Downsian competition with participatory democracy*
(VERY PRELIMINARY VERSION, DO NOT QUOTE)

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Abstract

We study a scenario where elections are held and two parties, a traditional downsian party and a party implementing Participatory Democracy, compete to win elections. We examine in detail the options of the party representing Participatory Democracy to win. The Participatory Democracy party will celebrate an assembly if taking-office, where any citizen-candidate can represent his ideal position. This will make evaluate the party as a lottery. We consider different assembly equilibria varying the number of candidates. We include the valence characteristic concept and show the different strategies of both parties according to the valence advantage over the opponent.

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1 Introduction

Nowadays, citizens are demanding new ways of taking part on political decisions. Movements such Occupy Wall Street (OWS) in the United States or 15-M in Spain include in their principles to engage in a more transparent and direct citizen participation in the decision-making process.

We present a model in which two parties with different institutional organizations compete to win elections. There will be a party representing a case of Participatory Democracy, which combines elements from both Direct and Representative Democracy.[1] In this party, citizens have the ultimate power to decide on who is going to assume the role of policy implementation. This party will be referred as Party B. Its opponent, Party A, represents the typical downsian model where the candidate location will be given by the strategy which guarantees its victory. Both parties share a common aim which is to obtain more votes than the opponent party and win elections.

In this paper, we evaluate the possibilities of a party representing Participatory Democracy to win elections. This party differs considerably of the typical decision process carried out by traditional parties. First, an assembly should be held, to take into account the different policy positions of the citizens. Second, it uses elements of Participatory Democracy. We use the *citizen-candidate* concept developed by Osborne and Slivinski (1996). Thus, we allow any citizen who wishes to state his point of view, his ideas and his ideal policy position, in which we will be focused on. Any citizen-candidate will assume a cost and will hold a benefit in case of taking office. Another element of Participatory Democracy is the fact that any citizen can vote to that citizen-candidate who best represents his policy preferences. Additionally, there is an element of Representative Democracy, as the citizen-candidates elected at the assembly will represent their ideal and sincere voting policy position. This will be the policy of Party B. Due to uncertainty about the final number of citizen-candidates, Party B will be evaluated by the citizens as a lottery L_B.

We start adding a variable which represents the valence characteristic of each of the parties, β_A and β_B, as used at the work of Aragonés and Palfrey (2002). We study which kind of policies will be offered by the parties according to their valence advantage over its opponent. We also take into account situations in which uncertainty in valence advantage will limit the strategies of both parties.

In general, we study two cases. Firstly, we are going to consider that citizens with right to vote will assist to the assembly. Therefore, the median voter of the assembly will be equal to the median voter of the electorate. Secondly, we study a more realistic case, in which only the citizens who wishes to, will assist to the assembly. Thus, we will have situations where the median voter of the assembly can be located at any position of the policy space.

[1] See Aragonès and Sanchez-Pagés (2009) for a more detailed description of some forms of participatory democracy.

We finish analyzing whether a multi-candidate representation of Party B can affect positively to its chances of winning elections. We study the particular case for three and four candidates.

The rest of the paper is organized as follows. In Section 2 we introduce the formal model. Section 3 provides the results for the case of two-leader assembly. Section 4 analyzes the case of two-leader assembly when the assembly median voter does not coincide with the median voter of the electorate. Section 5 analyzes the case of more than two-leader assembly. Finally, Section 6 contains some concluding remarks.

2 Model

A general election is going to be held, in which voters will elect one out of two political parties. The two competing political parties are denoted by Party A and Party B. These parties differ in the institutional structure they support. Party A is a traditional party that announces its political platform before the election and has a clear leader that represents the candidacy of the party. Party B is not a traditional party in the sense that it neither defends a clear political platform, nor a leader. The only statement that Party B proposes is the defense of participatory democracy. By participatory democracy, if Party B wins the elections, the party will organize an assembly to choose their policy and leader. This implies that, during the electoral campaign, there is uncertainty on the policy that Party B will implement in the case of winning the elections.

Policy space and voters

The policy space is continuous and one dimensional. We take without loss of generality the interval $[0,1]$. The continuum of voters have symmetric single-peaked preferences over the policy space. The ideal policies of voters are distributed over $[0,1]$ according to a continuous distribution F. Let $x_i \in [0,1]$ be the ideal policy of voter i and let $x_M \in [0,1]$ be the ideal policy of the median voter.

Political Parties

The two political parties competing to win elections are denoted by $j \in \{A, B\}$. Party A represents the "downsian" party which offers a particular policy $x_A \in [0,1]$ as to win the elections. Party B, on the contrary, defends Participatory Democracy. As a first step, we consider that all the citizens attend the assembly. We later analyze the case in which citizens decide whether or not to attend this. In the assembly, citizens can become leaders by proposing political positions. After which, the assembly will vote over these proposals and by plurality rule, the policy obtaining more votes wins. Voters are assumed to vote sincerely at the assembly, and preferences over policies are represented by

$$v_i(x_k) = -|x_k - x_i| \tag{1}$$

where x_k is the policy proposed by leader k at the assembly. Following Osborne and Slivinsky (1996), when citizen with ideal position x_k becomes leader, the payoff derived by her decision is represented by the following function:

$$\left\{ \begin{array}{l} b - c \text{ if she wins} \\ -\left|x_w - x_i\right| - c \text{ if she loses} \end{array} \right. \tag{2}$$

where $c > 0$ is the cost of becoming leader, $b > 0$ is the benefit of winning in the assembly and x_w is the ideal policy of the winner at the assembly.

An *assembly Nash-equilibrium* is a set of $m > 0$ policies, each of which proposed by a leader such that no other leader strictly improves either proposing a new policy or dropping out his policy proposal.

