Interactive display of 2D and 3D discrete quadrics with controlled topology

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Abstract In this demonstration, we are going to propose an interactive animation of analytically defined discrete conics (quadrics in 2D) and discrete quadrics in 3D. The digitization is performed on the 2D quadratic equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and the 3D quadric equation $Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$.We propose 4 and 8-connected discrete 2D conics (naive and standard discrete conics) defined analytically where the user can see the resulting discrete conic while interacting with the parameters A, B, C, D, E and F. In the same way, we propose 6-separating and tunnel free 3D quadrics (naive and standard 3D quadrics) defined analytically where the user can can interactively modify the parameters $A, B, C, D, E, F, G, H, I$ and J.

Keywords: Discrete primitives, 2D Quadric curves, 3D Quadric surfaces, Conics, 4-connectivity, 8-connectivity, 18-tunnels, Tunnel-Free.

1 Analytical definition of thin and separating discrete quadrics

The digitization scheme used to define the primitives is a morphological based digitization scheme [1]. The digitization $D_e(\mathcal{A})$ according to the structuring element e and centered on the continuous object $\mathcal{A}$ is defined by:

$$D_e(\mathcal{A}) = (\mathcal{A} \oplus e) \cap \mathbb{Z}^n.$$

where $\oplus$ denotes the Minkowski addition.

The structuring elements we are considering are defined as the inner diagonals of the unit balls of the ℓ^1 and ℓ^∞ norms. These structuring elements are called *digital flakes* [2]. It has been shown that this ensures that the resulting discrete object is respectively 8 and 4-connected in 2D, and 6-separating and tunnel free in 3D, as long as the Euclidean conic being digitized it is not too small (the size of a couple of pixels/voxels) [2].

In 2D, thz digitized curve is defined by the general 2D quadric: $Ax^2+Bxy+Cy^2+Dx+Ey+F = 0$. This equation allows to represent all types of Euclidean conics. There is therefore no restriction on the orientation or the type of the discrete conic as can be seen in the figures that follow. $\Delta = B^2 - 4AC$ is called the discriminant of the quadric and, if the conic is non-degenerate, then:

- if $\Delta < 0$, the equation represents an ellipse; if $A = C$ and $B = 0$, the equation represents a circle, which is a special case of an ellipse; See Figure 1.
- if $\Delta = 0$, the equation represents a parabola; See Figure 2.
- if $\Delta > 0$, the equation represents a hyperbola; if we also have $A + C = 0$, the equation represents a rectangular hyperbola. See Figure 3.

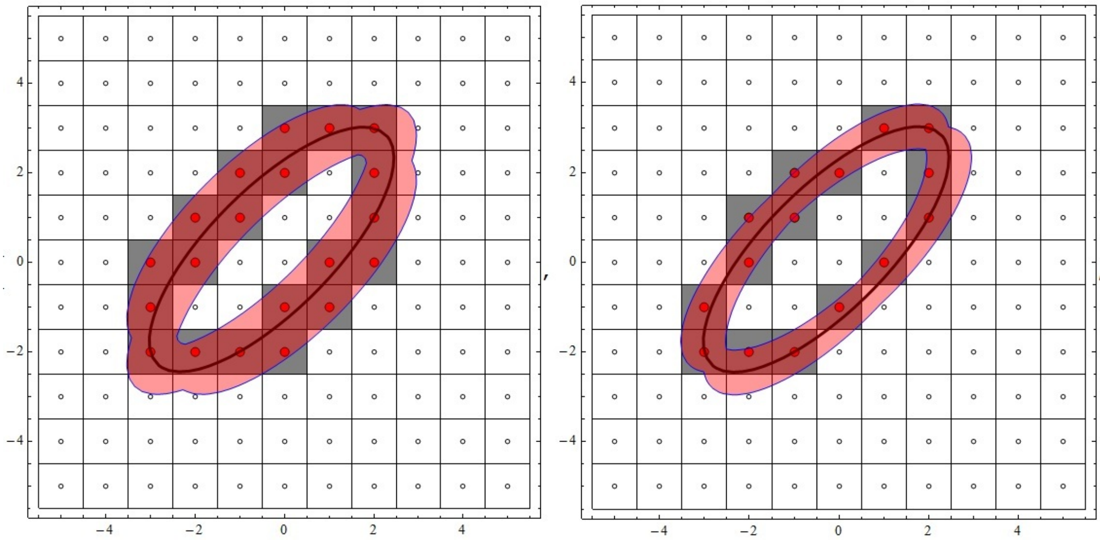

Figure 1: 4 and 8-connected digitized Ellipses of equation $-2x^2 + 3xy - 2y^2 - 2x + 2y + 6 = 0$.

