Optimal Zoning in the Circular and Linear City under different Regulatory Approaches

Hamid Hamoudi[*], Isabel Rodriguez

 Departamento Fundamentos del Análisis Económico, Universidad Rey Juan Carlos, Paseo de los Artilleros s/n, 28032 Madrid, Spain.

Abstract:

In this article we study zoning in a circular and linear city model where firms are not allowed to locate in certain areas. A biased regulator is then introduced in a Bertrand spatial Competition framework. We prove that city zoning depends on the regulator bias towards consumers or firms. Both city models show a formal equivalence in the results: a consumer-biased regulator pushes for strong competition whereas a firm-biased regulator induces weak competition.

* Corresponding author

E-mail address*: Isabel.rodriguez@urjc.es*

1. INTRODUCCIÓN

Life in the cities is full of externality effects caused by firms which are detrimental to the population. Public intervention can then become an internalization mechanism for firms that impose social costs on the rest of society. Authorities need to find regulating instruments which enable them to reduce any harmful external factors in the best possible way. Some of the possible tools to prevent negative externalities include tax related policies or urban planning. Authors like Mills (1989), Henderson (1991), Miceli (1992) and Wheaton (1993) have shown that zoning is a very popular urban planning policy. We analyse a regulation design in a duopolistic framework á la Hotelling under alternative political profiles.

We consider a regulator in charge of the design of an urban city space divided in two different zones: an exclusively residential area where consumers live and a mixed area where consumers and firms locate. The aim of the regulator when restricting certain areas is to provide a high-quality environment, reduce trouble and prevent delinquencies. One of the advantages from this approach is that it can be analysed in terms of industrial policies[1].

In the context of regulation it is well known that authorities try to satisfy consumers and firms. Consequently, it is common to define a welfare function as a sum of consumer´s and producer´s surplus. However, in this paper, we introduce the social welfare function as a linear combination of firm´s profit and consumer´s utility[2], in a similar way to Hamoudi and Risueño (2012). The interpretation for this welfare function is related to the political profile of the public authority. A "social" government or planner has more incentives to overvalue consumer surplus whereas a "liberal authority" values the surplus of firms more highly. In between the two types, a "centre or neutral oriented" government assigns similar weights to both firms and consumers.

---

[1] The analogies between location and differentiation models are well-known: geographic space reflects characteristic space; consumer location expresses the preferred variety; location of firms is associated to firms´ offered product variety and transportation cost is taken as a disutility.

[2] Such weighted approach was defined by Baron and Myerson (1982) and used in welfare function formulation with several regulated models by Armstrong et al. (1994).

An important amount of studies on zoning regulation can be found in the spatial competition literature. This research focuses mainly on optimal firm location, regulated zone dimensions, pricing, land usage[3] and social welfare effects. In this regard, Lai y Tsai (2004) examine Hotelling´s linear city model with restrictions on the location of firms. They show that maximum differentiation holds under Bertrand competition and social welfare is improved. Tsai et al. (2006) analyse how zoning affects firm´s location and land rents. Chen and Lai (2008) investigate the effects of symmetric zoning in the linear city and prove that firms locate in equilibrium at the extremes of the zoning area under Cournot competition. They conclude that introducing a regulated zoning area can be welfare improving. Matsumura and Matsushima (2011) study a duopoly model with restrictions on the location of firms. Their objective is to analyse the effects on consumer's welfare. The model shown is related to the issue of urban sprawl in order to determine the allowed dimension of economic activities. Lastly, Hamoudi and Risueño (2012) consider the effect of zoning regulation in duopolistic circular model with Bertrand competition where consumers and firms are situated in different city regions. They show that the optimal size of the shopping area depends on the regulator's political profiles.

The present research paper uses two standard spatial competition models: the circular and the linear city. Both models lead to similar results for Bertrand competition as in D'Aspremont et al (1979) and De Frutos et al.(1999) but different results for competition á la Cournot as in Hamilton et al. (1989), Anderson and Neven (1991) or Pal (1998)). Here, we intend to examine similarities and formal differences between both types of zoning models as well as highlighting the implications for urban and industrial policies. The results obtained for the circular and the linear case show that a firm-biased regulator favours dispersion in location and maximum differentiation in terms of the offered good. On the other hand, a consumer-biased regulator produces opposite results: agglomeration and minimum differentiation.

The rest of this article is organized as follows: part one analyses the circular model and part two the linear model. In both cases we describe the model first and then determine price

---

[3] See Fujita and Thisse´s model (1986).

and location equilibrium as well as optimal zoning policies. Each part concludes with the most relevant remarks on the effects of regulation on competition.

