A procedure to track vibration modes under changing external factors: application to a pedestrian bridge

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ABSTRACT

A low-cost vibration monitoring system has been developed and installed on an urban steel- plated stress-ribbon footbridge. The system continuously measures: the acceleration (using 18 triaxial MEMS accelerometers distributed along the structure), the ambient temperature and the wind velocity and direction. Automated output-only modal parameter estimation based on the Stochastic Subspace Identification (SSI) is carried out in order to extract the modal parameters, i.e., the natural frequencies, damping ratios and modal shapes.

Thus, this paper analyzes the time evolution of the modal parameters over a whole-year data monitoring. Firstly, for similar environmental/operational factors, the uncertainties associated to the time window size used are studied and quantified. Secondly, a methodology to track the vibration modes has been established since several of them with closely-spaced natural frequencies are identified. Thirdly, the modal parameters have been correlated against external factors. It has been shown that this stress-ribbon structure is highly sensitive to temperature variation (frequency changes of more than 20%) with strongly seasonal and daily trends.

Keywords: Modal analysis; Footbridge; Continuous dynamic monitoring; Environmental effects.

1. INTRODUCTION

The long-term vibration monitoring of civil engineering structures is increasingly used to monitor both, the structure integrity and the vibration serviceability. A number of examples of bridges and footbridges equipped with a monitoring system can be found [7, 9, 10, 16]. In these monitoring systems, the vibrations due to traffic or wind are measured and recorded. These data can be used to continuously extract the modal parameters (natural frequencies, damping ratios and modal shapes) which may be employed to assess the structural integrity since structural damages lead to changes in the modal parameters. Thus, vibration-based Structural Health Monitoring (SHM) systems using automated output-only modal identification (also known as Operational Modal Analysis, OMA) have been extensively proposed, for instance [8, 13]. One of the main problems of these systems is that they have

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to cope with changing environmental/operational conditions that often significantly affect the modal estimation.

The main obstacle for a more widespread adoption of these monitoring systems is the cost associated to their installation. Recently, a successful attempt to reduce the cost of a monitoring system has been carried out through the use of low-cost sensors to measure structural vibrations on a footbridge. Thus, a low-cost continuous vibration monitoring system using small MEMS (Micro Electro-mechanical Systems) accelerometers has been developed and installed on Pedro Gómez Bosque (PGB) footbridge (Valladolid, Spain) in order to track its long-term dynamic performance. The system continuously measures: the acceleration (using 18 triaxial MEMS accelerometers distributed along the structure), the ambient temperature and the wind velocity and direction [18]. PGB footbridge is a singular steel-plated stress-ribbon footbridge with a single span of 85 m. The dynamic behavior shows several low-frequency vibration modes with closely-spaced natural frequencies and low damping ratios.

A procedure to track vibration modes has been programmed to carry out an automatic modal parameter estimation. This procedure has to cope with vibration modes with closely-spaced natural frequencies. In this case, sorting vibration modes using frequency criteria and removing outliers that lay out of intervals of frequency variation might not be adequate [14, 16]. This paper describes the detailed procedure carried out to track vibration modes of PGB footbridge over time. The procedure uses three tolerances based on the Modal Assurance Criterion (MAC). This approach allows reliable monitoring of vibration modes that are maintained over time.

Regarding the use of modal parameter variations as possible damage detector in structures, it is necessary to observe the modal variations over time due to the influence of some external agents acting on the undamaged structure [12, 14]. It is important to evaluate the magnitude of this modal variation, as well as the main factors that influence the relationship between them. The temperature is usually the most significant environmental factor [20]. However, other factors, such as operational or boundary conditions, may significantly affect the modal estimates [1, 12]. Finite Element (FE) models are usually updated using the modal parameters [4, 15]. However, when the modal parameters show significant variability with changing environmental factors, the updated model might not be as accurate as required [2]. Hence, it is important to estimate uncertainties in the modal parameter identification in order to establish the degree of confidence of the updated model. This updated model can be useful to guarantee a successful structural intervention during structural rehabilitation [6, 11]. That is, non-destructive tests, such as an OMA, together with calibrated models greatly benefit reliable structural evaluations.

The paper continues with the description of the structure and its monitoring system. Section 3 analyzes the uncertainties associated to a single time history record. The output-only modal parameter estimation using different data blocks is carried out. Thus, uncertainties due to the selection of the data block are quantified. The results of continuous dynamic analysis are described in Section 4. A tracking method to follow the evolution of the persistent vibration modes over time is proposed. The influence of the environmental/operational factors on the modal estimates is studied. Finally, some conclusions are drawn and suggestions for future work are given.

2. THE FOOTBRIDGE AND ITS VIBRATION MONITORING

2.1. Structure description

PGB footbridge, sited in Valladolid (Spain), is a slender and lightweight structure that creates a pedestrian link over the Pisuerga River between a sport complex and the city centre (see Figure 1). This bridge, built in 2011, is a singular stress-ribbon footbridge born by a pretensioned catenary-shape steel band with a single span of 85 m that provides minimal impact on the surroundings. The structure mainly consists of a Corten steel band of 94 m long, 3.6 m wide and only 30 mm thick which is pre-tensioned and anchored to the two abutments. The complete steel band is fabricated by welding 8-meter long sheets and a number of 110 precast concrete slabs lay on the steel band [17]. These slabs do not have bearing capacity in such a way that the only structural element is the band. The structure is completed by rubber flooring and a stainless steel glass handrail.

Figure 1. *Pedro Gómez Bosque footbridge. Landscape view.*

2.2. Monitoring system

A structural vibration monitoring system was devised in order to continuously estimate the modal parameters of the structure and to assess their changes under varying environmental conditions. Therefore, apart from the accelerometers needed to perform a modal analysis, sensors for the wind and environmental temperature conditions were installed. The monitoring system comprises 18 triaxial accelerometers, 9 at each side of the deck, a temperature sensor and an anemometer with a vane. Wires and acceleration sensors were installed inside the handrail so the structure aesthetic was not modified in any way. This fact introduced additional complications: (i) the installation process was a laborious task, and (ii) additional angular transformations are required to obtain the acceleration vector in the structure axes for each accelerometer.