For the duration of the electoral campaign, voters share a common belief about the outcome of the assembly. This belief is represented by a set of m policies $X_B = \left\{x_B^1, ..., x_B^m\right\} \in [0,1]^m$ and a vector of probabilities, $p = (p_1, ..., p_m)$, where each $p_k \geqslant 0$, is the probability that voters assign to the policy of leader k, x_B^k, to be elected in the assembly. Therefore, $p_1 + ... + p_m = 1$. We order this set so that $x_B^1 < x_B^2 < ... < x_B^m$. Thus, because there is uncertainty on the outcome of the assembly, Party B is evaluated by voters according to the lottery $L_B = \{X_B, p\}$.

Besides, each party is associated with a valence characteristic $\beta_A, \beta_B > 0$, that represents the popularity and affinity that voters show towards Party A and Party B respectively.

Voting decisions and equilibrium

Let u_i be the von Neumann-Morgenstern (vNM) utility representation of the preferences of voter i over policy proposals and valence characteristics:

$$u_i\left(x_j, \beta_j\right) = \beta_j - \left|x_j - x_i\right|. \tag{3}$$

Given the policy of Party A, x_A, and the lottery of Party B, $L_B = \{X_B, p\}$, agent i votes at the general election for party A when:

$$u_i\left(x_A, \beta_A\right) > E\left[u_i\left(L_B, \beta_B\right)\right], \tag{4}$$

which is equivalent to

$$\beta_A - \left|x_A - x_i\right| > \beta_B - \sum_{k=1}^{m} p_k \left|x_B^k - x_i\right| \tag{5}$$

If the agent is indifferent between both parties, he abstains from voting. Otherwise, agent i votes for party B.

Party A aims at winning the elections, that is, Party A is a pure office-seeking political party. The following function represents the objective of Party A:

$$V\left(x_A, L_B, \beta_A, \beta_B\right) = \left\{ \begin{array}{l} 1 \text{ if Party A wins} \\ 0 \text{ otherwise} \end{array} \right. \tag{6}$$

Thus, Party A only derives benefits when winning the general election. Observe that Party B will have a similar objective, winning the elections, however, given that beliefs over the assembly outcome and the valence characteristic are exogenously given, Party B has no strategic tool to win the elections.

Next, we introduce the equilibrium concept that accounts for the strategic behavior of Party A to select its platform, and the strategic entry of leaders at the assembly proposed by Party B in the case of winning the general election.

Definition: A *political equilibrium* is a policy for Party A, x_A, and a lottery for Party B, $L_B = \{X_B, p\}$, such that:
i) X_B is an assembly Nash-equilibrium and p is the associated vector of winning probabilities.
ii) x_A is a best response to the expected assembly outcome of Party B represented by L_B.

Our notion of political equilibrium requires that beliefs on the assembly outcome be consistent with the citizen-candidate approach. This requirement does not provide unique equilibrium prediction regarding the assembly outcome, but discards those locations of platforms and their associated probabilities that cannot be sustained by the individual incentives derived from winning at the assembly.

3 Two-leader assembly

First, we analyze the case in which voters belief that just two leaders will show up at the assembly. According to the citizen-candidate model, we can describe those policy positions and winning probabilities that can be sustained as Nash-equibrium at the assembly.

Lemma 1 *(Osborne and Slivinsky 1996) In every two-leader assembly Nash-equilibrium the leaders choose symmetric positions with respect to the median voter, i.e.* $x_B^1 = x_M - \varepsilon$ *and* $x_B^2 = x_M + \varepsilon$*, where* $\varepsilon \in (0, \overline{\varepsilon})$ *and the winning probabilities coincide* $p_1 = p_2$.

The upper bound $\overline{\varepsilon}$ is defined as to avoid the entrance of a third leader in between the two others. Thus, $\overline{\varepsilon}$ depends upon the distribution of votes and guarantees that for all $\varepsilon < \overline{\varepsilon}$, there is no policy position in the interval $[x_M - \varepsilon, x_M + \varepsilon]$ such that, according to sincere voting, this position is strictly preferred by more than 1/3 of the total voters.

Next, we derive the electoral result at the general election depending on the valence characteristic of the political parties.

Proposition 1 *In every political equilibrium with two expected leaders at the assembly:*
if $\beta_A \geq \beta_B$ *Party A always wins the elections.*
if $\beta_A < \beta_B$ *Party A wins the elections if and only if* $\beta_B - \beta_A < \frac{\varepsilon}{2}$.

Proof. *Step 1: We show that when $\beta_A \geq \beta_B$, Party A wins locating at $x_A = x_M$.*
First, we show that when $\beta_A \geq \beta_B$ any strategy x_A guarantees the victory of Party A. We consider the case in which Party A locates at the median voter position $x_A = x_M$, and Party B candidates are located in $X_B = \left(x_B^1, x_B^2\right)$, with probability $p = \left(\frac{1}{2}, \frac{1}{2}\right)$. First, we start analyzing the voting decision of a citizen located in $x_i \in [0, x_M - \varepsilon]$. There, the utility of voting Party A is $U(x_A) = \beta_A - |x_M - x_i|$ and $U(x_B) = \beta_B - \frac{1}{2}u_i\left(x_B^1 - x_i\right) + \frac{1}{2}u_i\left(x_B^2 - x_i\right) = \beta_B - |x_M - x_i|$. As $\beta_A > \beta_B$, they vote for Party A. Second, if the voter position is $x_i \in [x_M - \varepsilon, x_M]$ then he obtains that $U(x_A) = \beta_A - |x_M - x_i| > \beta_B - \frac{1}{2}u_i\left(x_i - x_B^1\right) + \frac{1}{2}u_i\left(x_B^2 - x_i\right) = \beta_B - \varepsilon$ and will also vote for Party A as $\beta_A \geq \beta_B$ implies that $\beta_A \geq \beta_B - \varepsilon$. Likewise, a citizen located at the median voter position $x_i = x_M$, will also vote for Party A. Symmetrically, $x_i \in [x_M, 1]$ will also vote for Party A. Thus, Party A will win unanimously. We can apply this analysis to any strategy of Party A: $x_A \in \left[x_B^1, x_B^2\right]$. In case $\beta_A = \beta_B$, Party A wins only with the support of $x_i \in [x_M - \varepsilon, x_M + \varepsilon]$ as any citizen located in $x_i \in [0, x_M - \varepsilon]$ and $x_i \in [x_M + \varepsilon, 1]$ will be indifferent.

