UNIVERSIDAD DE SEVILLA

TESIS DOCTORAL

NETWORK DATA ENVELOPMENT ANALYSIS: MODELS AND APPLICATIONS

(Modelos y Aplicaciones de Análisis por Envoltura de Datos a Procesos en Red)

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Diciembre de 2013

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Quiero expresar mi agradecimiento a quienes han contribuido en la realización de esta tesis y me han apoyado durante todos estos años:

A Sebastián, por sus conocimientos, paciencia y consejos.

A Ester, que me introdujo en la investigación.

A mis padres, que se han sacrificado y me hicieron amar el mundo académico.

A mis hermanos, por su sincera amistad e incansables ánimos.

RESUMEN

El análisis por envoltura de datos (DEA) es una metodología no paramétrica que evalúa la eficiencia de unidades con capacidad de decisión (DMUs), teniendo en cuenta únicamente información relativa a las entradas y salidas de los procedimientos de transformación o producción que realizan. DEA ha sido utilizado ampliamente por académicos y profesionales en casi cualquier sector y se ha convertido en una técnica muy popular. Recientemente, DEA se ha extendido a procesos en red (Network DEA), permitiendo incorporar la estructura interna de las DMUs, que correspondería a los procesos internos y sus interrelaciones en los que se divide cualquier organización o industria.

Esta tesis ha profundizado en aplicaciones de Network DEA a nuevos sectores, desarrollando, por tanto, modelos que permitan identificar las ineficiencias de cada proceso. En primer lugar, un modelo con cinco etapas, incluyendo la adquisición de jugadores y los sistemas ofensivos y defensivos, ofrece un mayor conocimiento de la falta de rendimiento de los equipos de baloncesto de la NBA. Dicho trabajo se complementó con otro artículo que ha estimado la evolución de la productividad durante un período de cinco años mediante un índice Malmquist. Con respecto a la evolución temporal, una propuesta dinámica permite incluir actividades que influyen a sucesivos períodos de tiempos, como es el caso de la inversión en instalaciones y líneas de transmisión en las empresas de telecomunicaciones fijas en los Estados Unidos.

Asimismo, se ha considerado el efecto de salidas no deseables. Dicho estudio se basa en el sector de los aeropuertos, donde incorporar los retrasos de los aviones lleva a una evaluación más realista y justa de la eficiencia. Network DEA también posibilita la detección de ineficiencias en sistemas complejos, como la prestación de servicios por parte de los estados, mediante una estimación de las posibles reducciones en las partidas de gasto público, impuestos y deuda, sin disminuir el nivel actual de bienestar. Por último, se han propuesto aproximaciones de Network DEA para tratar con conjuntos de datos borrosos. En

resumen, a cambio de un mayor requerimiento de datos, Network DEA revela más ineficiencias que DEA tradicional, debido a un conocimiento más profundo de la estructura interna de las unidades, además de proporcionar la eficiencia de las diferentes etapas que componen el proceso productivo.

El documento de la tesis ha sido desarrollado siguiendo las pautas marcadas por la Universidad de Sevilla para un compendio de artículos. La introducción presenta los conceptos básicos de la metodología DEA, que junto a los primeros modelos que se propusieron de Network DEA, servirán como base para las aplicaciones desarrolladas en el ámbito de la tesis. A continuación se incluyen los objetivos de cada una de las publicaciones, así como las motivaciones que han llevado a su estudio. En la siguiente sección se resumen del análisis de los resultados de los diferentes modelos. Finalmente, se adjunta una síntesis de las conclusiones de la tesis, mientras que las publicaciones han sido incorporadas en el anexo que cierra el presente documento.

"Efficiency is doing better what is already done"

Peter F. Drucker

(*Father* of management education)

"There are only two qualities in the world: efficiency and inefficiency, and only two sorts of people: the efficient and the inefficient"

George Bernard Shaw

(Co-founder of the London School of Economics)

I. FOREWORD

The current document has been prepared by following the guidelines required by the University of Seville to submit the PhD thesis as a collection of journal papers and book chapters. It consists of the following sections:

1. An *introduction* to the basics of Data Envelopment Analysis, along with the foundations of Network Data Envelopment Analysis, which will be the basis for the development of models and applications

2. A section covering the *aim* and *motivations* of the published papers.

3. A brief description of the *results* and *discussion* of the proposed approaches.

4. The global *conclusions*, as well as further research.

5. The list of *references* which are cited in previous sections.

6. An *appendix*, containing the journal papers and book chapters, on which the PhD thesis is based. All journals where the papers have been published are indexed in JCR database (Journal Citation Reports), while the books have an international impact.

II. INTRODUCTION

II. INTRODUCTION

The aim of this section is at providing the foundations needed to develop the approaches proposed in the published works enclosed in the Appendix. First, an introduction to Data Envelopment Analysis is presented and afterwards a brief review of the main models of Network Data Envelopment Analysis is carried out. Although there are many other features, models and extensions on DEA, a review of all DEA methodologies is beyond the scope of this document.

II.1. DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis, henceforth DEA, is a non-parametric optimization technique first proposed by Charnes et al. (1976). DEA allows assessing the relative efficiency of *Decision Making Units* (DMUs), which must operate in a similar way, by taking into account data of the *inputs* and *outputs* regarding their production process. In fact, inefficiency is seen as excesses for the inputs and shortfalls for the outputs. Therefore, an efficient unit (efficiency = 1) cannot increase its current output levels without increasing its input consumption.

The relative efficiency depends on the distance of the observations to the efficient frontier (the closer to the frontier, the more efficient a DMU is), which is formed by the efficient DMUs, and there is no need to define its form in advance. Due to its versatility, DEA has been widely applied in many sectors by researchers, even experiencing a significant growth in the number of publications during years (Emrouznejad et al. 2008).

Although a deeper degree of insight is shown in any book about DEA, e.g. Cooper et al. 2011, the basic models which are the basis for Network DEA models are going to be presented in this section. The CCR model, named after its authors (Charnes et al. 1979), was a linearization of the DEA ratio model. Let consider there are n DMUs, each of one transforming m inputs into s outputs, being x_{ij} the observation of the input i consumed by DMU j and y_{kj} the observation of the output *k* produced by DMU *j*. Hence the efficiency of DMU *J* is obtained by the following linear program (1):

$$
\begin{aligned}
& E_J = Max \sum_{k=1}^{s} v_k y_{kJ} \\
& s.t. \\
& \sum_{i=1}^{m} u_i x_{iJ} = 1 \\
& \sum_{k=1}^{s} v_k y_{kj} - \sum_{i=1}^{m} u_i x_{ij} \leq 0 \qquad j = 1 \ldots n \\
& u_i, v_k \geq 0 \qquad \forall i \forall k
\end{aligned}
\tag{1}
$$

where u_i and v_k are the multipliers associated to the input *i* and the output *k*, respectively. The multipliers represent the importance given to each input and output, thus being the product between multipliers and variables the virtual inputs and outputs. This model is actually said to be a *multiplier* model.

Despite the LP in (1) being aimed at maximization and, in addition, the DMU under evaluation will choose the most beneficial values to its multipliers, there is a constraint in model (1) that guarantees that all DMUs show an efficiency lower than 1 when taking into account the values of the multipliers for DMU *J*, i.e. the DMU under assessment.

Model (1) is said to be radial, because the value of the efficiency for DMU *J* points out the proportional reduction in the current value of all inputs that would lead to a projection on to the efficient frontier. This interpretation can be deduced from the *dual* model of (1), called *envelopment* model, and presented in (2):

$$
\begin{aligned}
& E_J = Min\ \theta \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta \cdot x_{iJ} \qquad \mathrm{i} = 1 \ldots m \\
& \sum_{j=1}^{n} \lambda_j y_{kj} \geq y_{kJ} \qquad \mathrm{k} = 1 \ldots s \\
& \lambda_j \geq 0 \qquad \forall j
\end{aligned} \tag{2}
$$

where λ_j, also known as *lambdas*, represents the technology of the production system from the observations, allowing to reach any feasible points within this production process. By means of a convex linear combination of the observations of the efficient DMUs, a *projection* in the frontier for the DMU under assessment can be obtained. The values of the inputs and outputs for the projected DMU represent its *targets* inputs and outputs, respectively.

Furthermore, if the restriction that sum of lambdas equals 1 is introduced, the technology will shift from *constant returns of scale* (CRS) to *variable returns of scale* (VRS). VRS can also be implemented in multiplier models (Banker, 1984). The CRS efficiency (global efficiency) can be decomposed into VRS efficiency (*technical efficiency*) times *scale efficiency*. Finally, θ means the proportional reduction of the inputs needed for an inefficient DMU to reach the frontier.

The above model (2) is said to be *input-oriented*, since it aims at minimizing the value of the inputs while maintaining the value of outputs. An *output-oriented* model, which aims at maximizing the value of outputs, for both multiplier and envelopment models, can be also implemented. In addition to the proportional reduction, a second phase can be implemented, where additional input excesses and output shortfalls, also known as *slacks*, are sought, even along the frontier when the DMU has been radially projected.

Instead, there are other approaches to DEA which do not imply a radial optimization. For instance, Tone (2001) proposed a non-oriented model, Slacks-Based Measure (SBM), which seeks the maximization of all slacks. In other words, it aims at maximizing outputs and minimizing inputs. Furthermore, the SBM model, which is attached below, shows other desirable properties, such as both translation and unit invariant.

$$
\begin{aligned}
& E_J = Min \left(1-(1/m)\sum_{i=1}^{m} s_{iJ}/x_{iJ}\right) \Big/ \left(1+(1/s)\sum_{k=1}^{s} s_{kJ}/y_{kJ}\right) \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{iJ} - s_{iJ} \qquad \mathrm{i} = 1...m \\
& \sum_{j=1}^{n} \lambda_j y_{kj} \ge y_{kJ} + s_{kJ} \qquad \mathrm{k} = 1...s \\
& \lambda_j, s_{iJ}, s_{kJ} \ge 0 \qquad \forall j, \forall i, \forall k
\end{aligned} \tag{3}
$$

where s_{iJ} and s_{kJ} are the slacks for inputs and outputs, respectively. Please note that the metrics seeks the maximization of all slacks, while guaranteeing that an efficient unit will have efficiency equal to 1. Again the lambdas will define the target values of the variables for any inefficient DMUs.

Apart from consuming inputs and producing desirable outputs, there are applications where *undesirable outputs*, such as waste or noise, are inevitable generated, since they are the result of the production process (Ramli and Munisamy, 2013). Therefore, it is usually assumed the *joint weak disposability* of desirable and undesirable outputs, while the Directional Distance Function approach (DDF) proposed by Chung et al. (1997) is a common methodology to handle with these situations. Let consider there are *l* undesirable outputs, being u_{bj} the undesirable output *b* produced by DMU *j*. The DDF approach is presented below:

$$
\begin{aligned}
& Max\beta \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{iJ} - \beta \cdot g_i^x \qquad \mathrm{i} = 1...m \\
& \sum_{j=1}^{n} \lambda_j y_{kj} \ge y_{kJ} + \beta \cdot g_k^y \qquad \mathrm{k} = 1...s \\
& \sum_{j=1}^{n} \lambda_j u_{bj} \ge u_{bJ} - \beta \cdot g_b^u \qquad \mathrm{b} = 1...l \\
& \lambda_j \ge 0 \quad \forall j \qquad \beta \ \text{free}
\end{aligned}
\tag{4}
$$

where the vector $\left(g_i^x, g_k^y, g_b^u\right)$ represents the direction along which the observed DMU will move towards the frontier, being β the step size without abandoning the *Production Possibility Set* (PPS), i.e. the set of all feasible points. It can be seen that the larger the value of β, the more inefficient the DMU under assessment. The relation between DDF (4) and SBM (3) approaches was stated by Färe and Grosskopf (2010).

Finally, it is relevant to mention that there have been proposed several methodologies to measure the change of the efficiency over time. Amongst all approaches, the *Malmquist Productivity Index* (MPI) (Färe et al. 1992), measures the variation of the productivity efficiency between two adjacent periods of time, e.g. *t* and *t+1*. Given the data of inputs and outputs observed during periods t and t+1, if a input-oriented radial DEA model such as model (2) is considered, $DF_t^I\left(x_J^{t+1}, y_J^{t+1}\right)$ $\left(DF_{t+1}^I\left(x_J^t, y_J^t\right)\right)$ stands for the proportional reduction of the inputs of DMU *J* observed in period t+1 (t) assuming that the production technology is constructed from the observations in period t (t+1). Hence the input-oriented MPI for DMU *J* is defined as the geometric mean:

$$MPI^{I}_{t,t+1} = \left[\frac{DF^{I}_{t}\left(x^{t+1}_{J}, y^{t+1}_{J}\right)}{DF^{I}_{t}\left(x^{t}_{J}, y^{t}_{J}\right)} \cdot \frac{DF^{I}_{t+1}\left(x^{t+1}_{J}, y^{t+1}_{J}\right)}{DF^{I}_{t+1}\left(x^{t}_{J}, y^{t}_{J}\right)} \right]^{1/2} = EFFCH^{t}_{t,t+1} \cdot TECCH^{t}_{t,t+1}$$

where

$$EFFCH^{t}_{t,t+1} = \frac{DF^{I}_{t+1}\left(x^{t+1}_{J}, y^{t+1}_{J}\right)}{DF^{I}_{t}\left(x^{t}_{J}, y^{t}_{J}\right)} \tag{5}$$

$$TECCH^{t}_{t,t+1} = \left[\frac{DF^{I}_{t}\left(x^{t}_{J}, y^{t}_{J}\right)}{DF^{I}_{t+1}\left(x^{t}_{J}, y^{t}_{J}\right)} \cdot \frac{DF^{I}_{t}\left(x^{t+1}_{J}, y^{t+1}_{J}\right)}{DF^{I}_{t+1}\left(x^{t+1}_{J}, y^{t+1}_{J}\right)} \right]^{1/2}$$

An improvement in productivity corresponds to a MPI greater than unity. Otherwise, productivity has declined over time. As shown above in (5), MPI is commonly decomposed into efficiency change (EFFCH) and technical change (TECCH).

Efficiency change measures the magnitude of the change in technical efficiency between periods *t* and *t+1*. An improvement in EFFCH is an evidence of catching-up with the frontier for that DMU. Otherwise, production is moving away from the frontier. Concerning technical change, it measures the shift in the frontier over time. In other words, an improvement in TECCH implies progress in the technology, whereas a worsening in TECHH involves technological regress. When the case under study exhibits VRS, the FGNZ decomposition (Färe et al. 1994) can be implemented.

II.2. NETWORK DATA ENVELOPMENT ANALYSIS

Unlike traditional DEA, which considers each DMU as a *black box*, i.e. only data of exogenous inputs and outputs are used, Network DEA takes into account the internal structure of the DMU, i.e. the *internal processes* or interrelated stages which a DMU consist of. In the literature, the terms internal process, stage, division and sub-DMU are used equally.

Apart from the exogenous inputs and outputs, the *intermediate products* connecting sub-DMUs, i.e. output from one internal stage and input to other stage, are included. The only restrictions are that all intermediate products must be produced and consumed within the system and that there cannot be loops, e.g. an intermediate product being produced and consumed at the same stage. However, exogenous variables can be inputs to or outputs from any stage.

Although complex structures of internal processes are allowed, most of the research has been carried out on series systems, especially on 2-stage series system. A review of the approaches on 2-stage Network DEA was done by Cook et al. (2010).

So Network DEA allows revealing more sources of inefficiency, which are related to the under-performing of the internal processes. In fact, even a DMU assessed as efficient when traditional DEA is applied, could be regarded as inefficient according to a Network DEA approach, because all its processes must be efficient for a DMU to be network (or global) efficient.

First attempts on Network DEA laid the foundations for Network DEA PPS (Färe and Grosskopf, 1996a, 2000) and successfully modeled previous applications as a set of interrelated stages (Löthgren and Tambour, 1999, Seiford and Zhu, 1999). However, these approaches did not differ from applying traditional DEA to each of the stages in an independent way.

However, if the interconnection between stages is not taken into account, the target values of intermediate products computed for a stage could lead to inconsistencies in other stages. For instance, if a input-oriented DEA model is applied to a stage, the computation of its efficiency will imply a reduction in the value of an intermediate product, but this target value will have a negative impact on the calculation of the efficiency of the stage that produce the intermediate product. This issue was first addressed by Kao and Hwang (2008) and their *Relational Model* will be described in the following section.

There have been other methodologies defined as Network DEA, such as computing the efficiency of latter stages by using the target variables of former stages, depending on the orientation (Sexton and Lewis, 2003, Lewis and Sexton, 2004a, 2004b, Lewis et al. 2009) and the multi-activity approaches, which integrate *shared inputs* (Yu, 2008a, 2008b, 2010, Yu and Chen, 2011, Yu and Fan, 2009, Yu and Lee, 2009, Yu and Lin, 2008).

Despite shared inputs not being included in the published papers that make up this document, the term refers to variables that are divided among several divisions, usually without stating the ratios in advanced (Chen et al. 2010, Zha and Liang, 2010). For a more detailed classification of all possibilities when the internal structure of a DMU is considered, please refer to Castelli et al. (2010).

II.2.1. RELATIONAL MODEL

The Relational Model (Kao and Hwang, 2008, Kao, 2009) extended the multiplier model (1) to a multi-stage structure, but making the multiplier of a certain intermediate product have the same value for all processes regardless of being produced or consumed (the relative importance of an intermediate product should remain the same regardless of it being produced or consumed). Equivalences with previous approaches were pointed out by Chen et al. (2009).

The relational model by Kao and Hwang (2008) only included a 2-stage structure and will be reproduced in this section due to its simplicity. There will be inputs to the first stage, outputs from the second stage and intermediate products only linking both stages. Let consider there are R intermediate products, being z_{rj} be observation of the intermediate product r of DMU j. Thus the input-oriented CRS relational model for a two-stage series system is presented in (6).

$$
\begin{aligned}
&E_J = Max \sum_{k=1}^{s} v_k y_{kJ} \\
&s.t. \\
&\sum_{i=1}^{m} u_i x_{iJ} = 1 \\
&\sum_{k=1}^{s} v_k y_{kj} - \sum_{r=1}^{R} w_r z_{rj} \le 0 \qquad j = 1 \ldots n \\
&\sum_{r=1}^{R} w_r z_{rj} - \sum_{i=1}^{m} u_i x_{ij} \le 0 \qquad j = 1 \ldots n \\
&u_i, v_k, w_r \ge 0 \qquad \forall i \forall k \forall r
\end{aligned}
\tag{6}
$$

where w_r is the multiplier associated to the intermediate product r and there are two restrictions that guarantee that the efficiencies for stage 1 and 2 are equal or lower than 1. After solving model (6), the global efficiency can be decomposed into the efficiencies for every stage (7), which can be obtained by substituting the value of the multipliers (although a second optimization may be needed when there are multiple solutions). It can be proved that a DMU is overall efficient if and only if it is efficient for all divisions. Also notice that the product of the efficiencies of both stages will lead to the objective function in (6).

$$
E_1 = \frac{\sum_{r=1}^{R} w_r z_{rJ}}{\sum_{i=1}^{m} u_i x_{iJ}} \qquad\qquad E_2 = \frac{\sum_{k=1}^{s} v_k y_{kJ}}{\sum_{r=1}^{R} w_r z_{rJ}} \tag{7}
$$

After introducing the relational model for parallel systems (Kao, 2009b), a relational model for a combination of series and parallel systems was presented by Kao (2009a). Finally, Kao (2014) formulated a general relational Network DEA model where exogenous inputs and outputs can be introduced in any stage within the system, which is an improvement to Li et al. (2013) model that includes exogenous inputs only in the second stage.

One of the two main drawbacks of the model in (6) is that a VRS version would be highly non-linear, as long as it is computed as the product of the efficiency of the internal processes. Although this issue was partially addressed by Kao and Hwang (2011), it has been common to turn to the *Additive Decomposition* model when the internal stages exhibit VRS.

The Additive Decomposition model for a 2-stage system can be found in Chen et al. (2009) and a model for general structures is defined by Chen et al. (2010). Instead of a product of the efficiencies of the individual stages, Chen et al. (2009) proposed the weighted average of these efficiencies. For a certain value of the weights (the sum of the virtual inputs in a stage divided by the total sum of virtual inputs), the input-oriented VRS additive decomposition model is presented below:

$$
\begin{aligned}
& E_J = Max \left(\sum_{k=1}^{s} v_k y_{kJ} + \sum_{r=1}^{R} w_r z_{rj} + \eta_1 + \eta_2 \right) \\
& s.t. \\
& \sum_{r=1}^{R} w_r z_{rj} + \sum_{i=1}^{m} u_i x_{iJ} = 1 \\
& \sum_{k=1}^{s} v_k y_{kj} - \sum_{r=1}^{R} w_r z_{rj} + \eta_2 \le 0 \qquad j = 1 \ldots n \\
& \sum_{r=1}^{R} w_r z_{rj} - \sum_{i=1}^{m} u_i x_{ij} + \eta_1 \le 0 \qquad j = 1 \ldots n \\
& u_i, v_k, w_r \ge 0 \qquad \forall i \forall k \forall r \\
& \eta_1, \eta_2 \quad \text{free}
\end{aligned}
\tag{8}
$$

The other main drawback of the relational model is the inability to provide targets for the intermediate products and thus failing at stating the optimal operation points of all stages (Chen et al. 2010). This issue is solved when applying dual models, as reviewed in the following section.

According to other authors, the efficiency decomposition of the relational Network DEA can be seen as a centralized approach, since it optimizes the efficiencies of all stages at the same time. Instead, other options are proposed, like giving priority to one of the stages or introducing game theory approaches (Liang et al. 2006, 2008, Chen and Yan, 2011).

II.2.2. NETWORK SBM

Tone and Tsutsui (2010) proposed the Network SBM model, which added the SBM metrics from model (3) (Tone, 2001) to the envelopment form of the Relational Network DEA (model (6)). In addition, the Network SBM model was designed to admit any number of processes and any network structure.

By following the notation by Lozano (2011), let $P^{out}(r)\left(P^{in}(r)\right)$ the set of processes that generate (consume) the intermediate product *r*, and for each $p \in P^{out}(r)\left(p \in P^{in}(r)\right)$, let z_{rj}^{p} the observed amount of intermediate product *r* generated (consumed) by process *p* of DMU *j*.