The vibration sensor used for the monitoring system was the low-cost MEMS accelerometer ADXL327 (ANALOG DEVICES) able to measure the static acceleration of gravity. The ADXL327 is a very small, low power, 3-axis accelerometer with signal condition voltage output. The key properties of this sensor are: measurement range up to ±2.5g, sensitivity up to 500 mV/g and bandwidth up to 550 Hz. However, this sensor is not designed to transmit the signal over long distances. To overcome this problem, an ad-hoc conditioning circuit was designed to enhance its long-distance performance. First, three capacitors, one to each channel, were placed to fix the frequency bandwidth to 100 Hz. Second, since the accelerometer has to be supplied by 3.6 V to get its maximum nominal sensitivity of 500 mV/g, the power supply unit of 12 V and a voltage regulator to 3.6 V were integrated into each circuit board in order to avoid power losses by the long distance wires. Thirdly, an operational amplifier was used to reduce significantly the output impedance, achieving thus a good signal-to-noise ratio for acceleration parameter (constant sensitivity and low noise). The achieved signal-to-noise ratio was 25 μg/√Hz, which was considered to be enough for monitoring the structural vibrations. Finally, each circuit board with all its components was covered with a plastic coating to protect it from environmental conditions. Then, the accelerometers were ready to be installed inside the tube of the handrail. The sensing system was completed by a temperature sensor (model T0110 transmitter of Comet) and a wind sentry (model 03002L of R.M. Young Company) to measure the speed and direction of the wind. The temperature sensor and wind sentry were installed on the public light tower sited closed to the structure (see Figure 2).

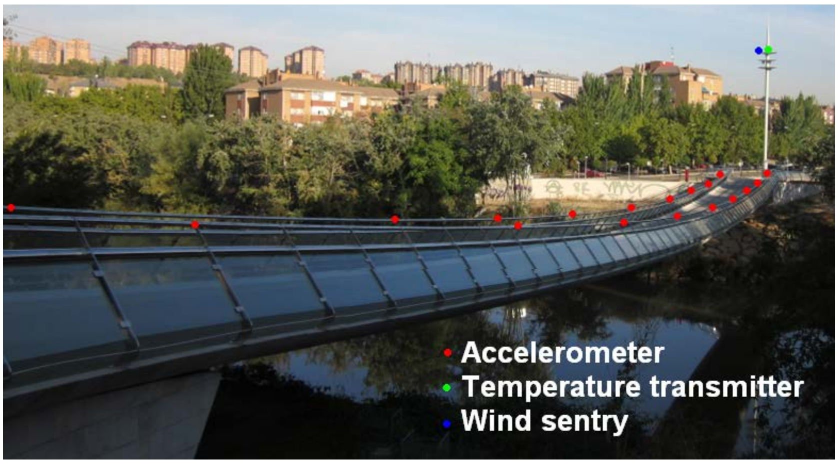


Figure 2. *Monitoring system. Distribution of sensors.*

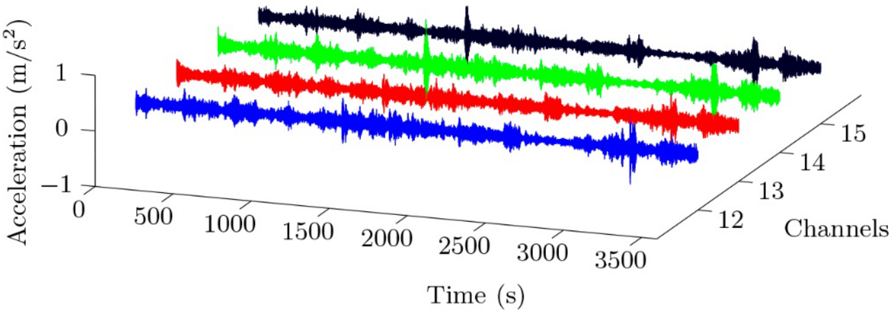
The monitoring system then comprises 57 voltage channels that are processed continuously. The data logger CompactRIO 9076 (National Instruments) with two NI 9205 with 32 analog input channels is used for real-time data acquisition. The frequency sampling for each channel was chosen to be 200 Hz, enough to identify the modal parameters of the structure and to avoid aliasing problems during the post-processing. The actual orientation of the accelerometers installed inside the handrail is unknown. However, the Euler angles between the accelerometer coordinate system and the structure coordinate system can be derived taking into account the following: (i) the longitudinal axis of each accelerometer matches with the longitudinal axis of the footbridge, and (ii) the accelerometers are able to measure acceleration due to gravity. Therefore, the transformation matrix between both coordinate systems is obtained, and then, the acceleration in the global axes can be finally calculated [18]. Before its final installation, a laboratory validation was carried out and after the installation, an in-situ validation was performed by comparing against conventional piezoelectric accelerometers.

3. PEERED ANALYSIS OF ONE TEST

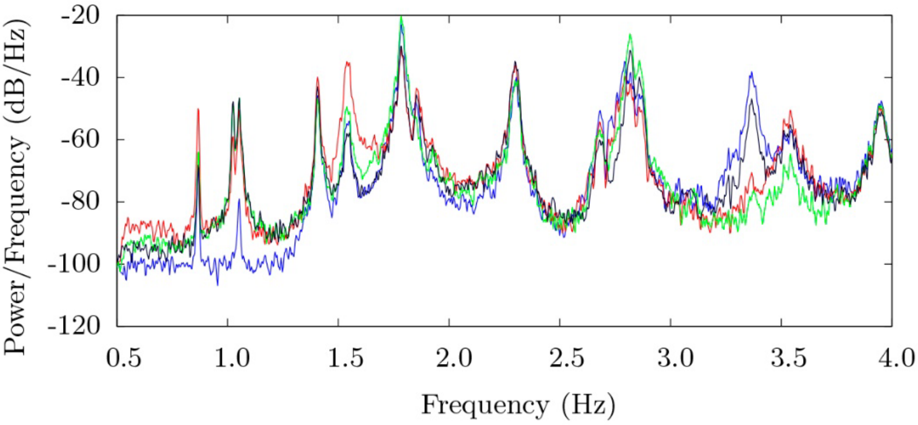
The uncertainties associated to the modal identification of one test, corresponding to the record measured on 01/05/2013 at 19:16 (60-minute test), are analyzed herein. The dynamic behavior of the structure is mainly governed by the vertical response (previous time-history analyses have shown that lateral and longitudinal accelerations are small and negligible, respectively, as compared to vertical ones). That is the reason why this analysis concentrates on the vertical vibration. The process followed is, firstly, the signal processing undertaken before the modal identification is presented. Afterwards, using SSI-cov technique is applied to different 20-minute time windows of the 60-minute test.

