

ABSTRACT

Title: Semilinear order property and infinite games

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In this thesis we analyze the determinacy of the Lipschitz and Wadge games, as well as the tightly related semilinear ordering principle, in the setting of second order arithmetic and of the program of Reverse Mathematics. Firstly, we obtain direct proofs, formalizable in second order arithmetic, of the determinacy of Lipschitz and Wadge games for the first levels of the Hausdorff's hierarchy of differences. Then we determine the set existence axioms needed to formalize such proofs within the classical subsystems of second order arithmetic (\mathbf{RCA}_0 , \mathbf{WKL}_0 , \mathbf{ACA}_0 , \mathbf{ATR}_0 , and $\prod_1^0\text{-CA}_0$). Finally, in some cases we show that these axioms of existence are optimal, proving that they turn out to be equivalent (over a suitable weak subsystem as \mathbf{RCA}_0 or \mathbf{ACA}_0) to the corresponding formalization of the principles of determinacy or semilinear ordering. The main results are:

Theorem A. The following assertions are pairwise equivalent over \mathbf{RCA}_0 :

- (1) \mathbf{ACA}_0 .
- (2) $(\Sigma_1^0)_2\text{-Det}_L^*$ (determinacy of Lipschitz games for subsets of the Cantor space which are differences of closed sets).
- (3) $(\Sigma_1^0)_2\text{-SLO}_L^*$ (Lipschitz semilinear ordering for subsets of the Cantor space which are differences of closed sets).

Theorem B. The following assertions are pairwise equivalent over \mathbf{RCA}_0 :

- (1) \mathbf{ATR}_0 .
- (2) $(\Sigma_1^0 \cup \prod_1^0)\text{-Det}_L$ (determinacy of Lipschitz games for open or closed subsets of the Baire space).

Theorem C. The following assertions are pairwise equivalent over \mathbf{ACA}_0 :

- (1) \mathbf{ATR}_0 .
- (2) $\Delta_1^0\text{-Det}_L$ (determinacy of Lipschitz games for clopen subsets of the Baire space).
- (3) $\Delta_1^0\text{-SLO}_L$ (Lipschitz semilinear ordering for clopen subsets of the Baire space).