2. THE CIRCULAR MODEL

2.1 THE MODEL.

We study a location model in which a regulating authority plans the design of a city in a unitary length circular space. Any point in the circle corresponds to a number from the interval [0,1]. The southeasters' point is 0 and we move anti clock wise from there. Points 0 and 1 will therefore coincide. The planner divides the circle in two regions: the first one is a commercial area bounded by points v_1, v_2 such that $0 \leq v_1 \leq v_2 \leq 1/2$ where firms and families locate. The second one is the residential area where only families locate (see figure 1). There are two firms located at x_1 and x_2 such that $x_1 \leq x_2$ and $x_1, x_2 \in [v_1, v_2]$ selling the same good in the commercial area at prices p_1 and p_2 respectively.
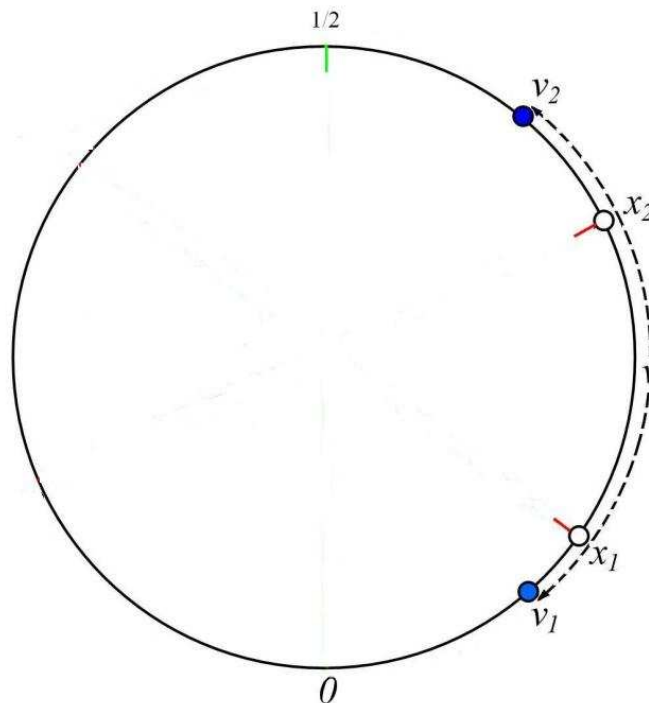

Figure 1: Circular Market

A continuum of consumers spread uniformly along the city. Each consumer buys one unit of good and pays the cost of transporting from the location of the firm from which it is bought to his/her own location. The transportation cost incurred by consumer is assumed to be a quadratic function of distance. Specifically, the function is taken as:

$$c(d_i(x)) = b\, d_i^2(x),\ b > 0,\quad i = 1,2\,,$$

where the distance $d_i(x) = |x - x_i|$ between location consumer x and the location firm x_i, is defined as the shortest distance on the circle between the two points x, x_i.

Let s be the gross surplus for an arbitrary consumer, x. We assume s is large enough ($s \gg 0$) to allow all consumers to buy. Utility for consumer x from buying the good from firm i is, therefore, given as: $u_i(x) = s - p_i - c(d_i(x))$. A consumer purchases the product from firm i when $u_i(x) < u_j(x),\ i = 1,2,\ i \neq j$.

The model is then formalized as a three-stage game. In the first stage, the regulator chooses the optimum size of the commercial area; in the second stage firms choose their locations simultaneously; in a third stage firms decide on their prices at the same time. The game is solved by backward induction. First, we discuss the equilibrium outcomes given the size of the commercial area.

2.2 PRICE AND LOCATION EQUILIBRIUM

The fact that the location space for firms is restricted does not affect in any way the location of the indifferent consumers[4]. Thus, the demand function expression remains the same as for the unrestricted space case (see de Frutos et al (1999)). We can then make the following change of variable: $z = x_2 - x_1$, where z represents the distance between both firms, that is, the difference between the chosen characteristics. The demand function is represented as follows:

---

[4] An indifferent consumer, α buys from firm 1 or 2 so that; $u_1(\alpha) = u_2(\alpha)$.

$$D_1(p_1, p_2, x_1, x_2, v) = \begin{cases} 1 & p_1 - p_2 \leq -bz(1-z) \\ \left[\dfrac{p_2 - p_1}{2bz(1-z)} + \dfrac{1}{2}\right] & -bz(1-z) \leq p_1 - p_2 \leq bz(1-z) \\ 0 & bz(1-z) \leq p_1 - p_2 \end{cases}$$

In addition, demand for firm 2 is: $D_2 = 1 - D_1$.

Once consumer demands are found, the profit functions can be calculated using the following relation: $B_i(p_i, p_j) = p_i D_i(p_i, p_j),\, para\ i = 1, 2 \quad j = 1, 2 \quad siendo\ i \neq j$

The existence of Nash equilibrium in prices is guaranteed for any size of the market v and any value of z, since profit functions are strictly concave (see De Frutos 1999). The solution corresponds to: $p_1^N(z) = p_2^N(z) = bz(1-z)$. Consequently, demand and profit functions can be written as:

$$D_1^N = D_2^N = \frac{1}{2}, \quad B_1^N(z) = B_2^N(z) = \frac{1}{2} b\, z\, (1-z)$$

Proposition 1:

