Taxes	0.02954	0.04578	0.03766
	(I)	*(III)*	*(V)*
Transfers	0.12931	0.11307	0.12119
	(II)	*(IV)*	*(VI)*
Total	0.15885	0.15885	0.15885
	(VII)	*(VII)*	*(VII)*

Source: own elaboration from Table 2.

The second repercussion is that for policy reforms we can distinguish two types of redistributive effects: a marginal one and a redistributive effect of the instrument. Indeed, let us suppose that we wish to assess the redistributive effect of a policy reform. If the reform consists of introducing a new instrument, we could calculate its redistributive effect according to boxes *II* or *III*. Now, the impact calculated in this manner measures the marginal incidence of its incorporation, but not the redistributive effect attributable to the new instrument within the whole policy. To measure the latter

[12] The possibility of there being arbitrary manipulation does not,evidently, imply that this is practised.

– that is, the redistributive effect of the instrument- the appropriate procedure is decomposition via Shapley´s value.

The difference is important, as the redistributive effect contains two effects: the marginal effect of the instrument itself and that which derives from the alteration that is produced in the effects of the remaining instruments. Let us imagine, for example, that social transfers do not exist and that they are created. The marginal effect of the creating of the transfers in 2004 would be 0.12931 (box II) but the redistributive effect of the transfers, as component of one policy, according to Shapley´s decomposition is 0.12119 (box VI). The difference, 0.00812, is the increase of the redistributive effect attributable to the taxes and is because its redistributive effect is reinforced when it is created together with the transfer instrument.

Finally, there exists an additional positive repercussion for the studies of redistributive impact. When we apply Shapley´s additive decomposition to the redistributive effect and its vertical and horizontal components, we can compare the relative weight of the effect of each instrument in percentage terms. This possibility improves the information of any compared study, not only the intertemporal comparison as carried out in this work, but also the policy comparison or that of tax systems of different countries and the comparison of alternatives of the composition of spending or income.

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