THE CYLINDRICAL CAPACITIVE MODEL FOR WATER TREEING DEGRADATION IN EXTRUDED HV CABLES

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INTRODUCTION

The initiation and growth of voids and microchannels filled with water and different ions in extruded polymeric insulation of HV cables is usually known in literature as 'water treeing', due to their fan and/or bush-like appearance. Water and electrical trees have revealed as some of the most important causes of cable breakdown detected from late sixties. Since then, many papers have been devoted to study the problem. Nevertheless, although large amounts of experimental results have been provided, there is a lack of physical-mathematical models which explain why those results are produced. In order to propose an adequate and specific model for extruded cables deteriorated by water treeing, we must take into consideration:

a) **Internal structure of water trees.**
b) **Cylindrical geometry of the device (cable) in which water trees are growing.**

INTERNAL STRUCTURE AND DIELECTRIC PROPERTIES

Inhomogeneity

Chen and Filippini [1] improved optical observation techniques of water tree voids, so that they could conclude every tree is made up of 'bouquets', which are alignments of microcavities (see Fig.1). From all this experimental evidence and complementary microscopic observations, it has been proved that internal structure of water trees is highly non uniform. There is an evident decrease in the number of microcavities as we move away from the base of the tree.

Dielectric behaviour

Several researchers had previously shown a dielectric behaviour for polyethylene (PE) affected by water trees (permittivity $\epsilon_r \sim 6$ at frequency $f=1500Hz$, Koo et al. [2]). Chen and Filippini [1] gave a further step by obtaining the dielectric permittivity, ϵ_r, as a function of the volume fraction of inclusions, A. It is absolutely basic for any physical model of water treeing the fact that permittivity ϵ_r increases with A. Besides, A grows as we approach the base of the water tree. All this implies that ϵ_r must be represented by a decreasing function versus distance as we move away from the base of the water tree on its central axis towards the PE unaffected by degradation. As the distance from the tip of the needle increases, A diminishes and permittivity decreases gradually from its maximum value to the minimum value corresponding to PE ($\epsilon_r=2.3$). We will see below the importance of the decreasing law of variation for ϵ_r, because the dielectric behaviour of water trees determines the electric field distribution in deteriorated cables. Several hypotheses for such a decreasing function will also be examined.

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CYLINDRICAL CAPACITIVE MODEL FOR WATER TREEING DEGRADATION IN A COAXIAL CABLE

Water treeing has been modelled in many ways. Summarizing, we will say that water trees were primarily considered as conductors by Ashcraft [3]. Then, Koo et al. [2] showed that the water tree behaviour was that of a dielectric, and finally Chen and Filippini [1] depicted them as inhomogeneous dielectric spheres of linearly decreasing permittivity, growing uniformly and radially from a small water spherical electrode. In the present work, we consider a multitude of vented water trees modelized as a dielectric cylindrical zone (Fig.2a). Such a situation can be found in cables failed in service [4].

Water tree degradation growing from the inner electrode

Geometric and physical properties of the model. We consider that the water-treed-degradated zone of a coaxial cable can be represented as a dielectric cylinder that grows uniformly and radially from a small cylindrical water electrode towards another electrode, a concentric conductor cylinder, as it is shown in Fig.2a. We supposed, in a first level model that the permittivity of the water-treed-region, $\epsilon_t(r)$ decreases linearly. The linear dependence for the decrease in $\epsilon_t(r)$ can be written mathematically by equation (1),

$$\epsilon_t(r)=\epsilon_t(r_1)-\frac{r-r_1}{r_2-r_1}[\epsilon_t(r_1)-\epsilon_1] \tag{1}$$

$\epsilon_t(r)$ ≡ permittivity of degradated cylinder at point M ($r_1 \leq r \leq r_2$)
r_1 ≡ radius of water electrode
r_2 ≡ *maximum radius of cylindrical degradated zone*
$\epsilon_t(r_1)$ ≡ permittivity at surface of water electrode (we assume $\epsilon_t(r_1)=3\epsilon_1$)
$\epsilon_t(r_2)$ ≡ permittivity of homogeneous PE non affected by treeing ($\epsilon_t(r_2)=\epsilon_1=2.3$)

Distribution of electric field, $E_t(r)$, due to water treeing. We will consider the electric field, $E_t(r)$, in every point M of the dielectric material, in or outside the region affected by treeing (if $r_1 \leq r \leq r_2 \rightarrow r \equiv r_{int}$; if $r > r_2 \rightarrow r \equiv r_{out}$). We will also refer the existing field in any point M of the dielectric material before the growing of trees (before the degradation of PE) as $E_i(r)$. From the distribution of permittivity shown in Fig.2a and equation (1), we can calculate the variations for the electric field $E_t(r)$ in every point M, during the growing of water trees.
Considering that the point M can be placed in or outside the degradated zone, the electric field $E_t(M)$ can be diminished or enhanced compared to its value $E_i(M)$ before the development of trees, we define the variation rate $k=E_t/E_i$. The capacitance per unit length of a new cable is represented by c_i while c_{ag} stands for the capacitance of the cable after the growing of water trees; c_t is the capacitance of the degradated region and c_e the capacitance of the remaining external zone.