There is a unique Nash location equilibrium for any commercial area given by: $[v_1, v_2]$,

$$x_1^N = v_1, \qquad x_2^N = v_2$$

Demonstration: (See Appendix). ∎

The perfect equilibrium subgame expressions for prices, demand and profits are:

$$p_i^N(v) = bv(1-v), \quad D_i^N = \frac{1}{2}, \quad B_i^N(v) = \frac{1}{2} bv(1-v), \quad i = 1, 2, \quad v = v_2 - v_1$$

Remarks

In the circular model under zoning regulation, the location pattern satisfies the maximum differentiation principle: $x_1^N = v_1$, $x_2^N = v_2$. Notice than if $v_2 - v_1 = 1/2$, the location equilibrium remains the same as in the circular model without zoning (see De Frutos et al. (1999)). Therefore, zoning half of the circular market does not affect the location strategies

of firms. The intuition behind the maximum differentiation result is straightforward: firms locate at $x_1^N = v_1$, $x_2^N = v_2$ in order to avoid competition and reap some spatial monopoly.

The demands are equal and independent of the size v. Thus, zoning does not affect the structure of demand in the location equilibrium. Prices and profits are also equal to each other and they are increasing to respect v.

$$\frac{\partial p_1^N}{\partial v} = \frac{\partial p_2^N}{\partial v} = b(1 - 2v) \geq 0, \qquad \frac{\partial B_1^N}{\partial v} = \frac{\partial B_2^N}{\partial v} = \frac{1}{2}b(1 - 2v) \geq 0 \ for \ \forall \ v \in \left[0, \frac{1}{2}\right].$$

If the regulator chooses a large value for v, competition decreases. Therefore, if firms were able to decide on their location, they will always be interested in the commercial area to be as large as possible in which case, $v = 1/2$. On other hand, the regulator can force both firms to undergo more competition which could yield to zero profits for them. Indeed, if v tends to zero, prices and benefits will tend to zero, which mean that both firms engage in Bertrand competition. Subsequently, zZoning regulation can be seen as an industrial instrument to limit firms' monopoly power.

Given that the location pattern in this zoning model still satisfies the maximum differentiation principle, the purpose of a regulator is to find the dimensions for the commercial area and the exclusively residential area.

2.3 OPTIMAL ZONING

Planners take their decisions according to the interests of firms and consumers and for this reason the objective function is usually defined as the sum of firm´s profit and consumer´s utility. Instead, at this point, we use an objective function for the regulator described as a linear combination from profits (firms) and utility (consumers). Consequently, we introduce the possibility to formally represent the regulator´s preferences in terms of the weight attached to profits or consumers. The welfare function can now be written as:

$$W(v) = \lambda B^N(v) + (1 - \lambda) U(v) \ .$$

Where:

- λ is the weight given by the regulator to firms. Thus, $(1 - \lambda)$ accounts for the weight given to consumers.

- $B^N(v) = B_1^N(v) + B_2^N(v)$ is firms' profit.

- $U(v) = S - \left[B^N(v) + C_T(v) \right]$, is the utility for all consumers.

- S is consumer´s gross surplus.

- $C_T(v)$ stands for total transport cost paid by consumers.

  Total transport cost is then formalized as: $C_T(v) = I_1 + I_2$

- I_1 corresponds to total transport cost paid by consumers when they buy the good from seller 1.

- I_2 is the total transport cost paid by consumers when they buy the product from seller 2.

$$I_1 = \int_0^{\alpha_1^N} \left[b(x - x_1^N)^2 \right] dx + \int_{\alpha_2^N}^1 b\left[(1-x) + x_1^N \right]^2 dx; \qquad I_2 = \int_{\alpha_1^N}^{\alpha_2^N} \left[b(x_2^N - x)^2 \right] dx$$

,where α_1^N, α_2^N are indifferent consumers in terms of buying from firm 1 or firm 2.[5]

We can then represent $C_T(v)$ as follows: $C_T(v) = \dfrac{b}{12}(3v^2 - 3v + 1)$.

- $C_T(v)$ decreases as the size of v increases. The mixed location area for consumers-firms is: $\dfrac{\partial C_T}{\partial v} = \dfrac{b}{4}(2v - 1) \leq 0$, $\forall v \in [0, 1/2]$. When the regulator´s objective is to minimize the total transport cost for consumers, $\lambda = 1/2$ in the objective function. The optimal size for this case is $v = 1/2$.

Utility for the total of consumers is given by: $U(v) = S - \dfrac{b}{12}\left(-9v^2 + 9v + 1 \right)$.

- $U(v)$ is decreasing and reaches a maximum for v equal zero. In contrast to firms consumers are interested in a minimum size for the mixed consumers-firms area which then turns into a single point and $v = 0$. The price of the good is equal to zero for this

---

[5] Indifferent consumers $\alpha_1^N = \dfrac{v_1 + v_2}{2}$, $\alpha_2^N = \dfrac{v_1 + v_2}{2} + \dfrac{1}{2}$.

value and transport cost reaches a maximum. Consumers pay a high price for transport cost but are compensated by a zero cost for the good. The intuition for this result is illustrated by the fact that consumers travel massively (ignore collateral costs) when a free good is offered.