Step 2: We show that when $\beta_A < \beta_B$ and $\beta_B - \beta_A < \frac{\varepsilon}{2}$, Party A wins locating at $x_A = x_M - \frac{\varepsilon}{2}$.
When $\beta_A < \beta_B$, Party A can win locating at $x_A = x_M - \frac{\varepsilon}{2}$. First, we consider that the voter position is $x_i \in [0, x_M - \varepsilon]$. Then, $U(x_A) = \beta_A - |x_A - x_i| = \beta_A - \left|x_M - \frac{\varepsilon}{2} - x_i\right|$ and $U(x_B) = \beta_B - \frac{1}{2}u_i\left(x_B^1 - x_i\right) + \frac{1}{2}u_i\left(x_B^2 - x_i\right) = \beta_B - |x_M - x_i|$. Comparing utilities, we find that $\beta_B - \beta_A < \frac{\varepsilon}{2}$ and x_i supports Party A. The same result is obtained by $x_i \in \left[x_M - \varepsilon, x_M - \frac{\varepsilon}{2}\right]$ and $x_i \in \left[x_M - \frac{\varepsilon}{2}, x_M\right]$, who will also vote for Party A. If the citizen locates at the median voter position $x_i = x_M$ he will vote for Party A as $U(x_A) = \beta_A + \frac{\varepsilon}{2} > U(x_B) = \beta_B$. Thus, Party A can win even with a less valence advantage leader.

Step 3: We show that when $\beta_A < \beta_B$ and $\beta_B - \beta_A \geq \frac{\varepsilon}{2}$, Party A cannot win at any location.
When $\beta_A < \beta_B$ Party A cannot win at any location if the condition $\beta_B - \beta_A \geq \frac{\varepsilon}{2}$ is applied. First, we start analyzing the case in which Party A decides to locate in $x_A = x_M$. For each citizen $x_i \in [0, x_M - \varepsilon]$, he obtains $U(x_A) = \beta_A - |x_M - x_i|$ and $U(x_B) = \beta_B - \frac{1}{2}u_i\left(x_B^1 - x_i\right) + \frac{1}{2}u_i\left(x_B^2 - x_i\right) = \beta_B - |x_M - x_i|$. The restriction $\beta_A < \beta_B$ implies that they will vote for Party B. For any citizen $x_i \in [x_M - \varepsilon, x_M]$ Party A gives $U(x_A) = \beta_A - |x_M - x_i|$ and Party B $U(x_B) = \beta_B - \varepsilon$. Thus, it is necessary to satisfy the condition $\beta_B - \beta_A \geq \frac{\varepsilon}{2}$ to vote for Party B. Finally, the same is applied if the citizen i is the median voter, $x_i = x_M$. Thus, Party A cannot win under the strategy $x_A = x_M$. Second, we take $x_A = x_M - \frac{\varepsilon}{2}$ as the strategy of Party A. If $x_i \in [0, x_M - \varepsilon]$, then $U(x_A) = \beta_A - |x_A - x_i| = \beta_A - \left|x_M - \frac{\varepsilon}{2} - x_i\right|$ and $U(x_B) = \beta_B - |x_M - x_i|$. This implies that $\beta_A - \left|x_M - \frac{\varepsilon}{2} - x_i\right| < \beta_B - |x_M - x_i|$ and $\beta_B - \beta_A \geq \frac{\varepsilon}{2}$ is satisfied. Meanwhile, when $x_i \in \left[x_M - \varepsilon, x_M - \frac{\varepsilon}{2}\right]$ citizen x_i obtains $\beta_A - \left|x_M - \frac{\varepsilon}{2} - x_i\right| < \beta_B - \varepsilon$, which implies that $\beta_B - \beta_A \geq \frac{\varepsilon}{2}$ and Party B obtains support. By symmetry, $x_i \in \left[x_M - \frac{\varepsilon}{2}, x_M\right]$ will also vote for

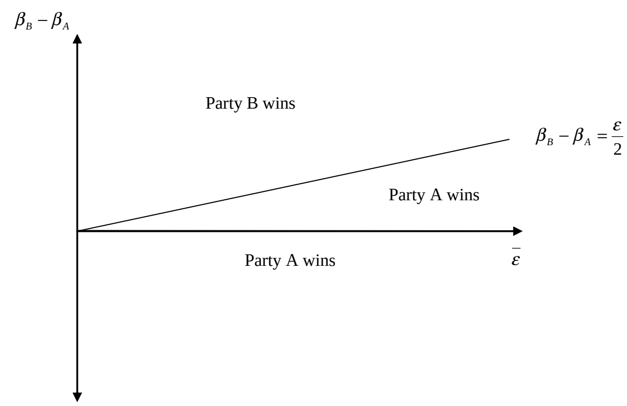

Figure 1: Results of Proposition 1.