$$c_i=\frac{C_i}{L}=\frac{2\pi\epsilon_0\epsilon_1}{\ln\frac{r_3}{r_1}} \; ; \; c_{ag}=\frac{c_t \cdot c_e}{c_t+c_e} \;\rightarrow\; c_t=\frac{C_t}{L}=\frac{2\pi}{\int_{r_1}^{r_2}\frac{dr}{r\epsilon_0\epsilon_t(r)}} \; ; \; c_e=\frac{C_e}{L}=\frac{2\pi\epsilon_0\epsilon_1}{\ln\frac{r_3}{r_2}} \tag{2}$$

Considerations about the electric charge contained *in the water tree lead to expressions for* k outside (k_{out}) and inside (k_{int}) the degradated region,

$$k_{out}=\left(\frac{E_t}{E_i}\right)=\frac{c_{ag}}{c_i}\quad ;\quad k_{int}=\left(\frac{E_t}{E_i}\right)_{int}=\epsilon_1\frac{k_{out}}{\epsilon_t(r)} \tag{3}$$

In figure 2b, we show the variation for k as a function of r/r_1 for a degradated region of length, $l=r_2-r_1=0.5r_1$ ($\Rightarrow r_2/r_1=1.5$) and permittivity at surface of water electrode, $\epsilon_t(r_1)=3\epsilon_1$, where $\epsilon_1=2.3$. Exploring figure 2b, we get:

a) Outside the deteriorated zone, the electric field E is amplified with a variation rate k, independent of the distance r and hence, of the distance to the degradated region.

b) Inside the deteriorated zone, k diminishes when approaching the inner electrode and, in the largest part of the degradated region, E is lower than in the absence of the tree ($k=1$), especially near the water electrode. It can be pointed out, even more precisely, that in the water tree zone near its border, the electric field equals and gets bigger than in the absence of treeing ($E_t(r)>E_i(r)$). Other functional dependencies were proved for the decreasing permittivity, $\epsilon_t(r)$. A decreasing exponential permittivity (i) and another function which we will refer as modified exponential permittivity (ii). For each one, taking into account the surrounding conditions at both ends of the water treed region, ($\epsilon_t(r_1)=3\epsilon_1=6.9$) and, ($\epsilon_t(r_2)=\epsilon_1=2.3$), we get:

$$(i)\quad \epsilon_t(r)=a\cdot\exp(-br)\Rightarrow a=\frac{\epsilon_1}{\exp\left(-\frac{r_2}{r_2-r_1}\cdot\ln\frac{\epsilon_t(r_1)}{\epsilon_1}\right)},\ b=\frac{\ln(\epsilon_t(r_1)/\epsilon_1)}{r_2-r_1}$$

$$(ii)\quad \epsilon_t(r)=a-\exp(br)\Rightarrow \exp(br_2)-\exp(br_1)=2\epsilon_1,\ a=\epsilon_1+\exp(br_2) \tag{4}$$

In figures 3a and 3b, we compare the permittivity dependencies and the amplification results for the geometric coefficient $r_2/r_1=1.5$.

Water tree degradation growing from the outer electrode

Features of the model. **A similar study can be performed when supposing that the degradated region grows from the outer semiconductor of the cable and when corresponding near symmetrical dependencies for permittivity, $\epsilon_t(r)$, are used if $r_3>r>r_2$. In Fig.4a we show the permittivity dependencies used. These dependencies would appear if the percentage value of degradated width for PE were $D.W.\equiv 75\%$. Taking into account the corresponding surrounding conditions used for the inner degradation, now we have, $\epsilon_t(r_3)=3\epsilon_1=6.9$, and $\epsilon_t(r_2)=\epsilon_1=2.3$; then, we find:**

$$a)\quad \epsilon_t(r)=a\cdot\log(br)\Rightarrow a=\frac{2\epsilon_1}{\log(r_3/r_2)},\ b=\sqrt{\frac{r_3}{2r_2}}\quad ;\ b)\quad \epsilon_t(r)=a+\exp(br)\Rightarrow \left|\begin{array}{l}\exp(br_3)-\exp(br_2)=2\epsilon_1\\ a=\epsilon_1-\exp(br_2)\end{array}\right.$$

$$c)\quad \epsilon_t(r)=ar+b\Rightarrow a=\frac{2\epsilon_1}{r_3-r_2},\ b=\frac{\epsilon_1(r_3-3r_2)}{(r_3-r_2)} \tag{5}$$

Electric field amplification. **As in the inner case, it can be shown that amplification factor, $k=(E/E_i)$ is the biggest for the modified permittivity and the lowest for the decreasing exponential permittivity, but for all of them, amplification is now directed towards the inner electrode. However, the most significant feature can be found in the fact that, due to the**