Given the expression for the welfare function $W(v)$ the following can be highlighted:

- When $\lambda > 1/2$, the regulator weights the interest of firms more than consumer´s. In this case we define the planner as conservative.

- However, when $\lambda < 1/2$, the opposite happens and we define the biased regulator as liberal.

- When $\lambda = 1/2$, we have a neutral case in which the same weight is given to both groups: consumers and firms. Furthermore, the welfare function for this case is depicted as:

$$W(v) = 1/2\,[S - C_T(v)]$$

The social welfare $W(v)$ is defined as the sum of the firms' profit and the utility for all consumers.

We now focus on the study of the welfare function when equilibrium in prices and location is taken into account. The optimal strategy for the regulator is given by:

$$v^O = Arg \underset{v}{Max}\ W(v),$$
$$s.t.\ 0 \le v \le (1/2)$$

By substituting the expression for $B^N(v)$ y $C_T(v)$ in the objective function we obtain the following: $W(v) = (2\lambda - 1)\,bv\,(1-v) + (1-\lambda)\left[S - \frac{b}{12}(3v^2 - 3v + 1)\right]$

Proposition 2:

For $0 \leq v \leq \dfrac{1}{2}$, the optimal size for the mixed firms-consumers area is given as:

$$
v_C^* = \begin{cases}
v_{C1}^* = 0, & si \quad 0 \leq \lambda \leq \dfrac{3}{7} \\[2mm]
v_{C2}^* \in \left[0, \dfrac{1}{2}\right], & si \quad \lambda = \dfrac{3}{7} \\[2mm]
v_{C3}^* = \dfrac{1}{2}, & si \quad \dfrac{3}{7} \leq \lambda \leq 1
\end{cases}
$$

Demonstration :(See appendix)■

Remarks:

Note that according to the bias of the regulator, it pushes for one or another type of zoning or other. Zoning, thus, changes the distribution of firms, in effect:

- If $0 \leq \lambda \leq \dfrac{3}{7}$, the regulator is consumer-biased. Agglomeration is then obtained, that is, the location space for firms is reduced to a single point, $v_{c1}^* = 0$. In this case, competition among firms is maximum (Bertrand type) which implies the price of all products is close to zero. These factors clearly benefit consumers as they can live in a larger area enjoying more welfare. On the other hand, in terms of industrial policy, the regulator allows only one characteristic of the good to be produced despite the possibility to produce two characteristics. In this case, we obtain the following results:

  i) utility for the total of consumers is: $\quad U(0) = S - b/12$,

  ii) the profit function of firms equals: $\quad B(0) = 0$,

  iii) the welfare function can be expressed as: $\quad W(0) = (1 - \lambda)(S - b/12)$.

- If $\lambda = 3/7$, social welfare remains constant and independent from the size of the commercial area. The regulator has no a priori preference on the dimension of the commercial area.

- When $\frac{3}{7} \leq \lambda \leq 1$, the regulator favours firms because the value of v, $v_{c3}^{*} = \frac{1}{2}$ is their preferred result. In this case we have dispersion, since firms locate in the end points of the interval. $x_1 = 0,\ x_2 = \frac{1}{2}$. This involves a clear support for a product variety industrial policy. In this case:

i) the utility for the total of consumers: $U(1/2) = S - 13b/48,$

ii) the profit of firms is: $\qquad\qquad\qquad B(1/2) = b/4,$

iii) the welfare function is: $\qquad\qquad\qquad W(1/2) = (1-\lambda)S + (b/48)(25\lambda - 13)$.

We can observe that a firm biased regulator, $(3/7 \leq \lambda \leq 1),$ diminishes the total utility for consumers and improves profit for firms $U(1/2) < U(0)$, .

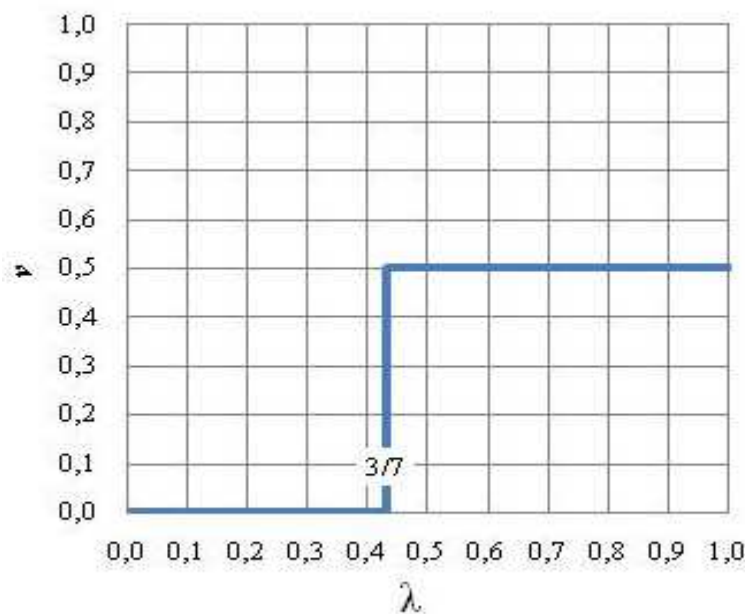Now, we can represent these results in figure 3 by drawing the optimum size for the commercial area:

Figure 2 : Optimum size for the commercial area.