3.1. Data processing

The vertical response of the structure for 1-hour test is analyzed (with an initial sampling frequency of 200 Hz). The signal is filtered by a low-pass *Butterworth* filter of order 4 with a cutoff frequency of 5 Hz. A decimation factor of 16 is applied obtaining a *Nyquist* frequency of 6.25 Hz. As an example, Figure 3a and b show the filtered and decimated response in time and frequency domain, respectively, corresponding to the four central channels for upstream side. Up to ten peaks below 4 Hz can be observed in Figure 3b.



(a) Time domain.



(b) Frequency domain.

Figure 3. *Processed raw data of 4 central channels for upstream side.*

3.2. Operational Modal Analysis using the same SSI technique

The modal parameters have been tracked over time to capture the dynamic behavior of the structure as best as possible. As a rule of thumb, the minimum duration of the measurement should be at least

1000 cycles of the lowest natural frequency is expected to be identified [19]. This duration is usually recommended as the minimum one for noise-contaminated signals and close vibration modes. Therefore, this duration, which corresponds approximately to 20 minutes, has been adopted here to carry out the modal tracking.

Three consecutive windows of 20-minute data are considered to identify the modal parameters using only SSI-cov. These results are compared with those obtained using the 60-minute record. Table 1 shows the estimates obtained from the four data blocks. A MAC correlation between the modal shapes estimated by three techniques has been carried out. Modes that exhibit a MAC value greater than 0.95 for all of the cross values are highlighted in bold. Modes with natural frequencies over 4 Hz (0.8 times the cutoff frequency of the filter) are not included. Modes that exhibit a MAC value greater than 0.95 for all of the cross values are highlighted in bold. The average temperature for each time window was 16.42, 13.84 and 11.56 °C, respectively, and 13.94 °C for the 60-minute record.

Table 1. Natural frequencies and damping ratios identified by SSI-cov and different time blocks.

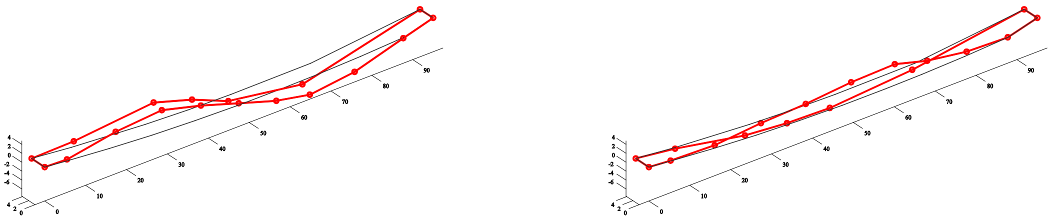
Mode	**60 min.**		**1st 20 min.**		**2nd 20 min.**		**3rd 20 min.**	
	f (Hz)	**ζ (%)**	**f (Hz)**	**ζ (%)**	**f (Hz)**	**ζ (%)**	**f (Hz)**	**ζ (%)**
-	0.8671	0.3352	0.8663	0.2526	-	-	0.8660	1.0824
-	-	-	1.0251	0.3793	1.0243	0.1433	1.0281	1.8147
1	**1.0548**	**0.9622**	**1.0536**	**0.3755**	**1.0518**	**0.3513**	**1.0535**	**0.4883**
2	**1.4077**	**0.3792**	**1.4073**	**0.3523**	**1.4059**	**0.5874**	**1.4093**	**0.2594**
3	**1.5408**	**0.6762**	**1.5408**	**0.5343**	**1.5395**	**0.7945**	**1.5431**	**0.7658**
4	**1.7869**	**0.4483**	**1.7884**	**0.5036**	**1.7882**	**0.6227**	**1.7833**	**0.4014**
5	**1.8571**	**0.6381**	**1.8567**	**0.6340**	**1.8574**	**0.6775**	**1.8569**	**0.4995**
-	-	-	1.9260	0.3870	-	-	-	-
6	**2.3020**	**0.4799**	**2.3022**	**0.4444**	**2.3020**	**0.5353**	**2.3014**	**0.4788**
-	2.6819	0.3206	-	-	2.6802	0.2709	2.6836	0.2546
-	-	-	-	-	-	-	2.7306	0.4594
-	-	-	-	-	2.7848	0.3825	-	-
-	2.8051	0.5159	2.8097	0.7520	-	-	2.8041	0.4918
-	-	-	-	-	2.8147	0.1976	-	-
-	2.8682	0.3238	-	-	-	-	2.8631	0.3248
-	-	-	2.9171	0.2887	-	-	-	-
7	**3.3674**	**0.4485**	**3.3680**	**0.4194**	**3.3659**	**0.4449**	**3.3709**	**0.4587**
8	**3.5375**	**0.8723**	**3.5356**	**0.6417**	**3.5361**	**0.9920**	**3.5343**	**0.8376**
9	**3.9487**	**0.5106**	**3.9443**	**0.4427**	**3.9458**	**0.5759**	**3.9552**	**0.3504**

Table 2 presents a summary of the selected modes. The table shows the mean values for the 20-minute tests and the maximum errors compared with the results obtained with the 60-minute test. In this case, the maximum errors obtained are 0.2855% and 61.09% for frequencies and damping ratios respectively. The errors in natural frequency estimates using 20 minutes as compared with 60 are negligible.

Table 2. Summary of identified mode frequencies and statistical comparison for the 20-minute against 1-hour time blocks: mean frequency, mean damping and corresponding errors.

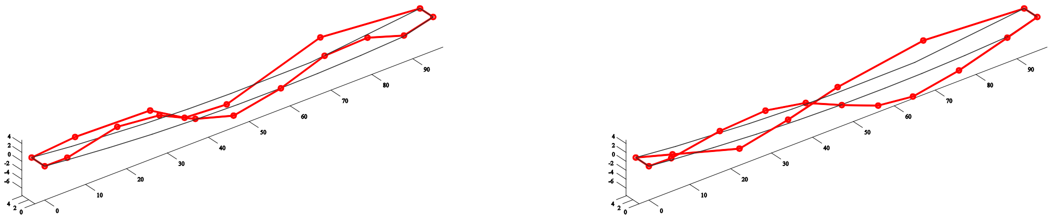
Mode	Frequency		Damping	
	$\bar{f}$ (Hz)	Error (%)	$\bar{\zeta}$ (%)	Error (%)
1	1.0530	0.2849	0.4050	**61.09**
2	1.4075	0.2416	0.3997	32.80
3	1.5411	0.2336	0.6982	26.02
4	1.7866	**0.2855**	0.5092	22.13
5	1.8570	0.0377	0.6037	17.80
6	2.3019	0.0348	0.4862	9.090
7	3.3683	0.1484	0.4410	3.930
8	3.5353	0.0905	0.8238	35.05
9	3.9484	0.2761	0.4563	22.55

Figure 4 shows an example of identified modal shapes corresponding to the lowest six modes obtained with SSI-cov for the 60-minute test.