For the scope of this work, the sets $P^{out}(r)$ and $P^{in}(r)$ only contain one process. In other words, every intermediate product is produced by one stage and consumed by another. Please note that for each exogenous input and output (and their slacks), the super-index *p* will point out which process consumes or generate it, respectively. The Network SBM model is presented in (9).

Apart from the inclusion of the processes, the main difference from (3) is the constraint that guarantees that, for every intermediate product, its generated amount is greater than the consumed amount within the system. This constraint turns out to be the dual to the imposition on the multipliers of intermediate products to have the same value regardless of the process. Sometimes the constraint is changed to set the generated amount equals the consumption.

$$E_J = Min \ \sum_{p=1}^{P}\left(1-(1/m)\sum_{i=1}^{m} s_{iJ}^{p} / x_{iJ}^{p}\right) \Big/ \sum_{p=1}^{P}\left(1+(1/s)\sum_{k=1}^{s} s_{kJ}^{p} / y_{kJ}^{p}\right)$$

$$\begin{aligned}
& s.t. \\
& \sum_{j=1}^{n} \lambda_j^p x_{ij}^p \le x_{iJ}^p - s_{iJ}^p && \mathrm{i} = 1 \ldots m \\
& \sum_{j=1}^{n} \lambda_j^p y_{kj}^p \ge y_{kJ}^p + s_{kJ}^p && \mathrm{k} = 1 \ldots s \\
& \sum_{j=1}^{n} \lambda_j^{p \in P^{out}(r)} z_{rj}^{p \in P^{out}(r)} \ge \sum_{j=1}^{n} \lambda_j^{p \in P^{in}(r)} z_{rj}^{p \in P^{in}(r)} && \mathrm{r} = 1 \ldots R \\
& \lambda_j^p, s_{ij}^p, s_{kj}^p \ge 0 \qquad \forall j, \forall i, \forall k, \forall p
\end{aligned} \tag{9}$$

Unlike the multiplier form of the relational model, the set of optimal lambdas from (9) allows to compute the projection for all inputs, outputs and intermediate products. In addition, it can be deduced from (9) that there is a different set of lambdas associated with each stage, that is, each process has its own technology. Therefore, the technology of each stage can be defined as CRS or VRS in an independent way.

Regarding the intermediate products, additional constraints can be imposed to force them to be greater than, equal to or lower than the observed amounts, depending on the nature of each intermediate product. These additional constraints over intermediate products also allow including their slacks in the objective function of (9). After model (9) being solved, the efficiency of a certain process is computed as below. It can be proved that a DMU is overall efficient if and only if it is efficient for all divisions.

$$E_J^p = \left(1-(1/m)\sum_{i=1}^{m} s_{iJ}^{p} / x_{iJ}^{p}\right) \Big/ \left(1+(1/s)\sum_{k=1}^{s} s_{kJ}^{p} / y_{kJ}^{p}\right) \tag{10}$$

Although Network DEA models differ on the metrics, which will be chosen to fit a certain application, a general PPS, i.e. the set of feasible points within the technology, can be defined. The overall PPS and the PPS of all processes are defined by Lozano (2011). In fact, the notation used in the published works enclosed in the Appendix is based on that of Lozano (2011).

There have been an increasing number of publications and models based on the envelopment form of the relational Network DEA, such as Avkiran (2009), Avkiran and McCrystal (2012) or Fukuyama and Matousek (2011). Previous concepts that were reviewed in section II.1 has been also extended to DEA, such as the MPI in two-stage systems (Kao and Hwang, 2014). Regarding undesirable outputs, Fukuyama and Weber (2010) applied their directional slacks-based measure of technical inefficiency (SBI) (Fukuyama and Weber, 2009) to a two-stage system with bad outputs.

Due to the similarities between intermediate products and *carry-over activities*, Tone and Tsutsui (2010) proposed a Dynamic DEA model based on its previous Network SBM model (Tone and Tsutsui, 2009). Carry-over activities, a.k.a. links, refer to connecting activities between two periods of time, i.e. activities from previous periods of time that have an effect on a later period (Chen, 2009). If periods of time are considered to be stages within a network structure, carry-over activities can be modeled as intermediate products and the Network SBM model applied. This concept has been also introduced into the multiplier relational model (Kao, 2013).

II.2.3. FUZZY NETWORK DEA

So far, all DEA and Network DEA models were supposed to work with crisp data. Since input and output data are sometimes ambiguous in the real world, many researches have proposed fuzzy methods in DEA to deal with imprecise data. A fuzzy number is a function whose domain is a set of real numbers and each value in that domain is assigned a grade of membership. In other words, it refers to a set of possible values, where each value has its own weight. Unfortunately, there is not a unique way to fit fuzzy data in DEA. For a review of the fuzzy DEA models please refer to Hatami-Marbini et al. (2011).

Concerning Network DEA, Kao and Liu (2011) extended the multiplier 2-stage relational model to a fuzzy environment, where all inputs, outputs and intermediate products are supposed to be LR-type fuzzy numbers. It can be seen as a modification of their previous work (Kao and Liu, 2000) to support a 2-stage structure.

The efficiency will also be a LR-type fuzzy number, whose membership function can be built by means of its alpha-cuts (an alpha-cut is the set of all elements that have a membership value greater than or equal to alpha), due to its nested structure. An alpha-cut is defined by two values, namely its lower and upper bounds. For the efficiency of DMU J, the lower and upper bounds of an alpha-cut are represented by $\left(E_J\right)_\alpha^L$ and $\left(E_J\right)_\alpha^U$, respectively.

The computation of the upper bound will consist in maximizing the calculation of the efficiency for the range of values of the alpha-cuts of each variable, whereas the lower bound consists in minimizing the efficiency considering that variables can take any value within their alpha-cuts. Therefore, both computations are two-level programs, where the outer and inner programs could be combined only when they have the same direction for optimization.

The program for computing the upper bound for the efficiency of DMU J is presented in (11), whereas the lower bound can be obtained by (12). Please notice that a multiplier model is used in (11) and an envelopment model in (12), since the direction for optimization have to be the same in the outer and inner programs, as stated above.

In addition, when computing the upper bound for the efficiency of DMU J, the most favorable conditions for DMU J and the most unfavorable conditions for the rest of DMUs have been selected, i.e. the largest values of outputs and the lower values of inputs within their ranges of values for DMU J and the lower values of outputs and higher value of inputs for the rest of DMUs. In the case of the lower bound for the efficiency of DMU J, it works the other way around. Regarding the intermediate products, these variables can take any value within the range corresponding to the domain of their alpha-cuts.

$$
\begin{aligned}
&\left(E_J\right)_\alpha^L = Max \sum_{k=1}^{s} v_k \left(Y_{kJ}\right)_\alpha^U \\
&s.t. \\
&\sum_{i=1}^{m} u_i \left(X_{iJ}\right)_\alpha^L = 1 \\
&\sum_{k=1}^{s} v_k \left(Y_{kJ}\right)_\alpha^U - \sum_{r=1}^{R} w_r z_{rj} \le 0 \\
&\sum_{k=1}^{s} v_k \left(Y_{kj}\right)_\alpha^L - \sum_{r=1}^{R} w_r z_{rj} \le 0 \qquad \forall j \ne J \\
&\sum_{r=1}^{R} w_r z_{rj} - \sum_{i=1}^{m} u_i \left(X_{iJ}\right)_\alpha^L \le 0 \\
&\sum_{r=1}^{R} w_r z_{rj} - \sum_{i=1}^{m} u_i \left(X_{ij}\right)_\alpha^U \le 0 \qquad \forall j \ne J \\
&\left(Z_{rj}\right)_\alpha^L \le z_{rj} \le \left(Z_{rj}\right)_\alpha^U \qquad \forall r \forall j \\
&u_i, v_k, w_r \ge 0 \qquad \forall i \forall k \forall r
\end{aligned}
\tag{11}
$$

Although the program below (12) is non-linear, nonlinearity is not a complex issue and it can be linearized. Concerning parallel systems, Kao and Lin (2012) and Lozano (2013) have provided methods to work with fuzzy data.

$$
\begin{aligned}
& E_J = Min\ \theta \\
& s.t. \\
& \lambda_J^1 \left(X_{iJ}\right)_\alpha^U + \sum_{j \neq J} \lambda_j^1 \left(X_{ij}\right)_\alpha^L \leq \theta \cdot \left(X_{iJ}\right)_\alpha^U && \mathrm{i} = 1...m \\
& \lambda_J^2 \left(Y_{kJ}\right)_\alpha^L + \sum_{j \neq J} \lambda_j^2 \left(Y_{kj}\right)_\alpha^U \geq \left(Y_{kJ}\right)_\alpha^L && \mathrm{k} = 1...s \qquad (12) \\
& \sum_{j=1}^{n} \lambda_j^1 z_{rj} \geq \sum_{j=1}^{n} \lambda_j^2 z_{rj} && \mathrm{r} = 1...R \\
& \left(Z_{rj}\right)_\alpha^L \leq z_{rj} \leq \left(Z_{rj}\right)_\alpha^U \qquad \forall r \forall j \\
& \lambda_j^p \geq 0 \qquad \forall j, \forall p
\end{aligned}
$$

III. AIM AND MOTIVATIONS

III. AIM AND MOTIVATIONS

This PhD thesis consists in a collection of published works regarding Network DEA, whose aim was to carry out applications of Network DEA and designing new models. The publications are enclosed at the appendix and will be referred to as [1] to [6]. In this section, brief reviews of the motivations of each publication, starting at [1], are presented.

III.1. DEA APPROACH TO NBA TEAMS

The motivation of the paper [1] was to implement a Network DEA model to measure the performance of NBA teams. For instance, if a team underperforms, stakeholders may be interested in studying which aspects of the team did not meet the expectations, but only by taking into account the internal structure of the DMU, it can be stated which stage has to be improved. Although there have been previous applications to evaluate efficiency of sports teams, the only Network DEA approaches were those of Lewis and Sexton (2004a, 2009), which did not involve the relational model.

Specifically, the proposed approach, which consists of five stages, is an application of the non-oriented Network SBM reviewed in previous sections. The first stage is related to the acquisition process, since the budget is used to sign up players who have certain skills. The manager has to distribute the resources between first-team players and the bench team, i.e. allocate the economic resources, in order to optimize their performance.

The rest of the stages can be seen as part of the production process. By means of the offensive and defensive systems, which the coach is in charge of, the skills of the players will be transformed into points (and avoidance of the points by rival teams). Finally, the points made and received should be converted into wins (the more wins, the better ranking). To sum up, the production process of a NBA team has been divided into acquisition, offensive, defensive and effectiveness stages.

III.2. DYNAMIC EFFICIENCY OF ILECS

The telecommunications sector has been on the most competitive industries since the liberalization of the market by the Telecommunications Act of 1996. In addition, in this context the long-term planning and investments in network elements and facilities are crucial for companies to succeed. So, instead of a static approach, it is fully justified to implement a Dynamic DEA approach to assess the global performance along the whole horizon, where the long-term activities, which have an effect on the following periods of time, as well as the customer base, are considered to be carry-over activities.

So the Dynamic DEA by Tone and Tsutsui (2010), where carry-over activities are considered are treated as intermediate products, was implemented in [2]. Every period of time will function as an internal process, being the operating expenses the single input and operating revenues (at the end of the period) the single output. With respect to the carry-over activities, the four links that are taken from one year to the following are the number of employees (free link), the number of total switched lines (good link), total assets (good link) and total liabilities at the end of the year (bad link).

Once the dynamic efficiency for every company and year have been computed, a regression on the external factors that might have influence the performance of the companies, such as regulatory policies and both local and intermodal competition, was carried out.

III.3. MALMQUIMST NETWORK DEA

Although a Network DEA approach was proposed in the article [1] to evaluate the efficiency of NBA teams, the motivation of publication [3] is that managers and coaches usually elaborate plans and rosters for several years, mainly due to the length of players' contracts, thus analyzing the productivity change of the teams become relevant.

In addition, a lookout took place before season 2011/12, where the owners proposed to reduce players' income, so an assessment of economic resources management through the recent seasons can reveal if there have been excesses in salaries preventing efficiency. A MPI has been implemented to estimate the productivity change of NBA teams from 2006/06 to 2012/13 seasons, as well as a FGNZ decomposition.

The actual Network DEA model consists in input-oriented envelopment Network DEA, based on the following network of processes: an acquisition process, where the teams use the budget to sing up players, the offensive and defensive subsystems, which transform the players' skills into points, and the final stage relative to effectiveness.

Finally, a regression is carried out to establish the influence of the budget, efficiency, technological and scale change on efficiency (the last three explanatory variables come from the FGNZ decomposition), and the approach by Lewis and Sexton (2004a, 2004b, 2009) is implemented to obtain efficiencies for each of the stages.

III.4. AIRPORTS WITH UNDESIRABLE OUTPUTS

The motivation of this paper is to develop a Network DEA approach to the modeling of airports operations considering that some or all of the processes generate undesirable outputs. Specifically, taking into account the undesirable effects of airport operations contributes to a fairer performance assessment, because some of the airports may be oversaturated and causing excessive pollution and noise to passengers.

The network structure consists in two processes: one related to the movement of the aircrafts and a second one related to the airplane's load factors. Hence, the intermediate product linking the two processes will be the annual aircraft movements in and out of the airport, since the movements are only a means of providing the service of transporting people and goods. In addition, the first stage generates the two undesirable outputs, namely the number of delayed flights and accumulated flight delays.

The paper includes the Production Possibility Set for the process and global structure of Network DEA when there are undesirable outputs. The proposed approach is based on the directional distance function, so that the movements of passengers and cargo can be maximized at the same time that the undesirable outputs are minimized.

III.5. FUZZY NETWORK DEA

Despite the variety of fuzzy DEA models, previous fuzzy Network DEA approaches only deal with two serial stages or parallel stages. So the motivation of the publication [5] was to provide a formulation of fuzzy Network DEA models that could work for any internal structure. As well as extending the fuzzy 2-stage DEA by Kao and Liu (2012), other fuzzy approaches, namely those of Saati et al. (2002) and Wang et al. (2005) are going to be extended to general network of processes.

III.6. NDEA APPROACH TO PUBLIC SERVICES

Due to the global economic crisis, the motivation of the research done in [6] was to identify the inefficiencies in the provision of public services, e.g. feasible reductions in taxes, debt and public expenditures, that could be removed from the public finances. An efficient public system would, in order to provide its public services, spends what is necessary but without burdening taxpayers more than is required.

The proposed network DEA approach consists of two stages. The first stage aims at collecting the revenues, basically from tax receipts, in order to finance the expenditures. Next stage transforms the expenditures into services to the people and economy. GDP and population are set to be the proxies of that provision. The proposed model optimizes the sum of multiple directional distance functions along the different inputs and outputs, involving the feasible reductions in taxes and debt.

IV. RESULTS AND DISCUSSION

IV. RESULTS AND DISCUSSION

In this section, a quick glance at the results of each publication and their interpretation, starting at [1], are presented. Please refer to the full publications [1] to [6] for further details.

IV.1. DEA APPROACH TO NBA TEAMS

The proposed approach was applied to measure the efficiency of the 30 NBA teams during regular season 2009/10. Results show that Network DEA has more discriminating power and provides more insight than traditional DEA, although rankings of the teams by both approaches were similar. In fact, even teams with a large number of wins, such as *Los Angeles Lakers*, may end up with low efficiency scores, provided their economic management of resources was not adequate.

Network DEA allows uncovering sources of inefficiency that can remain hidden in the traditional DEA approach. In this publication [1], Network DEA reveals additional slacks, i.e. feasible reductions for inputs and improvement for outputs, in the performance of teams that were considered to be almost efficient when traditional DEA was applied. The team with the highest Network DEA efficiency turned out to be *Oklahoma City Thunders*, which presents a well-balanced budget and number of victories. Finally, slacks for intermediate products are also included.

IV.2. DYNAMIC EFFICIENCY OF ILECS

The Dynamic DEA approach was applied to the main Incumbent Local Exchange Carriers (ILECs) in the U.S. telecommunication market from 1997 to 2007. Results show that there is no clear relation between size of the ILEC and efficiency. In addition, the approach allows analyzing the evolution of the carry-over activities along time: the operational revenues should have been larger than they were during the first years under study, while the operating expenses should have been reduced during the remaining periods of time. Regarding the assets, there was a significant lack of investment on facilities and switched lines during the last periods.

Finally, the multiple regression point outs that local competition, encouraged by the liberalization of the telecommunications sector, has had a negative impact on the efficiency of ILECs. In fact, a larger investment by local competitors on their own equipment and facilities, instead of leasing the ILECs' facilities, would have led to a sharper deterioration of ILECs' efficiency. On the other hand, the intermodal competition and incentive regulation seemed not to have an influence on efficiency.

IV.3. MALMQUIMST NETWORK DEA

Results reveal that there has been technological progress for the last seasons, consisting of a reduction in the budget of the efficient teams, excluding that of the lockout, and an increasing efficiency change. This means that best practices are improving and that most teams have been reducing their payrolls to catch up with these practices, thus backing up the owners' proposal to reduce players' income.

These conclusions also match up with regression results, which show that change in wins between seasons is mainly affected by the shift in scale efficiency, and thus managers should adjust their resources properly in order to operate in their most productive scale size. Regarding the efficiencies of the stages, the offensive and defensive stages show values very close to 1, revealing that the first and last stages are the most decisive to the overall efficiency.

IV.4. AIRPORTS WITH UNDESIRABLE OUTPUTS

The proposed approach was applied to a dataset of 39 Spanish airports for the year 2008, leading to the conclusion that the Network DEA approach has much more discriminatory power than the traditional one, specially revealing a much greater shortfall in the cargo handled by the airports and excesses in the flight delays.

IV.5. FUZZY NETWORK DEA

The publication [5] shows that extending the crisp Network DEA approaches to handle fuzzy data becomes natural and it is even possible to work with linear programs. Since Network DEA represents a more fine-grained level of analysis, it enhances the usefulness of the approach.

IV.6. NDEA APPROACH TO PUBLIC SERVICES

The proposed approach was applied to the U.S. states during the period 2007-2011, adopting a contemporaneous approach, since the budgetary circumstances in each year are different. Although 22 states have managed their finances in an efficient way, other have been taxing people excessively or financing expenditures through borrowing. In fact, the estimated total inefficiency of the states raises to 500 billion US$. Furthermore, the approach identifies the reductions or increases in expenditures that might have been made.

V. CONCLUSIONS

V. CONCLUSIONS

The running of most of the production processes, companies and activities are organized by following a certain structure, and thus providing a single measure of efficiency for the whole entity omits the operation of the internal stages and leaves out the chance to improve the weakest areas of the organization. Furthermore, there are many applications where a network structure makes more sense, because there are some variables that are not suitable to be set as inputs or outputs, but as intermediate products.

The scope of this thesis includes a mixed range of applications of Network DEA as well as theoretical models. First, a Network SBM approach consisting of five stages, e.g. acquisition and production stages, is applied to evaluate the potential reduction of team budget and increase of games won by the NBA teams. The network structure allows incorporating data regarding the performance of the players on the court and therefore analyzing the efficiency of the offensive and defensive systems, among others.

The Network DEA approach to NBA teams has been accompanied by another paper estimating the productivity change, since managers make the financial planning and coaches build the roster with a view to consecutive seasons. Therefore, a Malmquist Productivity Index has been applied, based on the network structure of NBA teams. Results backed up the teams owners' proposal to reduce players' income, because of the catching up with the best practices.

Apart from the estimation of productivity change between consecutive periods, there are sectors where activities from previous years have an influence on the performance of the following years, such as the US wireline telecommunications sector, where previous customer base and investment in network elements are crucial for the future performance. In this case, a Dynamic DEA model was implemented, by considering the carry-over activities as intermediate products.

Finally, a regression analysis was carried out to evaluate the impact of local and intermodal competition on carriers' efficiency.

There are some other processes which may generate not only desirable outputs but also undesirable outputs, such as airport operations, where airplane delays take place. Taking into account the effects of these undesirable outputs contributes to a fairer performance assessment. A two-stage model was implemented, which consisted of one stage related to the movement of airplanes and a second one related to the load factors, while a directional distance approach measured both the increase in desirable outputs and decrease in undesirable ones.

Network DEA allows not only to *open the black box* of a DMU, but also to integrate into a single model the computation of the efficiencies of several interrelated processes, e.g. optimal taxation and public expenditure. The feasible reductions in taxes, debt and public expenditures, while maintaining the current level of public services, have been computed for the US Stages by applying a two-stage approach.

Apart from working with crisp data, in the literature researchers have proposed DEA approaches to handle fuzzy data. Recently, fuzzy proposals were made recently for only two-stage and parallel productions networks. In this thesis, several fuzzy approaches have been extended to general network of processes, providing the foundations of the formulations of Network Fuzzy DEA applications.

To sum up, including the intermediate products and the network structure implies that Network DEA approaches require much more data. In return, Network DEA has proved to show a greater discrimination power than traditional DEA By means of the different applications and models attached in this document, it has been proved that the results obtained from Network DEA are more accurate and valid, as well as revealing additional sources of inefficiency related to the performance of the internal stages. Therefore, Network DEA can be seen as an improvement over DEA and has a brilliant future ahead of.

As for further research, there are some applications where a Network DEA approach has not been proposed yet, e.g. the complete running of a hospital, the performance of tennis players and the assessment of tertiary education agents. Another topic for further research that is attracting experts' attention is the Dynamic Network DEA (Tone and Tsutsui, 2014), which integrates their previous dynamic (Tone and Tsutsui, 2010) and network approaches (Tone and Tsutsui, 2009). The operation of a DMU for each period of time will be made up of a network structure, while these network structures from different periods of time will be connected by carry-over activities. In addition, the carry-over activities can link any of the internal stages with the equivalent stage in consecutive periods of time.

However, there are some critical issues concerning Network DEA that are pending to be solved out. Fukuyama and Mirdehghan (2012) claimed that the Network SBM model by Tone and Tsutsui (2009) did not provide a proper measure of the efficiency of the individual stages, and proposed a second phase where additional slacks for intermediate products were addressed at. The status of the efficiency of each stage depends on the existence of second-phase slacks. Chen et al. (2013) also pointed out the faults of the envelopment form of Network DEA models in assessing stage efficiency, whereas the multiplier model failed at providing targets for the intermediate products (and thus failing at establishing the efficient frontier). An unified primal-dual approach for Network DEA has been pursued by researchers for a long time.

VI. GENERAL REFERENCES

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VII. APPENDIX

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[1] Moreno, P., Lozano, S. (2012), "A network DEA assessment of team efficiency in the NBA", *Annals of Operations Research*, 1-26. Article in Press. DOI: 10.1007/s10479-012-1074-9.