B. In case $x_i = x_M$ the condition $\beta_B - \beta_A \geq \frac{\varepsilon}{2}$ is also satisfied and $x_i \in [0, x_M]$ will make Party B to win when Party A locates at $x_A = x_M - \frac{\varepsilon}{2}$. ■

This result gives a clear prediction of the party winning at the general election as a function of the difference in the valence characteristic. Figure 1 represents the results of Proposition 1. In this figure we clearly deduce how "a priori", Party A has more chances to win: whereas a valence advantage for party A over B (i.e., $\beta_B - \beta_A < 0$) assures the victory of Party A, the valence advantage for Party B over A (i.e., $\beta_B - \beta_A > 0$) cannot guarantee the victory of Party B. As Figure 1 shows, however, the smaller the value of ε is, the higher the chances of Party B to win at the general election. That is, the more moderate, in expected terms, are the leaders at the assembly, the higher the probability of Party B to win the general election.

We deduce, therefore, a necessary condition for Party B to win at the general election: Party B needs some advantage with respect to Party A in terms of valence characteristic, i.e. $\beta_B - \beta_A > 0$. Besides, the smaller this advantage, the more moderate (in expected terms) the leaders at the assembly should be to guarantee the victory of Party B. Hence, the higher this advantage, the more extremist (in expected terms) the leaders at the assembly can be without abandoning the electoral victory. This observation is highlighted in our following result.

Corollary 1: *Suppose that Party B has a valence advantage (i.e., $\beta_B - \beta_A > 0$). Then, there is a fixed distance between the two leaders at the assembly (measured by 2ε), below which Party B can guarantee its victory. Besides, the higher $\beta_B - \beta_A$ the higher is this distance, i.e. the more polarized the assembly proposals can be.*

Because the leaders at the assembly symmetrically locate around the ideal policy of the median voter, our result establish that closer locations around this median are needed when the advantage of Party B is low. Thus, too polarized positions (in expected terms) can generate electoral default when the advantage of Party B over Party A is not too large.

So far, we have just paid attention to describing which party can win the general election. Next, we want to describe the equilibrium location of Party A as a function of the valence characteristic of the political parties. As we show in our following result, when $\beta_B - \beta_A < 0$, Party A has some flexibility regarding the location that guarantees its victory.

Lemma 2 *If $\beta_A \geq \beta_B$, Party A can lose the elections locating its platform at either $x_A \in [0, x_M - \varepsilon)$ or $x_A \in (x_M + \varepsilon, 1]$. However, every location $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ guarantees its electoral victory.*

Proof. We take that the valence characteristic is equal for both Parties, i.e., $\beta_A = \beta_B$. We start supposing that $x_A \in [0, x_M - \varepsilon)$. Then, for every agent i with ideal policy $x_i \in [0, x_M - \varepsilon]$, x_i obtains $u(x_A, \beta_A) = \beta_A - |x_A - x_i| > u(x_B, \beta_B) = \beta_B - |x_M - x_i|$. Thus, Party A obtains the vote of these agents. For every agent with ideal policy $x_i \in [x_M - \varepsilon, x_M]$, we have that $u(x_A, \beta_A) = \beta_A - |x_i - x_A|$ whereas $u(x_B) = \beta_B - \varepsilon$. In this case, depending on the distance $|x_i - x_A|$ they may prefer one or the other political party. In particular, those agents which ideal policy x_i satisfies that $|x_i - x_A| > \varepsilon$ will opt for Party B. Finally, for any voter with ideal policy $x_i \in [x_M, 1]$, Party B is preferred to Party A. This implies that if $\beta_A \geq \beta_B$ Party A can lose in $x_A \in [0, x_M - \varepsilon)$ and $x_A \in (x_M + \varepsilon, 1]$. Conversely, if $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ and $\beta_A = \beta_B$ Party A wins. We show that using $x_A = x_M$, every voter which ideal policy located at $x_i \in [0, x_M - \varepsilon]$ and $x_i \in [x_M + \varepsilon, 1]$ abstain from voting as $u(x_A, \beta_A) = \beta_A - |x_A - x_i| = u(x_B, \beta_B) = \beta_B - |x_M - x_i|$. For those voters with ideal policy $x_i \in [x_M - \varepsilon, x_M]$, they prefer Party A because $|x_A - x_i| < |-\varepsilon|$. Symmetrically, $x_i \in [x_M, x_M - \varepsilon]$ will also vote for Party A. ■

When Party A does not have valence advantage, i.e., when $\beta_B - \beta_A > 0$, Party A can only win by supporting certain political positions within the interval $[x_M - \varepsilon, x_M + \varepsilon]$. As shown in Proposition 1, there are some cases in which, no matter the location of Party B, Party A cannot win at the general election.

Lemma 3: *If $\beta_A < \beta_B$, Party A lose the elections locating its platform at either $x_A \in [0, x_M - \varepsilon)$ or $x_A \in (x_M + \varepsilon, 1]$. If $\beta_B - \beta_A < \frac{\varepsilon}{2}$, then either $x_A = x_M - \frac{\varepsilon}{2}$ or $x_A = x_M + \frac{\varepsilon}{2}$ can guarantee the victory of Party A.*