The *horizontal* axis shows the value of parameter λ whereas the vertical axis shows the value of parameter v. The thick line refers to the optimum size of the commercial area.

3. THE LINEAR CITY MODEL.

3.1 THE MODEL

Similarly to the circular case, we introduce a regulator which constraints the production area to the segment (v_1, v_2). Inside this interval the location of firms is given by (x_1, x_2), so that $0 \leq v_1 \leq x_1 \leq x_2 \leq v_2 \leq 1$.[6] Consumers also distribute uniformly among the linear city of length one [0,1] where the areas [0, v_1) y (v_2, 1] are only residential. Again, this model is formalised as a three stage game. In the first stage, a regulator chooses the size of the commercial area. In the second and third stages firms simultaneously decide on locations and prices.
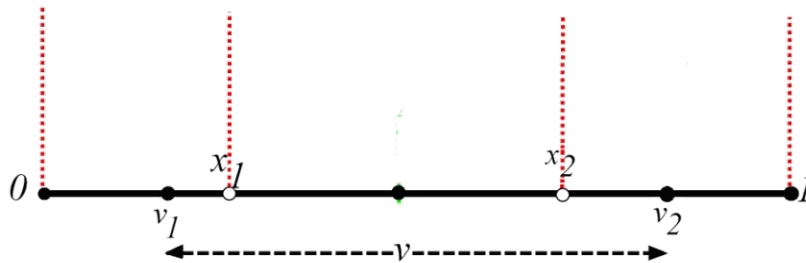
We can represent the model as follows:

Figure 3: Linear Market

Using a quadratic function as transport cost defined as:

$$c(d_i(x)) = b\, d_i^2(x),\ b > 0,\quad i = 1,2\,,$$

where the distance $d_i(x) = |x - x_i|$ between the location for consumer x and the location of firm x_i.

The consumer indifferent is: $\alpha = \dfrac{p_2 - p_1}{2b(x_2 - x_1)} + \dfrac{(x_2 + x_1)}{2}$.

---

[6] Lai y Tsai (2004) have metioned this type of zoning in their article withouth analysing it. They have studied the linear city model in [0, 1] and assumed the area [0, z) to be only residential.

Given the uniform distribution of consumers along the linear city, the results obtained for the regulated model are the same as in D'Aspremont et al. (1979) except for the location equilibrium which is given by: $x_1^N = v_1$; $x_2^N = v_2$. In this context, we can deduce the following results:

$$p_1^N(v) = \frac{1}{3}b(v_2 - v_1)(2 + v_1 + v_2), \quad p_2^N(v) = \frac{1}{3}b(v_2 - v_1)(4 - v_1 - v_2)$$

$$\alpha^N(v) = \frac{1}{3} + \frac{(v_2 + v_1)}{6}$$

$$B_1(p_1, p_2) = \frac{b}{18}(v_2 - v_1)(2 + v_2 + v_1)^2, \quad B_2(p_1, p_2) = \frac{b}{18}(v_2 - v_1)(4 - v_2 - v_1)^2,$$

In order to compare profits between the firms, we compute $B_1^N(v) - B_2^n(v)$ and we obtain:

$$B_1(p_1, p_2) - B_1(p_1, p_2) = \frac{2b}{3}(v_2 - v_1)(1 - (v_2 + v_1)) \geq 0 \quad if \quad v_1 + v_2 \leq 1$$

Given the equilibrium locations: $x_1^N = v_1$; $x_2^N = v_2$, the firm closer to the center of the commercial area obtains higher profits. Under symmetric zoning, $v_1 + v_2 = 1$ firms obtain the same profit. The regulator has no a priori preferences for any of the firms since both are private profit maximizing entities. In turn, we focused on the study of optimal zoning for the commercial area in the symmetric zoning case where: $v_1 + v_2 = 1$.

3.2 OPTIMAL ZONING.

We proceed to solve the last stage of the game given prices and locations in equilibrium. We determine the optimal size of the commercial area by restricting the study to the symmetric case in which $v_1 + v_2 = 1$, where $v_2 - v_1 = v$.

Equilibrium locations under this condition are also symmetric to the extremes of the market. The above obtained results can then be expressed as:

$$x_1^N = \frac{1}{2} - \frac{v}{2}, \quad x_2^N = \frac{1}{2} + \frac{v}{2}, \quad \alpha^N = \frac{1}{2}, \quad p_1^N(v) = p_1^N(v) = bv, \quad B_1^N(v) = B_2^N(v) = \frac{1}{2}bv.$$

Prices and profit, thus are increasingly dependent to v. Firms ideally prefer the maximum size for the commercial, $v=1$.