[2] Moreno, P., Lozano, S., Gutiérrez, E. (2013), "Dynamic performance analysis of U.S. wireline telecommunication companies", *Telecommunications Policy*, 37(6-7), 469-482.

[3] Moreno, P., Lozano, S. (2014), "Estimation of productivity change of NBA teams from 2006/07 to 2012/13 seasons", International Journal of Sport Finance. Article just accepted.

[4] Lozano, S., Gutiérrez, E., Moreno, P. (2013), "Network DEA approach to airports performance assessment considering undesirable outputs", *Applied Mathematical Modelling*, 37(4), 1665-1676.

BOOK CHAPTERS:

[5] Lozano, S., Moreno, P., "Chapter 10: Network Fuzzy Data Envelopment Analysis", *Performance Measurement with Fuzzy Data Envelopment Analysis*. Springer-Verlag. ISBN: 978-3-642-41371-1.

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A network DEA assessment of team efficiency in the NBA

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Abstract In this paper, a Network DEA approach to assess the efficiency of NBA teams is proposed and compared with a black-box (i.e. single-process) DEA approach. Both approaches use a Slack-Based Measure of efficiency (SBM) to evaluate the potential reduction of inputs consumed (team budget) and outputs produced (games won by the team). The study considers the distribution of the budget between first-team players and the rest of the payroll. The proposed network DEA approach consists of five stages, which evaluate the performance of first-team and bench-team players, the offensive and defensive systems and the ability for transforming the points made by itself and by the opponents into wins. It has been applied to the 30 NBA teams for the regular season 2009–2010. The results show that network DEA has more discriminating power and provides more insight than the conventional DEA approach.

Keywords Sports efficiency · NBA · Network DEA · SBM · Inefficiency sources

1 Introduction

NBA (National Basketball Association) has become one of the biggest sports businesses all around the world, because of broadcasting rights, advertising and merchandising sales. However, the popularity and number of fans of a NBA team will depend on its results. Therefore, it is useful to perform an efficiency analysis and evaluate if the team manage properly the current budget in order to win the maximum number of games from the regular season.

Since the development of DEA methodology (Data Envelopment Analysis), by Charnes et al. (1978) it has been widely used to assess the relative efficiency of Decision Making

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Published online: 03 February 2012

Units (DMUs). These DMUs must be homogeneous, working in an identical way: transforming the same inputs into the same outputs. DEA itself is a non-parametric tool that infers the production frontier without assuming any particular functional frontier form and the relative efficiency of every DMU is assessed based only in the observed inputs and outputs of the DMUs.

Although DEA has been applied in many different fields, DMUs has traditionally been seen as *black boxes* with its inputs and outputs, but without any consideration of what is happening inside the DMU. However, Färe and Grosskopf (2000) argue that if DMUs are modeled taking into consideration their internal processes, the efficiency assessment can be achieved in a more accurate way and additional information can be obtained.

The aim of this paper is to use a network DEA approach to perform an efficiency analysis of the 30 teams in the NBA. Apart from getting further insight into the resources management of every team, network DEA has never before been applied to basketball. First-team wages together with bench-team wages are both considered as inputs, because every team has to decide how to allocate the input resources, choosing between signing up better first-team players and having more competitive players in the bench. The proposed approach considers five stages, corresponding to the performances of the players, the offensive and defensive systems, as well as the effectiveness in winning games. Furthermore, since reducing payroll is as important as winning more games, the proposed approach provides a non-oriented efficiency measure, based on inputs and output slacks.

The structure of the paper is the following. Section 2 makes a literature revision of the applications of DEA to sports. Section 3 introduces the proposed approach (variables, model, data and methodology). Section 4 presents the results of the relative efficiency of the NBA teams, by using data from the regular season 2009/2010. Finally, Sect. 5 summarizes and concludes.

2 Literature review

Anderson and Sharp (1997) published one of the first papers that implemented a DEA approach to analyze sports performance, namely that of baseball batters. However, most of the previous works on DEA applied to sports deals with soccer leagues. Thus, Haas (2003a) presented an input-oriented DEA model, both VRS and CRS, that takes total wages and salaries as inputs, plus population of the clubs' home town as a non-discretionary input variable. The outputs include points awarded during the season and the total revenue figures which serve as an indicator for a team's success in international competitions. Haas (2003b) studied the technical efficiency of the Major Soccer League in the United States considering players' wages and head coach's wage as inputs and awarded points, number of spectators and revenues as outputs.

Espitia-Escuer and García-Cebrián (2006) studied the potential of the teams in the Spanish soccer league between the years 1998 and 2005, analyzing each year separately. The evaluation is carried out from an output-oriented perspective: the efficiency in obtaining better results given the available resources on the field of play. In order to perform that task, they considered a system which takes as inputs the attacking and defensive moves against the opposing team and the total points awarded as the single output.

Boscá et al. (2009) measured offensive and defensive efficiency of teams in Italian soccer league, from the attacking and possession moves made by the team and by the opposing team as inputs and the goals as outputs. Once the home and away efficiencies are evaluated, they performed a regression analysis to explain the points obtained by teams with regard to different indicators, so they could decide if it is more important to be efficient offensively or defensively to obtain a high ranking in the league. Picazo-Tadeo and González-Gómez (2010) used a radial version of DEA, as in the other papers commented in this section so far. However, it focused on the fact that participating in other official competitions consumes resources, predictably reducing a team's potential in the league. They proposed the addition of further restrictions that force the performance of an observation to be assessed by comparing its productive plan with a plan corresponding to an efficient team (or a linear combination), which has to play at least the same number of extra games in all competitions.

As in their previous work, Espitia-Escuer and García-Cebrián (2010) described a production process with two stages but again they focused on the second one: how every team manage the on-field performance during matches. Their paper considers teams that play in the Champions' League as well as in their own national competition and both efficiency and superefficiency models are used, the latter for discriminating among efficient units. Guzmán and Morrow (2007) used the Malmquist productivity index to measure the change in productivity over a 6-year period (the seasons 1997–1998 to 2002–2003) of the English Premier League. Barros and Douvis (2009) also studied the changes in total productivity, but this time of the football clubs of Greece and Portugal and for the seasons 1999–2000 through 2002–2003.

Barros et al. (2010) and Barros and Garcia-del-Barrio (2011) implemented a DEA-bootstrapping procedure to analyze the technical efficiency of Brazilian first soccer league and Spanish first division soccer league, respectively. The procedure consists of two stages. In the first stage, a bootstrapped DEA is used to estimate relative efficiency scores whose bias has been corrected. Then, in the second stage, the Simar and Wilson's procedure is applied to bootstrap the DEA scores with a truncated regression, to further explain the influence of variables in the efficiency results.

There are also some works in this area using imprecise data and fuzzy linear programming. Thus, Aoki et al. (2009) proposed a model that classifies the DMU with imprecise data into four groups. The method is applied to 25 soccer teams that participated in Japan *Robocup* 2008. Cadenas et al. (2010) use fuzzy linear programming models applying DEA to the Spanish Football League 2006/2007. The inputs of their offensive model are the offensive on-field production: balls kicked into the area, attacking plays, minutes of possession and shots-on-goal. Whereas the inputs of the defensive model has been defined as the inverse of the above variables by the opposing team. The output is the number of goals.

Chen and Johnson (2010) studied the dynamics of the performance space of Major League Baseball by means of a DEA approach. Concerning other sports, Fried et al. (2004) evaluated the technical efficiency of golf players, by using game statistics as inputs and earnings per event as the single output. They used a radial, output-oriented model that computes earnings targets for each player as well. Finally, DEA has also been applied to the Olympic Games (e.g. Lozano et al. 2002; Lins et al. 2003; Li et al. 2008; Soares de Mello et al. 2009; Zhang et al. 2009, and Wu et al. 2010).

To the best of our knowledge, there are only a couple of papers on DEA applied to basketball. Namely, Cooper et al. (2009) applied the multiplier DEA formulation to evaluate the effectiveness of basketball players of the Spanish Basketball League (ACB), considering the stats of the player (i.e. adjusted field goal, free throw, rebounds, assists, steals, etc.) as outputs. AR-I type constraints on the output weights are included to incorporate the views of the basketball experts. The lower and upper bounds of the weight constraints depend on the type of basketball player, allowing the weights to vary across players in order to reflect their different characteristics. The procedure proposed by Cooper et al. (2009) added a second step that involves a choice of weights from among the alternative optimal solutions of the extreme efficient DMUs. Cooper et al. (2011) use a novel cross-efficiency approach that reduces the differences in the weights chosen by the DMUs and apply the model to the ranking of basketball players who play in the position of center, by considering the weight profiles of some model players.

A final group of studies that are relevant to this research is the one formed by network DEA applications to sports. Thus, Sexton and Lewis (2003) presented a network DEA approach to the Major League Baseball (MLB). Each DMU is described as a two-stage acquisition and production operation. In Stage 1, the team's front office uses resources (total player salaries) to acquire talent (offensive and defensive production). In Stage 2, the talent produces games won on the field. They identified two intermediate products, total bases gained and total bases surrendered as offensive and defensive production, and selected a radial output orientation for the overall organization. However, that approach differs from ours, because it establishes separate efficient frontiers for every sub-DMU and computes the organizational efficiency based on the Stage 2 frontier using the intermediate product levels that would have arrive at the Stage 2 if the Stage 1 had been efficient. So-called reverse variables are dealt-with the same way as in Lewis and Sexton (2004a), in which the efficiency of the 30 MLB organizations during the 1999 regular season was analyzed.

Lewis and Sexton (2004b) generalized their previous work, splitting up the front-office stage in two stages, one of which consumes player salaries to produce position player talent while the other consumes salaries to produce pitching talent. The on-field sub-DMU consist of three stages, one of which represents the offensive process, another the defensive stage and a third one consumes the output of the previous offensive and defensive stages (runs gained and surrendered) to produce games won. In a subsequent paper, Lewis et al. (2009) take only the three sub-DMUs from the on-field competition, and evaluate their single and total efficiencies in the same way as in Sexton and Lewis (2003).

Finally, García-Sánchez (2007) uses a three-stage approach to evaluate the efficiency in Spanish Football League, but it can be hardly considered as network DEA, because every stage takes the efficiency of the previous stage as an input. His proposed first stage measures the operating efficiency, by taking offensive and defensive movements as inputs and goals as output. The second and third stages measures operating and social effectiveness, whose outputs are the classification of the team and the outcome, respectively.

3 Evaluation of NBA teams' efficiency

NBA (National Basketball Association) is the main basketball league in the USA and the most important basketball competition all around the world. There are 30 teams in the NBA, grouped into two conferences (East and West) and six divisions (Atlantic, Central, Southeast, Southwest, Northwest and Pacific). Every year, NBA consists of two phases: regular season and playoffs. The top eight teams from each conference go to the conference playoffs and the two winners from each conference (East and West) play for the title in the last playoff. Concerning regular season, every team play 82 games and it is mandatory to achieve a good place in the ranking to gain access to the playoffs.

NBA is not only one of the biggest sport entertainments but it also has become an important business. It involves a great deal of money in TV broadcasting rights, pay-per-view, sponsorship, and tickets, sports clothes and merchandising sales. The income from all those sources will depend on the performance on every team though: the better the team performs in the league, the more revenues it will get, since there would be a greater number of fans for that team. However, the teams have a limited number of economic resources every season,

Table 1 DMU's inputs

Input	Name	Label
x_1	First team budget	FTBudget
x_2	Bench team budget	BTBudget

Table 2 DMU's output

Output	Name	Label
y_1	Number of team victories	Wins

so it is appropriate to study how efficiently the teams manage their resources in order to win games.

3.1 Single-process model

The first model we are going to work with is a single-process production model that transforms input resources into outputs. We are going to take players salaries as input, since payroll is the main expense in NBA teams. Although other previous works on DEA applied to sports (Sexton and Lewis 2003; Guzmán and Morrow 2007) also choose salaries as input, we differentiate between the budget related to first and bench teams. The first team consists of the five players who have played more regularly during the season and the bench team refers to the rest of the players.

Therefore, the most important decision in every team is how to allocate the economic resources, so some teams usually decide to set a great part of the budget to sign up top-performing players to build a powerful first team, whereas other teams tend to spread the total payroll between all the players in order to have more competitive players in the bench, since every game is 48 minutes and first-team players cannot play the whole game. Therefore, there are two inputs in our model, and are presented in Table 1.

With regards to the selection of the outputs, note that the outputs chosen in DEA applications to sports depend on the sport that is being considered. For instance, the research works focused on baseball (e.g. Lewis et al. 2009; Lewis and Sexton 2004a) usually take the number of victories as an output, whereas works on other sports in which the ranking depends on the number of points (e.g. soccer) assign the number of points won as an output (e.g. Espitia-Escuer and García-Cebrián 2010; Picazo-Tadeo and González-Gómez 2010). As shown in Table 2, in our case, the number of team victories is the single output considered. This is so because the ranking in the regular season depends on the number of victories. A better rank means easier matches during the playoffs, due to the fact that the best team will be paired off with the eighth team to access the playoffs, the second team will be paired off with the seventh and so on. Furthermore, in each playoff, the team with the highest rank will play more matches at home, which is an obvious advantage.

In order to measure the efficiency of each NBA team, the SBM DEA model (Tone 2001) is used. The SBM model, which is equivalent to the Enhanced Russell Measure (ERM) independently proposed by Pastor et al. (1999), is non-radial and tries to minimize the ratio of average inputs reductions and outputs increases, instead of making a radial reduction of inputs or radial increase of outputs. Since every NBA team tries to obtain as many wins as possible (in order to qualify for play-offs in a high rank thus getting an easier first round) and simultaneously to reduce its team's payroll (in order to move away from league salary cap and be able to sign players next seasons), the use of a non-oriented SBM model is fully justified.

Let x_{iJ} be the i-th input of the model, according to Table 1, of the DMU J and y_{1J} the output of the model for the DMU J.

The proposed SBM efficiency score will take into account the output slacks s_J^{wins} (number of victories) and the slack in the total budget s_J^{budget} (first team budget plus bench team budget), since every team seeks to maximize the number of victories while reducing its total budget. Although the SBM model is non-linear, it can be linearized as

$$Efficiency_J^{SBM} = \text{Min}\, t - \frac{s_J^{\prime budget}}{x_{1J} + x_{2J}} \tag{1}$$

s.t.

$$t + \frac{s_J^{\prime wins}}{y_{1J}} = 1 \tag{2}$$

$$\sum_j \lambda_j^{\prime} \cdot x_{ij} = x_{iJ}^{\prime} \quad i = 1, 2 \tag{3}$$

$$x_{1J}^{\prime} + x_{2J}^{\prime} = t \cdot (x_{1J} + x_{2J}) - s_J^{\prime budget} \tag{4}$$

$$\sum_j \lambda_j^{\prime} \cdot y_{1j} = t \cdot y_{1J} + s_J^{\prime wins} \tag{5}$$

$$\sum_j \lambda_j^{\prime} = t \tag{6}$$

$$\lambda_j^{\prime} \geq 0 \tag{7}$$

$$s_J^{\prime budget}, s_J^{\prime wins} \geq 0 \tag{8}$$

Equation (1) accounts for the objective function, which corresponds to maximizing the total budget slack while (2) imposes the linearization constraint. Equation (3) computes the inputs of the target operation point. These input targets are related to the observed values by means of the input slack computed in (4). Note that this input slack is aggregated and includes both first and bench team budgets. Equation (5) computes the target value for the single output, which is equal to the observed value plus the corresponding output slack. Note that, in (6), Variable Returns to Scale (VRS) are considered since an increase in the budget will not necessarily mean a fixed, proportional increase in the number of victories.

3.2 Proposed network DEA model

Before formulating the proposed network DEA approach a brief introduction to network DEA in general will be presented. The main differences with respect to the more traditional single-process DEA will be highlighted. The notation proposed in Lozano (2011) will be used.

3.2.1 Network DEA methodology

Beyond traditional DEA models, new approaches have been developed to evaluate the efficiency of DMUs. Thus, for example, Färe and Grosskopf (2000) suggests that instead of considering a DMU as a black box, a network DEA approach taking into consideration the internal configuration of the DMUs is preferable. Castelli et al. (2010) has made a review of different shared-flow, multilevel and network DEA models. The number of network DEA models that have been proposed in the literature is increasing rapidly, e.g. the relational network DEA approach of Kao and Hwang (2008, 2010), Kao (2009a, 2009b), the weighted additive efficiency decomposition approach of Chen et al. (2009), Cook et al. (2010), the SBM-NDEA approach of Tone and Tsutsui (2009, 2010), Avkiran (2009), Yu (2010), the Network Slacks-Based Inefficiency (NSBI) approach of Fukuyama and Weber (2010) and the network DEA scale and cost efficiency approach of Lozano (2011), to name a few.

The main difference between network DEA and conventional DEA is that while conventional DEA considers a single process that consumes all the inputs and produces all the outputs, network DEA considers the existence of several stages each of which consumes its owns set of inputs and produce its own set of outputs, in addition to consuming and producing intermediate products. These intermediate products are defined as inputs for some stages are outputs for others.

For each process p of DMU j, denote x_{ij}^{p} as the observed amount of input i consumed and let y_{kj}^{p} be the observed amount of output j produced. Let $z_{rj}^{in,p}$ be the observed amount of intermediate product r consumed by process p of DMU j and $z_{rj}^{out,p}$ denote the observed amount of intermediate product r generated by process p of DMU j. Let $P_I(i)$ the set of processes that consume the input i and $P_O(k)$ the set of processes that generate the output o. In order to model the composition of intermediate flows inside the network, let $P^{out}(r)$ be the set of stages that produce the intermediate product r and $P^{in}(r)$ the set of processes that consume the intermediate product r. Also, let $I(p)$ the set of exogenous inputs used in process p and $O(p)$ the set of final outputs of process p.

A radial, input-oriented VRS network DEA model can be formulated as (see Lozano 2011)

$$\text{Min}\,\theta \tag{9}$$

subject to

$$\sum_{p\in P_I(i)}\sum_{j}\lambda_j^p x_{ij}^p \le \theta x_{i0} \quad \forall i \tag{10}$$

$$\sum_{p\in P_O(k)}\sum_{j}\lambda_j^p y_{kj}^p \ge y_{k0} \quad \forall k \tag{11}$$

$$\sum_{p\in P^{out}(r)}\sum_{j}\lambda_j^p z_{rj}^p - \sum_{p\in P^{in}(r)}\sum_{j}\lambda_j^p z_{rj}^p \ge 0 \quad \forall r \tag{12}$$

$$\sum_{j}\lambda_j^p = 1 \quad \forall p \tag{13}$$

$$\lambda_j^p \ge 0 \quad \forall j\ \forall p\ \theta \text{ free} \tag{14}$$

First of all note that a key feature of Network DEA models is that each stage or process has its own production possibility set, which implies that the model must have a distinct set of lambda multipliers λ_j^p for each process p. This leads to a larger overall production possibility set which increases the discriminate power of the DEA model. Because of that, it is very common in Network DEA that none of the DMUs is found efficient, since if a DMU must be efficient it has to be on the efficient frontier of all its processes, something which does not happen often.

Constraints (10) compute the radial reduction in the total amount of exogenous inputs. Constraints (11) guarantee maintaining at least the total amount of final outputs. Constraints (12) are a type of free-links constraints that guarantee that the total amount internally produced of each intermediate product is at least equal to the amount that is internally consumed. Finally, constraints (13) are the convexity constraints associated to assuming VRS for each process p.

From the optimal solution to this model, denoted with an asterisk superscript, the target operation point for each process p can be computed as

$$\hat{x}_i^p = \sum_j \left(\lambda_j^p\right)^* x_{ij}^p \quad \forall i \in I(p) \tag{15}$$

$$\hat{y}_k^p = \sum_j \left(\lambda_j^p\right)^* y_{kj}^p \quad \forall k \in O(p) \tag{16}$$

$$\hat{z}_r^p = \sum_j \left(\lambda_j^p\right)^* z_{rj}^p \quad \forall r \in R^{in}(p) \cup R^{out}(p) \tag{17}$$

and, from that, corresponding total input and output targets for the whole system

$$\hat{x}_i = \sum_{p \in P_I(i)} \hat{x}_i^p \leq x_{i0} = \sum_{p \in P_I(i)} x_{i0}^p \quad \forall i \tag{18}$$

$$\hat{y}_k = \sum_{p \in P_O(k)} \hat{y}_k^p \geq y_{k0} = \sum_{p \in P_O(k)} y_{k0}^p \quad \forall k \tag{19}$$

The use of a network DEA model is fully justified in our application. For instance, if a team does not meet the expectations, both fans and sponsors will try to study which aspects of the team went wrong and may not agree in the solutions. From a traditional analysis it is not easy to decide if there is a need to change to the coach, the roster make-up or the strategies and tactics of play. Although one of the drawbacks of network DEA is the need of internal operations data, in our case data availability is not a problem since NBA freely provides all the game statistics in their official website www.nba.com.

3.2.2 Network DEA approach for NBA teams

The network DEA model we propose in this paper can be seen in Fig. 1. It is a generalization of a two-stage production process that has been applied to evaluate the DEA efficiency of some sports (e.g. Espitia-Escuer and García-Cebrián 2006; Lewis and Sexton 2004b). These two stages can be named as acquisition stage and the production stage itself.

The acquisition stage (1) consumes budget to sign up basketball players, who will have a certain performance during the matches. However, with regard to inputs, a distinction between first and bench teams is made, because of the reasons we stated in Sect. 3.1. Therefore, as in the single-process case, there are two inputs related to the corresponding budgets, as shown in Table 1. The outputs of this first stage will be the attacking and defensive moves against the opposing team, as described in previous works (e.g. Boscá et al. 2009; Espitia-Escuer and García-Cebrián 2010). The number of moves can be taken as a representative of the players' skills and performance in the field and are measured in absolute figures. Since these variables will be output of the first stage and input of the following ones, they are considered to be intermediate products, and are shown in Table 3. This stage can be named: Team-work Performance. It can be seen as a stage where the manager and their management staff have to take decisions about the composition of the roster. Every year, there are a limited amount of economic resources to spend on salaries, and the manager has to properly distribute the resources between first-team players and bench team, in order to get the best performance of the whole team in the field. As we stated before, some managers choose to sign up top-rated players for the first team while having ordinary players in the bench, whereas other managers decide to have a more well-balanced team.