Proof. The first part was proved in Lemma 2, where we showed that even with valence advantage $\beta_A \geq \beta_B$ Party A can lose in $x_A \in [0, x_M - \varepsilon)$ and $x_A \in (x_M + \varepsilon, 1]$. Indeed, we deduce that when Party A has no valence advantage over Party B $\beta_A < \beta_B$ Party A cannot win either. If we focus on the second part of Lemma 3, we obtain that it is possible for Party A to win locating at $x_A = x_M - \frac{\varepsilon}{2}$ or $x_A = x_M + \frac{\varepsilon}{2}$ if and only if $\beta_B - \beta_A < \frac{\varepsilon}{2}$. We assume that Party A locates at $x_A = x_M - \frac{\varepsilon}{2}$. Then, any citizen $x_i \in [0, x_M - \varepsilon]$ will obtain that $u(x_A, \beta_A) = \beta_A - \left|x_M - \frac{\varepsilon}{2} - x_i\right|$ and $u(x_B, \beta_B) = \beta_B - |x_M - x_i|$. If we compare utilities, we obtain that $\beta_B - \beta_A < \frac{\varepsilon}{2}$ is satisfied and they vote for Party

A. Taking $x_i = x_M$ we get that $u(x_A, \beta_A) = \beta_A - \left|x_M - \left(x_M - \frac{\varepsilon}{2}\right)\right| = \beta_A - \frac{\varepsilon}{2}$ and $u(x_B, \beta_B) = \beta_B - \varepsilon$, so $\beta_B - \beta_A < \frac{\varepsilon}{2}$ is also satisfied. Thus, the median voter also vote for Party A. We can spread the voting decision to $x_A \in [x_M - \varepsilon, x_M]$. The proof for the case in which $x_A = x_M + \frac{\varepsilon}{2}$ is analogue. ■

In order to derive a unique equilibrium prediction regarding the political platform of Party A, we suppose that Party A is uncertain about the advantage that gathers with respect to Party B. In this case, Party A will locate in that political position that maximizes its expected probability of winning.

Proposition 2 *If Party A is uncertain about its advantage over Party B, then in every two-leader political equilibrium either* $x_A = x_M - \frac{\varepsilon}{2}$ *or* $x_A = x_M + \frac{\varepsilon}{2}$.

Proof. The objective function of Party A is defined by Expression (6) hence, Party A only derives benefits from winning the elections. As shown in Lemma 2, the strategies $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ make Party A win for $\beta_A \geq \beta_B$. Moreover, by Lemma 3 $x_A = x_M - \frac{\varepsilon}{2}$ or $x_A = x_M + \frac{\varepsilon}{2}$ guarantees the electoral victory of Party A when $\beta_A < \beta_B$ with $\beta_B - \beta_A < \frac{\varepsilon}{2}$. Since $x_M - \frac{\varepsilon}{2} \in (x_M - \varepsilon, x_M + \varepsilon)$ and $x_M + \frac{\varepsilon}{2} \in (x_M - \varepsilon, x_M + \varepsilon)$, the political positions $x_A = x_M - \frac{\varepsilon}{2}$ and $x_A = x_M + \frac{\varepsilon}{2}$ guarantee the victory of Party A at the general election when there is uncertainty about the valence advantage. By Proposition 1, Party A cannot win the election when $\beta_B - \beta_A \geq \frac{\varepsilon}{2}$ and, in this case, Party A is indifferent between every policy position according to its objective function as it is not going to win. We conclude that $x_A = x_M - \frac{\varepsilon}{2}$ and $x_A = x_M + \frac{\varepsilon}{2}$ are therefore, the positions that maximize the probability of winning of Party A. ■

4 Two-leader assembly: the *assembly* median voter

In the previous section, we analyzed the case in which voters belief that just two leaders will show up at the assembly and leaders were symmetrically located around the median voter (Lemma 1). In addition, we took for granted that all the citizens attended the assembly to elect the Party B candidate.

Now, we want to consider a more realistic scenario where not all the voters assist to the assembly. We still maintain the assembly Nash-equilibrium concept of Lemma 1. Hence, the previous section is a particular case in which the median voter position of both the assembly and the electorate were the same. Now, we make a distinction between both. We now define the *assembly* median voter position as x_M^a. From now on, x_M will be referred as the *electorate* median voter position. Through this section, we consider that the parties' valence characteristic coincide, i.e., $\beta_A = \beta_B$.

Lemma 4: *In every two-leader assembly Nash-equilibrium leaders choose symmetric positions with respect to the assembly median voter, i.e.* $x_B^1 = x_M^a - \varepsilon$ *and* $x_B^2 = x_M^a + \varepsilon$, *where* $\varepsilon \in (0, \overline{\varepsilon})$ *and the winning probabilities coincide* $p_1 = p_2$.

$\beta_A = \beta_B$	$x^a_M \in (0, x_M - \varepsilon)$	$x^a_M \in (x_M - \varepsilon, x_M)$	$x^a_M = x_M$	$x^a_M \in (x_M, x_M + \varepsilon)$	$x^a_M \in (x_M + \varepsilon, 1)$
$x_A \in (0, x_M - \varepsilon)$	**A/B**	B	B	B	
$x_A \in (x_M - \varepsilon, x_M)$	A	**A/B**	A	A	A
$x_A = x_M$	A	A	A	A	A
$x_A \in (x_M, x_M + \varepsilon)$	A	A	A	**A/B**	A
$x_A \in (x_M + \varepsilon, 1)$		B	B	B	**A/B**

Figure 2: Results of the election according to the *assembly* median voter position.

Next, we derive the electoral result at the general election depending on the location of the *assembly median voter position,* x^a_M.

Proposition 3 *In every political equilibrium with two expected leaders at the assembly:*
i) if $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ *Party A always wins the elections.*
ii) if $x_A \leq x_M$, *Party A wins the elections if and only if* $x_A \geq x^a_M$.
iii) if $x_A \geq x_M$, *Party A wins the elections if and only if* $x_A \leq x^a_M$.