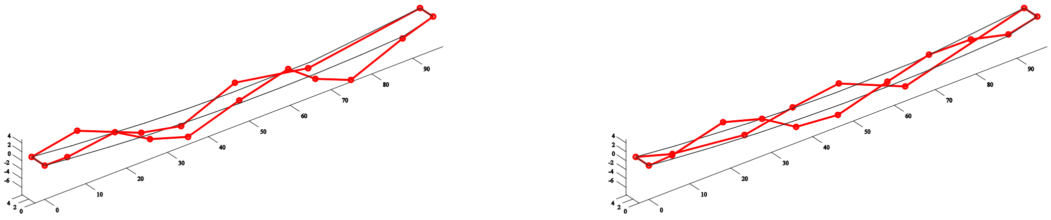
(a) f = 0.87 Hz

(b) f = 1.05 Hz



(c) f = 1.41 Hz

(d) f = 1.55 Hz



(e) *f = 1.79 Hz*

(f) *f = 1.86 Hz*

Figure 4. *First six modes shapes obtained with SSI-cov.*

4. CONTINUOUS DYNAMIC ANALYSIS

The procedure carried out to track the vibration modes is depicted in Figure 5. The results obtained from 1-year of continuous dynamic monitoring are described from now on. A method to track the evolution of the main vibration modes over time is described. Then, the influence of the environmental/operational factors on the modal estimates is studied.

4.1. Tracking of modal properties

For a single test, no special treatment on SSI itself to separate closely-spaced frequencies is considered. However, in a dynamic continuous monitoring framework, the vibrations modes must be carefully tracked since their natural frequencies can be similar and even can change their order. Therefore, a procedure to link modal parameters identified from each data set that are associated with the same vibration mode is needed. A criterion based on selecting a mode within a fixed frequency range could lead to wrong results due to the variation of frequency ranges between different tests. As a consequence, it proposes a tracking method in order to follow the evolution of the main vibration modes over time. From the acquired data, an automated OMA has been implemented using SSI-cov. The procedure carried out is depicted in Figure 5. The process can be divided into the following steps:

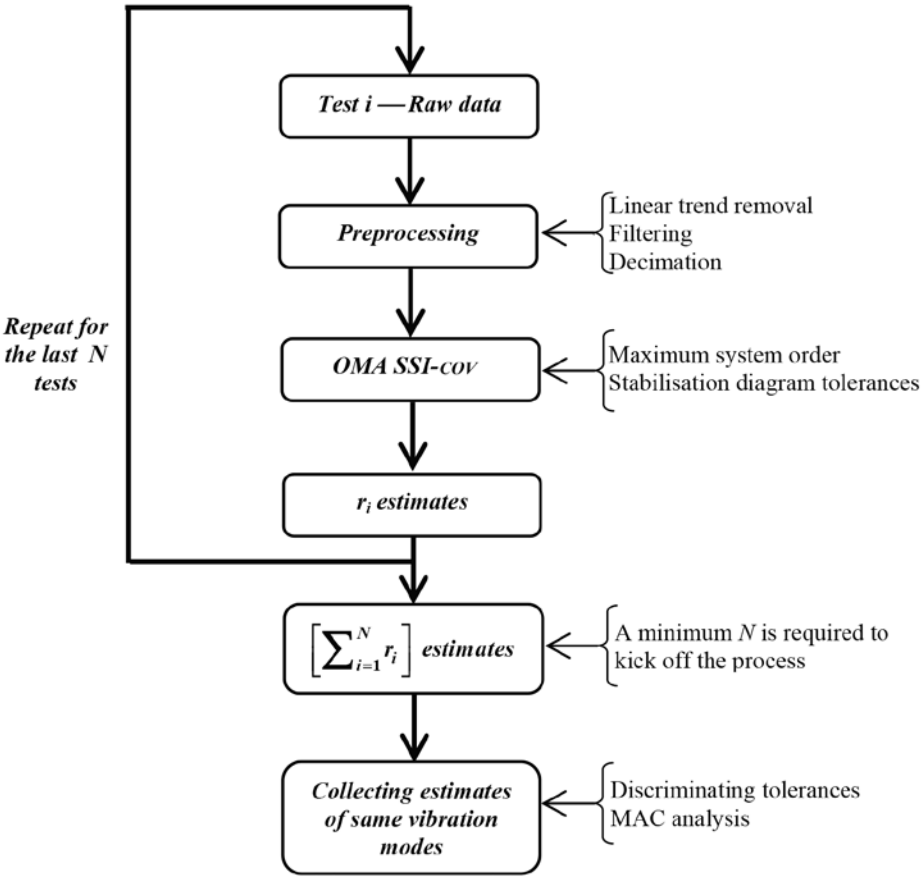


Figure 5. *Tracking procedure for modal parameters.*

- Take the last *N* tests, *N* being the number of tests. *N* should be representative of the variability of the modal estimates.
- For each test, the OMA is carried out. Thus, a number of modes for each test is identified and denoted as r_i (with $i = 1 \ldots N$).
- The objective is to find modal estimates that correspond to the same persistent mode over time. Each modal shape estimate is compared with all the remainder estimates within the *N* tests and a counter increases each time that two mode shapes match. The counter indicates the repeatability of a vibration mode. This is done for all the modal shape estimates. To carry out this step, the MAC value is used. That is, a number of $\left[\sum_{i=1}^{N} r_i\right]$ estimates are compared using the MAC value. Three tolerances have been defined for this purpose:
    - tol_1: the first one is a MAC value that allows to group estimates corresponding to the same vibration mode. The repeatability is the number of estimates of a group.
    - tol_2: the second one is included in order to reject groups that are not repeated sufficiently to be considered as significant modes. This tolerance is the lower limit of success ratio, that is, groups with low repeatibility are automatically rejected.
    - tol_3: the third one is a MAC value finally included to detect groups of estimates that are actually estimations of the same mode. Then, if more than one group correspond to the estimation of the same mode, the one with higher repeatability is selected.
- From above procedure, the most significant modes of the dynamic response of the structure are detected for the *N* tests.
- Variations of these selected modes are statistically studied and corrected modal parameters are derived.

