

A Hybrid Procedure Including Subtransmission Systems and

Substations for Reliability Assessment

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Abstract

A new procedure focused on reliability analysis of subtransmission systems supported by the state enumeration technique is presented. This new methodology is conducted in three stages. First, a classical state enumeration reliability assessment is performed for the branch-node model of a subtransmission system, assuming that substations are perfectly reliable. Second, a detailed model of the subtransmission system is considered and the reliability of each substation is assessed by considering them in a “one-by-one” process, supposing perfect operation for the branch-node model. Finally, the reliability indices calculated in the first and second stages are analytically combined to obtain the reliability indices for the subtransmission system (system reliability indices) and for the load nodes of the distribution system (load-node reliability indices). Test results show that the proposed methodology is suitable for both planning studies and 24 hours-ahead security assessment.

Key words: State enumeration, power system reliability, subtransmission systems, substation configurations.

I. INTRODUCTION

“Reliability” in power systems is primarily concerned about the interruptions to the utility’s customers, so improving it is a key issue to enhance customer’s supply [1][2][3][4]. Before restructured power systems, subtransmission systems had been included in the reliability analysis of composite power systems [5][6][7].

Nowadays, the subtransmission level has the overall responsibility to manage power exchange and load curtailment bids to fulfill reliability commitments and customers’ requirements [8]. Moreover, substations are significant supply points because they are located where the meshed network ends and the radial supply system begins (i.e., feeders) [2][3]. Hence substations must be properly modelled in the reliability assessment to obtain customer interruption indices and evaluate their impact on the reliability of subtransmission systems [4][8].

Efforts on substation reliability analysis have been mainly focused on methods to compare the performance of various substation configurations (layouts) [9], and the trade-off between economy and reliability [10]. Furthermore, several methods have considered the substation as a separate part of the power system and employed an equivalent model to obtain acceptable computational times [11].

This paper deals with the detailed model of substations and their influence on the reliability assessment of supply points [5] [6] [12]. This influence poses the following question: is the worst case from a reliability system viewpoint the same as that from a load-bus viewpoint for a specific system? A new approach, taking into account correlated load-shedding, is presented here to help answering this question, and to improve the reliability of subtransmission systems.

In this paper, various comparative studies are also presented to illustrate how the proposed methodology can be used to assess the reliability of substations and subtransmission systems. In these studies, the proposed procedure is applied to the

138 kV subtransmission network of the well-known IEEE-RTS [13], and a regional 132 kV Spanish subtransmission network. The errors associated to the proposed methodology are presented for both test networks, and these show accurate results in both planning and 24 hours-ahead security assessment.

The paper is organized as follows. First, the basic concepts of the proposed probabilistic technique are exposed. Second, the mathematical problem and the proposed probabilistic technique are presented, followed by the numerical simulations and the subsequent analysis of the results. Finally, some conclusions derived from this work are summarized.

II. PROBLEM DESCRIPTION

At present, reliability evaluation techniques developed for conventional systems are being reviewed or modified to suit their application in the reliability evaluation of restructured power systems [8] [14]. Two main techniques are used in probabilistic reliability analysis: Monte Carlo and state enumeration. Monte Carlo technique presents better performance in the reliability analysis of composite generation and transmission systems (bulk power systems). It is computationally faster and there are no limitations in the number of components involved in simultaneous outages. However, in the reliability analysis of subtransmission and/or distribution systems the state enumeration technique is a simple and usually fast evaluation process that can gather much information [3] [15] [16] [17] [18].

The procedure presented here focuses on the reliability assessment problem of electrical utilities considering meshed subtransmission networks and the supply points of the distribution-facilities, with a reduced number of generation plants. Because of this, the probabilistic analysis of the proposed procedure has been based on the state enumeration technique.

A complete analysis of subtransmission networks including a detailed model of each substation would provide more accurate results but it would require more computational time. To deal with this issue, this paper presents a new technique which assesses “one-by-one” the substations’ reliability and calculates the correlated load/shedding between substations. By combining this assessment with a previous branch-node model reliability analysis of the subtransmission system, accurate results for the detailed model of the subtransmission system can be calculated. The steps of the proposed strategy are shown in Figure 1.

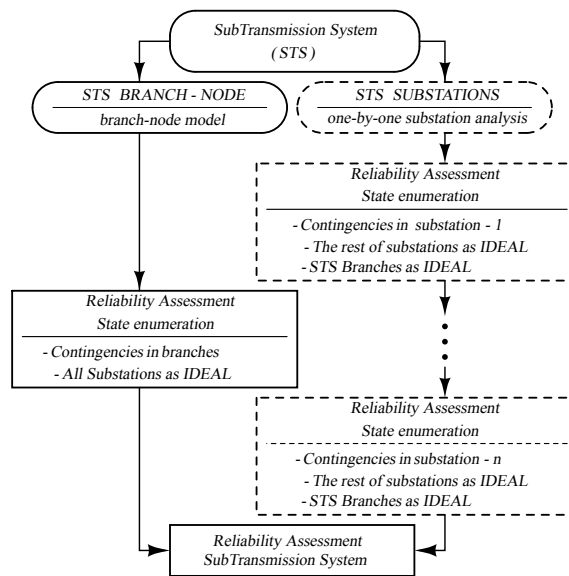


Fig. 1. Diagram of the proposed methodology.

These stages are gathered in two different structures (Figure 1) and the reliability indices are computed combining the results of both paths. The branch depicted with continuous lines (*STS BRANCH - NODE*) corresponds to the reliability analysis of the subtransmission system branch-node model, and the branch depicted with dashed lines (*STS SUBSTATIONS*) corresponds to the analysis of the substations considered “one-by-one” and supposing perfect operation for the branch-node model of the subtransmission system. The proposed reliability methodology includes four modules: Topological Analysis, DC Load Flow, DC Optimal Power Flow and Statistical Evaluation. The Topological Analysis is activated whenever the topology of the system changes as a result of a contingency. The DC Load Flow determines the existing overloads and, when an overload occurs, a DC Optimal Power Flow is applied. The DC approach has been adopted in order to reduce the computational time, taking into account that the assumed DC simplifications are acceptable in subtransmission systems analysis [19][20].

The DC-OPF is based on linear programming and its objective function is to obtain corrective actions extended to generation rescheduling and to minimize load/shedding [21]. Linear analysis approximate the full nonlinear power-flow solution to produce the “long-term” steady-state conditions after an outage, and they are used primarily to verify that thermal line limits are not exceeded after the outage. Once the linear analysis is performed then an AC power flow (or even an AC-OPF) can be used to obtain more complete and accurate results [22] [23]. The AC analysis is not included in this paper.