Proof. First, we show that any strategy $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ guarantees the victory of Party A if *ii* and *iii* of Proposition 4 are satisfied. In Figure 2 we obtain the results of the elections depending on the location of the *assembly* median voter position x^a_M and the location of the candidate of Party A, x_A. ■

In Figure 2, we can see the results of the elections for any possible locations of Party A, x_A, and Party B, $X_B = (x^a_M - \varepsilon, x^a_M + \varepsilon)$. At each row, we show the different strategies of Party A, x_A, respect to the electorate median voter position, x_M. On the contrary, all the possible locations of the *assembly* median voter, x^a_M, are shown in columns. For instance, if the assembly-median voter is located at $x^a_M \in (x_M - \varepsilon, x_M)$ the best strategy for Party A is to offer a policy located at the electorate-median voter position $x_A = x_M$ or any at $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ satisfying condition ii). Thus, A stands for the victory of Party A and B the defeat . In the cases represented by A/B, both parties can win. Its victory will depend upon the exact location of x_A with respect to x^a_M. Specifically for that strategy, Party A wins if $x_A \geq x^a_M$, and B otherwise. For $x_A \in (0, x_M)$ Party A wins if $x_A \geq x^a_M$. Symmetrically, for any policy location of Party A such that $x_A \in (x_M, 1)$ Party A cannot be defeated if $x_A \leq x^a_M$. Empty cells in Figure 2 indicates a tie between both parties.

Step 1: Show that B represents the cases where Party B wins.

We consider the case in which Party A locates at $x_A \in (0, x_M - \varepsilon)$ and Party B candidates are located in $X_B = [x^a_M - \varepsilon, x^a_M + \varepsilon]$, with $x^a_M \in (x_M - \varepsilon, x_M + \varepsilon)$ and probability $p = \left(\frac{1}{2}, \frac{1}{2}\right)$. We start analyzing the voting decision of the *electorate* median voter position, x_M as its decision will be the same of those

candidates located at $x_i \in (x_M + \varepsilon, 1)$. Considering the case in which $x_A = x_B^1 = x_M - \frac{3\varepsilon}{2}$. If the electorate median voter votes for Party A, he obtains $U(x_A) = \beta_A - |x_M - (x_M - \varepsilon)| = \beta_A - \frac{3\varepsilon}{2} < U(x_B) = \beta_B - \frac{1}{2}|x_M - (x_M^a - \varepsilon)| - \frac{1}{2}|x_M - (x_M^a + \varepsilon)| = \beta_B$
$-\frac{1}{2}\left|x_M - \left(x_M - \frac{3\varepsilon}{2}\right)\right| - \frac{1}{2}\left|x_M + \frac{\varepsilon}{2} - (x_M)\right| = \beta_B - \varepsilon$. Therefore, any citizen $x_i \in (x_M + \varepsilon, 1)$ will also vote for Party B. We can extrapolate this analysis to any *assembly* median voter location $x_M^a \in (x_M - \varepsilon, 1)$. Additionally,the strategy $x_A \in (x_M + \varepsilon, 1)$ show the same results and Party A cannot win either.

Step 2: Show that A represents the cases where Party A wins.

We consider the case in which Party A locates at the *electorate* median voter position $x_A = x_M$ and Party B candidates are located in $X_B = (x_M^a - \varepsilon, x_M^a - \varepsilon)$ with $x_M^a = x_M$ and probability $p = \left(\frac{1}{2}, \frac{1}{2}\right)$. We study the voting decision of the *electorate* median voter position, $x_i = x_M$. x_M will vote for Party A as $U(x_A) = \beta_A - |x_M - (x_M)| = \beta_A > U(x_B) =$
$\beta_B - \frac{1}{2}|x_M - (x_M^a - \varepsilon)| - \frac{1}{2}|x_M^a + \varepsilon - x_M| = \beta_B - \frac{1}{2}|x_M - (x_M - \varepsilon)| - \frac{1}{2}|x_M + \varepsilon - x_M| = \beta_B - \varepsilon$. As the *electorate* median voter supports Party A, Party A wins obtaining the support of $x_i \in (x_M - \varepsilon, x_M + \varepsilon)$ as citizens out of this space will be indifferent between both parties. This analysis is equivalent to any case denoted by A in Figure 2. That is, for any $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ where condition II of Proposition (4) is satisfied.

Step 3: Show that A/B represents the cases in which Party A wins satisfying condition II.

We consider one of the cases in which we find A/B in Figure 2. Suppose Party A locates at $x_A \in (x_M - \varepsilon, x_M)$ and Party B candidates are located in $x_M^a \in (x_M - \varepsilon, x_M)$, with probability $p = \left(\frac{1}{2}, \frac{1}{2}\right)$. We start analyzing a situation in which $x_A = x_M^a$. For instance, $x_A = x_M^a = x_M - \frac{\varepsilon}{2}$. In this case the *electorate* median voter x_M will vote for Party A as $U(x_A) = \beta_A - \left|x_M - \left(x_M - \frac{\varepsilon}{2}\right)\right| = \beta_A - \frac{\varepsilon}{2} > U(x_B) = \beta_B - \frac{1}{2}|x_M - (x_M^a - \varepsilon)| - \frac{1}{2}|x_M - (x_M^a + \varepsilon)| =$
$\beta_B - \frac{1}{2}\left|x_M - \left(x_M - \frac{3\varepsilon}{2}\right)\right| - \frac{1}{2}\left|x_M + \frac{\varepsilon}{2} - x_M\right| = \beta_B - \varepsilon$. This decision will be the same as those candidates located at $x_i \in (x_M^a - \varepsilon, x_M^a + \varepsilon)$. Conversely, any citizen $x_i \in (x_M^a + \varepsilon, 1)$ and $x_i \in (0, x_M^a - \varepsilon)$ will be indifferent between both parties as the utility obtained by each agent is equal, as $x_A = x_M^a$.