As in the circular case, the objective function for the regulator is given as:

$$W(v) = \lambda\, B^N(v) + (1-\lambda)\left[S - B^N(v) - C_T(v)\right]$$

In this function λ, $B^N(v)$, S, $C_T(v)$, are respectively the weight given to firms by the regulator; the total profit, consumers´ total surplus, and total transportation cost which is:

$$C_T(v) = \int_0^{\alpha^N} b\left[x - 1/2 + v/2\right]^2 dx + \int_{\alpha^N}^1 b\left[x - 1/2 - v/2\right]^2 dx,$$
$$= b(3v^2 - 3v + 1)/12$$

Observe that transportation cost coincides with the expression for the circular city. Nevertheless, in this case $v \in [0,1]$, which for $C_T(v)$ implies that:

- is decreasing for $\forall v \in [0, 1/2]$, is increasing $\forall v \in [1/2, 1]$, and reaches its minimum value for $v = 1/2$, which will oblige firms to locate in the following points: $x_1^N = 1/4$, $x_2^N = 3/4$, for which $C_T(1/2) = 1/48$. This is an identical result to the optimal social value stemming from the unrestricted linear city model.

- On the other hand, the total price $\left[B^N(v) + C_T(v)\right] = b(3v^2 + 9v + 1)/12$ paid by consumers reaches a minimum for v equal zero: $\partial\left[B^N(v) + C_T(v)\right]/\partial v = b(2v + 3)/4 \geq 0$ *for* $\forall v \in [0,1]$.

Due to the above argument, oppositely to firms consumers are interested in the size of the commercial area being reduced to a single point, $v = 0$, which involves firms locating at exactly the same point: $x_1^N = x_2^N = 1/2$. The price of the good is equal to zero for this value and transport cost reaches a maximum. Therefore, it is worth for consumers to pay a high total

14

transport cost since that will mean a zero price $p_1^N(0) = p_1^N(0) = 0,$ for the good as in the circular case.

Given the expressions of $B^N(v) \ y \ C_T(v)$, the regulator´s objective function is written as: $W(v) = (2\lambda - 1)\,bv + \dfrac{b}{12}(1 - \lambda)\,(S - 3v^2 + 3v - 1).$

Proposition 7:

For $0 \leq v \leq 1$, the optimal size of for the mixed consumers-firms area is given by:

$$
v_L^* =
\begin{cases}
0, & si \ \ 0 \leq \lambda \leq \dfrac{3}{7} \\[2mm]
\dfrac{7\lambda - 3}{2(1 - \lambda)}, & si \ \ \dfrac{3}{7} \leq \lambda \leq \dfrac{5}{9} \\[2mm]
1, & si \ \ \dfrac{5}{9} \leq \lambda \leq 1
\end{cases}
$$

Demonstration :(See appendix)∎

Remarks.

- If the regulator is consumer-biased $(\lambda \leq 3/7 \approx 0{,}42)$ and the commercial area is restricted to $v_L^* = v_{L1}^* - v_{L2}^* = 0,$ the end points of the restricted interval coincide with $v_{L1}^* = v_{L2}^* = 1/2$. Firms locate then in the same point, $x_1^N = x_2^N = v_{L1}^* = v_{L2}^* = 1/2$. They engage in competition "à la Bertrand", with null prices ($p_1^N(v) = p_2^N(v) = 0$). We then have agglomeration in urban policy terms and minimum differentiation in product variety (industrial policy). We can then calculate $U(0)$, $B(0)$ and $W(0)$:

i) utility for the total of consumers is: $\qquad U(0) = S - b/12,$

 ii) the profit function of firms equals: $\qquad B(0) = 0,$

iii) the welfare function can be expressed as: $\qquad W(0) = (1 - \lambda)\,(S - b/12)$

If the regulator´s bias (λ) takes some value between 3/7 and 5/9; the optimal size v_L^* reaches a minimum in the lower end of the interval for $\lambda=3/7$, and a maximum in the upper end for $\lambda=5/9$, i.e for $v_L^*(3/7) = 0$, $v_L^*(5/9) = 1$. In this case $0 \le v_L^* \le 1$ is increasing so that when λ is higher the value of v grows. We then move from agglomeration to dispersion (from minimum differentiation to maximum differentiation). When the social planner assigns a value of $\lambda = 1/2$, the optimal size of the commercial area is reached for $v_L^* = 1/2$. In this case, $U(0)$, $B(0)$ and $W(0)$ are given by:

i) utility for the consumers is: $U(\dfrac{7\lambda - 3}{2(1 - \lambda)}) = S - \dfrac{b}{48(1 - \lambda)^2}(25\lambda^2 + 46\lambda - 23)$,

ii) profit function of firms is : $B(\dfrac{7\lambda - 3}{2(1 - \lambda)}) = b\dfrac{7\lambda - 3}{2(1 - \lambda)}$

iii) welfare function is: $W(\dfrac{7 - \lambda}{2(1 - \lambda)}) = (1 - \lambda)S - \dfrac{b}{48(1 - \lambda)}(143\lambda^2 - 118\lambda + 23)$.