The above tracking method is applied to consecutive time-history records every 20 minutes using SSI-cov. A number of $N = 21643$ tests have been taken for the whole year 2013. If computational burdens appear, the procedure explained above must be modified slightly. In that case, the tests should be divided into sub-blocks ($N = 500$) to solve the problem. That done, you just have to join each sub-block with the next sub-block using the tol_1.

The tolerances chosen to carry out the tracking were: $tol_1 \geq 0.95$, $tol_2 \geq 40\%$ and $tol_3 \leq 0.80$. Up to nine vibration modes below 4 Hz have been tracked. Table 3 shows the following statistics of the estimation: mean, standard deviation, absolute percentage variation and their repeatability (the success ratio is included between brackets). Note that the fourth mode, with a frequency around 1.79 Hz (and with a damping ratio of only 0.42%) is prone to be excited by pedestrian walking.

Table 3. Summary of identified natural frequencies and damping ratios for one year monitoring and their statistics: mean frequency, mean damping, standard deviation and corresponding variation.

Mode	**Frequency**			**Damping**			**Repeatability**
	$\bar{f}$ **(Hz)**	**Std**	υ **(%)**	$\bar{\zeta}$ **(%)**	**Std**	υ **(%)**	
1	1.0482	0.0152	14.23	0.3665	0.1710	147.89	9667 (44.7%)
2	1.4145	0.0107	35.26	0.3381	0.1513	110.74	10619 (49.1%)
3	1.5440	0.0181	27.63	0.6498	0.2357	133.62	9886 (45.7%)
4	1.7937	0.0291	20.27	0.4192	0.1502	221.88	**13817 (63.8%)**
5	1.8594	0.0168	6.87	0.5718	0.1605	234.74	9936 (45.9%)
6	2.3117	0.0425	15.01	0.3753	0.1474	128.54	8746 (40.4%)
7	3.3821	0.0549	42.95	0.3868	0.1191	103.96	12210 (56.4%)
8	3.5512	0.0524	51.87	0.7226	0.1884	157.48	9237 (42.7%)
9	3.9619	0.0624	8.95	0.3853	0.1185	230.82	10183 (57.8%)

The frequency distribution for each mode is shown in Figure 6. There are some modes (3, 5 and 8) whose frequencies have a very narrow distribution, indicating that: (i) they do not change significantly and (iii) they might be already used for SHM. Figure 7 shows the damping ratio distribution for each mode. It can be observed that the damping of these modes of the structure are very low, except for modes 3, 5 and 8. These modes are the same which also have a very narrow distribution frequency.

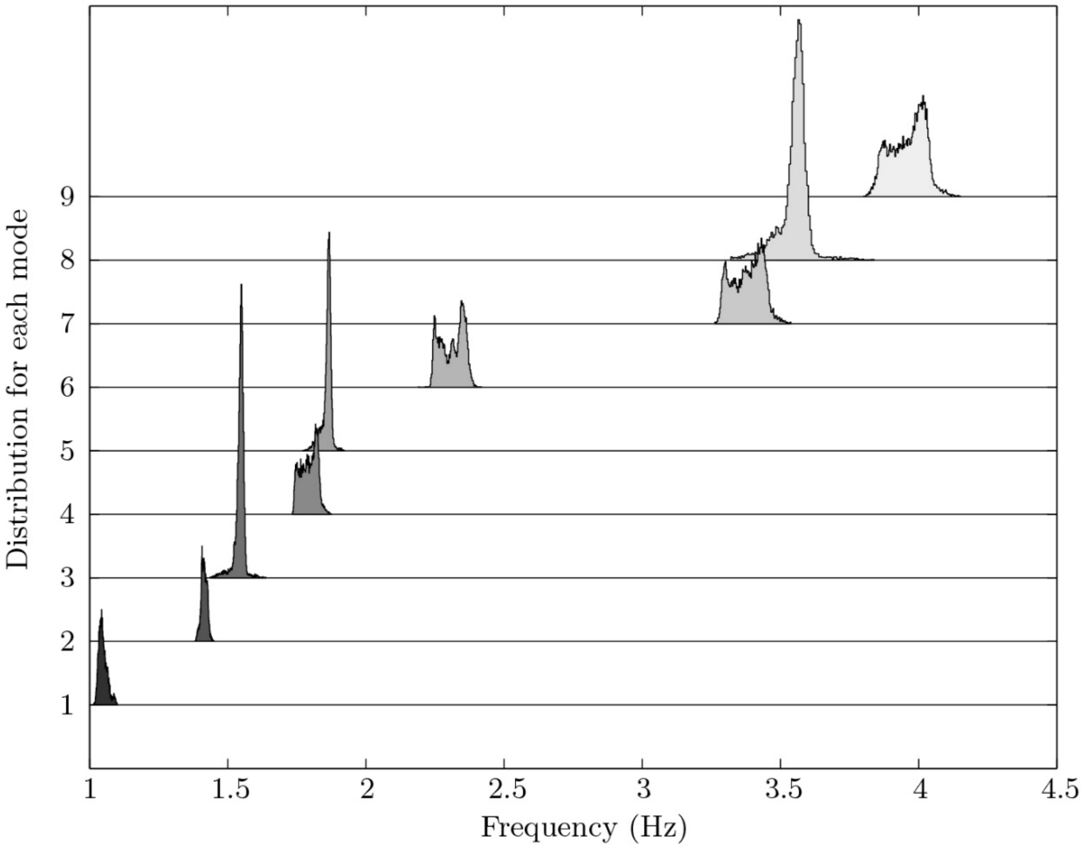


Figure 6. *Overlaid distributions of the identified natural frequencies.*

It can be seen that the repeatability of modes is quite lower than total number of tests. Figure 8 shows the time history and PSD results of two tests at day and night time corresponding with 15:00 and 3:00 hours respectively. Although, vibration levels are similar, the frequency contents show significant differences between both tests. We should comment that the signal to noise ratio and the sensitivity of the conditioned sensors is not high, anyway, when the structure is in use, structural vibrations arise and the monitoring system has success in the estimation. Remind that the monitoring system is much cheaper than a conventional one.