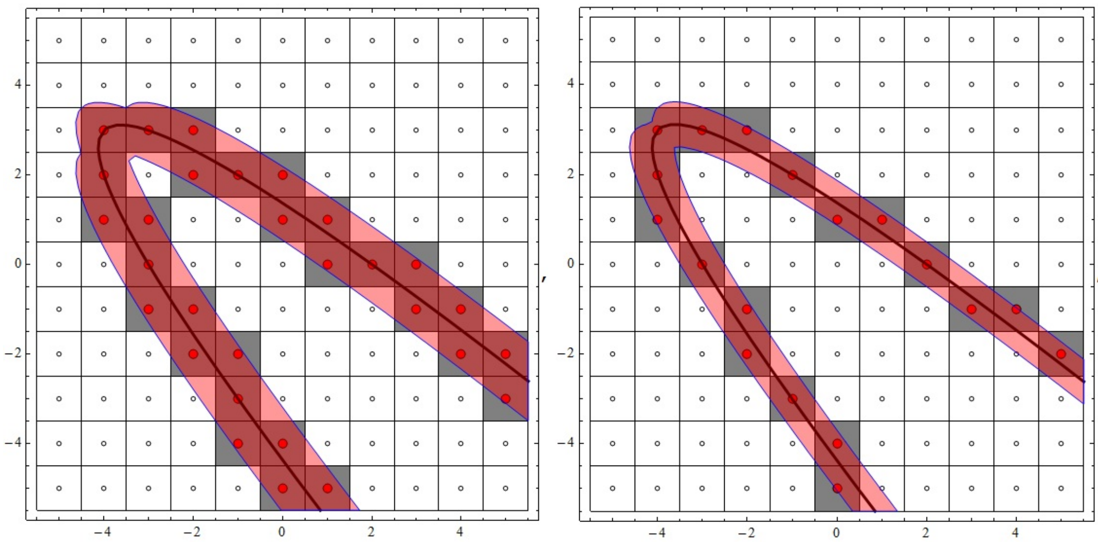

Figure 2: 4 and 8-connected digitized Parabolas of equation $x^2 + 2xy + y^2 + x + 3y - 6 = 0$.

2 Content of the Demonstration

The viewer of the demonstration will be able to interact with the parameters A, B, C, D, E and F and observe the resulting discrete conic interactively. The discrete conic is simply defined by an analytical equation deduced from the digitization scheme and the graphical output is performed with the software Mathematica. See Figure 4.

The same goes for 3D quadrics, where this time we propose a digitization of the quadric surface defined by $Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$. See the Figure 5 for an example of 6-separating hyperbolic paraboloid.

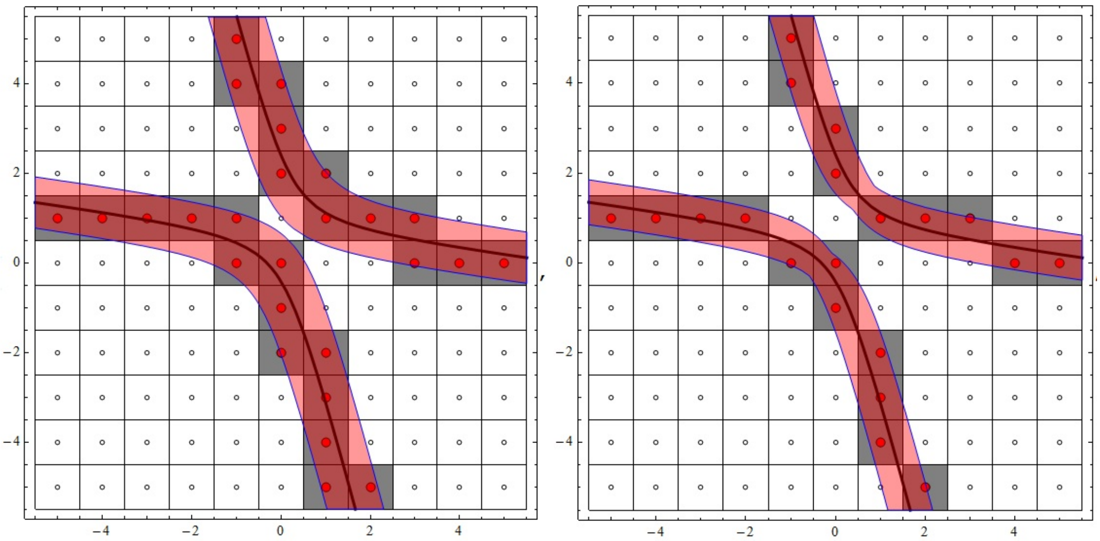

Figure 3: 4 and 8-connected digitized Hyperbolas of equation $x^2 + 8xy + 2y^2 - 6x - 4y - 2 = 0$.