- Finally, if the regulator is firms-biased, $\lambda \ge 5/9 \approx 0,55$, then $v_L^* = 1$, or identically, $x_1^N = v_{L1}^* = 0$, $x_2^N = v_{L2}^* = 1$. The mixed consumers-firms area corresponds to the interval $[0, 1]$; firms can locate in the whole market and they choose maximum differentiation in terms of product variety. This corresponds to agglomeration when interpreted in terms of location patterns. Now, $U(0)$, $B(0)$ and $W(0)$ correspond to:

i) the utility for the total of consumers: $U(1) = S - 13b/12$,

ii) profit of firms is: $B(1) = b$,

iii) welfare function is: $W(1) = (1 - \lambda)S + (b/12)(25\lambda - 13)$.

Similar to the circular model when the regulator is firms-biased meaning that, $(3/7 \le \lambda \le 1)$, the total utility for consumers decreases and $U(1/2) < U(0)$, firms profits improve, $B(1/2) > B(0)$.

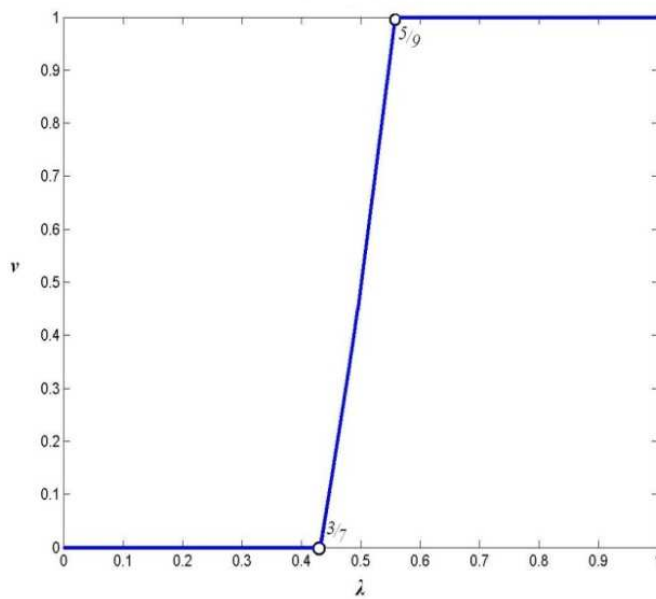We can easily represent these analytical results in the following figure:

Figure 4: Linear Model.

Notably, the results for the optimal size of the mixed commercial-residential are the same for the linear and the circular model.

4. CONCLUSIONS

In this article we analyse spatial competition in a regulated market in which consumers locate freely along the market space, whereas firms are obliged to locate in a restricted area. First, we study the influence of regulation on competition in a circular space and the same question is then analysed for the circular space. The contribution from the present article is the analysis of a regulator with different biases (weights in the objective function). Under these premises we obtain agglomeration or dispersion results which can be interpreted in terms of urban policies. Moreover, these results can also be scrutinized from an industrial policy perspective as minimum or maximum differentiation cases. On the other hand, the question of the influence from the regulators´ bias on competition is also investigated. In this respect it can be stated that strong competition is triggered in the consumer-biased regulator case whereas

weak competition arises in the case of a firms-biased regulator. For a neutral regulator we find moderate competition. Lastly, we find no relevant differences between the results for the circular zoned market and the linear one.

REFERENCES

Anderson, SP., Neven, DJ (1991).*Cournot competition yields spatial agglomeration.* International Economic Review 32 (4):pp 793-808.

Armstrong, M., Cowen, S., Vickers, J. 1994. *Regulatory Reform: Economic   Analysis   and the British Experience*. MIT Press.

Baron, DP., Myerson, RB. 1982. *Regulating a Monopolist with Unknown Costs.* Econometrica, Vol. 50, Nº. 4, pp. 911-930.

Chen, Ch.-Sh., Lai, F.-Ch.. 2008. *Location Choice and Optimal Zoning under Cournot Competition*. Regional Science and Urban Economics 38, pp. 119-126.

D'Aspremont, C., Gabszewicz, J.J. Thisse, J-F. 1979. *On Hotelling's Stability in Competition"* Econometrica 47,pp1145-1150.

De Frutos, M.A., Hamoudi, H. Jarque, X. 1999. *Equilibrium Existence in the Circle model with linear quadratic transport cost.* Regional Science and Urban Economics 29, pp 605-615.

Hamilton, JH., Thisse JF, Weskamp, A. 1989. *Spatial discrimination: Bertrand versus Cournot in a model of location choice.* Regional Science and Urban Economics 59(3), pp 87-615.

Hamoudi, H. and Risueño, M. 2012. *The effect of zoning in Spatial Economics.* Journal of Regional Science Vol 52, Nº2, pp 361-374

Henderson, J. 1991. *Optimal regulation of land development through price and fiscal controls.* Journal of Urban Economic 30: pp. 64-82.