The reliability indices of the system (global indices) and of the load-nodes (individual indices) are calculated by means of a Statistical Evaluation module that includes a non-sequential state enumeration process [24], and that updates the reliability indices considering the load curtailment obtained from the DC OPF and the probability of each particular state. The reliability indices of all the load-nodes are calculated in the distribution side of substations (reliability indices from load-bus viewpoint), but this work does not face the reliability assessment of distribution systems. The calculated indices in the down side of substations are the reliability indices in the origin of the distributions feeders and can be used for distribution utilities in a later reliability analysis. Primary distribution systems consist of radial feeders departing from substations, and a huge background in the reliability literature exists related to very efficient techniques focussed to the reliability analysis of these systems [25][26].

Finally, it is important to note that the computation cost of state enumeration technique may become very high if the number of generators in subtransmission networks is significant, and Monte Carlo simulation can be more competitive than the proposed procedure. To overcome this problem, the proposed method can be extended to consider different contingency levels for branches (lines and transformers) and generators.

III. METHODOLOGY AND FORMULATION OF THE PROBLEM

Figure 2 shows the branch-node model of the three-node system considered in the formulation of the problem and, in a simplified form, the configuration (layout) of its step-down substations. Traditional reliability studies assess reliability of the subtransmission system and reliability of substations separately, but this paper considers them together.

The presented procedure assesses the reliability of a specific state of the system with a load level. The uncertainty of the load forecasting can be easily included as a component with different load states and probabilities associated to the uncertainty of each load state. Additionally to the load forecasting error, the load is subject to uncertainties associated with load fluctuations in the study period, but this subject is not faced in this work.

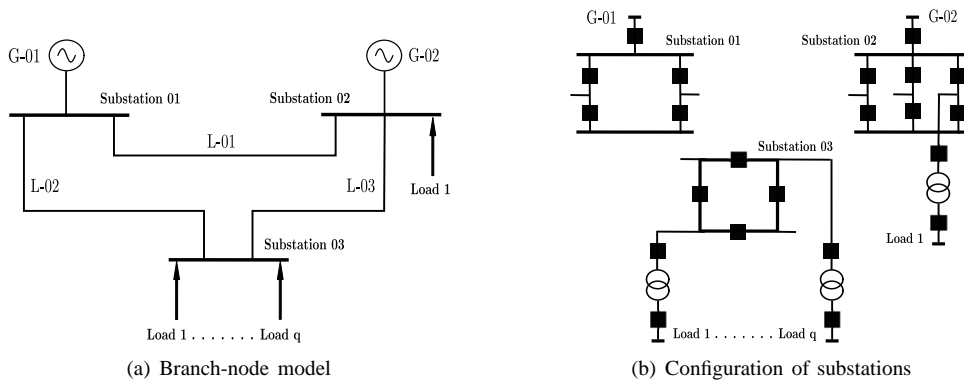


Fig. 2. Proposed branch-node model and configuration of its substations.

Bear in mind that the complete branch-node model and the substations of the subtransmission system are also included in the following explanations. It is described next how, by considering both the subtransmission system and substations, it is possible to calculate the impact that the failure of substation's equipments have on the reliability of the rest of substations of the subtransmission system. This correlated impact of failures between substations can not be obtained with the traditional techniques, but it can be gathered by means of the proposed "one-by-one" reliability analysis of the substations of the subtransmission system.

A. First stage: the branch-node model reliability analysis

This stage corresponds to the assessment of the branch-node model's reliability of the subtransmission system. This analysis is performed supposing that the substations are ideal, hence the states of failure of the substations are not included in the analysis.

However, in this stage, the configuration of each substation is taken into consideration, because each load-node (Load 1 to Load q , Figure 2) belonging to a substation of the subtransmission system has its own cost of load-shedding and, hence, each load-node has its own reliability indices. Consequently, the reliability indices of each load-node (down-side of each substation) are obtained considering that the failure probability of the equipments of all substations is equal to zero. In this way, only the branches of the subtransmission system (HV-Line, Figure 2) and their failure probabilities are considered in the proposed branch-node model reliability analysis.

Once a generation-load scenario has been assigned to the subtransmission system, it is possible to define the corresponding state "s" of the subtransmission system. Considering that the probability of a system state depends not only on the failed elements, but also on the non-failed ones (ideal equipment). Then, according to the enumeration technique, the probability $p(s)$ of this system state is formulated as follows:

$$p(s) = \prod_{e=1}^{n_u} U_e \cdot \prod_{e=1+n_u}^N (1 - U_e) \quad (1)$$

where N and n_u , respectively, are the total number of elements and the number of unavailable elements of the branch-node model of the subtransmission system. In this system, only the branches (HV-Line, Figure 2) are enumerated, and U_e is the unavailability of the "e" element.

The state enumeration process calculates the Loss Of Load Probability (LOLP) as an individual index in each load-node of the substation k (Figure 2) according to

$$\text{LOLP}_{(k,1)} = \sum_{s \in s_{(k,1)}} [1 \cdot p_B(s)]; \quad \dots \quad ; \quad \text{LOLP}_{(k,q)} = \sum_{s \in s_{(k,q)}} [1 \cdot p_B(s)] \quad (2)$$

where $s_{(k,1)}$ to $s_{(k,q)}$ are subsets of the subtransmission system states. These subsets include the states that lead to a load-shedding in the load-nodes (Load 1 to Load q) of substation k , and $p_B(s)$ is the probability of the state of the branch-node model that causes a load-shedding in the load-nodes of the substation k . Notice that each reliability index $\text{LOLP}_{(k,j)}$ is an individual-node reliability index and represents the probability of a load-shedding in the load-node j of substation k . For simplicity, the LOLP (Loss Of Load Probability) reliability index has only been calculated, but similar expressions are obtained when the EPNS (Expected Power Not Served) and other usual reliability indices are considered.