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RegionPlot[{
  a*x^2 + b*x*y + c*y^2 + d*x + e*y + f ≤ Max[Abs[a*x + b*y/2 + d/2] - a/4, Abs[c*y + b*x/2 + e/2] - c/4] &&
   a*x^2 + b*x*y + c*y^2 + d*x + e*y + f >= -Max[Abs[a*x + b*y/2 + d/2] + a/4, Abs[c*y + b*x/2 + e/2] + c/4]
 },
 {x, -5.5, 5.5}, {y, -5.5, 5.5},
 BoundaryStyle → Blue, PlotStyle → {Directive[Red, Opacity[0.4]], Directive[Red, Opacity[0.4]]}
], Frame → True]
```

Figure 4: Plot of the offset region describing analytically the discrete 8-connected conic in Mathematica.

References

[1] Eric Andres, Tristan Roussillon: Analytical Description of Digital Circles. DGCI 2011: Nancy, France, April 6-8, 2011. Proceedings. Lecture Notes in Computer Science 6607 Springer, pp.235-246, 2011

[2] Jean-Luc Toutant, Eric Andres, Tristan Roussillon, Digital Circles, Spheres and Hyperspheres: From Morphological Models to Analytical Characterizations and Topological Properties. Submitted to the journal "'Discrete Applied Mathematics"'.

[3] Wikipedia web page on $2D$ conics. *http://en.wikipedia.org/wiki/Conic$_s$ection*

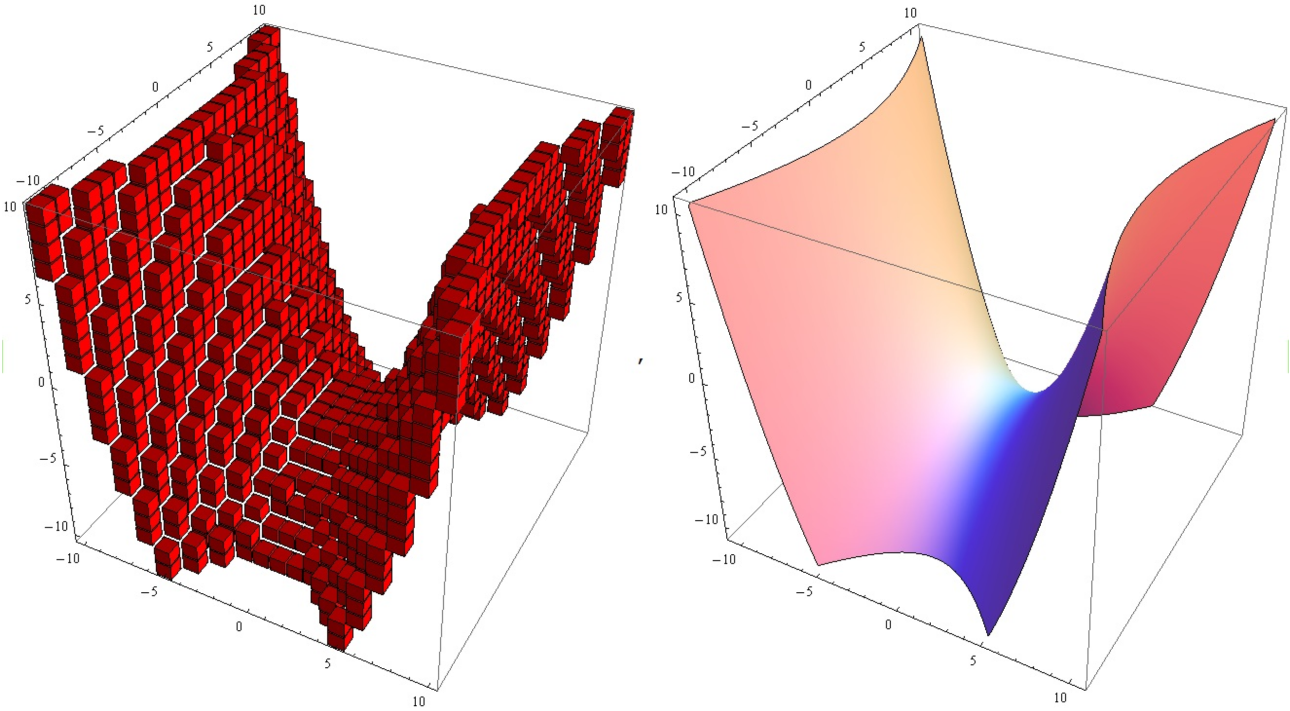

Figure 5: Naive Hyperbolic Paraboloid of equation $\frac{x^2}{4} - \frac{y^2}{9} - (z+5) = 0$.