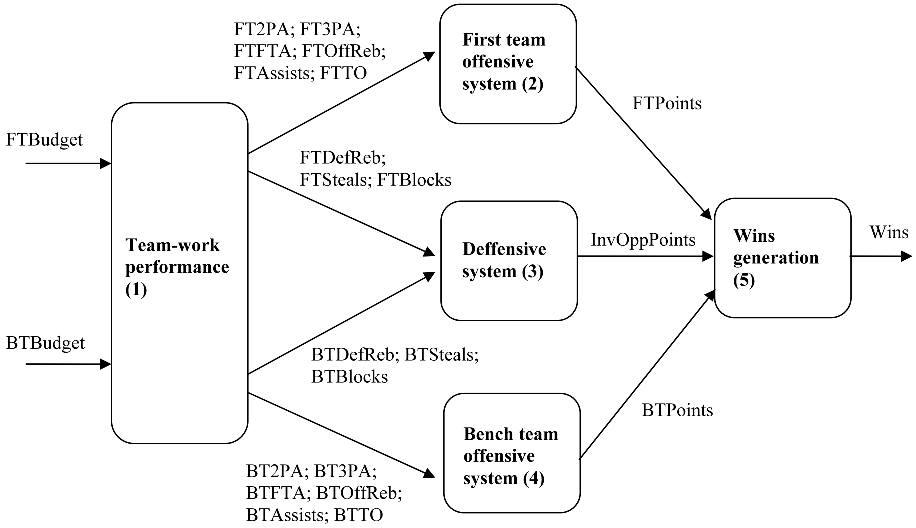


Fig. 1 DMU as a network of processes

Table 3 Intermediate products when DMU is seen as a network of processes

Intermediate product	Name	Label
z_1	2-Point shots attempted by First Team	FT2PA
z_2	3-Point shots attempted by First Team	FT3PA
z_3	Free throws attempted by First Team	FTFTA
z_4	Offensive Rebounds by First Team	FTOffReb
z_5	Assists by First Team	FTAssists
z_6	Inverse of Turnovers by First Team	FTTO
z_7	Defensive rebounds by First Team	FTDefReb
z_8	Steals by First Team	FTSteals
z_9	Blocked shots by First Team	FTBlocks
z_{10}	2-Point shots attempted by Bench Team	BT2PA
z_{11}	3-Point shots attempted by Bench Team	BT3PA
z_{12}	Free throws attempted by Bench Team	BTFTA
z_{13}	Offensive Rebounds by Bench Team	BTOffReb
z_{14}	Assists by Bench Team	BTAssists
z_{15}	Inverse of Turnovers by Bench Team	BTTO
z_{16}	Defensive rebounds by Bench Team	BTDefReb
z_{17}	Steals by Bench Team	BTSteals
z_{18}	Blocked shots by Bench Team	BTBlocks
z_{19}	Points by First Team	FTPoints
z_{20}	Points by Bench Team	BTPoints
z_{21}	Inverse of Points by Opponent	InvOppPoints

Among the intermediate products which are outputs of this first stage, note that the turnovers made by a team is a variable that represents worse performance when it takes higher numerical values. Traditionally, although this kind of outputs can be dealt treated as reverse variables (Lewis and Sexton 2004a) we found it easier to take the inverse of the quantity, as other authors have done previously (e.g. Cooper et al. 2009 in their application to basketball).

The resulting output attacking moves of stage (1) will be inputs to the offensive systems (2) and (4), which evaluate the efficiency of the team at transforming the available offensives resources on the field into points. The corresponding output defensive moves of stage (1) will be inputs to the defensive system (3) that evaluate how the team manages its defensive resources to minimize the points made by opponents. Again, the inverse of the points made by the opponent is considered as output, since a greater number of points surrendered will mean less efficiency.

Note that it makes sense to have two stages related to offensive systems, because the coach usually makes players from first team play together and plan specific offensive strategies and movements for them, depending on their skills. The inputs for the first-team offensive system involve the main indicators of the attacking play:

- FT2PA: number of two-point shots attempted by first team.
- FT3PA: number of three-point shots attempted by first team. Jointly with FT2PA, this statistic can be interpreted as the number of attacking moves made by the first team.
- FTFTA: number of free-throw shots attempted by first team. This is also a measure of personal faults made by the opponent in order to stop offensive actions.
- FTOffReb: number of offensive rebounds grabbed by first Team, allowing the team to perform an additional offensive action.
- FTAssists: number of assists by the first team, which is a measure of the cooperation among the players.
- FTTO: number of turnovers by the first team, allowing the opponent to perform a counter-attack.

There are similar intermediate products referred to the bench team (with the corresponding variable names starting with BT) which are inputs of the corresponding bench-team offensive system. Each of these two offensive subsystems has as single output the points made by the corresponding team (FTPoints, BTPoints).

Although there are two offensive systems, for both first and bench team, there is only one defensive system, because the points made by the opponent are not differentiated for its first and bench teams. The inputs for the sole defensive system are:

- FTDefReb: number of offensive rebounds grabbed by first team, allowing the team to stop the opponent's offensive action and start an attack.
- FTSteals: number of steals by the first team, usually allowing the team to subsequently perform a counter-attack.
- FTBlocks: number of blocks by first team, allowing the team to stop the opponent's offensive action and start an attack.

There are similar intermediate products referred to the bench team (again with the corresponding variable names starting with BT). The single output of this stage is the inverse of the number of points by the opponent teams (InvOppPoints).

Moreover, the stages related to the offensive and defensive systems are under the control of the coach. In fact, the coaches have to manage the production of their players (to maximize the number of points scored and minimize the number of points made by opponents) by means of strategies, tactics and planned moves. For instance, an appropriate offensive system will allow a better shot selection, which means greater shot percentage and more points.

The final stage (5) evaluates the ability of the team to manage the points made and received in order to win matches, in a similar way as the integration stage in Lewis et al. (2009). The stages related to the offensive and defensive systems and the final stage in our five-stage approach can be seen as equivalent to their on-field sub-process, i.e. as the second stage in a two-stage production system.

As in the single-process DEA model, in the proposed network DEA approach the SBM efficiency score is maximized. This takes into consideration the total budget slack and the number of victories slack. However, additional equations have to be included, since the amount of every intermediate product produced by a stage must be equal to or greater than the amount of that intermediate product consumed by the following stage. Also, a very important difference with respect to the single process DEA approach is that a different set of lambda multipliers exist for each process, since each process has its own Production Possibility Set (PPS).

Stages (1) and (5) assume VRS. Thus, on the one hand, not always an increase in the budget will generate a proportional increase in performance. On the other hand, the output of the stage (5) (i.e. the number of wins) has an upper limit independently of how much its inputs increase, i.e. that stage has a saturation limit. Stages (2), (3) and (4) assume Constant Returns to Scale (CRS) because the number of points made is not limited and will not have scale effects.

The proposed Network SBM Model (NSBM) is the following, where the λ_j^p are the variables that, for each process p, allow for the computation of a target operation point within its corresponding Production Possibility Set. As also occurred in the single-process case, the network SBM model is not linear, so the corresponding linearized model is shown:

$$Efficiency_J^{NSBM} = \operatorname{Min} t - \frac{s_J^{\prime budget}}{x_{1J} + x_{2J}} \tag{20}$$

s.t.

$$t + \frac{s_J^{\prime wins}}{y_{1J}} = 1 \tag{21}$$

$$\sum_j \lambda_j^{\prime 1} \cdot x_{ij} = x'_{iJ} \quad i = 1, 2 \tag{22}$$

$$x'_{1J} + x'_{2J} = t \cdot (x_{1J} + x_{2J}) - s_J^{\prime budget} \tag{23}$$

$$\sum_j \lambda_j^{\prime 1} \cdot z_{lj} - \sum_j \lambda_j^{\prime 2} \cdot z_{lj} \geq 0 \quad l = 1 \ldots 6 \tag{24}$$

$$\sum_j \lambda_j^{\prime 1} \cdot z_{lj} - \sum_j \lambda_j^{\prime 3} \cdot z_{lj} \geq 0 \quad l = 7 \ldots 9 \tag{25}$$

$$\sum_j \lambda_j^{\prime 1} \cdot z_{lj} - \sum_j \lambda_j^{\prime 4} \cdot z_{lj} \geq 0 \quad l = 10 \ldots 15 \tag{26}$$

$$\sum_j \lambda_j^{\prime 1} \cdot z_{lj} - \sum_j \lambda_j^{\prime 3} \cdot z_{lj} \geq 0 \quad l = 16 \ldots 18 \tag{27}$$

$$\sum_j \lambda_j^{\prime p} \cdot z_{lj} - \sum_j \lambda_j^{\prime 5} \cdot z_{lj} \geq 0 \quad l = 19, 20, 21 \; p = 2, 3, 4 \tag{28}$$

$$\sum_j \lambda_j'^5 \cdot y_{1j} = t \cdot y_{1J} + s_J'^{wins} \tag{29}$$

$$\sum_j \lambda_j'^1 = t \tag{30}$$

$$\sum_j \lambda_j'^5 = t \tag{31}$$

$$\lambda_j'^p \geq 0 \quad \forall p \tag{32}$$

$$s_J'^{budget}, s_J'^{wins} \geq 0 \tag{33}$$

Constraints (21)–(23) and (29) are equivalent to those of the single-process DEA model except for the fact that each process p has its own set of variables λ_j^p. Constraints (24)–(28) impose that the amount of a certain intermediate product consumed within the system has been previously produced by other stage. For instance, (24) guarantees that the offensive stats generated as output from stage (1) are enough to fulfill the requirements in stage (2), where those intermediate products are used as inputs. Note this type of constraints use two different sets of lambda multiplier, one of them corresponding to the technology of the stage which produces the product and the other one corresponding to the stage which consumes it. Equations (30) and (31) assume VRS technology for stages (1) and (5).

4 Results

In this section the results of the proposed network DEA approach are presented and compared with those of the single-process DEA. The data for all teams were obtained from the official statistics of NBA for regular season 2009/10 available in their official website (www.nba.com). The data for the two inputs and the single output considered are shown in Table 4. The budget figures are in million US$. For the sake of completeness and replicability of the results, the data for the 21 intermediate products used in the network DEA approach are shown in Table 9 in the Appendix.

As for the efficiency assessment, the single-process SBM and network SBM efficiency scores are shown for every team in Table 5. It can be noted that network DEA efficiency is lower than the corresponding single efficiency for every team. In addition, Table 6 shows the ranking for NBA teams ordered by single-process and by network DEA scores. Note that the Spearman's rank correlation coefficient between the SBM and NSBM scores is 0.886, which means that there is a strong positive correlation between both rankings.

Figure 2 shows the SBM efficiency score of each team versus its number of wins. Note that, in the case of single-process DEA, a substantial number of wins is usually enough to achieve a high efficiency score. On the contrary, in the case of network DEA, even teams with a large number of wins may end up with low efficiency scores.

The box-plots of the efficiency scores of both approaches are shown in Fig. 3, grouping the teams according to their division. Note that the network DEA efficiency is lower in every division. On one hand, Southeast and Pacific divisions experience the greatest reduction in their efficiencies when the network approach is considered, whereas the rest of the divisions do not undergo such a pronounced change. On the other hand, Northwest division has high single-process efficiency and remains the division with the highest network DEA efficiency.

Going back to Table 5, there are 3 teams that are fully efficient when the single-process approach is considered: Cleveland Cavaliers (CLE), Oklahoma City Thunder (OKC) and Sacramento Kings (SAC). From Table 4 it can be noted that CLE and OKC have a high

Table 4 Input-output data for the regular season 2009/10

			Inputs		Outputs
			FTBudget	BTBudget	Wins
NBA Eastern	Atlantic	Bolton Celtics	61.83	25.57	50
		New Jersey Nets	15.64	36.49	12
		New York Knicks	39.86	28.93	29
		Philadelphia 76ers	42.99	22.18	27
		Toronto Raptors	38.61	28.59	40
	Central	Chicago Bulls	28.53	27.68	41
		Cleveland Cavaliers	49.71	35.09	61
		Detroit Pistons	25.52	32.22	27
		Indiana Pacers	27.29	32.44	32
		Milwaukee Bucks	20.01	46.76	46
	Southeast	Atlanta Hawks	43.80	22.11	53
		Charlotte Bobcats	38.12	31.63	44
		Miami Heat	53.24	23.72	47
		Orlando Magic	57.74	22.51	59
		Washington Wizards	34.36	26.36	26
NBA Western	Southwest	Dallas Mavericks	56.52	34.72	55
		Houston Rockets	19.34	31.51	42
		Memphis Grizzlies	31.14	21.04	40
		New Orleans Hornets	52.33	16.91	37
		San Antonio Spurs	53.28	22.38	50
	Northwest	Denver Nuggets	54.26	18.65	53
		Minnesota Timberwolves	24.99	29.40	15
		Portland Trail Blazers	22.14	27.40	50
		Oklahoma City Thunder	20.09	24.76	50
		Utah Jazz	51.75	20.38	53
	Pacific	Golden State Warriors	23.79	34.44	26
		Los Angeles Clippers	34.15	18.17	29
		Los Angeles Lakers	51.24	30.87	57
		Phoenix Suns	47.83	15.80	54
		Sacramento Kings	15.05	29.32	25

number of victories, CLE has allocated the main part of its budget to the first team, and that OKC has a more balanced team, i.e. more money goes to the bench team salaries. On the other hand, SAC is labeled efficient without having a substantial number of victories and that it is due to its low players' salaries, which SAC mainly allocates to the bench team.

In Table 7, the budget and wins slacks obtained from both models are shown. These slacks represent the amount that the team budget has to be reduced and the increase in the number of victories that the team should obtain in order to become efficient. For instance, according to the single-process approach, Los Angeles Lakers (LAL) obtained a great number of victories during the regular season, but it consumed a very significant amount of budget

Table 5 Efficiency scores for single-process and network DEA approaches

			SBM efficiency	Network SBM efficiency
Atlantic	Bolton Celtics	BOS	51.31	41.71
	New Jersey Nets	NJ	20.65	16.79
	New York Knicks	NYK	37.81	30.74
	Philadelphia 76ers	PHI	37.16	30.21
	Toronto Raptors	TOR	53.39	43.40
Central	Chicago Bulls	CHI	65.42	53.18
	Cleveland Cavaliers	CLE	100.00	52.45
	Detroit Pistons	DET	41.94	34.10
	Indiana Pacers	IND	48.06	39.07
	Milwaukee Bucks	MIL	61.79	50.24
Southeast	Atlanta Hawks	ATL	84.57	58.63
	Charlotte Bobcats	CHA	56.58	46.00
	Miami Heat	MIA	54.77	44.53
	Orlando Magic	ORL	96.61	53.61
	Washington Wizards	WAS	38.40	31.22
Southwest	Dallas Mavericks	DAL	69.06	43.96
	Houston Rockets	HOU	74.09	60.23
	Memphis Grizzlies	MEM	68.75	55.89
	New Orleans Hornets	NO	47.93	38.96
	San Antonio Spurs	SAS	59.27	48.19
Northwest	Denver Nuggets	DEN	76.46	53.01
	Minnesota Timberwolves	MIN	24.74	20.11
	Portland Trail Blazers	POR	90.54	73.60
	Oklahoma City Thunder	OKC	100.00	81.30
	Utah Jazz	UT	77.28	53.58
Pacific	Golden State Warriors	GSW	40.05	32.56
	Los Angeles Clippers	LAC	49.72	40.42
	Los Angeles Lakers	LAL	85.59	50.62
	Phoenix Suns	PHO	93.32	61.88
	Sacramento Kings	SAC	100.00	45.79

and therefore it would need to reduce it by 11.83 millions in order to attain efficiency. Other important teams that got a high number of victories, like Boston Celtics (BOS) and Dallas Maverick (DAL) also would have to reduce their budgets according to the single-process approach. On the other hand, again according to the single-process approach, New Jersey Nets (NJ) and Minnesota Timberwolves (MIN) should have got more victories given the salaries they pay to their players.

Regarding the results of the proposed network DEA approach, it can be seen in the last column from Table 5 that there isn't any efficient team. This is not uncommon in network DEA and clearly differentiates it from conventional DEA. The team with the highest network DEA efficiency is OKC, which was efficient according to single-process DEA. According

Table 6 Efficiency ranking by both approaches

Teams	Ranking	
	SBM	NSBM
Bolton Celtics	20	20
New Jersey Nets	30	30
New York Knicks	27	27
Philadelphia 76ers	28	28
Toronto Raptors	19	19
Chicago Bulls	14	9
Cleveland Cavaliers	2	11
Detroit Pistons	24	24
Indiana Pacers	22	22
Milwaukee Bucks	15	13
Atlanta Hawks	8	5
Charlotte Bobcats	17	15
Miami Heat	18	17
Orlando Magic	4	7
Washington Wizards	26	26
Dallas Mavericks	12	18
Houston Rockets	11	4
Memphis Grizzlies	13	6
New Orleans Hornets	23	23
San Antonio Spurs	16	14
Denver Nuggets	10	10
Minnesota Timberwolves	29	29
Portland Trail Blazers	6	2
Oklahoma City Thunder	2	1
Utah Jazz	9	8
Golden State Warriors	25	25
Los Angeles Clippers	21	21
Los Angeles Lakers	7	12
Phoenix Suns	5	3
Sacramento Kings	2	16

to the network DEA slacks in Table 7, OKC has a well-balanced budget and number of victories, and only has to reduce its budget by 0.37 millions and get 11 more victories to become efficient. However, the other single-process efficient teams are very inefficient when a network DEA approach is considered. Thus, CLE would have to reduce its budget by 40.32 millions and SAC would have to increase its number of wins by almost 30. Despite that fact, according to the network DEA approach, CLE has no wins slack and SAC does not need to reduce its budget.

In Fig. 4, the slacks obtained from both models are graphically represented and grouped by divisions. The first column, for each approach, corresponds to total budget slack whereas the second column corresponds to wins slack. In every case, the slacks obtained from network DEA model are higher, which means that the network DEA formulation identifies more sources of inefficiency than the single-process approach. Thus, when the internal struc-

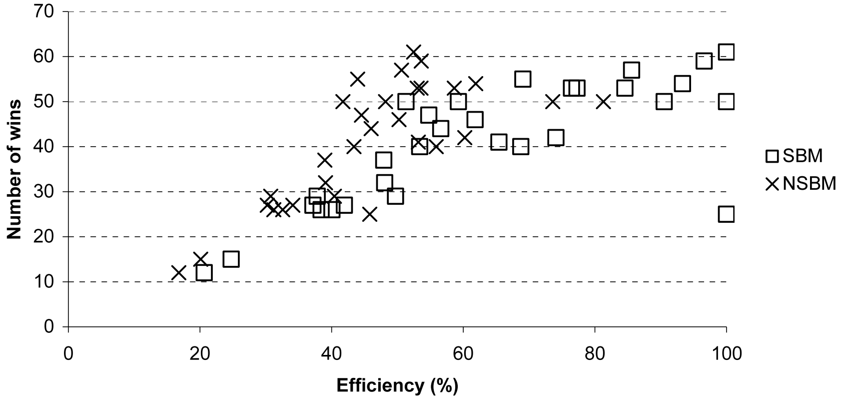
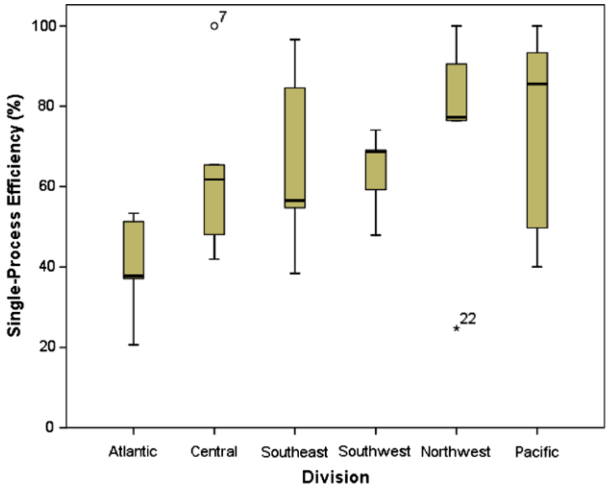


Fig. 2 Single-process and network DEA SBM efficiency versus the number of wins of each team

Fig. 3 Box-plots of single-process and network DEA SBM results for NBA Divisions



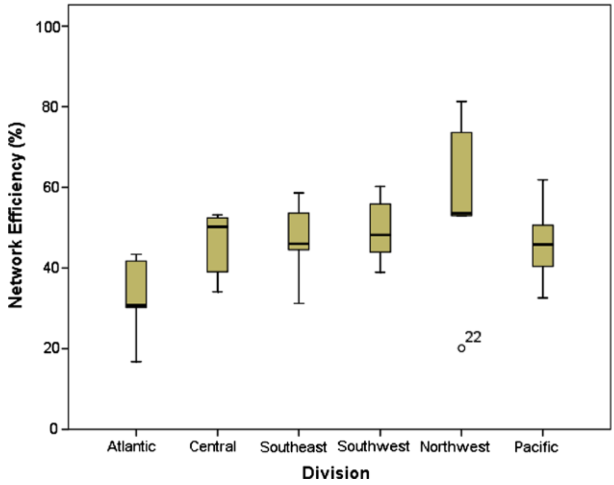


Table 7 Input and output slacks for both approaches

			Slacks			
			Single		Network	
			Budget	Wins	Budget	Wins
NBA Eastern	Atlantic	Bolton Celtics	42.56	0.00	42.93	11.00
		New Jersey Nets	7.28	38.00	7.65	49.00
		New York Knicks	23.94	21.00	24.31	32.00
		Philadelphia 76ers	20.32	23.00	20.69	34.00
		Toronto Raptors	22.36	10.00	22.72	21.00
	Central	Chicago Bulls	11.37	9.00	11.73	20.00
		Cleveland Cavaliers	0.00	0.00	40.32	0.00
		Detroit Pistons	12.89	23.00	13.26	34.00
		Indiana Pacers	14.88	18.00	15.24	29.00
		Milwaukee Bucks	21.92	4.00	22.29	15.00
	Southeast	Atlanta Hawks	10.17	0.00	21.43	8.00
		Charlotte Bobcats	24.90	6.00	25.27	17.00
		Miami Heat	32.12	3.00	32.49	14.00
		Orlando Magic	2.72	0.00	35.77	2.00
		Washington Wizards	15.88	24.00	16.25	35.00
NBA Western	Southwest	Dallas Mavericks	28.23	0.00	46.76	6.00
		Houston Rockets	6.00	8.00	6.36	19.00
		Memphis Grizzlies	7.34	10.00	7.71	21.00
		New Orleans Hornets	24.39	13.00	24.76	24.00
		San Antonio Spurs	30.81	0.00	31.18	11.00
	Northwest	Denver Nuggets	17.17	0.00	28.43	8.00
		Minnesota Timberwolves	9.54	35.00	9.91	46.00
		Portland Trail Blazers	4.69	0.00	5.05	11.00
		Oklahoma City Thunder	0.00	0.00	0.37	11.00
		Utah Jazz	16.39	0.00	27.65	8.00
	Pacific	Golden State Warriors	13.39	24.00	13.75	35.00
		Los Angeles Clippers	7.47	21.00	7.84	32.00
		Los Angeles Lakers	11.83	0.00	37.62	4.00
		Phoenix Suns	4.25	0.00	19.15	7.00
		Sacramento Kings	0.00	0.00	0.00	29.60

ture of the units is taken into account, it can be seen that some teams having high single-process efficiency like CLE, Portland Trail Blazers (POR), ORL, LAL, Phoenix Suns (PHO) and SAC, need to increase their number of victories or to significantly reduce their roster wages to become efficient, i.e. those teams have small slacks according to single-process approach but there are larger hidden inefficiencies that are revealed by network DEA. Thus, for example, SAC was single-process efficient, with one of the smallest budgets, but should have to increase the number of victories to become network DEA efficient. Other teams with lots of victories, like CLE, DAL, LAL and PHO, would have to reduce their wages to