We have shown that for $x_A = x_M^a$, Party A wins when both parties shares the same platform. Therefore, for $x_A \in (0, x_M)$ a strategy $x_A \geq x_M^a$ will guarantee the victory of Party A. We are going to show it by contradiction. Suppose the same prior case where $x_A \in (x_M - \varepsilon, x_M)$ and $x_M^a \in (x_M - \varepsilon, x_M)$ but now $x_A < x_M^a$. We can take $x_A = x_M - \frac{2\varepsilon}{3}$ and $x_M^a = x_M - \frac{\varepsilon}{2}$. In this case the *electorate* median voter x_M will vote for Party B as $U(x_A) = \beta_A - \left|x_M - \left(x_M - \frac{2\varepsilon}{3}\right)\right| = \beta_A - \frac{2\varepsilon}{3} < U(x_B) = \beta_B - \frac{1}{2}\left|x_M - \left(x_M - \frac{3\varepsilon}{2}\right)\right| - \frac{1}{2}\left|x_M + \frac{\varepsilon}{2} - x_M\right| = \beta_B - \varepsilon$. Therefore, any citizen $x_i \in (x_M + \varepsilon, 1)$ will vote for Party B as they will also obtain more utility for voting Party B rather than Party A. Thus, we have shown the importance of satisfying condition II and III: for any strategy of Party A at platform $x_A \in (0, x_M)$ it is necessary to satisfy $x_A \geq x_M^a$ to guarantee the victory of Party A and for Party A platform

$x_A \in (x_M, 1)$ Party A needs to locate $x_A \leq x_M^a$ to win.

Step 4: Show that empty cells of Figure 2 represents a tie between Party A and Party B.

We consider the case in which Party A locates at $x_A \in [0, x_M - \varepsilon]$ and Party B candidates are located in $X_B = [x_M + \varepsilon, 1]$, with probability $p = \left(\frac{1}{2}, \frac{1}{2}\right)$. As the aim of both parties is to win elections, Party A will locate at the limit of the platform, $x_A = x_M - \varepsilon$ and we consider that the *assembly* median voter is also located at the limit of its actual platform, $x_M^a = x_M + \varepsilon$. The voting decision of the *electorate* median voter position, x_M. We obtain that $U(x_A) = \beta_A - |x_M + \varepsilon - x_M| = \beta_A - \varepsilon = U(x_B) = \beta_B - \frac{1}{2}|x_M^a - \varepsilon - x_M| - \frac{1}{2}|x_M^a + \varepsilon - x_M| = \beta_B - \frac{1}{2}|x_M - x_M| - \frac{1}{2}|x_M + 2\varepsilon - x_M| = \beta_B - \varepsilon$. As the *electorate* median voter is indifferent between voting one or the other Party, Party A and Party B will tie as $x_i \in (x_M, 1)$ will vote for Party B and $x_i \in (0, x_M)$ will vote for Party A. Thus, if parties' locations are at the same distance from the *electorate* median voter, they tie. However, if one of the parties decide to implement a more extreme policy (moving away from x_M) then, the other party can win. So, the assumption we have used related to locate x_A and x_M^a at the limit of its platforms is adequate. Symmetrically, the results obtained can be applied to the platforms where Party A is located at $x_A \in (x_M + \varepsilon, 1)$ and Party B at $x_M^a \in (0, x_M - \varepsilon)$.

5 More than 2-leader equilibrium

In this section, we want to show that no matter the number of candidates elected at the assembly and their locations, Party B cannot win. We continue considering that the valence advantage of both parties is equal $\beta_A = \beta_B$. We want to analyze whether the number of candidates elected at the assembly can influence on the results of the elections. We focus our study on the best response of Party A, i.e. $x_A = x_M$.

Lemma 5: *In every n-leader assembly Nash-equilibrium, leaders choose symmetric positions with respect to the assembly median voter. We take as a three-leader assembly Nash-equilibrium the following locations,* $x_B^1 = x_M^a - \varepsilon$, $x_B^2 = x_M^a$ *and* $x_B^3 = x_M^a + \varepsilon$, *where* $\varepsilon \in (0, \overline{\varepsilon})$ *and the winning probabilities coincide* $p_1 = p_2 = p_3$. *As a four-leader assembly Nash-equilibrium we take* $x_B^1 = x_M^a - \varepsilon$, $x_B^2 = x_M^a - \frac{\varepsilon}{2}$, $x_B^3 = x_M^a + \frac{\varepsilon}{2}$ *and* $x_B^4 = x_M^a + \varepsilon$, *where* $\varepsilon \in (0, \overline{\varepsilon})$ *and the winning probabilities coincide* $p_1 = ... = p_4$.

Proposition 4 *No matter the number of candidates elected at the assembly* $X_B = \left\{x_B^1, ..., x_B^m\right\}$ *and the location of the assembly median voter* x_M^a, *the strategy* $x_A = x_M$ *will assure the victory of Party A.*

Proof. For $x_A = x_M$, we start considering a three-leader assembly equilibrium: $X_B = \left\{x_B^1, x_B^2, x_B^3\right\}$. Let the *assembly* median voter location be $x_M^a = x_M$.