Once the individual-node reliability indices from a load-node viewpoint are obtained, the global-node LOLP_k reliability index of node k (from subtransmission system viewpoint) is computed. The LOLP_k index is calculated as follows:

$$\text{LOLP}_k = \text{LOLP}_{(k,1)} + \dots + \text{LOLP}_{(k,q)} = \sum_{j=1}^q \text{LOLP}_{(k,j)} \quad (3)$$

As it happens in contingency analysis, the state enumeration technique can be applied to various “ $N - i$ ” contingency levels. The probability $P(s)$ of each “ $N - i$ ” contingency level is calculated as follows

$$P(s) = \sum_{s \in (N-i)} p(s) = \sum_{s \in (N-i)} \left(\prod_{e=1}^{n_u} U_e \cdot \prod_{e=1+n_u}^N (1 - U_e) \right) \quad (4)$$

where $i \in \{0, 1, \dots, N\}$ and $n_u \in \{0, 1, \dots, i\}$.

When the state enumeration technique is extended from the “ $N - 0$ ” contingency level to the “ $N - \ell$ ” contingency level, then the accumulated probability $P(s)^{acu}$ is

$$P(s)^{acu} = \sum_{(N-0)}^{(N-\ell)} P(s) = \sum_{(N-0)}^{(N-\ell)} \left(\sum_{s \in (N-i)} p(s) \right) \quad (5)$$

Thus the probability $P(s)_R$ of being in any of the remaining states is obtained as

$$P(s)_R = 1 - P(s)^{acu} = 1 - \left[\sum_{(N-0)}^{(N-\ell)} \left(\sum_{s \in (N-i)} p(s) \right) \right] \quad (6)$$

B. Second stage: assessing substations reliability “one-by-one”

In this section the second stage of the proposed methodology is explained in detail. This is an individual reliability analysis focused on the configuration of each substation. It is important to note that each individual analysis includes the branch-node model of the subtransmission system because the branch-node model stores the interconnections between the analyzed substation and the rest of the substations. Each substation is assessed considering its configuration jointly with the branch-node model of the subtransmission system. In this way, the proposed analysis of substations reflects the configuration (substation) influence not only in the load nodes of the studied substation, but also on the load nodes of the rest of substations.

Note that the proposed reliability analysis considers the configuration of each substation as something included in the subtransmission system and not as something isolated. Hence, this is not a classical analysis to determine how good or bad the configuration of the substation is. The next explanations are supported by Figure 3, which shows the application of the proposed “one-by-one” process to the system of Figure 2.

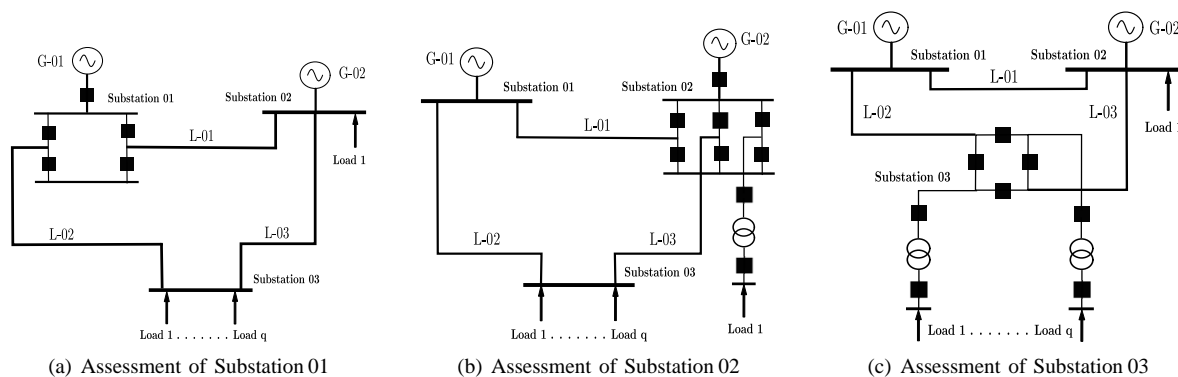


Fig. 3. Proposed one-by-one reliability analysis.

This second stage of the proposed procedure analyzes “one-by-one” the substations of the subtransmission system and this analysis is based on the state enumeration process. The “one-by-one” process is performed under the next assumptions: first, the elements of the branch-node model of the subtransmission system are considered as ideal elements, and second, the equipments (transformers, breaks and buses) of the analyzed substation (in the so-called “one-by-one” process) are the only ones included in the enumeration process, so the elements of the rest of the substations are considered to be ideal. If the other substations and the branch-node model of the subtransmission system are considered as ideal elements, they are perfectly reliable and not included in the enumeration procedure. Nevertheless their operational and thermal limits are used in the analysis.

For each particular contingency state, the correlated load-shedding between substations is determined by using a DCOPF (Direct Current Optimal Power Flow) in the proposed “one-by-one” process. The DCOPF is carried out to obtain remedial actions (generation re-dispatch and/or load shedding) to bring the system back to normal state when a contingency occurs. The formulation of the optimization problem is not included in the paper although this is properly referenced in the introduction.

Let us assume that the substation k (with q load-nodes) is the assessed substation. In the individual reliability analysis of the k substation, the proposed technique calculates the reliability indices of the load nodes of substation k , and the coefficients of influence on the load nodes reliability indices of the rest of substations belonging to the subtransmission system. Obviously the calculated k substation reliability indices and coefficients of influence, called hereafter correlated load-shedding, are only due to the failure of equipments in substation k .

Considering \hat{n}_u as the number of unavailable components and \hat{N} as the total number of components, both in the analyzed k substation, according to the enumeration technique the probability $\hat{p}(s)$ of a substation state is

$$\hat{p}(s) = \prod_{e=1}^{\hat{n}_u} U_e \cdot \prod_{e=1+\hat{n}_u}^{\hat{N}} (1 - U_e) \quad (7)$$

where U_e is the unavailability of equipment e in the k analyzed substation. Consequently, the loss of load probability in each j load node (Node 1 to Node q) of substation k , due to the failure of equipments in substation k , is obtained as follows:

$$\widehat{\text{LOLP}}_{(k,1)_k} = \sum_{s \in s_{(k,1)_k}} [1 \cdot \widehat{p}_k(s)]; \quad \cdots \quad ; \quad \widehat{\text{LOLP}}_{(k,q)_k} = \sum_{s \in s_{(k,q)_k}} [1 \cdot \widehat{p}_k(s)] \quad (8)$$

where $s_{(k,1)_k}$ to $s_{(k,q)_k}$ are the subsets of all those states of substation k leading to a load shedding on load-nodes of substation k , and $\widehat{p}_k(s)$ represents the substation state probability that implies a load-shedding in substation k . The $\widehat{\text{LOLP}}_{(k,1)_k}$ to $\widehat{\text{LOLP}}_{(k,q)_k}$ reliability indices are the loss of load probabilities corresponding to states of the k substation that entails a load shedding, respectively, in Node 1 to Node q of the k substation.