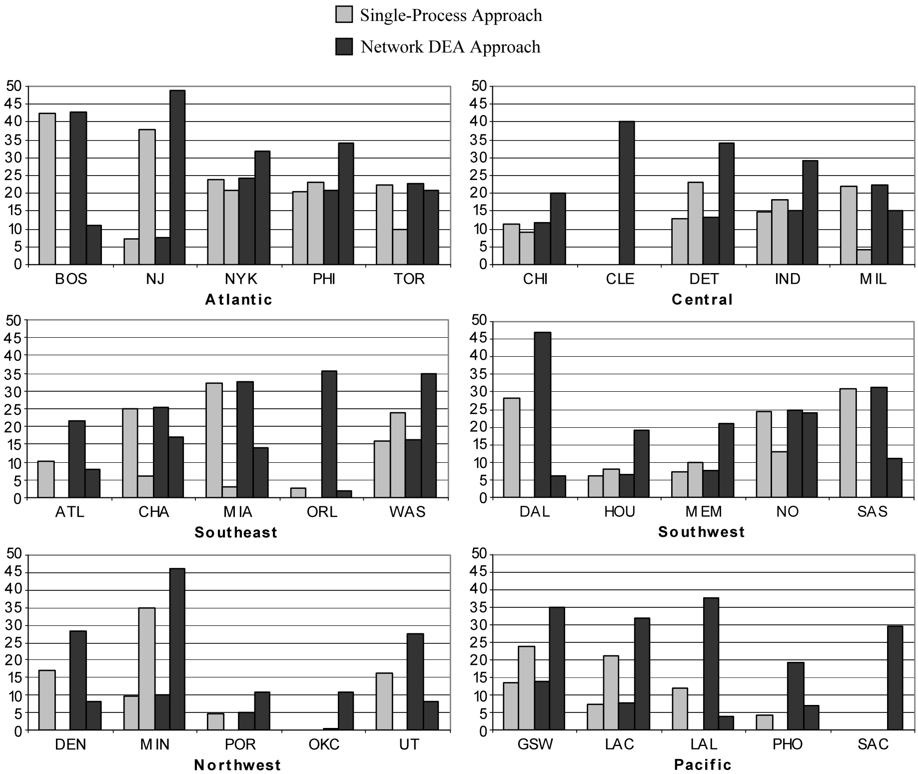


Fig. 4 Budget and wins slacks for all teams grouped by NBA Division

become network DEA efficient, even though the traditional DEA model did not identify any budget slack.

Furthermore, in Fig. 5 network DEA efficiency versus single-process efficiency are plotted for every team, tagging the teams according to the division they belong to. As all network DEA efficiencies are lower than their single-process counterparts, all points are below the diagonal. Notice that there are a significant number of teams very close to the diagonal, whose efficiencies are mostly between 20 and 80 percent. However, there are several teams relatively far from the diagonal, with single-process efficiencies very close to 100. In particular, it can be noticed the three teams with 100% single-process efficiency that are not completely efficient according to network DEA.

Finally, the difference in the efficiency scores computed by the two approaches for each team is shown in Fig. 6, in increasing order of that difference. It can be noted that 2 of the 3 efficient teams by the single-process approach (CLE and SAC) have the biggest difference, which means that network DEA shows sources of inefficiency hidden to the traditional approach. Moreover, looking back to Table 7, those inefficiencies can be traced to the corresponding input and output slacks.

With respect to the target values for the intermediate products, it must be taken into account that in those network DEA models, like the one proposed, in which intermediate products are considered as free-links (as opposed to fixed-links, see Tone and Tsutsui 2009) the computed targets for intermediate products do not need to dominate the observed values. That prevents computing a specific efficiency score for each process. Table 8 shows the

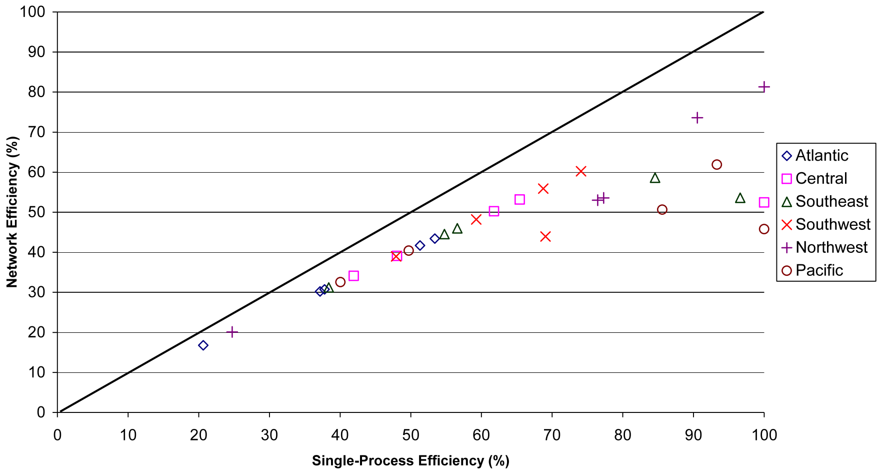


Fig. 5 Network DEA vs single-process efficiency

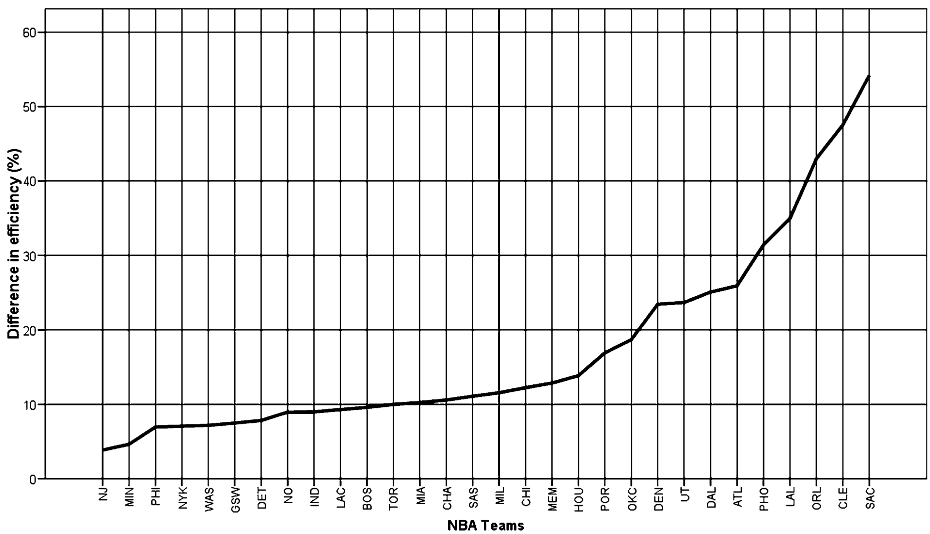


Fig. 6 Single-process efficiency minus network DEA efficiency

differences between the computed target values for the intermediate products and the corresponding observed values. The analysis of these differences should allow the coach to detect which offensive and defensive aspects to strengthen or rectify. For example, the difference for FT2PA is less than zero for most teams, which means that first team offensive systems must work in order to select better shooting positions because the teams could achieve the same number of points by attempting fewer shots. On the other hand, in case that the difference is greater than zero, for instance BT3PA for OKC, the team should be able to boost that game feature to take advantage of its resources. In that way, teams can be aware of its defects and realize which feature needs more attention.

Table 8 Difference between target and observed values for intermediate products

	Intermediate products																				
	FT 2PA	FT 3PA	FT FTA	FT OffReb	FT assists	FTTO	FT DefReb	FT steals	FT blocks	BT 2PA	BT 3PA	BT FTA	BT OffReb	BT assists	BTTO	BT DefReb	BT steals	BT blocks	FT points	BT points	Inv OppPoints
BOS	−402	−191	−445	22	−540	−13	−416	−150	−49	39	−136	12	140	−21	−73	47	−73	−33	−1171	−102	−230.39
NJ	−2692	−405	−1152	−409	−644	−131	−934	−252	−198	−1597	−515	−465	−278	−568	−142	−925	−207	−118	−3750	−2178	−950.51
NYK	−1072	−642	−355	−181	−490	−114	−835	−117	−42	−1050	−862	−443	−145	−488	−72	−435	−191	−77	−2069	−2324	−544.73
PHI	−1674	−299	−494	−449	−511	−72	−986	−273	−207	−1222	−484	−419	−18	−473	−137	−312	−135	−59	−2244	−2064	−506.64
TOR	−998	−404	−666	−97	−271	−67	−639	−1	−69	−638	−108	−126	−7	−437	−81	−204	−85	−58	−2026	−1017	−315.46
CHI	−1499	35	−335	−306	−367	−53	−738	−60	−146	−571	−194	−268	89	−214	−69	−272	−79	−69	−1534	−831	−372.94
CLE	699	−339	−206	204	−141	−7	39	34	36	−21	107	47	77	−24	−23	−193	−14	−70	0	0	0
DET	−1335	−105	−481	−370	−363	−146	−476	−178	−38	−1648	−487	−622	−209	−489	−71	−643	−159	−102	−1496	−2507	−665.60
IND	−850	−905	−448	−187	−384	−78	−899	−176	−164	−1199	−283	−511	−36	−470	−69	−388	−103	−69	−2121	−1749	−505.80
MIL	−43	−458	113	−106	−113	−72	−303	−65	−74	−1004	−337	−264	−51	−368	−40	−343	−75	−16	−128	−1567	−308.54
ATL	−667	−222	−170	−194	−299	−41	−536	−106	−97	88	−61	12	155	−39	−104	289	21	24	−1007	−56	−148.40
CHA	−794	−363	−485	−134	−385	−25	−576	−186	−60	−223	6	−237	48	−69	−59	−84	−25	−104	−1455	−319	−379.61
MIA	−789	−205	−366	−31	−121	−56	−298	−89	−55	−75	−182	−8	−18	−140	−60	−300	−66	−101	−1208	−255	−311.14
ORL	676	−528	−356	7	143	−4	−416	18	−61	479	−408	135	219	−143	−40	134	34	−16	−197	−131	−46.08
WAS	−976	−457	−269	−38	−397	−119	−325	−80	−2	−2160	−192	−765	−472	−449	−65	−1052	−164	−252	−1548	−2775	−661.79
DAL	267	8	97	33	−87	−59	−327	−38	−33	−581	−289	−145	99	−324	−10	29	−60	−63	265	−1079	−78.01
HOU	44	−747	−105	−163	−387	−51	−568	−148	−25	−1351	−162	−525	−70	−253	−60	−156	−33	−24	−1268	−1362	−308.50
MEM	−2157	−375	−871	−393	−376	−32	−867	−249	−70	−100	240	−5	25	−71	−100	35	−13	−72	−3191	278	−335.99
NO	−695	−264	−209	−166	−206	−126	−519	−91	−52	−1246	−490	−225	−36	−609	−35	−392	−180	−11	−1146	−1995	−413.50
SAS	17	129	10	−21	48	−79	−259	74	−2	−631	−570	−322	12	−508	0	−282	−112	−59	244	−1693	−220.86

Table 8 (*Continued*)

	Intermediate products																				
	FT 2PA	FT 3PA	FT FTA	FT OffReb	FT assists	FTTO	FT DefReb	FT steals	FT blocks	BT 2PA	BT 3PA	BT FTA	BT OffReb	BT assists	BTTO	BT DefReb	BT steals	BT blocks	FT points	BT points	Inv OppPoints
DEN	−453	−282	−720	−79	−77	−30	−301	−105	8	59	−63	−32	120	−191	−38	0	−71	−85	−1317	−137	−82.81
MIN	−2557	−548	−740	−290	−661	−109	−991	−306	−108	−1835	−301	−689	−408	−554	−108	−941	−148	−102	−3448	−2540	−817.75
POR	−593	−91	−337	−62	−86	−74	−89	−12	84	26	−191	−39	27	−219	−41	−214	−33	−114	−822	−360	−238.23
OKC	−1110	−344	−656	−131	−267	−10	−523	−172	−74	208	221	103	49	−3	−77	−4	−4	−87	−1841	382	−198.63
UT	−188	−134	−383	−24	−442	−13	−396	−67	1	−413	99	−94	82	−294	−27	12	−101	−61	−804	−468	−124.68
GSW	−1969	−825	−855	−104	−683	−69	−613	−297	11	−1099	−286	−369	−193	−445	−150	−703	−215	−184	−3609	−1744	−541.15
LAC	−1406	−733	−549	−129	−492	−76	−521	−120	−107	−1129	−83	−357	−288	−524	−75	−774	−134	−173	−2427	−1442	−588.20
LAL	−282	−190	−315	−122	−6	−18	−187	−61	−21	96	−111	219	149	−163	−53	−104	−6	−14	−673	158	−65.37
PHO	−367	−566	−307	−11	−400	−8	−505	19	−14	207	−10	−20	49	−33	−50	135	18	−58	−1619	−8	−28.38
SAC	−1837	−336	−612	−285	−618	−76	−748	−169	−133	−1151	−414	−455	−210	−293	−106	−617	−137	−66	−2524	−1853	−605.02

5 Conclusions and further research

In this paper, an efficiency analysis of NBA teams has been performed by means of conventional and network DEA. Since the latter approach has never been applied to basketball, we have developed a framework of the internal operation of a NBA team, by structuring the production process of a DMU into acquisition, offensive, defensive and effectiveness stages. SBM have been implemented instead of a radial adjustment in order to cover both contraction of wages and expansion of outputs. Another main feature of the proposed approach is to consider the importance of managing properly the economical resources, in terms of allocating the total wages salaries to the first and bench teams.

NBA is a demanding competition, where significant investments are involved. Hence, it becomes desirable to identify additional improvement guidelines within a team. In this paper it has been shown that Network DEA has provided a stronger assessment of the teams' performances uncovering sources of inefficiency that can remain hidden in the conventional DEA approach. In fact, some teams have moved from a high single-process efficiency to average network DEA efficiency. This is not surprising since for a team to be evaluated as network DEA efficient all its internal processes need to be efficient. It is often the case that a team has a well-performing First Team or Bench Team or perhaps a very efficient defense system, but it does not have all of them simultaneously then its overall efficiency decreases.

In addition to the target inputs and outputs, the intermediate products projections can also analyzed. However, when carrying out this analysis of intermediate products targets it must be taken into account that these intermediate products projections are not unique, i.e. there are usually more than one configuration of the operation points of the internal processes than lead to the same target inputs and outputs of the whole system.

With respect to topics for further research there are several. Thus, since NBA teams do not plan for only one season but usually elaborate a project for several years (including investment in players from draft, contracts' updates and players' transfers to other teams in case of not achieving short-time objectives), one topic of further research can be to evaluate the change in efficiency for every team in a four or five periods' time span. Another area of research is the application of statistical methods to reduce the number of intermediate products when network DEA approach is considered. Some of the intermediate products may have a high degree of correlation and by means of methodologies like Factor Analysis a new group of reduced variables could be obtained. Another possible path is to assign different weights to the different variables, thus reflecting their relative importance. In that case, an extension of the Tsutsui and Goto (2009) approach to network DEA together with an objective method for choosing these weights would be needed. Yet another feasible methodological development would be to adapt the SBM super-efficiency approach in Tone (2002) to the network DEA framework, so that in case several DMUs were deemed efficient in network DEA (something that does not occur easily) they can be ranked. Finally, the fact that Network DEA always provides lower efficiency scores than single-process DEA has been observed by other authors, e.g. Kao (2009a), and Tone and Tsutsui (2009) when analyzing the free-link case, but it has not been proved to hold for all scenarios. Thus, a mathematical proof would be a subject of further research.

Acknowledgements The authors would like to thank the reviewers for their constructive comments and suggestions, which have helped to improve the paper.

Appendix

Table 9 Intermediate products

	Intermediate products																				
	FT 2PA	FT 3PA	FT FTA	FT OffReb	FT assists	FTTO	FT DefReb	FT steals	FT blocks	BT 2PA	BT 3PA	BT FTA	BT OffReb	BT assists	BTTO	BT DefReb	BT steals	BT blocks	FT points	BT points	Inv OppPoints
BOS	3288	750	1469	417	1487	123.92	1607	431	256	1573	683	621	299	443	242.72	842	270	146	5491	2645	1276.16
NJ	3385	539	1398	514	871	157.48	1220	319	248	1984	646	617	383	669	182.82	1138	254	145	4787	2788	1201.49
NYK	2746	966	949	436	1039	178.57	1526	280	162	1985	1179	810	400	733	170.07	951	306	143	4574	3799	1151.28
PHI	3233	601	1047	686	1022	132.63	1629	425	319	2092	780	761	255	701	228.31	792	242	120	4577	3437	1071.35
TOR	3307	851	1485	448	1029	156.49	1592	226	235	1927	546	633	358	775	216.92	915	243	149	5482	3052	1152.07
CHI	3866	423	1175	666	1144	143.88	1714	290	316	1893	643	787	271	560	208.33	1001	241	162	5076	2917	1230.47
CLE	2822	1021	1455	332	1296	142.45	1414	309	217	1987	561	725	459	539	229.89	1278	255	208	5270	3103	1275.84
DET	2894	407	1034	607	874	206.61	1119	330	150	2518	783	964	446	717	162.34	1123	266	163	3829	3880	1230.31
IND	2697	1263	1103	468	990	149.25	1661	356	297	2230	633	916	317	740	177.94	957	229	142	4886	3377	1175.09
MIL	2698	972	829	510	984	174.22	1399	323	265	2487	841	846	455	756	195.69	1161	257	120	4102	3907	1270.65
ATL	3726	814	1255	659	1303	159.49	1798	404	317	1620	641	659	310	486	283.29	653	188	96	5586	2752	1256.91
CHA	3334	855	1386	520	1218	123.46	1624	433	242	1641	476	794	338	440	208.33	866	199	204	5256	2557	1299.88
MIA	3502	730	1328	444	1011	160.26	1417	353	250	1590	696	603	431	537	219.78	1136	252	207	5268	2646	1294.16
ORL	2730	1187	1564	511	974	135.32	1821	313	306	1423	1054	612	299	641	240.38	915	199	150	5294	3132	1280.08
WAS	2477	748	801	266	889	177.30	944	226	110	2998	477	1094	700	668	152.67	1514	267	311	3794	4098	1207.15
DAL	2908	607	1029	450	1129	181.49	1637	347	261	2354	891	841	384	788	196.85	949	277	188	4487	3877	1228.35
HOU	2380	1216	965	532	1182	144.09	1568	384	199	2705	622	1057	439	608	202.43	903	199	119	4897	3498	1186.94
MEM	4466	822	1690	744	1134	121.36	1820	474	236	1389	198	512	326	409	235.85	676	171	163	6647	1757	1172.61
NO	2831	677	967	491	907	208.77	1400	299	205	2439	895	694	361	921	160.26	1050	326	95	4343	3877	1187.37
SAS	2869	430	1014	460	899	190.11	1450	207	209	2243	1117	955	427	930	169.49	1171	309	172	4076	4236	1266.62

Table 9 (*Continued*)

	Intermediate products																				
	FT 2PA	FT 3PA	FT FTA	FT OffReb	FT assists	FTTO	FT DefReb	FT steals	FT blocks	BT 2PA	BT 3PA	BT FTA	BT OffReb	BT assists	BTTO	BT DefReb	BT steals	BT blocks	FT points	BT points	Inv OppPoints
DEN	3512	874	1805	544	1081	147.93	1563	403	212	1649	643	703	345	638	217.39	942	280	205	5896	2833	1191.33
MIN	3423	716	1047	422	945	142.05	1348	390	170	2319	465	879	540	681	158.98	1208	207	136	4744	3307	1131.48
POR	3479	650	1361	501	1033	185.87	1280	293	123	1586	738	672	412	641	210.97	1103	230	227	5142	2903	1286.34
OKC	3996	903	1680	570	1214	121.65	1714	453	281	1404	326	530	390	425	246.91	893	201	200	6161	2161	1244.40
UT	3247	726	1468	489	1446	131.06	1658	365	219	2121	481	765	383	741	207.04	930	310	181	5383	3164	1233.20
GSW	3470	1116	1387	332	1175	127.39	1232	443	97	1937	571	698	421	664	238.66	1165	318	243	5855	3067	1084.95
LAC	3080	1057	1143	384	1041	141.04	1212	283	227	2064	400	724	543	769	173.61	1290	249	239	4932	2917	1194.74
LAL	3572	827	1482	622	1086	144.93	1545	381	257	1741	735	503	351	644	246.31	1117	231	143	5597	2742	1257.55
PHO	3484	1169	1413	485	1423	128.04	1791	284	238	1534	601	704	425	489	233.10	825	195	180	6284	2755	1157.81
SAC	3389	620	1129	526	1139	139.86	1380	312	245	2123	763	840	451	540	195.69	1138	252	121	4782	3418	1168.50

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Telecommunications Policy 37 (2013) 469–482

Contents lists available at SciVerse ScienceDirect

Telecommunications Policy

URL: www.elsevier.com/locate/telpol

Dynamic performance analysis of U.S. wireline telecommunication companies

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ARTICLE INFO

Available online 8 May 2013

Keywords:
Telecommunications
ILEC
Dynamic DEA
Network DEA
SBM

ABSTRACT

Assessing the changes over time in the efficiency of firms participating in competitive markets has always been a major concern to researchers and experts alike. With respect to the US wireline telecommunications sector, recent changes in unbundling regulations, as well as intermodal competition and mergers, have just increased uncertainty in a sector still marked by the Telecommunications Act of 1996. Although Data Envelopment Analysis (DEA) has become a methodology commonly used in many efficiency assessment applications, in the telecommunications context there is a need to implement an approach that takes into account carry-over activities between consecutive years; because of a wide customer base, financial long-term planning and investments in network elements and facilities are crucial for Local Exchange Carriers (LECs) to succeed. To that end, a Dynamic DEA application is formulated in this paper to evaluate the Incumbent LECs' (ILECs) performance from 1997 to 2007. Finally, a regression analysis has been carried out to establish the impact of competition and regulatory schemes upon carriers' efficiency. The results show that local competition has worsened efficiency, whereas neither intermodal competition nor incentive regulation has such a clear influence.