TABLE 1 - Field amplifications for different cases of treeing cable degradation

$\epsilon_r(r)$	DW(%) / Location	25% k / F.A.	50% k / F.A.	75% k / F.A.
modified permittivity	int.	1.18 / 18%	1.42 / 42%	1.71 / 71%
	ext.	1.09 / 9%	1.23 / 23%	1.44 / 44%
linear permittivity	int.	1.17 / 17%	1.37 / 37%	1.62 / 62%
	ext.	1.09 / 9%	1.22 / 22%	1.41 / 41%
decreasing exp. permittivity	int.	1.15 / 15%	1.32 / 32%	1.52 / 52%
	ext.	1.09 / 9%	1.20 / 20%	1.35 / 35%

cylindrical shape of the device, inner degradation zones (and hence inner water trees) could be more dangerous than outer degradation zones (outer water trees). In fact, amplification factor k is bigger when the water tree zone departs from r_1 towards r_3 (dotted line in Fig.4b) than when the water tree zone departs from r_3 towards r_1 (continuous line in Fig.4b), for the same degradated width (*D.W.=75%*). This evaluation has been done for the entirely symmetrical case, the linear case. In Table.1, amplification factor, k, and its percentage value of field amplification, *F.A.(%)*, are printed for different degradation widths and different permittivity dependencies for both, internal and external degradation cases.

FINAL CONCLUSIONS AND UTILITY OF THE MODEL

The cylindrical capacitive model for HV cable deterioration shows in an easy way that water trees have an important influence on the distribution of electric field in a dielectric material affected by the type of degradation known as 'water treeing'. In a more extended work, we will show that an even more realistic model, considering the existence of water trees growing from both the inner and outer electrodes, lead to next conclusion: it is more dangerous an internal degradation zone of double width (*50%* of PE width) than two smaller degradation regions, internal (*25%* of PE width) and external (*25%* of PE width), which sum of widths is the same, *50%=25%+25%*, that the first one.

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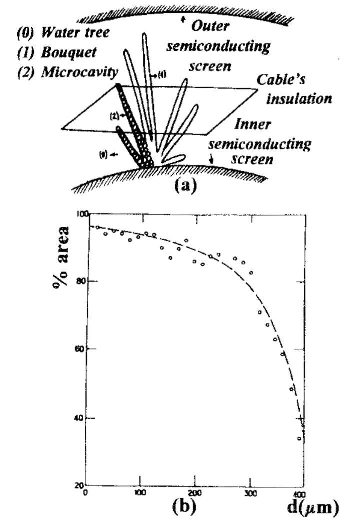

Figure 1: (a) Water tree structure. (b) Microc. percentage in cross microsection.

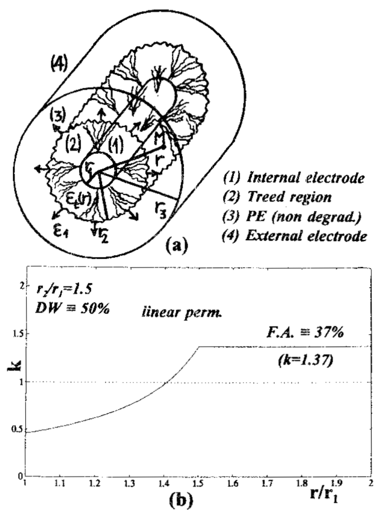

Figure 2: (a) Inner degradation model. (b) Amplification curve $k(r/r_1)$.

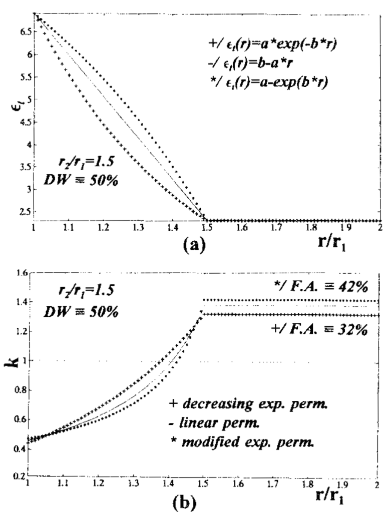

Figure 3: Different permitt. dependencies (a) and corresponding k curves (b).

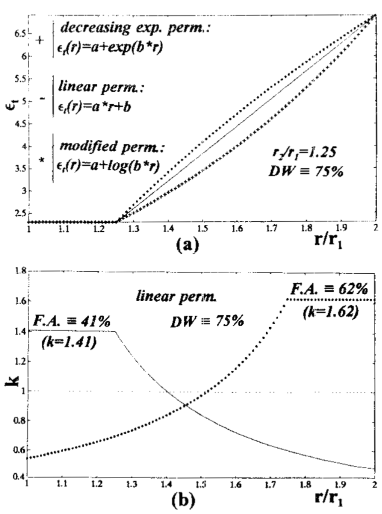

Figure 4: (a) Permit.dep. for outer model. (b) Comparing inner and outer models.