Lai, F.-Ch. and Tsai, J.-F. 2004. *Duopoly Location and Optimal Zoning in a Small Open City.* Journal of Urban Economics 55: pp.614-626.

Matsumura, T., Matsushima, N. Article first published online: 3 aug 2011. *Locating Outside a Linear City Can Benefit Consumers*. Journal of Regional Science

Miceli, T., 1992. *Optimal Fiscal Zoning when the Local Government is a Discriminating Monopolists*. Regional Science and Urban Economics Vol, Nº 22, pp. 579-504.

Mills, D.,1989. *Is zoning a negative –sum game?* Land Economics Vol, Nº 65, pp. 490-596.

Pal , D., 1998. *Does Cournot competition yields spatial agglomeration*? Economics Letters 60 (1) :pp 49-53.

Tsai, Jyh-Fa, Peng, Shin-Kun, Lai, Fu-Chuan. 2006. *Spatial Duopoly with Zoning*. Ann Reg Sci 40: pp. 515-530

Wheaton, W. 1993. *Land Capitalisation, Tiebout Mobility, and the Role of Zoning Regulation*. Journal of Urban Economics 34: pp. 102-117.

APPENDIX.

Demonstration of Proposition 1:

Given the expressions for the price equilibrium profit functions; the Nash equilibrium locations can be calculated by using the first order condition: $\frac{\partial B_1}{\partial x_1} = 0, \quad \frac{\partial B_2}{\partial x_2} = 0,$ We obtain that $x_2 - x_1 = \frac{1}{2}$

Since $0 \le v_1 \le x_1 \le x_2 \le v_2 \le 1/2,$ for any value of $x_1, x_2,$ we then have that $x_2 - x_1 \le 1/2$, so that the necessary condition $\frac{\partial B_1}{\partial x_1} = 0, \quad \frac{\partial B_2}{\partial x_2} = 0,$ is only fulfilled for locations:

$$x_1^N = v_1, \quad x_2^N = v_2 . \blacksquare$$

Demonstration of Proposition 2:

For clarity reasons, the welfare function is rewritten as follows:

$$W(v) = \frac{b}{4}(7\lambda - 3)\ v\ (1-v) + (1-\lambda)(S - \frac{b}{12})$$

By using the first order condition we find that: $\dfrac{\partial W}{\partial v} = \dfrac{b}{4}(1 - 2v)(7\lambda - 3)$. Considering

that this condition depends on the value of parameter λ, in order to determine the maximum a

second order condition is needed: $\dfrac{\partial^2 W}{\partial v^2}$.

Taking into account that: $\dfrac{\partial^2 W}{\partial v^2} = -\dfrac{b}{2}(7\lambda - 3) = 0$, we can deduct the following results:

$-$ If $\quad \lambda \geq \dfrac{3}{7} \quad \Rightarrow \dfrac{\partial^2 W}{\partial v^2} \leq 0 \quad$ the social welfare function is concave and reaches a

maximum for $v = v^*_{c3} = \dfrac{1}{2}$

$-$ If $\quad \lambda = \dfrac{3}{7} \quad \Rightarrow \dfrac{\partial^2 W}{\partial v^2} = 0 \quad$ the social welfare is constant which means it takes the

same value for any value of v between 0 y ½. This means the maximum is reached for $v = v^*_{c2}$

so that $0 \leq v^*_{c2} \leq \dfrac{1}{2}$.

$-$ If $\quad \lambda \leq \dfrac{3}{7} \quad \Rightarrow \dfrac{\partial^2 W}{\partial v^2} \geq 0$, in this case the social welfare function is convex and the

solution for the first order condition corresponds to a minimum, so that a maximum is obtained

for $v = v^*_{c1} = 0$. ∎

Demonstration of Proposition 3:

By using the first order condition, $\dfrac{\partial W}{\partial v} = \dfrac{b}{4}[(7\lambda - 3) - 2v(1 - \lambda))] = 0$ and for $\lambda \neq 1$,

it is found that $v^*_L = \dfrac{7\lambda - 3}{2(1 - \lambda)}$. Assuming that the second order condition is:

$\frac{\partial^2 W}{\partial v^2} = -\frac{b}{2}(1-\lambda) < 0$. Given that: $0 \le v_L^* \le 1$, v_L^* is the maximum of the objective function iif:

$\frac{3}{7} \le \lambda \le \frac{5}{9}$.

- If $\lambda \le \frac{3}{7} \Rightarrow v_L^* \le 0 \quad y \quad \frac{\partial W}{\partial v} \le 0 \quad para \ \forall v \in [0,1]$, el máximo se alcanza, para

$v_L^* = 0$.

- si $\lambda \ge \frac{5}{9} \Rightarrow v_L^* \ge 1 \quad and \quad \frac{\partial W}{\partial v} \ge 0 \quad for \ \forall v \in [0,1]$, the maximum is reached for

$v_L^* = 1$.■