1. Introduction

Many authors have pointed out that productivity efficiency can be considered as a key element for obtaining greater operating revenues and improved market position in competitive markets (Pentzaropoulos & Giokas, 2002; Tsai, Chen, & Tzeng, 2006). The telecommunications sector has been one of the most competitive industries since the liberalization of the market in 1996. Competitiveness requires operating efficiency. From among the different efficiency assessment methods, Data Envelopment Analysis (DEA) is the one that has been most commonly applied in a wide range of industries, due to its versatility.

DEA is a well-known non-parametric method that estimates the relative efficiency of similar Decision Making Units (DMUs) (see, for example, Cooper, Seiford, & Tone, 2006; Cooper, Seiford, & Zhu, 2011; Thanassoulis, 2001; Zhu, 2002). DEA evaluates the DMUs' observed inputs and outputs, in order to determine which DMUs make up the efficient frontier, and provides efficiency estimations for all units. Best-practices units are identified and become the reference sets for the

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http://dx.doi.org/10.1016/j.telpol.2012.12.001

less efficient DMUs. In the case of inefficient DMUs, DEA identifies the reduction in inputs or increase in outputs (with respect to the observed values) that these units have to carry out in order to reach the efficient frontier.

There are a number of DEA applications to the telecommunications sector in the literature. Thus, Lien and Peng (2001) examined the production efficiency of telecommunications in 24 OECD countries from 1980 to 1995, by applying DEA to every year separately. In their study, total revenue is chosen as the output measure and three inputs are considered, namely the number of telephone lines, the number of employees and the total amount of investment. With respect to the use of investment as an input, capital expenditure and total assets are included as alternatives. Pentzaropoulos and Giokas (2002) studied the situation of the European telecommunications market by comparing the DEA efficiency of the main European operators. One of their conclusions is that operational efficiency can be achieved by organizations with large and small revenues alike. Similar results were presented by Tsai et al. (2006) in their comparative analysis for global telecommunication companies. More recently, Sadjadi and Omrani (2010) estimated the efficiency of telecommunication companies in Iran by implementing a bootstrapped robust DEA model.

The statutory framework for the U.S. communications policy is based on the Telecommunications Act of 1996, and is aimed at opening the local and long distance telephone markets, which were previously being operated as monopolies, to competition, by removing barriers to entry for new incumbents. In other words, Competitive Local Exchange Carriers (CLECs) could gain access to unbundled network elements in order to provide telecommunication services. The influence of this deregulatory environment on the efficiency of Incumbent Local Exchange Carriers (ILECs) from 1988 to 2000 was investigated to some extent by Resende (2008) via DEA. Sastry (2009) also used DEA to study the links between these major changes in competition and the performance attributes of telecommunications providers, focusing on service quality.

There are a number of DEA models that have been developed to cope with changes in time. Thus, Charnes, Clark, Cooper, and Golany (1984) presented a Window-analysis (WA) approach that takes into account data from several years when assessing efficiency. This WA approach was used in Yang and Chang (2009) to measure telecommunication firms' efficiencies in Taiwan over the period 2001 to 2005.

An alternative approach is the Malquimst Productivity Index (MPI) that allows analysis of the productivity change of a certain industry over time (Färe, Grosskopf, Lindgren, & Roos, 1992). In addition, the MPI allows decomposing this productivity change into an efficiency change between adjacent periods of time (relative to the efficient frontier of each period) and an efficient frontier shift (a.k.a. technological change). In the literature there are some studies regarding productivity growth in telecommunications industry. Thus, Uri (2000) calculated the performance changes and shifts in technology of 19 LECs for the period 1988 to 1998 and concluded that growth was due mainly to technological innovation rather than improvements in relative efficiency. In contrast, more recent evidence (Seo, Featherstone, Weisman, and Gao 2010) shows that ILECs underperformed over the period 1996 to 2005 in terms of average productivity growth.

More recently, Sung (2012) also applied a MPI approach to evaluate the total factor productivity (TFP) of ILECs and estimated the effects of regulatory schemes and competitive pressure on the slowdown in productivity growth of ILECs by means of a TFP-level regression analysis. It was found that intermodal competition and incentive regulation have induced a positive technical change but have worsened the ILECs' performance. Other attempts have been made with the purpose of estimating technological progress in the U.S. wireless services industry (Banker, Cao, Menon, & Natarajan 2010), which were motivated by the expanding market share of mobile telecom firms.

Nevertheless, despite the MPI approach being able to evaluate the change effect, MPI only measures distance to the efficient frontier in single periods of time (or at most between adjacent periods of time) and does not consider the carry-over activities between consecutive periods of time. In most industries with economies of scale, such as the telecommunications sector, long-term planning and investments in network infrastructure and technology are critical to gain better positions in the market. In fact, the entry barriers in telephone markets, that the Telecommunications Act of 1996 was intended to remove, are related to the huge amounts of money that new firms had to invest in network elements to be able to compete with the ILECs. Some authors (Ai & Sappington, 2002; Jung, Gayle, & Lehman 2008) have included infrastructure from previous periods as an explanatory variable in their dynamic data panel models. This lagged investment influences the network modernization in future periods. Cambini and Jiang (2009) have thus considered the influence of investment on competition.

In order to take into account the connecting activities along multiple periods, the Dynamic DEA approach was proposed (Tone & Tsutsui, 2010). In this paper, Dynamic DEA is used to assess the performance of wireline telecommunications firms from 1997 to 2007, and afterwards a regression is carried out to evaluate the effects of local competition, unbundling regulation, intermodal competition, incentive regulation, and mergers upon the carriers' efficiency. The paper is divided into six sections. Section 2 reviews the current state of the industry in the U.S. In Section 3 the Dynamic DEA methodology is described. The description of the data used is presented in Section 4 with the discussion of the results Section 5. Finally, conclusions are drawn in Section 6.

2. Background

This section addresses the features of the U.S. telecommunication sector and the issues that have arisen in the last years. Specifically, the current state of the unbundling deregulation, the recent trends in telecommunications usage leading to intermodal competition and the influence of price-cap regulation are reviewed. Therefore, the evaluation of the long range performance of firms in the wireline segment market is fully justified because of the topics listed. The last subsection shows a brief summary of mergers amongst Regional Bell Operating Companies (RBOCs).

2.1. Unbundling deregulation in the U.S.

After the Bell System divestiture, when AT&T split its local operating system into seven independent local exchange operating companies known as RBOCs, the main step to deregulation was the passing of the Telecommunications Act of 1996. It was intended to open the local market, which was dominated by incumbent carriers, to new competitive carriers known as CLECs. The Act mandated that network elements owned by ILECs had to be unbundled and the CLECs could gain access to these Unbundled Network Elements (UNEs) under a wholesale pricing system established by the regulators.

There are two leasing types for UNEs, namely UNE-Platform (UNE-P) which accounts for the leasing of all ILEC network elements simultaneously and UNE-Loop (UNE-L), which means leasing just a part of the local loop network. In other words, under UNE-L entry, CLECs lease only the wires and install their own switching equipment. In exchange for the leasing of their infrastructure, RBOCs were allowed to provide an interLATA long distance service, i.e. service across different Local Access Transport Areas (LATAs), because, after the break-up of AT&T, RBOCs were restricted to providing an intraLATA service only.

The telecommunications network has a specific cost structure of high fixed cost and low marginal cost which leads to economics of scale and scope. That is why the CLECs could not bear the huge cost of entering into market competition before the passing of the Telecommunications Act. In contrast, the Act also encouraged the CLECs to make their own investments in the network after a period of leasing. However, Quast (2008) reported that facilities-based entry (UNE-L entry) into the telecommunications market had actually been negatively affected by the low cost of UNE-P entry.

Unsurprisingly, ILECs were not satisfied with UNEs rules because local-loop unbundling may dampen ILECs' incentives to upgrade their networks and discourage CLECs to move from leasing lines to building their own infrastructure (Cambini & Jiang, 2009). After many court appeals, the Federal Communications Commission (FCC) has been forced to adjust the UNE policy in several ways. Thus, as far as we are concerned, ILECs are no longer required to give CLECs access to their broadband facilities, fiber-optic networks or the transmission component of the Internet access service. Last but not least, Dai and Tang (2009) pointed out that the switching function was removed from the list of unbundled eligible elements; thus, leasing of all the portions of the incumbents' network in order to provide a phone service is no longer available.

2.2. Intermodal competition

Intermodal competition accounts for the increasing importance of wireless telephony and high-speed connections. According to FCC reports (*Statistical Trends in Telephone Service*), there has been a deterioration in the number of ILECs' lines over recent years. Loomis and Swann (2005) first warned about the intermodal competition in the U.S. telecommunications market, apart from competition between ILECs and CLECs. Both wireless development and substitution of a high-speed service for dial-up lines have had a negative impact on incumbent line based reduction.

Zimmerman (2007) discussed the decline in overall telephone service penetration rates, naming the substitution between wireline and wireless telephony (i.e., consumers prefer wireless services) and the high penetration of high-speed and cable connections as the reasons for such falling trends in wireline services. Special attention should be given to cable companies due to their ability to offer bundled voice, video and Internet access over their own networks.

In their study of the mobile market, Banker et al. (2010) highlighted the significant growth of mobile telephony, despite the economic environment. While assessing the effects of network unbundling in telecommunications, Ware and Dippon (2010) found that facility investment could have declined due to mandatory unbundling and pointed out the significance of intermodal competition between video, wireless and telephone providers. Therefore, ILECs should perform as efficiently as possible in order to be able to compete against all their rivals.

2.3. Price-cap regulation

At the beginning of the 1990s, most States changed the regulatory regime for US telecommunications firms from traditional Rate of Return regulation (RoR) to other incentive regulations, Price Cap regulation (PCR) being the most widespread type of incentive regulation. However, even nowadays there is a wide range of rate regulation plans of ILECs across the different states and ILECs usually provides telecommunication services in more than one state.

Resende (2000), after employing a regression analysis of DEA efficiencies scores, determined that ILECs can achieve a higher level of efficiency under PCR. In addition, Ai and Sappington (2002) found lower operating costs and network modernization where incentive regulation was implemented. However, Uri (2001) did not find an enhancement of technical efficiency over the period 1988 to 1998, when the incentive regulation was adopted.

The most recent studies have shed some light on this issue. In their review, Sappington and Weisman (2010) summarized that PCR promotes network modernization and increases productivity rates. Majumdar (2010) reached the same conclusions. Apart from the U.S. telecom sector, Hisali and Yawe (2011) showed how PCR implemented in Uganda's telecom market has advanced the technical efficiency frontier.

2.4. Mergers

Since the break-up of AT&T in 1984, the RBOCs have undergone several processes of mergers and acquisitions amongst them. For instance, SBC Communications acquired Pacific Telesis Group in 1997, Southern New England Telecommunications Corporation in 1998 and Ameritech in 1999, although kept them as separate reporting entities. One of the main processes was the merger of Bell Atlantic Corporation and GTE Corporation into one company, Verizon Communications. However, Verizon Communications kept the two separate holding companies intact; Bell Atlantic and GTE Corporation.

The two latest mergers in the period under assessment were between SBC and AT&T (December 2005) and between AT&T and Bell South (December 2006). Although the decisions of mergers have been motivated by synergies and could enable a share of assets and a reduction of costs, Majumdar, Yaylacicegi, and Moussawi (2012) and Seo, Featherstone, Weisman, and Gao (2010) have reported that the performance levels of ILECs has deteriorated after mergers. In addition, Loomis and Swann (2005) and Zimmerman (2007) highlighted the significance of mergers between wireline and wireless carriers, because of synergies and strategic implications.

3. Methodology

The two variants used in this paper are implementations of the Dynamic DEA approach proposed by Tone and Tsutsui (2010), which takes into account carry-over activities between consecutive periods of time. Dynamic DEA extends the slacks-based measure of efficiency (SBM) framework presented by Tone (2001), which means that there is no radial measure of efficiency but the model includes variable slacks. Therefore, instead of providing equi-proportional changes applicable to the whole set of inputs or outputs, this approach computes in a separate way the excesses and shortfalls in inputs and outputs, respectively. Dynamic DEA is also based on the SBM Network DEA of Tone and Tsutsui (2009) considering a series of stages, i.e. as many as time periods are included in the analysis.

Let m be the number of inputs, p the number of outputs and n the number of DMUs. Let T be the number of periods of time to be included in the analysis. Let x_{ij}^t be the value of the input i consumed by DMU j during period t, and y_{kj}^t the value of the output k produced by DMU j during period t. There is one slack variable associated with each input and output, which accounts for the difference between the target and observed values, i.e., the inefficiency in every single input and output. These slacks represent margins for improvement in each input and output and they will be denoted as s_i^t and s_k^t, i.e. the slacks during period t in the input i and output k, respectively.

As can be seen in Fig. 1, apart from the inputs and outputs in every period t, the Dynamic DEA approach considers carry-over activities from one period to the next. The carry-over activity r produced at the end of period t, which is consumed at the beginning of period $t+1$, is denoted by z_{rj}^t. Tone and Tsutsui (2010) claimed that there should be a distinction drawn between the characteristics of the connecting activities so that they can be classified into desirable, undesirable and free links. The number of connecting activities of each type is denoted *zgood*, *zbad* and *zfree*, and the different links will be marked with a superscript pointing out the type. The slack variable corresponding to a connecting activity r produced at the end of period t will be referred to as s_r^t.

It is logical to seek maximization in good links while bad links are minimized simultaneously. That is to say, desirable links are treated as outputs and bad links as inputs, leading to the objective function shown in (1), which is an extension of the SBM efficiency score proposed in Tone (2001) but incorporating T periods of time. The result of the metrics in (1) is an estimation of the efficiency for every DMU. As the specific production function under assessment in this application is considered to exhibit Constant Returns to Scale (CRS) (see Section 4), only the CRS formulation will be introduced here.

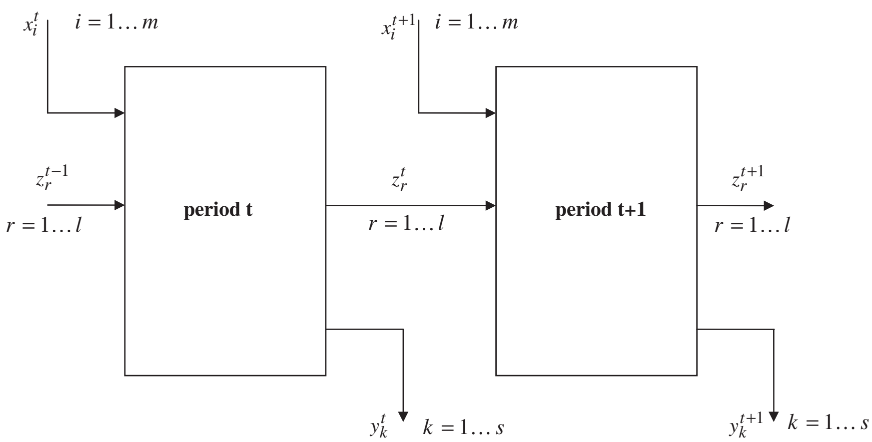

Fig. 1. General Dynamic DEA structure.

Constraint (2) guarantee that the targets for inputs are lower than the observed values. Similarly, constraint (3) require that outputs targets are larger than observed ones. The difference between these targets and observed values are computed as the corresponding input and output slacks. In these equations, λ_j^t represents the set of intensity weights defining the linear combination of the observed DMUs. The DMU under assessment is henceforth labeled DMU 0. If λ_j^t is greater than zero for a certain j, DMU j is said to be a member of the reference set for the DMU 0.

$$Efficiency = \min \frac{\frac{1}{T}\sum_{t=1}^{T}\left[1-\frac{1}{m+zbad}\left(\sum_{i=1}^{m}\frac{s_i^t}{x_{i0}^t}+\sum_{r\in zbad}\frac{s_r^t}{z_{r0}^t}\right)\right]}{\frac{1}{T}\sum_{t=1}^{T}\left[1+\frac{1}{p+zgood}\left(\sum_{k=1}^{p}\frac{s_k^t}{y_{k0}^t}+\sum_{r\in zgood}\frac{s_r^t}{z_{r0}^t}\right)\right]} \tag{1}$$

s.t.

$$\sum_{j=1}^{n} x_{ij}^t\lambda_j^t = x_{i0}^t - s_i^t \quad i=1\ldots m \;\; t=1\ldots T \tag{2}$$

$$\sum_{j=1}^{n} y_{kj}^t\lambda_j^t = y_{k0}^t + s_k^t \quad k=1\ldots p \;\; t=1\ldots T \tag{3}$$

$$\sum_{j=1}^{n} z_{rj}^t\lambda_j^t = z_{r0}^t + s_r^t \quad r\in zgood \;\; t=1\ldots T \tag{4}$$

$$\sum_{j=1}^{n} z_{rj}^t\lambda_j^t = z_{r0}^t - s_r^t \quad r\in zbad \;\; t=1\ldots T \tag{5}$$

$$\sum_{j=1}^{n} z_{rj}^t\lambda_j^t = z_{r0}^t + s_r^t \quad r\in zfree \;\; t=1\ldots T \tag{6}$$

$$\sum_{j=1}^{n} z_{rj}^t\lambda_j^t = \sum_{j=1}^{n} z_{rj}^t\lambda_j^{t+1} \quad \forall r \;\; t=1\ldots T-1 \tag{7}$$

$$\lambda_j^t \ge 0 \quad j=1\ldots n \;\; t=1\ldots T \tag{8}$$

$$s_i^t \ge 0 \quad i=1\ldots m \;\; t=1\ldots T \tag{9}$$

$$s_k^t \ge 0 \quad k=1\ldots p \;\; t=1\ldots T \tag{10}$$

$$s_r^t \ge 0 \quad r\in zgood \;\; t=1\ldots T \tag{11}$$

$$s_r^t \ge 0 \quad r\in zbad \;\; t=1\ldots T \tag{12}$$

$$s_r^t \quad free \quad r\in zfree \;\; t=1\ldots T \tag{13}$$

Concerning carry-over activities, they are treated as intermediate products in Network DEA (Färe and Grosskopf, 2000). Network DEA consider DMUs as a network of processes instead of 'black boxes', with intermediate products connecting the internal processes. In this case, every period will function as an internal process. Network DEA provides greater insight into the analysis and reveals more sources of inefficiency (Moreno and Lozano, in press).

Constraints (4)–(6) allow computing the slacks for all types of links, by establishing the target value for each link. All slacks are defined positive as per (11) and (12), except for free links (13). Note the similarity among constraints (4) and (5) and (2) and (3). Finally, constraint (7) reflects the fact that Dynamic DEA has been defined on the basis of the Network DEA. Thus, these constraints force the amount of links produced in every period to be the same as the amount consumed in the following period, which is the main constraint related to intermediate products in Network DEA. This idea can be seen in Fig. 1, which shows that connecting activities are outcomes for period t but are part of the resources used in period $t+1$. The following constraints (14) and (15) impose the initial conditions, i.e., the carry-over activities values that are input for the first period:

$$\sum_{j=1}^{n} z_{rj}^t\lambda_j^{t+1} \ge z_{r0}^t \quad r\in zgood \;\; t=0 \tag{14}$$

$$\sum_{j=1}^{n} z_{rj}^t\lambda_j^{t+1} \le z_{r0}^t \quad r\in zbad \;\; t=0 \tag{15}$$

Going back to Eq. (1), the objective function is non-linear, but it can be linearized by multiplying both the numerator and the denominator by an auxiliary variable, equating the resulting denominator to unity and redefining the products of variables as new variables (Tone, 2001). Once the optimization problem has been solved, the overall efficiency throughout all the periods under assessment is given by (1). In addition, Eq. (16) enables us to have a specific measure of the efficiency of period t. Moreover, the target values for all variables are given by Eqs. (2′)–(6′), where the superscript * refers to optimal values of model (1)–(15).

$$Efficiency^t = \frac{1 - \frac{1}{m+zbad}\left(\sum_{i=1}^{m}\frac{s_i^{*t}}{x_{i0}^t} + \sum_{r \in zbad}\frac{s_r^{*t}}{z_{r0}^t}\right)}{1 + \frac{1}{p+zgood}\left(\sum_{k=1}^{p}\frac{s_k^{*t}}{y_{k0}^t} + \sum_{r \in zgood}\frac{s_r^{*t}}{z_{r0}^t}\right)} \tag{16}$$

$$x_i^{t*} = x_{i0}^t - s_i^{*t} \quad i = 1\ldots m \ \ t = 1\ldots T \tag{2′}$$

$$y_k^{t*} = y_{k0}^t - s_k^{*t} \quad k = 1\ldots p \ \ t = 1\ldots T \tag{3′}$$

$$z_r^{t*} = z_{r0}^t + s_r^{*t} \quad r \in zgood \ \ t = 1\ldots T \tag{4′}$$

$$z_r^{t*} = z_{r0}^t - s_r^{*t} \quad r \in zbad \ \ t = 1\ldots T \tag{5′}$$

$$z_r^{t*} = z_{r0}^t + s_r^{*t} \quad r \in zfree \ \ t = 1\ldots T \tag{6′}$$

Since SBM computes the deviation for every variable involved in the model instead of a radial proportion, a Factor Efficiency Index (*FEI*) can be provided for each input and output (Tone & Tsutsui, 2010). *FEI* is defined in Eq. (17) and it implies an input excess, if positive, and an output shortfall, if negative.

$$FEI = \frac{observed\_data}{projection} - 1 \tag{17}$$

4. Data

In this paper, data for the main ILECs from 1997 to 2007 are included. The data are taken from the Automated Reporting Management Information System (ARMIS) reports (http://transition.fcc.gov/wcb/armis/) of the Federal Communications Commission (FCC). ARMIS aims to collect financial and operational data from the largest carriers. The list of ILECs that were required by the FCC to fill in all ARMIS reports, during the period under assessment, is included in Table 1. For every ILEC, its corresponding label, which refers to the Company Study Area (COSA) code used in FCC reports, is also shown. A similar choice of LECs was included in studies by Resende (2008) and Seo et al. (2010). Because of the intense competition in the sector (see Section 2) and the fact that the firms considered are the largest carriers, it can be assumed that ILECs operate in an optimal scale size and thus the production function exhibits CRS.