We find that the *electorate* median voter position will vote for Party A, as the utility of $x_i = x_M$ is $u_i(x_A, \beta_A) = \beta_A - |x_M - x_M| = \beta_A > E[u_i(L_B, \beta_B)] = \beta_B - \frac{1}{3}|x_M - (x^a_M - \varepsilon)| - \frac{1}{3}|x_M - (x^a_M)|$
$-\frac{1}{3}|x^a_M + \varepsilon - (x_M)| = \beta_B - \frac{2\varepsilon}{3}$. As $x^a_M = x_M$, any citizen located at $x_i \in (0, x_M - \varepsilon)$ and $x_i \in (x_M + \varepsilon, 1)$ will be indifferent as they will obtain the same utility of voting. We, thus, study the utility of $x_i \in (x_M - \varepsilon, x_M + \varepsilon)$. Taking, for instance, the agent $x_i = x_M - \frac{2\varepsilon}{3}$, he obtains $u_i(x_A, \beta_A) = \beta_A - \left|x_M - \left(x_M - \frac{2\varepsilon}{3}\right)\right| = \beta_A - \frac{2\varepsilon}{3} > E[u_i(L_B, \beta_B)] =$
$\beta_B - \frac{1}{3}\left|x_M - \frac{2\varepsilon}{3} - (x^a_M - \varepsilon)\right| - \frac{1}{3}\left|x^a_M - \left(x_M - \frac{2\varepsilon}{3}\right)\right| - \frac{1}{3}\left|x^a_M + \varepsilon - \left(x_M - \frac{2\varepsilon}{3}\right)\right| = \beta_B - \frac{8\varepsilon}{9}$. Thus, Party A wins with the support of $x_i \in (x_M - \varepsilon, x_M + \varepsilon)$. This result is equivalent to any *assembly* median voter location $x^a_M \in (0, x_M)$ as Party A have the support of at least $x_i \in [x_M, 1]$. Symmetrically, if the *assembly* median voter location is $x^a_M \in (x_M, 1)$ at least any citizen located at the left of the *electorate* median voter position $x_i \in [0, x_M]$ will support Party A, as $u_i(x_A, \beta_A) > E[u_i(L_B, \beta_B)]$. ■

This can be spread to any n-candidate assembly equilibrium. Applying the same case to a four-leader assembly equilibrium: $X_B = \left\{x^1_B, x^2_B, x^3_B, x^4_B\right\}$, with $x_A = x^a_M = x_M$, the *electorate* median voter $x_i = x_M$ obtains $u_i(x_A, \beta_A) = \beta_A - |x_M - x_M| = \beta_A > E[u_i(L_B, \beta_B)] = \beta_B - \frac{1}{4}|x_M - (x^a_M - \varepsilon)|$
$-\frac{1}{4}\left|x_M - \left(x^a_M - \frac{\varepsilon}{2}\right)\right| - \frac{1}{4}\left|x^a_M + \frac{\varepsilon}{2} - x_M\right| - \frac{1}{4}|x^a_M + \varepsilon - (x_M)| = \beta_B - \frac{3\varepsilon}{4}$.

We can conclude that Figure 2 can show the results for any n-leader assembly equilibrium when Lemma 1 is satisfied, which only consider symmetric candidates around the *assembly* median voter.

6 Conclusions

In this paper, we have studied the options of a party implementing Participatory Democracy (Party B) to win elections against a traditional downsian party (Party A). Our results are based on the valence advantage of the parties over the opponent and the location of the assembly median voter position. Very concisely, we make an introductory analysis to see whether the options of Party B to win elections are increased by the number of final elected candidates at the assembly.

We have started analyzing the different strategies of both parties according to their valence advantage over the opponent. Under the belief that the party with valence advantage wins, we have shown that this is not the case when Party B has a small valence advantage over Party A, as we found two strategies in which Party A can win: $x_A = x_M - \frac{\varepsilon}{2}$ and $x_A = x_M + \frac{\varepsilon}{2}$. On the contrary, valence advantage of Party A guarantees its victory at any location $x_A \in [0, 1]$. In those cases in which valence characteristic of both parties are equivalent, Party A can win in strategies situated between the two candidates location of Party B $x_A \in [x_M - \varepsilon, x_M + \varepsilon]$. Under $\beta_A = \beta_B$, this implies that the distance from the median voter will be lower than $\overline{\varepsilon}$. Conversely, when Party B has an important valence advantage over Party A, this allow candidates to locate far from the median voter position (see Figure 1), which can lead to stimulate polarized or extremist policies.

We have also taking into consideration the case in which there is uncertainty about the valence advantage over the opponent party. In this case, parties will locate in that political position that maximizes its expected probability to win, that is, in that location where the party wins if it has valence advantage and where it has more chances to win when it has not valence advantage and the other party decides to carry out a "risky" strategy. Thus, uncertainty will limit the strategies of both parties as Party A will locate at $x_A = x_M - \frac{\varepsilon}{2}$ or $x_A = x_M + \frac{\varepsilon}{2}$ and Party B will minimize its distance with respect to the median voter, $\varepsilon < \overline{\varepsilon}$.

Under $\beta_A = \beta_B$, we also evaluate the results of the elections when the median voter of the assembly is different to the median voter of the electorate (Figure 2). We obtain that Party A will locate around the *electorate* median voter $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$. The location of the *assembly* median voter x_M^a will define a close strategy of Party A to the *electorate* median voter x_M according to Proposition 3. We leave for further studies the results of the elections for the cases in which there is valence advantage over the opponent, $\beta_A \lesseqgtr \beta_B$.

Finally, we make an introductory analysis to the fact that more than 2 leaders are elected in the assembly. We study the particular case of a 3 and a 4 leader assembly equilibrium to see whether an odd or an even number of candidates will influence in the results of the elections. We conclude that, a priori, the number of candidates do not influence the final result, as the *assembly* median voter is mantained. We obtain the same results as in Figure 2. This will be due to the imposition of Lemma 1, in which candidates has to locate symmetrically around the median voter position, x_M. Further study should be focused on the results with asymmetric multi-candididate locations for any valence advantage situation $\beta_A \lesseqgtr \beta_B$.

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