In the case of any “ a ” substation (with r load-nodes), the correlated load-shedding due to the k substation corresponds to the reliability index of each load node of substation “ a ”, when the states of substation k are only considered. Using the LOLP reliability index the yielded expression is

$$\widetilde{\text{LOLP}}_{(a,1)_k} = \sum_{s \in s_{(a,1)_k}} [1 \cdot \widetilde{p}_k(s)]; \quad \cdots \quad ; \quad \widetilde{\text{LOLP}}_{(a,r)_k} = \sum_{s \in s_{(a,r)_k}} [1 \cdot \widetilde{p}_k(s)] \quad (9)$$

where $s_{(a,1)_k}$ to $s_{(a,r)_k}$ are the subsets of the states of substation k that carry out a load-shedding in substation “ a ”, $\widetilde{p}_k(s)$ shows the probability of these states of substation k , and $\widetilde{\text{LOLP}}$ is the loss of load probability due to correlated load-shedding.

When the one-by-one reliability analysis focuses on the a substation, the influence of this substation on the reliability indices of the load-nodes of the k substation, in terms of the LOLP index, is

$$\widetilde{\text{LOLP}}_{(k,1)_a} = \sum_{s \in s_{(k,1)_a}} [1 \cdot \widetilde{p}_a(s)]; \quad \cdots \quad ; \quad \widetilde{\text{LOLP}}_{(k,q)_a} = \sum_{s \in s_{(k,q)_a}} [1 \cdot \widetilde{p}_a(s)] \quad (10)$$

where $s_{(k,1)_a}$ to $s_{(k,q)_a}$ are the subsets of the states of substation a that carry out a load-shedding in substation “ k ”, $\widetilde{p}_a(s)$ shows the probability of these states of substation a , and $\widetilde{\text{LOLP}}_{(k,1)_a}$ to $\widetilde{\text{LOLP}}_{(k,q)_a}$ are the loss of load probability due to correlated load-shedding between the load-shedding in the k substation load nodes and the failures of the a substation equipments. Obviously the branch-node model of the subtransmission system and the equipments of substation k and the equipments of the rest of substations are supposed to be ideal in the reliability analysis of the a substation.

If the subtransmission system has “ n_s ” substations, except substation k , then the repercussion on the substation k (correlated load-shedding), as consequence of the failures in the rest of substations, is calculated as follows in terms of the LOLP index:

$$\widetilde{\text{LOLP}}_{(k,j)_{n_s}} = \sum_{\substack{a=1 \\ \forall a \neq k}}^{n_s} \widetilde{\text{LOLP}}_{(k,j)_a} = \sum_{\substack{a=1 \\ \forall a \neq k}}^{n_s} \left(\sum_{s \in s_{(k,j)_a}} [1 \cdot \widetilde{p}_a(s)] \right) \quad (11)$$

where $\widetilde{\text{LOLP}}_{(k,j)_{n_s}}$ is the loss of load probability in the node j of substation k , as consequence of the failures in the equipments of the other n_s substations of the subtransmission system. Bear in mind that n_s does not include substation k .

C. Third stage: the subtransmission system reliability assessment

Let us assume that node j of substation k is the critical node. The LOLP index of this node (reliability index from load-node viewpoint) is calculated as follows:

$$\overline{\text{LOLP}}_{(k,j)} = \text{LOLP}_{(k,j)} + \widehat{\text{LOLP}}_{(k,j)_k} + \widetilde{\text{LOLP}}_{(k,j)_{n_s}} = \text{LOLP}_{(k,j)} + \widehat{\text{LOLP}}_{(k,j)_k} + \sum_{a=1}^{n_s} \widetilde{\text{LOLP}}_{(k,j)_a} \quad (12)$$

where $\text{LOLP}_{(k,j)}$ expresses the repercussion on the node j of the states of the branch-node model, $\widehat{\text{LOLP}}_{(k,j)_k}$ represents the impact on the node j of the states of substation k , and $\widetilde{\text{LOLP}}_{(k,j)_{n_s}}$ is generated by the states of the other n_s substations of the subtransmission system (except substation k).

Equation(12) gives an approximate value of the individual reliability index LOLP of the node j of substation k . Anyway this is a sufficient value for a preliminary reliability analysis of the subtransmission system.

A more accurate analysis can be obtained when equation(13) is applied in the reliability assessment.

$$\begin{aligned} \widehat{\text{LOLP}}_{(k,j)} &= \text{LOLP}_{(k,j)} \cdot P(s)^{acu} \cdot \widehat{P}(\widehat{N}-0) \cdot \widetilde{P}(\widetilde{N}-0) + \widehat{\text{LOLP}}_{(k,j)_k} \cdot \widetilde{P}(\widetilde{N}-0) \cdot P(N-0) \\ &+ \sum_{a=1}^{n_s} \widetilde{\text{LOLP}}_{(k,j)_a} \cdot \left(\left(\widetilde{P}(\widetilde{N}-0)_a \right)^{-1} \cdot \widetilde{P}(\widetilde{N}-0) \right) \cdot \widehat{P}(\widehat{N}-0) \cdot P(N-0) \end{aligned} \quad (13)$$

where:

- $P(s)^{acu}$ is the branch-node model probability of not being in any state s with a contingency level greater than the $(N-i)$ level applied to the enumeration process. This probability verifies that: $P(s)^{acu} = 1 - P(s)_R$
- $P(N-0)$ is the branch-node model probability of being in a non-failure state
- $\widehat{P}(\widehat{N}-0)$ is the substation k probability of being in a non-failure state
- $\widetilde{P}(\widetilde{N}-0)_a$ is the substation a probability of being in a non-failure state
- $\widetilde{P}(\widetilde{N}-0)$ is the substations n_s probability of being in a non-failure state; $\widetilde{P}(\widetilde{N}-0) = \prod_{a=1}^{n_s} \widetilde{P}(\widetilde{N}-0)_a$

Taking common factor terms in equation(13), the yielded equation is:

$$\begin{aligned} \widehat{\text{LOLP}}_{(k,j)} &= \left[\frac{\text{LOLP}_{(k,j)} \cdot P(s)^{acu}}{P(N-0)} + \frac{\widehat{\text{LOLP}}_{(k,j)_k}}{\widehat{P}(\widehat{N}-0)} + \sum_{a=1}^{n_s} \frac{\widetilde{\text{LOLP}}_{(k,j)_a}}{\widetilde{P}(\widetilde{N}-0)_a} \right] \\ &\cdot P(N-0) \cdot \widehat{P}(\widehat{N}-0) \cdot \widetilde{P}(\widetilde{N}-0) \end{aligned} \quad (14)$$

Finally, it is important to comment that equation(12) only provides an approximate value of the LOLP index, but it is accurate enough to enable a selection of the critical supply-points and the critical substation in a 24 hours-ahead plan (operation, reinforcement and maintenance). Thanks to equation(12), critical substations (under a reliability point of view) are located, then by means of equation(14) more accurate reliability indices are obtained.