As can be seen in Fig. 2, where the model for the case under study is shown, the initial carry-over activities that are taken into account are from 1996. Note that each process converts the input from its own period and the links from the previous period into the output and links at the end of the period. To sum up, there are four stages corresponding to years 1997 to 2007 inclusive, each of them transforming one input into one output, while connecting activities from previous years also influence the process of production of the single output and links at the end of that year. The names of all variables are included in Fig. 2.

The single input considered is operating expenses; these are the costs incurred in developing the telecommunication services, including network operations, facility maintenance and customer operations expenses. The single output corresponds to operating revenues at the end of the period, i.e. the income that companies receive from their normal operating activities, such as local network service revenues and long distance revenues. Both variables are taken from the Income Statement Accounts of the largest ILECs (ARMIS Report 43-02 Table I-1). Our choice of input and output is well supported by previous works related to DEA as applied to the telecommunications sector (e.g. Banker et al., 2010; Yang & Chang, 2009).

With respect to carry-over activities, the four links that are taken from one year to the following are: number of employees (Income Statement Accounts, ARMIS Report 43-02 Table I-1), number of total switched lines terminating at the customer end (Operating Data Report, ARMIS Report 43-08 Table III), total assets, which are basically the Total Telecommunications Plant in Service (TPIS) minus accumulated depreciation (Balance Sheet Accounts, ARMIS Report 43-02 Table B-1.A) and total liabilities at the end of the year (Balance Sheet Accounts, ARMIS Report 43-02 Table B-1.A). Although these links have been considered by other authors as inputs for the models, such as Lien and Peng (2001) and Pentzaropoulos and Giokas (2002), it is clear that the four variables are a result of the operation of previous periods and have influence in the following periods, thus must be set as carry-over activities instead of inputs or outputs. Note that capital expenditure is implicitly included in the difference amongst assets from consecutive years. Ai and

Table 1
List of ILECs.

FCC label	Name of the company or study area
USTR	Qwest Corporation
SWTR	AT&T/Southwestern Bell Telephone
PTCA	Pacific Bell—California
PTNV	Nevada Bell
SNCT	AT&T/Southern New England Telephone
LBIL	Illinois Bell
NBIN	Indiana Bell
MBMI	Michigan Bell
OBOH	Ohio Bell
WTWI	Wisconsin Bell
BSTR	AT&T/BellSouth Corporation
CDDC	Verizon Washington D.C.
CMMD	Verizon-Maryland
CVVA	Verizon Virginia
CWWV	Verizon West Virginia
DSDE	Verizon Delaware LLC
PAPA	Verizon Pennsylvania
NJNJ	Verizon New Jersey
NETC	Verizon New England
NYNY	Verizon New York Telephone
GTGC	Verizon California
GTFL	Verizon Florida LLC
GTMW	Verizon North, Inc.
GTNW	Verizon Northwest, Inc.
GTSO	Verizon South, Inc.
GTSW	GTE of The Southwest, Inc. (dba Verizon Southwest)

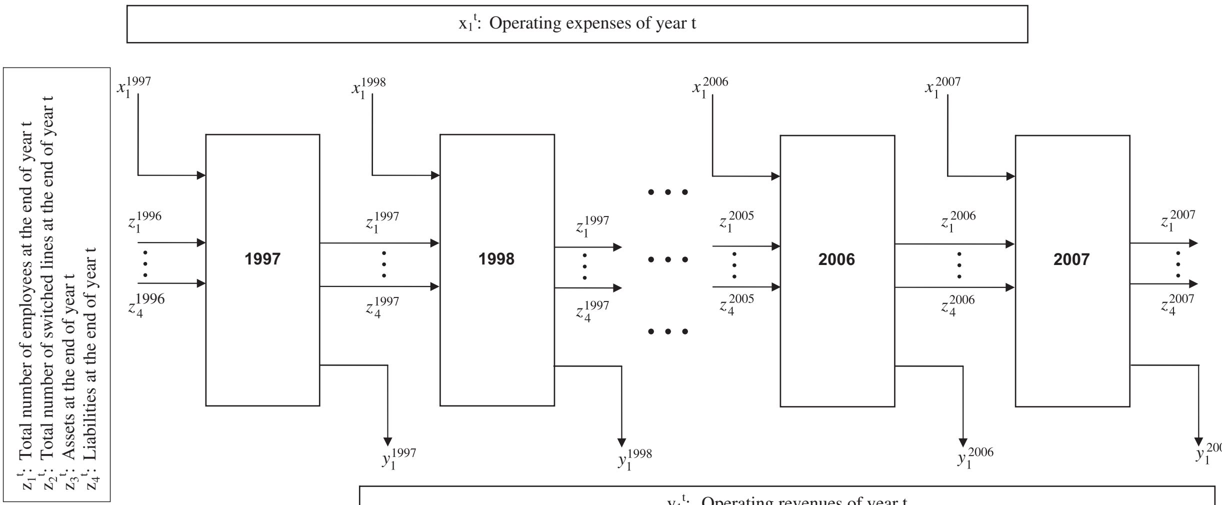

Fig. 2. Dynamic DEA model for telecom efficiency assessment.

Sappington (2002) and Jung et al. (2008) actually established the network infrastructure from previous periods as a variable influencing the outputs of future periods.

All data for operating expenses, revenues, assets and liabilities have been deflated; hence data shown throughout the paper are in real terms, using 2005 as the base year. Basic statistics of all variables for the period under assessment are included in Table 2.

As stated in Section 3, there is a need to determine the characteristics of each link (i.e. whether it is a good, bad or free link) so that the model can be completed, depending on the specific features of the carry-over activities. First of all, the number of employees cannot be easily classified as desirable or undesirable, thus it is established as a free link (constraint (6)), allowing the optimal value for the size of the staff to be greater or smaller than the observed one. In other words, the number of employees is bounded to constraint (6).

On the one hand, a first attempt for the rest of links could set the number of switched lines and total assets as desirable links (subject to constraint (4)), which means that both variables are to be maximized. It is logical to think that the more

Table 2
Descriptive statistics of data set.

	Mean	Stand. dev. (SD)	Max.	Min.
Input				
OPEX[a]	3113.29	3292.62	13,732.07	181.06
Output				
OPREV[a]	4090.68	4366.04	19,872.55	212.85
Intermediate products				
Employees	12,881.29	14,434.05	61,622.00	565.00
Customers[b]	5387.19	5524.69	25,087.03	310.74
Assets[a]	7301.35	8023.09	37,457.38	397.46
Liabilities[a]	5137.84	5588.03	23,094.25	253.69

[a] Dollar amounts in millions.
[b] Total switched access lines in thousands.

Table 3
Description of independent and explanatory variables involved in the regression.

Variable	Description	Cases	Mean (SD)
Dependent variable			
Efficiency	Efficiency scores estimated by proposed Dynamic DEA approach	176	0.872 (0.095)
Explanatory variables			
Mergers	=1 If the ILEC was involved in a merger, otherwise=0	176	0.006 (0.075)
ROR	=1 If the ILEC was operating under Rate of Return regulation, otherwise=0	176	0.023 (0.149)
PCR	=1 If the ILEC was operating under Price Cap regulation, otherwise=0	176	0.864 (0.344)
DER	=1 If the ILEC was operating under Rate Deregulation, otherwise=0	176	0.040 (0.196)
HighSpeed	Number of high speed subscriptions (in millions)	176	3.590 (4.620)
Mobile	Number of mobile telephony subscriptions (in millions)	176	12.769 (11.936)
CLECs	Number of CLEC switched access lines (in millions)	176	1.915 (1.635)
CLECpercent	Percentage of CLEC switched access lines	176	15.712 (4.723)
UNEpercent	Percentage of CLEC access lines provisioned by UNE	176	50.070 (16.826)
Year_i	=1 If the sample corresponds to the year *i* (where *i* varies from 1998 to 2007), otherwise=0	176	0.148 (0.356)

switched access lines there are connecting customers with their end-office, the better. In the same way, due to TPIS grouping of central office equipment, switching systems, cable and wire facilities and other assets, it can also considered to be beneficial and its maximization sought. In this sense, total assets can be seen as indicative of the long-term investment in equipment (e.g. Jung et al., 2008). However, since liabilities oblige a company to give up economic benefits from past transactions, this link could be set as an undesirable link to be minimized (subject to constraint (5)).

On the other hand, while ILECs investments have slowed down in recent years, some studies have questioned the benefits of investment in TPIS by local exchange carriers. Jung et al. (2008) concluded that it was not at all clear that competition from CLECs introduced by the unbundling leasing policy in the sector has stimulated ILECs to invest in new infrastructure. According to Quast (2008), the price reductions in the cost of leasing the whole infrastructure necessary to provide phone service to customers have discouraged CLECs from making their own network investments, even in switching equipment. In addition, Cambini and Jiang (2009) argued that the incentive regulation has had an impact on investment related to the price cap, whereas mandatory unbundling regulation can discourage firms from investing. The same conclusion about reducing network investment because of mandatory unbundling was reached by Ware and Dippon (2010).

Thus, since previous studies had not agreed on regarding investment as a positive decision in order to improve efficiency, all carry-over activities are going to be considered free in this study (subject to constraint (6)). Please notice that no link would be bounded to constraints (4) or (5) because all links can take any value. In theory, this configuration enhances the previous configuration by providing greater flexibility for finding the sources of inefficiency because the target values of intermediate products will be computed according to the needs for optimizing the objective function, regardless of whether this implies a reduction or an increase in the number of switched lines or in assets, or in the liabilities, for that matter. In addition, van Kranenburg and Hagedoorn (2008) pointed out that investments of European incumbents were mainly restrained by their huge debt. By setting free the liabilities incurred by ILECs, it is possible to determine if more financial flexibility would allow ILECs to expand their business.

In a second step, once the efficiency scores of the ILEC under study have been computed, a multiple regression analysis will be conducted to establish the impact of the regulatory policies and both local and intermodal competition on ILECs' efficiency. The explanatory variables, their description and basic descriptive statistics are shown in Table 3. As for the number of cases, cases with missing data points have been excluded listwise. The regression model selects the efficiency score computed by the Dynamic DEA approach for a certain ILEC in a time period as the dependent variable, while the regulatory scheme in which that ILEC had to operate and the competition the ILEC had to face during that year are chosen to be the explanatory variables. Note that the time variation has been coded by using a group of dummy variables, so each case will only have one of these dummy variables equal to one, depending on which year the corresponding observation was taken.

The information related to the explanatory variables corresponding to the incentive regulation, namely ROR, PCR and DER, come from the studies of Abel and Clements (1998) and Perez-Chavolla (2007), while the information about the number and percentage of CLEC access lines, the percentage of CLEC switched access lines obtained from ILEC local loops at cost-based UNE rates and the mobile telephony subscribers was obtained from the *Local Telephone Competition* reports provided by the FCC. In a similar way, data regarding the high speed subscriptions were obtained from the *High-Speed Services* reports provided by the FCC and the information about mergers from the Corporate History of the ARMIS database.

5. Results and discussion

The efficiency scores obtained for all ILECs from applying the approach of Dynamic DEA explained in Section 4 for the period from 1997 to 2007 (see Fig. 2) are shown in Table 4. Moreover, the specific performance scores calculated for every single year are also shown (see Eq. (16)). However, note that the score for a single year is less significant than the global score since the former is computed with the slacks obtained in the optimization of the latter. It is important, however, to note that a globally efficient ILEC must be efficient in every period.

An interesting observation to emerge from the analysis of Table 4 is that there is a great differentiation amongst ILECs scores, with Indiana Bell remaining the only efficient ILECs. The explanation behind the discrimination power of the proposed approach is the consideration of carry-over activities as free links so ILECs can compute more ambitious, efficient targets and find more sources of inefficiency. Fig. 3 shows, in decreasing order, the efficiency of the different ILEC computed by the proposed Dynamic DEA approach.

Fig. 4 plots Dynamic DEA efficiency versus the average number of switched lines during the assessment period, which can be considered to be representative of a firm's size. Note that the largest firms do not follow the same pattern: three of five largest ILECs are close to the average efficiency score, whereas AT&T/BellSouth Corporation can be considered to be quite efficient and Verizon New York Telephone is inefficient. Medium and smaller sized firms have average efficiency scores except for Indiana Bell and Wisconsin Bell which are efficient and close to efficiency, respectively, and Nevada Bell and Verizon New England which are rather inefficient. Anyway, there is no clear relation between size and efficiency, which is consistent with the results of Pentzaropoulos and Giokas (2002), which claimed that operational efficiency could equally be achieved by firms with large revenues and by others with smaller revenues.

Table 4
Efficiencies estimated by the proposed Dynamic DEA approach.

	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	**Global**	
USTR	0.770	0.818	0.787	0.766	0.797	0.891	0.934	0.886	0.893	1	1	0.861	USTR
SWTR	0.810	0.811	0.751	0.810	0.807	0.885	0.968	0.912	0.948	0.904	0.832	0.854	SWTR
PTCA	0.689	0.777	0.727	0.780	0.817	0.863	0.941	0.868	0.926	0.881	0.850	0.820	PTCA
PTNV	0.779	0.731	0.635	0.650	0.659	0.649	0.783	0.799	0.860	0.797	0.769	0.730	PTNV
SNCT	0.787	0.706	0.692	0.740	0.776	0.847	1	0.902	0.974	0.860	0.773	0.812	SNCT
LBIL	0.872	0.915	0.944	0.959	0.920	0.871	0.961	0.860	0.922	0.872	0.836	0.903	LBIL
NBIN	1	1	1	1	1	1	1	1	1	1	1	1	NBIN
MBMI	0.924	0.926	0.980	0.961	0.973	0.909	0.925	0.871	0.889	0.825	0.790	0.906	MBMI
OBOH	0.858	0.894	0.891	0.881	0.943	0.929	0.983	0.902	0.950	0.872	0.837	0.903	OBOH
WTWI	0.965	0.959	1	1	1	1	1	1	1	1	1	0.993	WTWI
BSTR	0.860	0.954	0.927	0.969	0.918	0.907	1	0.916	0.907	0.862	0.911	0.920	BSTR
CDDC	0.802	0.895	0.822	0.861	0.825	0.812	0.898	0.883	0.893	0.818	0.849	0.850	CDDC
CMMD	0.886	0.868	0.873	0.865	0.856	0.886	0.927	0.888	0.926	0.843	0.818	0.876	CMMD
CVVA	0.856	0.879	0.879	0.898	0.881	0.871	0.916	0.844	0.871	0.792	0.798	0.862	CVVA
CWWV	0.910	0.862	0.900	0.944	0.961	0.983	0.962	0.906	0.902	0.849	0.837	0.910	CWWV
DSDE	0.855	0.888	0.851	0.816	0.834	0.978	1	0.909	0.927	0.812	0.753	0.873	DSDE
PAPA	0.836	0.826	0.810	0.797	0.764	0.884	0.834	0.848	0.861	0.797	0.753	0.819	PAPA
NJNJ	0.883	0.853	0.853	0.849	0.807	0.895	0.883	0.845	0.887	0.804	0.735	0.845	NJNJ
NETC	0.785	0.812	0.780	0.782	0.720	0.775	0.760	0.709	0.742	0.668	0.645	0.744	NETC
NYNY	0.743	0.764	0.701	0.718	0.639	0.712	0.703	0.665	0.653	0.567	0.524	0.667	NYNY
GTGC	0.868	0.926	0.939	0.909	0.939	1	1	1	1	0.965	0.979	0.955	GTGC
GTFL	0.958	0.847	0.865	0.781	0.811	0.816	0.858	0.850	0.903	0.826	0.715	0.839	GTFL
GTMW	1	1	1	1	1	1	1	1	1	0.944	0.921	0.988	GTMW
GTNW	0.945	1	1	1	0.965	0.936	0.907	0.911	0.952	0.904	0.817	0.939	GTNW
GTSO	0.887	0.897	0.869	0.849	0.863	0.951	0.826	0.865	0.919	0.869	0.831	0.875	GTSO
GTSW	0.836	0.807	0.816	0.808	0.746	0.835	0.826	0.801	0.835	0.767	0.730	0.800	GTSW

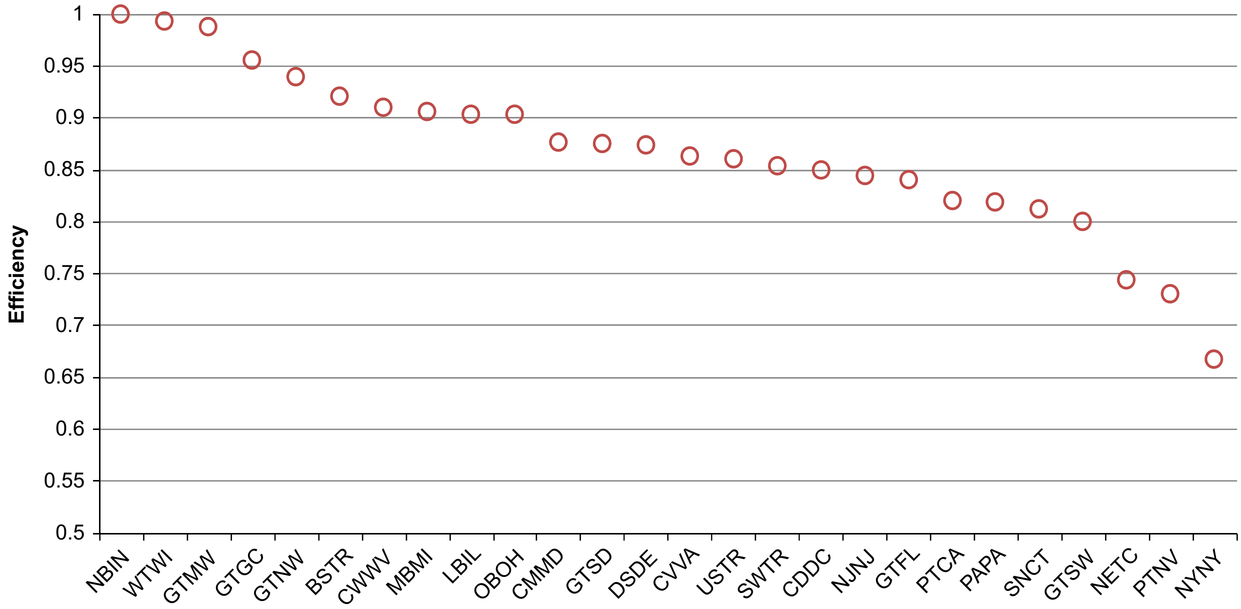

Fig. 3. Efficiency estimated by the proposed Dynamic DEA approach.

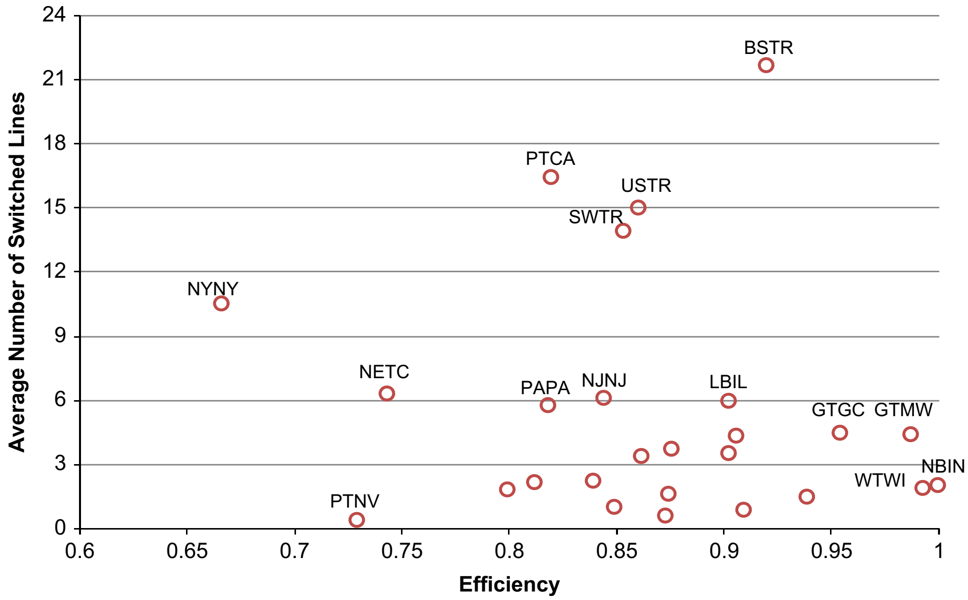

Fig. 4. Dynamic DEA efficiency VS average number of switched lines (in millions).

The methodology Dynamic DEA provides further insight into the sources of inefficiency via the slacks computed for all variables. Since the restrictions for carry-over activities have been lifted, it becomes more demanding for a DMU to be efficient, so this approach must detect a large number of inefficiencies, i.e. a large number of non-zero slacks, both for each ILEC and in total.

Regarding the slacks in OPEX and OPREV provided by Dynamic DEA, that is, the changes which ILECs should implement in the input and output in order to be efficient, Dynamic DEA reveals the need for increasing OPREV mainly in the period from 1997 to 2001, while OPEX should have been decreased after 2001. This trend is particularized in Fig. 5 for Nevada Bell, where the *FEI* of the input and output variables (see Eq. (17)) are shown. Note the shift from an OPREV shortfall in first years (*FEI* negative) to an OPEX excess (*FEI* positive) in the following years. The same trend is followed by most ILECs. It is confirmed again that Dynamic DEA provides great discriminatory power in assessing the efficiency and further insight into OPEX and OPREV slacks.