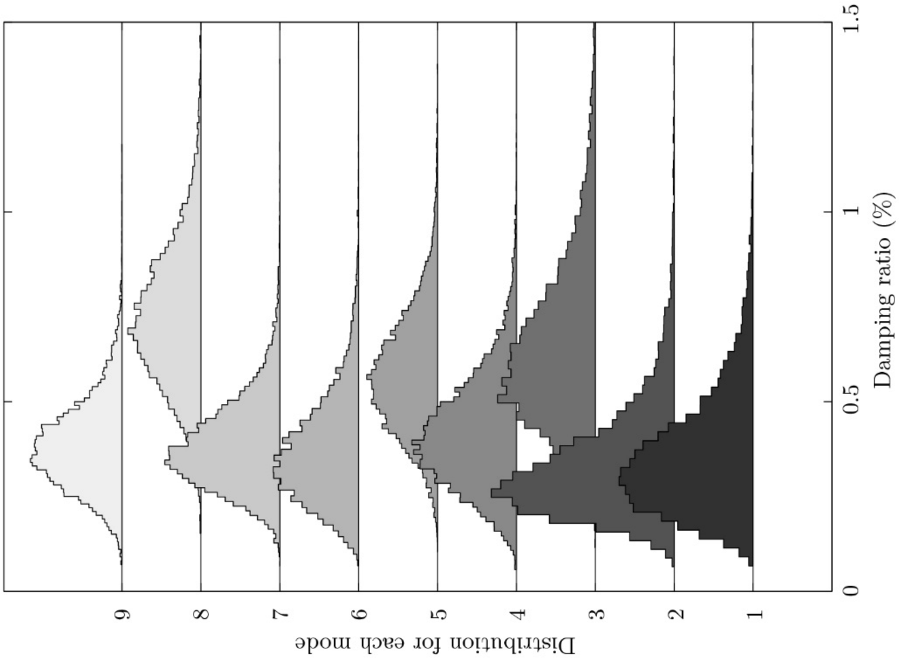
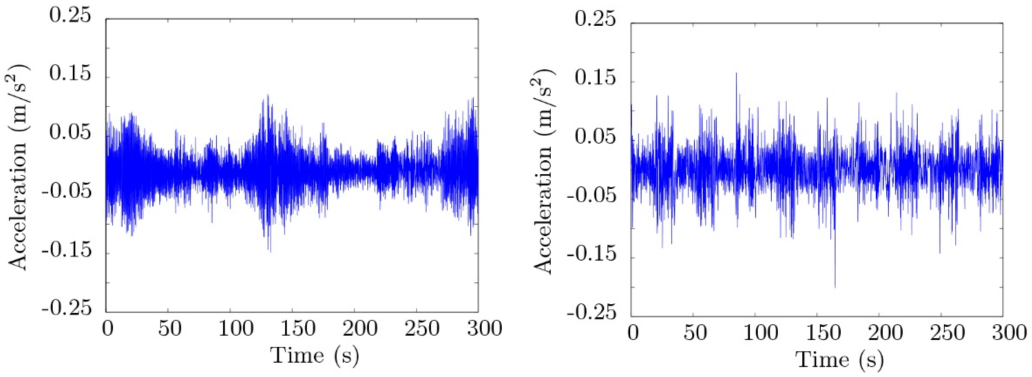
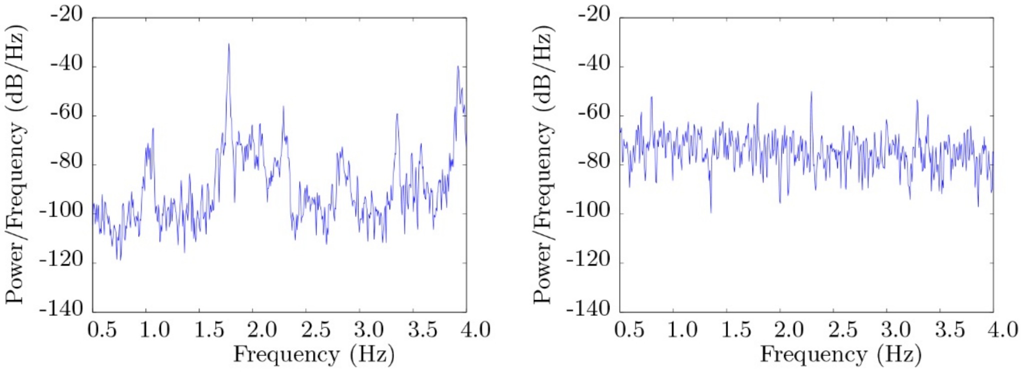


Figure 7. *Overlaid distributions of the identified natural damping ratios.*



(a) Time history day test.

(b) Time history night test.



(c) PSD day test.

(d) PSD night test.

Figure 8. *Results at day and night test.*

This is due to the fact that some of them are performed under very low vibration conditions leading to wrong estimates that, obviously, are not tracked by the method. The pedestrian traffic over the structure during the day hours induces on the structure the necessary environmental energy that allows successful identification. However, the lack of this excitation during the night hides the real structural response inside the signal noise. Figure 9 shows hourly distribution of successful identifications obtained for the fourth mode selected (see Table 3).