IV. NUMERICAL SIMULATIONS AND RESULTS

The proposed formulation is handled by using Matlab and follows the approach described in Figure 1. The same software has been used to handle the formulation corresponding to the traditional reliability study applied in the simulations. All tests have been performed on a 2.01-GHz, 2-GB RAM, PC AMD-Athlon.

In this point, it is important to note that the subtransmission system and substations are only considered together in this work for comparison purposes. For this case, the size of the problem to solve implies an excessive computation cost. This work shows that the proposed procedure avoids this problem.

In this Section a comparative analysis on the influence of the proposed technique on the reliability assessment of the 138 kV subtransmission network of the well-known IEEE-RTS is presented. The configuration of the substations and the IEEE-RTS data have been obtained from reference [13]. The 230 kV transmission system has been assumed perfectly reliable (unavailability equal to zero), and the load peak equal to 1755 MW in the 138 kV subtransmission area.

A state enumeration analysis for the 138 kV subtransmission system, including all substations represented by its own configuration, has been used as reference for the comparative analysis. In this enumeration process, the probabilities obtained for a “ $N - 3$ ” contingency level are shown in Table I, and the calculated reliability indices LOLP and EPNS have been included in Table II.

TABLE I
PROBABILITIES CONSIDERED AS REFERENCE

Contingency	States	$P(s)$ in pu	$P(s)$ in pu
Level	Number	Level	Accumulated
N-0	1	0.5266725	0.5266725
N-1	132	0.3515756	0.8782481
N-2	8646	0.1025276	0.9807757
N-3	374660	0.0172268	0.9980025
$N > 3$	—	0.0019975	1.0000000

In Table II the demand in each substation is also presented. As it can be noticed, the probability of having states with a higher contingency level than “ $N - 3$ ” is 0.0019975 pu (Table I). This probability is associated to the accuracy of the enumeration process and it also corresponds to the probability of the system state of being in any of the remaining states that are not analyzed in the enumeration process.

TABLE II
RELIABILITY INDICES CONSIDERED AS REFERENCE

BUS	Load	EPNS	LOLP
	(MW)	(MW)	(%)
01	161.7	0.0358	0.0387
02	154.9	0.3367	0.8505
03	206.5	0.0446	0.0599
04	140.6	0.0674	0.0685
05	138.7	0.0524	0.0626
06	179.1	0.3020	5.2946
07	172.3	0.0462	0.0559
08	182.0	0.2362	0.3642
09	203.4	0.0428	0.0293
10	215.8	0.3600	0.7241

The first step of the technique proposed in this paper (Sec. III-A) is to obtain the reliability indices using a single-node model for the 138 kV subtransmission system. Table III includes the probabilities $P(s)$ of the different contingency levels, obtained by an enumeration process. Note that the probability of the remaining states “ $N > 3$ ” (Table III) is only 0.0019122 pu, and this means that 0.19122 % states are not considered in the reliability analysis when the single-node model is been used.

For the up-side of the transformers, from system viewpoint, the bus reliability indices obtained in the single-node analysis are shown in Table IV. At this stage, and in terms of the LOLP index, the single-node analysis reveals that the most critical

substation corresponds to Bus 06, whose LOLP index is 5.1412% (Table IV). The substation associated to this bus has one step-down transformer, and the electrical bus scheme is a Ring Bus System [13].

TABLE III
PROBABILITY OF THE CONTINGENCY LEVELS USING THE BRANCH-NODE MODEL

Contingency Level	States Number	$P(s)$ in pu Level	$P(s)$ in pu Accumulated
N-0	1	0.5293238	0.5293238
N-1	39	0.3506874	0.8800112
N-2	741	0.1012761	0.9812873
N-3	9139	0.0168005	0.9980878
N > 3	—	0.0019122	1.0000000

TABLE IV
RELIABILITY INDICES USING BRANCH-NODE MODEL

BUS	Load (MW)	EPNS (MW)	LOLP (%)
01	161.7	0.0038	0.0155
02	154.9	0.2936	0.8060
03	206.5	0.0001	0.0016
04	140.6	0.0008	0.0045
05	138.7	0.0020	0.0073
06	179.1	0.2322	5.1412
07	172.3	0.0014	0.0094
08	182.0	0.1727	0.2934
09	203.4	0.0001	0.0002
10	215.8	0.3139	0.6891

Next, according to the proposed methodology (Sec. III-B), the subtransmission system (138 kV) is analyzed including the substations in a “one-by-one” basis, considering their respective configuration. This analysis of all the substations is performed again by an enumeration process, and the results of the simulations, the reliability indices LOLP and EPNS, are shown in Table V and Table VI, respectively. The non-diagonal elements (Table V and Table VI) represent the impact that the failures of equipments of a substation have in the rest of substations of the subtransmission system, and the diagonal elements reflect how the reliability indices of a substation are affected by the failure of equipments of the mentioned substation. For simplicity, the results obtained for the critical substation (Bus 06) are the only ones shown in Table VII. This table also includes (second column) the scheme of the substations’ configuration of the 138 kV subtransmission level.

TABLE V
LOLP(%). CORRELATION BETWEEN SUBSTATIONS (138 kV IEEE-RTS).

Outages Sub	01	02	03	04	05	06	07	08	09	10
01	0,0193	0,0083		0,0058		0,0200				
02		0,0310				0,0350				
03			0,0310			0,0117			0,0058	
04				0,0251						
05					0,0251					
06						0,0251				
07		0,0117					0,0193			
08								0,0426		
09			0,0292			0,0467			0,0251	
10				0,0350	0,0292	0,0233	0,0292	0,0292		0,0193

TABLE VI
EPNS (MW). CORRELATION BETWEEN SUBSTATIONS (138 kV IEEE-RTS).

Outages Sub	Correlated Sub									
	01	02	03	04	05	06	07	08	09	10
01	0,0312	0,0050		0,0013		0,0087				
02		0,0327				0,0021				
03			0,0422			0,0011			0,0119	
04				0,0353						
05					0,0348					
06						0,0650				
07		0,0079					0,0332			
08								0,0552		
09			0,0009			0,0152			0,0416	
10				0,0298	0,0178	0,0067	0,0112	0,0038		0,0416

TABLE VII
BUS 06 RELIABILITY INDICES ASSESSING SUBSTATIONS SEQUENTIALLY

BUS	Scheme	$P(N - 0)$ <i>pu</i>	EPNS (MW)	LOLP (%)
01	One&Half breaker	0.9990571	0.0087	0.0200
02	One&Half breaker	0.9990571	0.0021	0.0350
03	Ring	0.9995738	0.0011	0.0117
04	One&Half breaker	0.9997488	—	—
05	One&Half breaker	0.9997488	—	—
06	One&Half breaker	0.9997488	0.0450	0.0251
07	Ring	0.9996321	—	—
08	Ring	0.9995738	—	—
09	Ring	0.9993405	0.0152	0.0467
10	Ring	0.9993405	0.0067	0.0233

In Table VII, the $P(N - 0)$ column expresses the probability of the substations of the 138 kV level of being in a non-failure state. Notice that in this stage of the analysis, the number of tables obtained is the same than the number of substations belonging to the 138 kV subtransmission system. These tables are based on the relation between the failure of the equipments of a substation and their correlated influence on the reliability indices of the rest of substations.

We analyze the following example: for Bus01 with a load of 161.7 MW, $P(N - 0)$ is 0.9990571 pu (Table VII). To obtain this probability, the states of the substation corresponding to Bus01 are only analyzed. Consequently, the branch-node model and the substations from Bus02 to Bus10, both included, are assumed to be in a non-failure state.

The failure states of the substation corresponding to Bus01 resulted in a EPNS of 0.0087 MW and a LOLP of 0.0200 % (Table VII). It is important to note that these two reliability indices are calculated in the load-node of the critical substation (Bus06) and they are due to the repercussion on Bus06 of the failure states of equipments of the substation corresponding to Bus01.

It can be noticed in Table VII that the equipments failure of substations located at Bus04, Bus05, Bus07 and Bus08 have no influence in the reliability indices of Bus06, and that the failures of the equipments of the substation corresponding to Bus09 have the higher repercussion (0.0467 %) on the reliability indices of Bus06.

Just like it was exposed in Secc. III-C, a sufficiently accurate result for the reliability indices of the load-node (down side of the transformer) of the critical substation (Bus06) can be obtained with equation (14) as follows:

$$\overbrace{\text{LOLP}}_{(06)} = \left[\frac{5.1412 \cdot 0.9980878}{0.5293238} + \frac{0.0251}{0.9997488} + \left(\frac{0.0200}{0.9990571} + \frac{0.0350}{0.9990571} + \frac{0.0117}{0.9995738} + \frac{0.0467}{0.9993405} + \frac{0.0233}{0.9993405} \right) \right]$$

$$\cdot 0.5293238 \cdot 0.9997488 \cdot 0.9950829 = 5.1968 \%$$

$$\overbrace{\text{EPNS}}_{(06)} = \left[\frac{0.2322 \cdot 0.9980878}{0.5293238} + \frac{0.0450}{0.9997488} + \left(\frac{0.0087}{0.9990571} + \frac{0.0021}{0.9990571} + \frac{0.0011}{0.9995738} + \frac{0.0152}{0.9993405} + \frac{0.0067}{0.9993405} \right) \right]$$

$$\cdot 0.5293238 \cdot 0.9997488 \cdot 0.9950829 = 0.2730 \text{ MW}$$

Regarding the reference values shown in Table II (LOLP = 5.2946 %, and EPNS = 0.3020 MW), the calculated $\overbrace{\text{LOLP}}$ (5.1968 %) and $\overbrace{\text{EPNS}}$ (0.2730 MW) reliability indices present the errors:

$$\text{LOLP}_{(06)} |_{\text{Error}} \% = \frac{(0.9980025 \cdot 5.2946) - 5.1968}{(0.9980025 \cdot 5.2946)} \cdot 100 = 1.64 \%$$

$$\text{EPNS}_{(06)} |_{\text{Error}} \% = \frac{(0.9980025 \cdot 0.3020) - 0.2730}{(0.9980025 \cdot 0.3020)} \cdot 100 = 9.61 \%$$

The errors obtained are perfectly acceptable because the key issue is to show a new procedure to identify the critical load buses to schedule the preventive actions in a 24 hours-ahead plan, and the accuracy of the results are a secondary matter. Note that the most important preventive action corresponds to security reinforcement by putting into service line (or lines) initially included in a scheduled maintenance (planned outage), which has to be postponed.

Finally, Table VIII shows a comparative of the computational time for each simulation process. These simulations are: number 1 for the reliability analysis used as a reference, number 2 for the reliability analysis considering the branch-node model of the system, and number 3 for the one-by-one reliability analysis of all the substations, including the later analytical process.

TABLE VIII
COMPARATIVE OF COMPUTATIONAL TIMES

Simul Type	States Number	Computational Time
1	374660	37.00 · t
2	9139	1 · t
3	2249	0.42 · t

* t = 2.50 h

Simulation type 3 begins in the critical substation, e.g. Bus06 in IEEE RTS, but it is later extended to all the substations of the sub-transmission system, then an analytical process combines the results of this simulation with the results of the simulation type 2. The analytical process calculates the reliability indices of all the load-nodes. In consequence, the computation burdens are the same for the simulations in Table VIII, and hence the comparisons of the computational time listed are fair.

If the critical substation is used as the starting point of the proposed one-by-one process, it is easier to obtain intermediate information associated to the reliability analysis. This information reflects how the outages of the critical substation would affect the reliability of the rest of substations of the sub-transmission system. In the simulations only one critical substation

has been detected, e.g. Bus 06 in IEEE RTS, but it is possible that a higher number of critical substations appear in another generation-load scenario of this test network.

Another important aspect is that the simulations for the IEEE-RTS reveal that the worst case from a reliability system viewpoint is the same as that from a load-bus viewpoint.