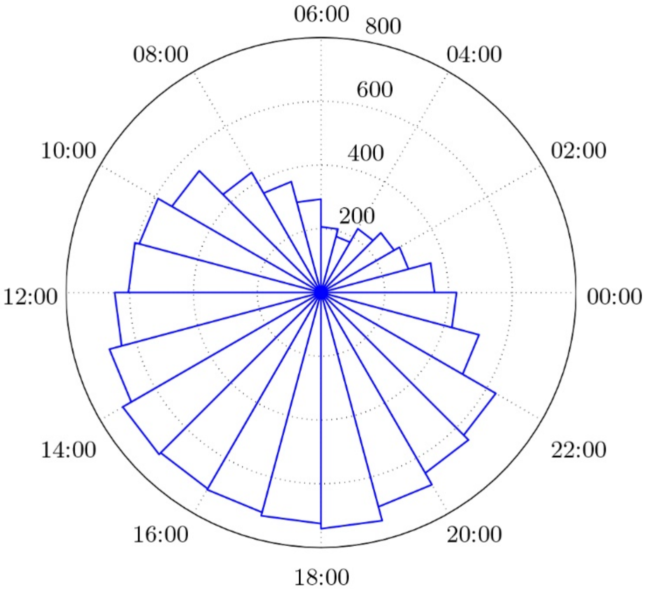
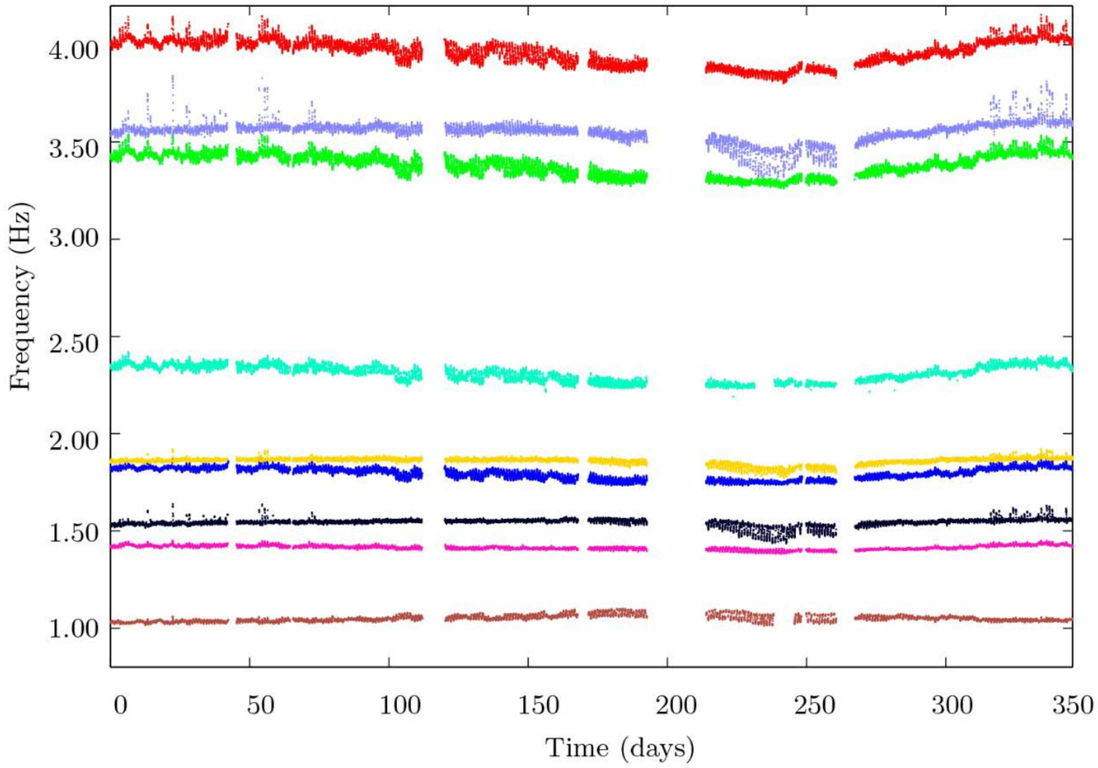
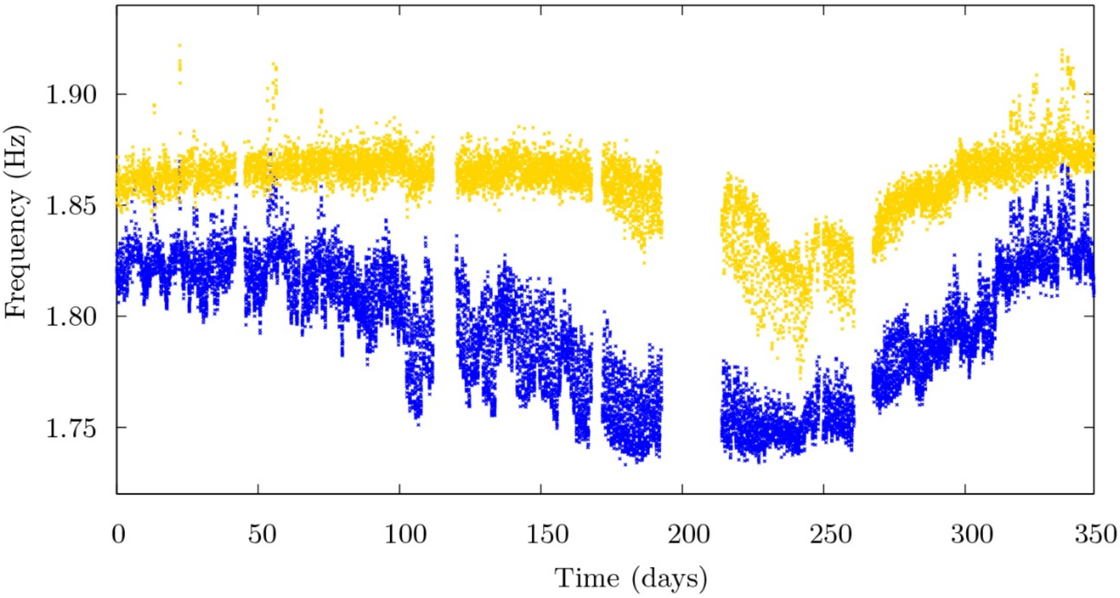


Figure 9. *Distribution per hour of the repeatability for mode 4.*

The time variation of the natural frequency estimates over a year for the lowest ninth modes is shown in Figure 10a. Figure 10b presents a zoom showing that the tracking method is able to identify two closely-spaced natural frequencies. Although the monitoring has been operating since January 1st 2013 up to now, some occasional stops due to minor technical problems and maintenance tasks are observed in the figure.



(a) Complete view.

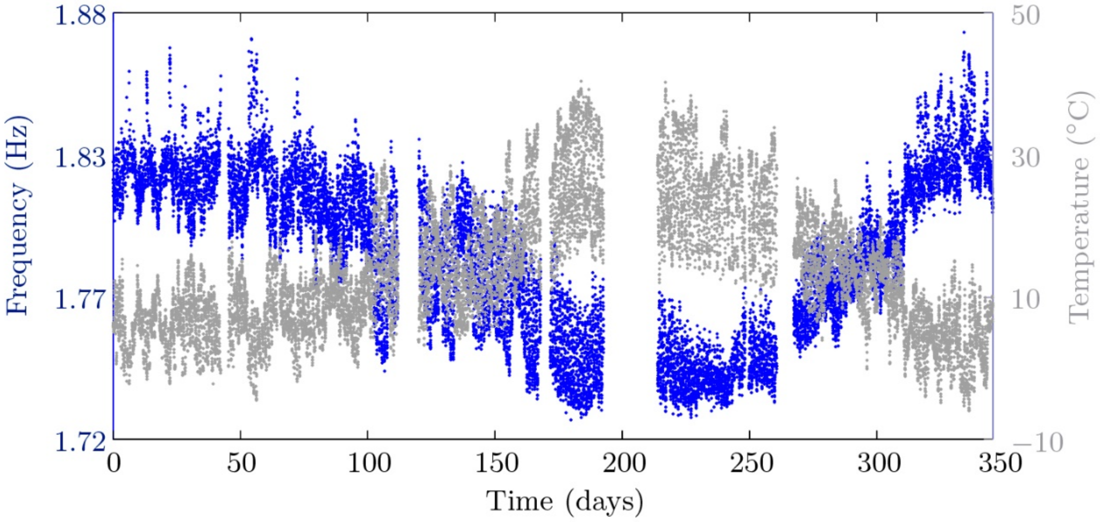


(b) Zoom view of the fourth and fifth vibration modes.