B. Spanish Regional 132/66 kV Subtransmission Network

A 132/66 kV Spanish regional subtransmission network (Figure 4) has also been analyzed with the proposed approach. The subtransmission system is supplied from Bus 09 at 400 kV and from Bus 10 at 220 kV, which are the standard voltage levels of the transmission system in Spain. Additionally, internal generating units are located at Bus 09, and the 132 kV Bus 01 also receives generation from local co-generation plants.

In the 132 kV level all the buses to supervise (02, 03, 04, 06 and 07) have local loads supplied through radial feeders, the total being load 117.60 MW. A meshed external equivalent was used to model the 220 kV and 400 kV transmission network. Moreover, this external equivalent has been assumed to be perfectly reliable, hence its unavailability (probability of failure) is zero.

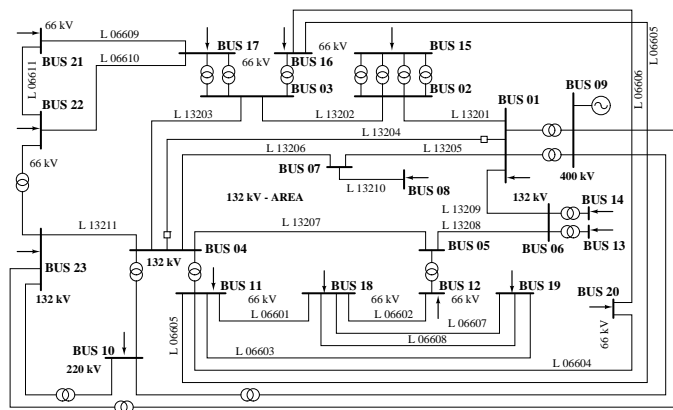


Fig. 4. Regional 132/66 kV Spanish Subtransmission Network.

As in the study of the IEEE-RTS network, the reference for the comparative analysis has been obtained from a state enumeration process extended to the 132 kV subtransmission system, including the configuration of all the substations. The calculated probabilities are shown in Table IX for a pre-selected contingency order “ $N - 2$ ”. For this real subtransmission system, the states with a contingency level higher than “ $N - 2$ ” have a 0.0000175 pu (Table IX). This probability means that 0.00175 % of the states are not considered in the reliability analysis.

TABLE IX
PROBABILITIES CONSIDERED AS REFERENCE

Contingency Level	States Number	$P(s)$ in pu Level	$P(s)$ in pu Accumulated
N-0	1	0.9272510	0.9272510
N-1	185	0.0710172	0.9982682
N-2	17020	0.0017143	0.9999825
$N > 2$	—	0.0000175	1.0000000

In Table X, EPNS and LOLP reliability indices of the 132 kV monitored buses are shown for the simulation mentioned above (load-node viewpoint). These reliability indices have been taken as the reference in the comparative study presented below. Also the load at each node has been included in Table X.

TABLE X
RELIABILITY INDICES CONSIDERED AS REFERENCE

BUS	Load (MW)	EPNS (MW)	LOLP (%)
02	67.4	3.1865	7.2731
03	14.4	0.0472	0.4814
04	15.2	0.0715	0.4764
06	20.2	0.0556	0.5524
07	0.4	0.0043	0.8043

According to the proposed methodology (Sec. III-A), in the first stage the reliability analysis of the branch-node model of the 132 kV subtransmission system is performed by applying a state enumeration process.

For this first analysis, the probabilities $P(s)$ that have been obtained by means of a “ $N - 2$ ” contingency level are shown in Table XI, and the EPNS and LOLP reliability indices (from a system viewpoint) that have been calculated for the 132 kV monitored buses appear in Table XII. This state enumeration process has a 0.0000103 pu neglected probability, shown in Table XI for the “ $N > 2$ ” contingency level, meaning that a 0.00103 % of states are not considered in the reliability analysis.

TABLE XI
PROBABILITY OF THE CONTINGENCY LEVELS USING THE BRANCH-NODE MODEL

Contingency Level	States Number	$P(s)$ in pu Level	$P(s)$ in pu Accumulated
N-0	1	0.9316703	0.9316703
N-1	15	0.0669256	0.9985959
N-2	105	0.0013938	0.9999897
$N > 2$	—	0.0000103	1.0000000

TABLE XII
RELIABILITY INDICES USING BRANCH-NODE MODEL

BUS	Load (MW)	EPNS (MW)	LOLP (%)
02	67.4	2.9689	6.8320
03	14.4	0.0314	0.3445
04	15.2	0.0523	0.3450
06	20.2	0.0370	0.3680
07	0.4	0.0027	0.6393

At this stage of the analysis and in terms of the LOLP index (Table XII) from the system viewpoint analysis reveals that the most critical substation corresponds to Bus 02 with a LOLP index of 6.8320 %. The substation associated to this bus has four 132/66 kV step-down transformers to supply the 66 kV network level, and the electrical bus scheme is a Double Bus System.

Once the critical substation is detected (in terms of the indices of the system), a second reliability analysis is performed assessing substations “one-by-one” in order to obtain a more accurate reliability indices (see Sec. III-B). The regional subtransmission network (132 kV) is analyzed including substations “one-by-one” considering their respective configuration.

This analysis of all substations is performed following an enumeration process and the results of the simulations are shown in Table XIII and Table XIV for the reliability indices LOLP and EPNS, respectively. As in the study of the IEEE-RTS network, the non-diagonal elements of these tables represent the impact that the failures of equipments of a substation have in the rest of substations of the subtransmission system, and the diagonal elements reflect how the reliability indices of a substation are affected by the failure of equipments of the mentioned substation. The substations located at Bus 01 and Bus 05 do not have load nodes connected, but the failure of their equipments affects their own reliability indices and the reliability indices of the rest of substations (correlated load-shedding).

TABLE XIII
LOLP (%). CORRELATION BETWEEN SUBSTATIONS (132 kV REGIONAL NETWORK)

Outages Sub	Correlated		Sub		
	02	03	04	06	07
01	0,0710	0,0237	0,0237	0,0336	0,0355
02	0,1016				0,0237
03	0,1054	0,0175			0,0118
04	0,1030	0,0237	0,0354	0,0354	0,0177
05	0,0256	0,0254	0,0254	0,0508	0,0254
06	0,0449	0,0297	0,0297	0,0746	0,0297
07	0,0238	0,0180	0,0180		0,0237

TABLE XIV
EPNS (MW). CORRELATION BETWEEN SUBSTATIONS (132 kV REGIONAL NETWORK)

Outages Sub	Correlated		Sub		
	02	03	04	06	07
01	0,0348	0,0021	0,0036	0,0024	0,0001
02	0,0507				0,0001
03	0,0447	0,0025			0,0001
04	0,0481	0,0027	0,0047	0,0036	0,0001
05	0,0172	0,0037	0,0039	0,0052	0,0001
06	0,0265	0,0043	0,0045	0,0074	0,0001
07	0,0101	0,0005	0,0027		0,0001

The reliability indices in Table XIII and Table XIV have been calculated by assessing the substations one-by-one jointly with the branch-node model of the 132/66 kV subtransmission system that is assumed ideal, and for a “ $N - 2$ ” contingency level. The most critical substation in this analysis corresponds to Bus 02. For simplicity, the reliability indices obtained for the critical substation (Bus 02) are the only ones shown in Table XV.

TABLE XV
BUS 02 RELIABILITY INDICES ASSESSING SUBSTATIONS SEQUENTIALLY

BUS	Scheme	$P(N - 0)$ <i>pu</i>	EPNS (MW)	LOLP (%)
01	Main&Transfer Bus	0.9992904	0.0348	0.0710
02	Main&Transfer Bus	0.9989844	0.0507	0.1016
03	Main&Transfer Bus	0.9989455	0.0447	0.1054
04	Main&Transfer Bus	0.9989699	0.0481	0.1030
05	Double Breaker	0.9997440	0.0172	0.0256
06	Ring	0.9995512	0.0265	0.0449
07	Double Breaker	0.9997618	0.0101	0.0238

These indices (Table XV) show how the outages of the substations of the 132 kV area affects the reliability indices of the

substation located in Bus02. Table XV also contains (second column) the bus system configuration of the substations of the 132 kV regional network.

For example, when only the failure of the equipments of the substation located in Bus01 are considered, the reliability indices EPNS and LOLP in Bus02 are, respectively, 0.0348 MW and 0.0710 % (Table XV). These indices are affected by a 0.9992904 *pu* probability $P(N = 0)$ (Table XV), which is the probability that this correlation does not occur.

A sufficiently accurate result for the reliability indices of the critical substation (Bus02) and for its load-node (down side of the transformers) can be obtained by using equation (14) (Secc. III-C). The results are exposed below for the LOLP and EPNS reliability indices.

$$\begin{aligned} \widehat{\text{LOLP}}_{(02)} &= \left[\frac{6.8320 \cdot 0.9999897}{0.9316703} + \frac{0,1016}{0.9989844} + \left(\frac{0,0710}{0.9992904} + \frac{0,1054}{0.9989455} + \frac{0,1030}{0.9989699} + \frac{0,0256}{0.9997440} + \frac{0,0449}{0.9995512} + \frac{0,0238}{0.9997618} \right) \right] \\ &\cdot 0.9316703 \cdot 0.9989844 \cdot 0.9962683 = 7.2406 \% \\ \widehat{\text{EPNS}}_{(02)} &= \left[\frac{2.9689 \cdot 0.9999897}{0.9316703} + \frac{0.0507}{0.9989844} + \left(\frac{0.0348}{0.9992904} + \frac{0.0447}{0.9989455} + \frac{0.0481}{0.9989699} + \frac{0.0172}{0.9997440} + \frac{0.0265}{0.9995512} + \frac{0.0101}{0.9997618} \right) \right] \\ &\cdot 0.9316703 \cdot 0.9989844 \cdot 0.9962683 = 3.1702 \text{ MW} \end{aligned} \quad (15)$$

By comparing these two indices (LOLP and EPNS) with the reference ones (Table X), the errors are:

$$\text{LOLP}_{(02)} \mid_{\text{Error}} \% = \frac{(0.9999825 \cdot 7.2731) - 7.2406}{(0.9999825 \cdot 7.2731)} \cdot 100 = 0.44 \%$$

$$\text{EPNS}_{(02)} \mid_{\text{Error}} \% = \frac{(0.9999825 \cdot 3.1865) - 3.1702}{(0.9999825 \cdot 3.1865)} \cdot 100 = 0.51 \%$$

It is important to note that from an engineering viewpoint, both errors are acceptable in a 24 hours-ahead plan for maintenance, reinforcement and operation of the subtransmission network.

Finally, a comparative of the computational time is shown in Table XVI. Just like in Table VIII of the IEEE-RTS section, in Table XVI the showed simulations are: number 1 for the reliability analysis of reference, considering the complete model of the system, number 2 for the reliability analysis considering the single-node model of the system, and number 3 for the individual reliability analysis of all the substations, considering Bus02 as critical.

TABLE XVI
COMPARATIVE OF COMPUTATIONAL TIMES

Simul Type	States Number	Computational Time
1	5357	9.00 · <i>t</i>
2	154	1.00 · <i>t</i>
3	211	0.50 · <i>t</i>

* *t* = 1.00 minute

The simulations presented in this section show that, as in the case of the IEEE-RTS, the proposed one-by-one process allows

an easier calculation of the information associated to how the outages of the critical substation affect to the reliability of the other substations of the subtransmission system. Furthermore, in the simulations only one critical substation has been detected, e.g. Bus 02, but is possible that a higher number of critical substations appear in another generation-load scenario of the test network.

Finally, as we saw in the previous case (IEEE-RTS network), for the Spanish regional subtransmission network the simulations show that the worst case from a reliability system viewpoint is the same as that from a load-bus viewpoint.

V. CONCLUSIONS

This paper focuses on the need to include the configuration of substations in the reliability assessment of meshed subtransmission networks in order to determine, in a first stage, the critical load-points, and to obtain, in a second stage, accurate reliability indices for the critical load-points. The state enumeration approach has been used in both stages. In the first one, the branch-node model of the system is analyzed while in the second the configuration of the substations are assessed in a “one-by-one” process. Finally, the reliability indices of the critical load-points are calculated by using an analytical process.

The proposed methodology allows to deal with realistic subtransmission systems, including substations’ configuration, with acceptable computational times for short-term reliability assessment. The advantages of the proposed enumeration technique are, first, determining the correlated impact of failures between substations, that cannot be obtained by traditional reliability studies and, second, the assessment of both subtransmission system’s reliability and substations’ reliability in a faster way than the one performed by traditional reliability studies.

Test results have remarked the importance of including substation configuration of critical buses, specially when the system is not close to overload states, such as the 138 kV IEEE-RTS subtransmission system. For both systems studied in this paper, the worst case from a reliability system viewpoint is the same that from a load-bus viewpoint.

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