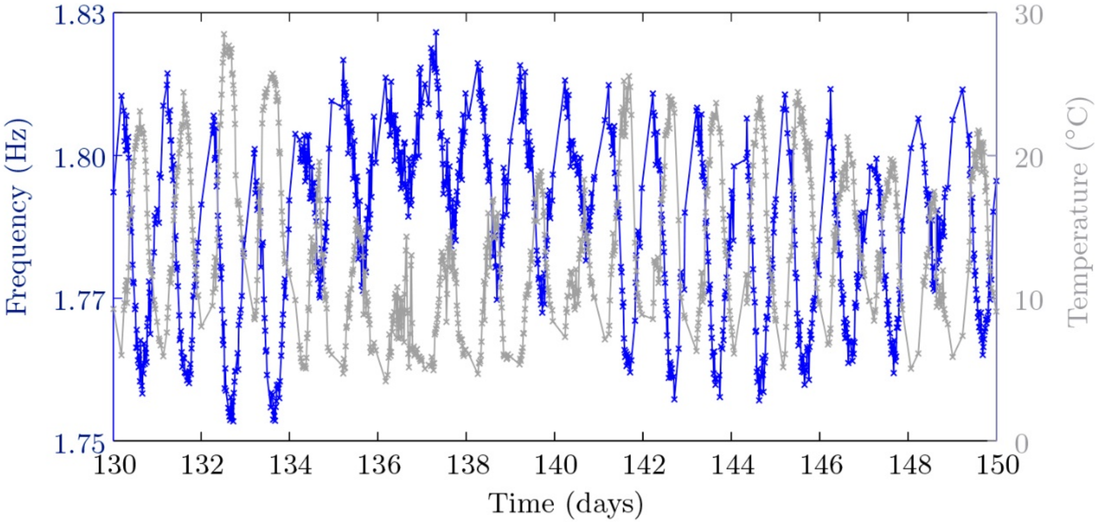
Figure 10. *Tracked frequency estimates for the whole year.*

4.2. Effects of external factors

A strong seasonal and daily trends with temperature have been identified. Figure 11 shows the time evolution of the frequency estimates of the fourth mode, in the left axis, and temperature, in the right axis. It is clearly observed the seasonal (Figure 11a) and daily trend (Figure 11b).



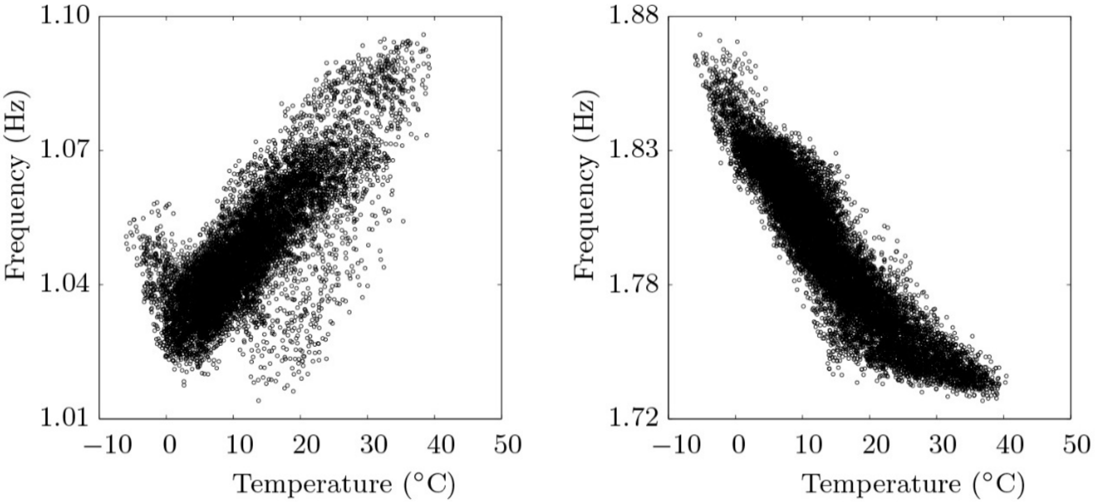
(a) Complete view.



(b) Zoom view of 20 days.

Figure 11. *Frequency estimates and temperature recorded for mode 4.*

Figure 12 shows the frequency estimates versus temperature for mode 1 (Figure 12a) and mode 4 (Figure 12b) of Table 3. It is noted that pure vertical response behaves similarity to the equivalent suspended cable as well as its thermal behavior [3]. Therefore, for these modes (such as mode 4), increasing temperature leads to an increase into the ribbon sag producing a reduction of band tension and leading to a decrease of their natural frequencies [5]. However, mode 1 (of Table 3 and Figure 4b) does not follow this pattern since this is a torsional mode.



(a) Mode 1.

(b) Mode 4.

Figure 12. *Frequency estimates versus temperature for modes 1 and 4.*

No other visual evidence of the correlation between the frequency estimates and other factors, different from the temperature, have been found (neither with the wind velocity nor the operational values). Regarding damping ratios, no clear visual dependencies with any external factor have been found.

5. CONCLUSIONS

A low-cost vibration monitoring system based on MEMS accelerometers has been successfully installed on a singular stress-ribbon footbridge and it is currently providing live data to be analyzed. It has been demonstrated that these low-cost sensors, carefully conditioned, can be a competitive alternative to traditional ones. Thus, using this innovate system, this paper has focused mainly on a procedure to track the modal parameters of structure and their time evolution over whole-year. For this particular stress-ribbon structure, it has been demonstrated that is highly-sensitive to temperature variation (frequency changes of more than 20%). Natural frequencies are quite close and time-varying (mainly due to temperature for the test structure). Thus, a tracking method based on three different tolerances and that makes use of the MAC value has been proposed.

Future work will consider the development of a FE model for this strongly non-linear structure and its model updating. Results obtained in this paper are essential for the development of a reliable model that might be used with a SHM system.

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