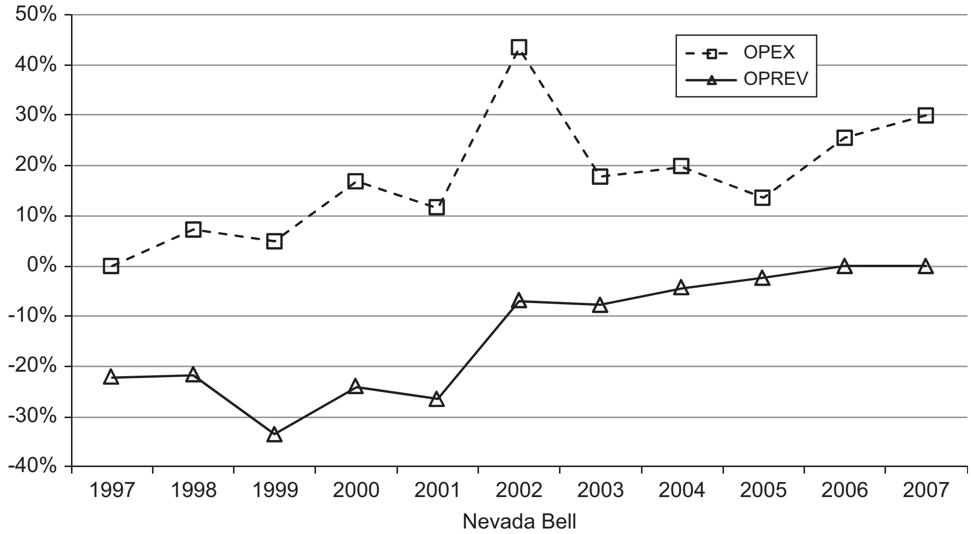

Fig. 5. Evolution of input and output *FEI*s according to Dynamic DEA for Nevada Bell.

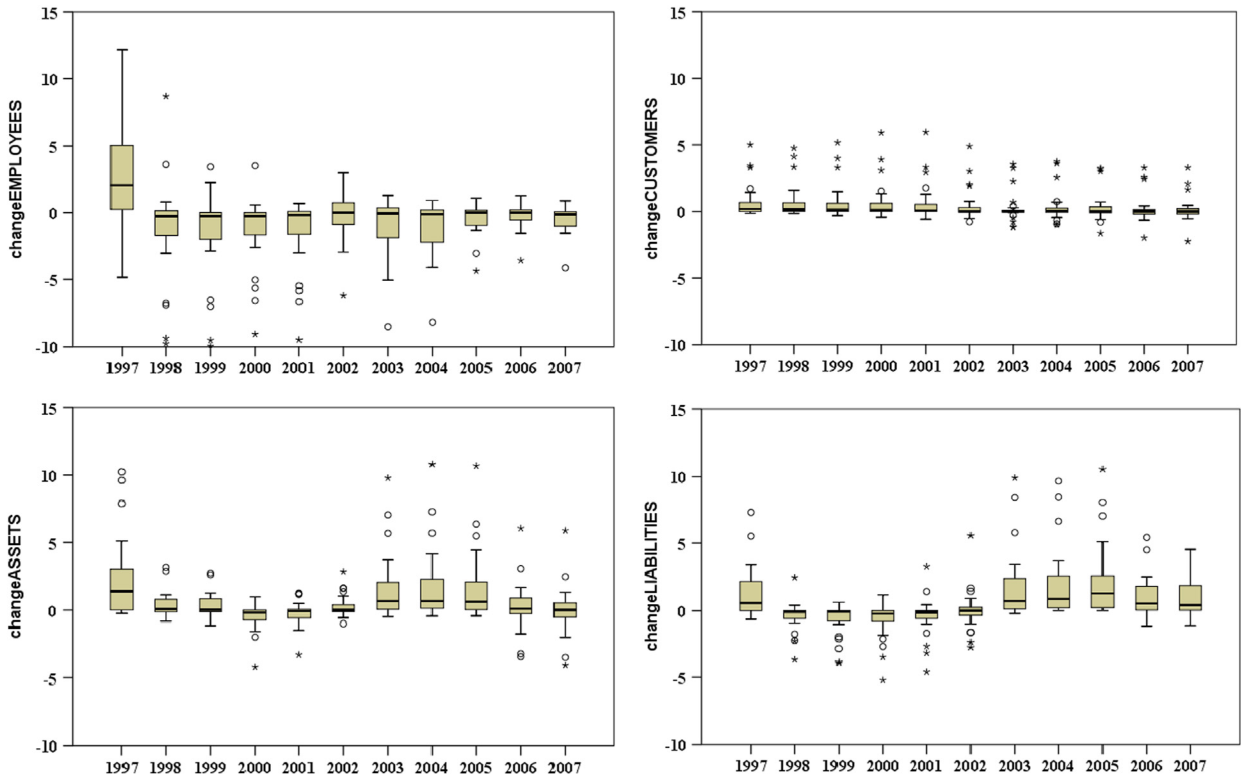

Fig. 6. Boxplots of changes in carry-over activities estimated by Dynamic DEA. (Dollar amounts in billions. Total switched access lines in millions. Employees in thousands.)

Finally, changes in carry-over activities are shown in Fig. 6 by boxplot graphics. Since all links, namely employees, customers, assets and liabilities, have been defined as free links in this application, there can be positive and negative changes in the carry-over activities.

Note also that ILECs should reduce their staff numbers, except for the first year after the Telecommunication Act was passed. That is, most ILECs employ many more workers than necessary. In contrast, although the customer base could remain the same in most cases, it should be expanded for some ILECs. These changes in both employees and access lines

Table 5
Summary of multiple regression model with correction of heteroscedasticity.

Explanatory variable	Standardized coefficient
CLECpercent	−0.474** (0.172)
UNEpercent	0.281** (0.035)
Mobile	0.250** (0.044)
y2005	0.263** (1.159)
y2003	0.213** (1.872)
y2004	0.148* (1.759)
R-squared	0.368
Adjusted *R*-squared	0.345
F-ratio	16.376**
No. observations	176

White-corrected standard errors for the unstandardized coefficients are reported in parentheses.
* Indicates statistical significance at the 95% level.
** Indicates statistical significance at the 99% level.

are mainly due to the decreasing trend observed in the decline of incumbent line based, having a negative effect on business size.

Regarding the assets and liabilities' boxplot, two trends can be identified: one from 1998 to 2002 and the other from 2003 to 2007. During the former period, the observed values for the network infrastructure remained very close to the optimal ones while only some ILECs should have cut their debt. However, during the latter period, the computed targets for assets suggest a significant increase for most ILEC, which means that, in general, much more investment should have been done in the telecommunications network and equipment for the last years under assessment. Logically, if the investment in assets increases, the liabilities necessary to fund the investment would have to increase as well. Apart from these two periods, the year 1997 stands alone, in that there was also a need for more investment in assets.

To sum up, it can be concluded that there is a continuous shortage in investment during last years, mainly because UNEs rules could have discouraged ILECs to upgrade their network infrastructure, and that ILECs should start to reverse this decreasing trend in capital expenditure in order to become more efficient. These results are useful to answer the questions raised by many authors (Cambini & Jiang, 2009; Jung et al., 2008; Ware & Dippon, 2010) about the need for investment in ILECs.

Results from the stepwise multiple regression analysis commented in previous section are included in Table 5. The second column of Table 5 shows the standardized regression coefficients and the White-corrected robust standard errors for the unstandardized coefficients. The overall *F* test suggests that the regression is meaningful in the sense that the efficiency of the ILEC does actually depend on the set of explanatory variables. Regression diagnostic indicated that residuals are normally distributed (Shapiro–Wilk statistic: 0.991; *p*-value: 0.341) with no serial autocorrelation (Durbin–Watson statistic: 2.083)

Note that local competition, expressed through the percentage of CLEC switched access lines out of total switched access lines, has a negative effect on ILEC efficiency, which is consistent with results from previous studies. That is, the increasing CLEC market share implies a worsening in the incumbent carrier efficiency.

At the same time, the provision of services by CLECs by means of leasing of the Unbundled Network Elements has a positive effect on ILEC efficiency, meaning that, given a certain percentage of switched lines provided by CLECs, if CLECs had made more investment in their own network and equipment instead of equipping ILEC UNE loops as CLEC switched access lines, ILECs would have decreased their efficiency. From other point of view, if there was an increase in the percentage of switched lines held by CLECs and those lines were provisioned by leasing UNE loops, there would be a net negative effect on ILEC efficiency. However, if the same increase in switched lines held by CLECs was due to CLEC-owned local loops, ILEC efficiency would undergo a more significant drop. These conclusions can be seen as incentives for CLECs to build their own infrastructure.

The rest of the regression results point out that intermodal competition with mobile telephony does not have a negative influence, suggesting that mobile and wireline telephony are becoming complementary markets. Also, it can be seen that the period from 2003 to 2005 was especially beneficial to efficiency. Surprisingly, none of the explanatory variables related to incentive regulation was found to be able to predict any variance of the efficiency scores, which can be connected with the results of Uri (2001), whose study did not find an improvement of carrier efficiency when PCR was adopted. Mergers did not have any influence on technical efficiency either.

6. Summary and conclusions

The telecommunications sector is one of the most dynamic and competitive industries. That is why aiming at efficiency is a must for the involved companies. In this paper, a Dynamic DEA approach has been used to assess the efficiency of the 23 largest ILECs during the period 1997 to 2007. The main feature of this approach is that carry-over activities from one period to the next are taken into account so that a global assessment of the performance along the whole horizon is carried out. The results show that the Dynamic DEA application has quite discriminatory power, assessing just one company, namely Indiana Bell, as being efficient in the whole period under study. The proposed approach also computes challenging input and output targets and uncovers existing sources of inefficiencies. In particular, in general, the first years (from 1997 to 2001) revenues (OPREV) should have been larger than they were, while in the remaining years (from 2001) it was the operating expenses (OPEX) which should have been reduced.

Another advantage of the implemented approach is that its increased modeling flexibility allows for computing target values for the carry-over activities without being constrained by the observed values. That allows determining the optimal values of these carry-over activities, thus confirming that network investments have a positive influence in the performance of ILECs and are constrained by the liabilities incurred. In general, it seems that there was an excess of employees and a significant lack of investment in TPIS during the last period. This is consistent with what has been reported in the literature.

In addition, a regression analysis has been conducted to determine the impact of the regulatory policies and both local and intermodal competition on the ILECs' efficiency. On the one hand, the regression analysis has pointed out the adverse impact of local competition from CLECs in the ILECs' efficiency, which could have been worse if CLECs had made more investments in their own infrastructure instead of leasing UNE loops. On the other hand, the broadband deployment and incentive regulation policies do not seem to have had a clear influence on the dynamic performance of wireline companies.

To sum up, had the ILECs reduced their workforce and made additional investments during the last few years, they may have maintained their number of customers, which is being threatened by the increasing local competition from CLECs. Those changes would have put the ILECs on a strong position to be able to face the financial crisis that came afterwards.

Concerning further research, future work could explore the dynamic performance of CLECs by applying a similar methodology to that presented here, so the significance of investments in networks by CLECs and the relevance of the latest regulation to their behavior might be revealed. Furthermore, due to the fact that Dynamic DEA, based on SBM, allows incorporating weights in the objective function, another issue could be to take into account the different relative significance of carry-over activities when assessing efficiency. Another continuation of this research can be to extend it to the telecommunications sector of other countries.

Finally, the analysis could include an adjustment in the costs faced by the ILECs depending on the different territorial conditions. That is, the population density of the areas served by every ILEC is not homogenous and there are some ILECs whose serving territory is mainly rural or mountainous, implying higher costs on local loops, among other costs. The territorial conditions could also be considered as environmental variables, which cannot be changed but may have an important effect on efficiency.

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Estimation of productivity change of NBA teams from 2006/07 to 2012/13 seasons

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Abstract

The aim of this work is to evaluate the productivity change of the NBA teams during the last seven seasons (from 2006/2007 to 2012/13). Within that period of time, a new collective bargaining agreement (CBA) of the National Basketball Association (NBA) was ratified before season 2011/12, ending a 161-day lockout. The Malmquist Productivity Index (MPI) has been used to measure the total factor productivity, while an input-oriented Network DEA approach is used to compute the distance of each observation to the corresponding frontier. The results reveal that there has been technological progress for the last few seasons, excluding that of the 2011 lockout, and an increasing efficiency change. This means that best practices are improving and that most teams have been reducing their payrolls to catch up with these practices, thus backing up the owners' proposal to reduce players' income. Also regression results show that changes in the number of wins are more dependent upon scale efficiency change than upon budget or efficiency changes.

Keywords: NBA; productivity change; Malmquist Productivity Index; Network DEA

Estimation of productivity change of NBA teams from 2006/07 to 2012/13 season

1. Introduction

The National Basketball Association (NBA) is one of the most important sport franchises in the world and every NBA team is able to generate substantial revenues through merchandising, tickets sales, TV rights, etc. NBA actually handles billions of dollars every year. But, at the same time, costs are increasing, mainly because players' contracts are becoming more and more expensive every year.

During the first five seasons covered in this study (2006/2007 to 2010/2011) the NBA lived under the 2005 Collective Bargaining Agreement (CBA). The CBA is the contract between the NBA (the commissioner and the 30 team owners) and the NBA Players' Association that states the business rules about players' contracts, trades, revenue distribution, salary caps, etc. The 2005 CBA expired on 30th June 2011, leading the NBA to a 'lockout', where the owners proposed to reduce players' income. Later that year, on 8th December 2011, the NBA Board of Governors ratified a new 10-year CBA after the played had accepted less money (Berri, 2012).

Some researchers have been concerned about the economic losses due to sports' lockouts (e.g. Coates and Humphreys, 2001). On the one hand, even before the lockout and according to the NBA commissioner, the owners claimed that NBA teams had lost more than $1 billion dollars during the validity of the 2005 CBA, mainly due to increasing players' salaries and guaranteed contracts, which are the most relevant points having been discussed in the negotiations to set up the 2011 CBA.

On the other hand, it has been questioned the need and incentives of the new agreement and its impact on competitive balance (Berri, 2012). Therefore, the negative criticism against the 2011 CBA justifies the need for an assessment of the teams' efficiency in economic resources management, since an analysis of the productivity change before and after the 2011 CBA will prove if owners had reasonable grounds for requesting a salary cut.

With that objective in mind, this paper estimates productivity change evolution of NBA teams during the five seasons prior to and the two seasons after the 2011 CBA. It makes sense to evaluate the changes in performance along a number of seasons, since players' contracts last for several years, and managers make the financial planning and coaches build the roster with a view to future seasons.

The productivity change between two periods can be estimated through the Malmquist Productivity Index (MPI), which is decomposed into two components: efficiency change and technology change (Färe et al., 1992). Färe et al. (1994) further include a third component (related to scale change) in what is known as FGNZ decomposition.

To project the observations onto the corresponding efficient frontier, Data Envelopment Analysis (DEA) has been used (e.g. Cooper et al., 2007). DEA has been applied to many different sectors, sports among them, e.g. Spanish soccer teams' efficiency assessment (González-Gómez et al., 2010; Barros and Garcia-del-Barrio, 2011), estimation of efficiency scores for Germany's premier league football players depending on their playing positions (Tiedemann et al., 2011), ranking of professional tennis players by deriving a common set of weights (Ramón et al., 2012), efficiency assessment of local entities in the provision of public sports facilities (Benito et al., 2012) or performance evaluation of each country in the 2008 Beijing Summer Olympic Games (Wu et al., 2010).

Moreover, DEA has been recently used to reveal that most Portuguese football clubs are spending more money in players' wages than they need to (Ribeiro and Lima, 2012). In relation to basketball, Aizemberg et al. (2011) use DEA to analyze the efficiency of NBA teams. Also the effectiveness of basketball players (Cooper et al., 2009) and their ranking (Cooper et al., 2011) have been studied using DEA.

In order to gain a better understanding of the sources of inefficiency, Network DEA (e.g. Färe and Grosskopf, 2000) has been applied so that the internal processes can be identified and the internal links (a.k.a. intermediate products) included in the model.

Network DEA has been applied to the study of efficiency in sports. Thus, Sexton and Lewis (2003), Lewis and Sexton (2004a, 2004b) and Lewis et al. (2009) and study the performance of baseball teams. Moreno and Lozano (in press) study the performance of NBA teams in the regular season 2009/2010 distinguishing between first and bench teams. The Network DEA approach proposed in Moreno and Lozano (in press) is the starting point for this research work, where the model itself has been revised and refined to consider several NBA seasons so that productivity changes (computed through MPI) could be estimated. NBA teams usually elaborate plans for several years, mainly due to the length of players´ contracts, thus analyzing the productivity changes in the different periods becomes relevant.

The structure of the paper is the following. The Malmquist Productivity Index, its decomposition and main Network DEA concepts are reviewed in section 2. Section 3 presents the processes and variables considered in the proposed approach together with the specific input-oriented Network DEA model used. Section 4 presents the results obtained using data from regular seasons 2006/2007 to 2012/13. Finally, section 5 summarizes and concludes.

2. Methodology

This section first reviews the Malmquist Productivity Index (MPI) and how it can be decomposed into the usual two components, namely technical and efficiency change, plus a scale change component. This decomposition can be used when there exist Variable Returns to Scale (VRS). Also, the main concepts of Network DEA are introduced.

2.1. MPI and FGNZ decomposition

MPI has been used to measure the variation of productive efficiency between two periods of time (Färe et al., 1992). The input-oriented MPI of a certain Decision Making Unit (DMU) labeled 0 is defined as the geometric mean

$$M^{I}_{t,t+1}(x^{t}_{0}, y^{t}_{0}; x^{t+1}_{0}, y^{t+1}_{0}) = \left[\frac{DF^{I}_{t}(x^{t+1}_{0}, y^{t+1}_{0})}{DF^{I}_{t}(x^{t}_{0}, y^{t}_{0})} \cdot \frac{DF^{I}_{t+1}(x^{t+1}_{0}, y^{t+1}_{0})}{DF^{I}_{t+1}(x^{t}_{0}, y^{t}_{0})}\right]^{\frac{1}{2}} \quad (1)$$

where x^{t1}_{0} and y^{t1}_{0} represent respectively the inputs and outputs of DMU 0 observed in period t1, while $DF^{I}_{t1}\left(x^{t2}_{0}, y^{t2}_{0}\right)$ stands for the proportional reduction of the inputs of DMU 0 observed in period t2, assuming that the production technology is constructed from the observations (of the different DMUs) in period t1. Note that t1 can correspond to period t or to period t+1 and the same applies for t2. Normally $DF^{I}_{t1}\left(x^{t2}_{0}, y^{t2}_{0}\right)$ is computed using a radial, input-oriented DEA model (Charnes et al., 1978). However, in this paper, instead of a conventional, single-process DEA, a Network DEA model, as formulated below, will be used.

MPI is commonly decomposed into efficiency change (EFFCH) and technical change (TECCH) as

$$M^{I}_{t,t+1}(x^{t}_{0}, y^{t}_{0}; x^{t+1}_{0}, y^{t+1}_{0}) = EFFCH^{I}_{t,t+1} \cdot TECCH^{I}_{t,t+1} \quad (2)$$

where

$$EFFCH^{I}_{t,t+1} = \frac{DF^{I}_{t+1}(x^{t+1}_{0}, y^{t+1}_{0})}{DF^{I}_{t}(x^{t}_{0}, y^{t}_{0})} \quad (3)$$

$$TECCH^{I}_{t,t+1} = \left[\frac{DF^{I}_{t}(x^{t}_{0}, y^{t}_{0})}{DF^{I}_{t+1}(x^{t}_{0}, y^{t}_{0})} \cdot \frac{DF^{I}_{t}(x^{t+1}_{0}, y^{t+1}_{0})}{DF^{I}_{t+1}(x^{t+1}_{0}, y^{t+1}_{0})}\right]^{\frac{1}{2}} \quad (4)$$

The first term, efficiency change (3), measures the magnitude of the change in technical efficiency between periods t and t+1. An improvement in $EFFCH^{I}_{t,t+1}$ can be interpreted as evidence of catching-up with the frontier for that DMU. In other cases, production is moving farther from the frontier. Concerning the second term, technical change (4) measures the shift in the frontier over time. In that way, an improvement in $\mathrm{TECCH}^{I}_{t,t+1}$ implies progress in the technology under study and a worsening in $\mathrm{TECCH}^{I}_{t,t+1}$ implies technological regress.

An improvement in productivity corresponds to a Malmquist index greater than unity. In case MPI is less than unity, productivity has declined over time. Analogously, improvements and worsening in its two components are also associated with values greater and less than unity, respectively.

When the problem under study exhibits VRS, the following FGNZ decomposition (Färe et al., 1994) can be used

$$M^{I}_{t,t+1}(x_0^t, y_0^t; x_0^{t+1}, y_0^{t+1}) = \mathrm{EFFCH}^{I,VRS}_{t,t+1} \cdot \mathrm{TECCH}^{I}_{t,t+1} \cdot \mathrm{PURESCACH}^{I}_{t,t+1} \quad (5)$$

where

$$\mathrm{EFFCH}^{I,VRS}_{t,t+1} = \frac{DF^{I,VRS}_{t+1}(x_0^{t+1}, y_0^{t+1})}{DF^{I,VRS}_{t}(x_0^{t}, y_0^{t})} \quad (6)$$

$$\mathrm{TECCH}^{I}_{t,t+1} = \left[\frac{DF^{I}_{t}(x_0^t, y_0^t)}{DF^{I}_{t+1}(x_0^t, y_0^t)} \cdot \frac{DF^{I}_{t}(x_0^{t+1}, y_0^{t+1})}{DF^{I}_{t+1}(x_0^{t+1}, y_0^{t+1})}\right]^{\frac{1}{2}} \quad (7)$$

$$\mathrm{PURESCACH}^{I}_{t,t+1} = \frac{\dfrac{DF^{I}_{t+1}(x_0^{t+1}, y_0^{t+1})}{DF^{I,VRS}_{t+1}(x_0^{t+1}, y_0^{t+1})}}{\dfrac{DF^{I}_{t}(x_0^{t}, y_0^{t})}{DF^{I,VRS}_{t}(x_0^{t}, y_0^{t})}} \quad (8)$$

In the above expressions, $DF^{I,VRS}_{t1}\left(x_0^{t2}, y_0^{t2}\right)$ corresponds to the radial efficiency of DMU 0 in period t2 evaluated by using the VRS production technology of the period t1. Note that the basic difference between (1) and (5) is that the efficiency change is divided into a VRS efficiency change term and a pure scale efficiency change component (PURESCACH). The former measures the change in technical efficiency assuming VRS technology, while the latter detects differences over time in the distance between the efficient frontiers of the VRS and CRS technologies.

2.2. Network DEA

In this section, an input-oriented Network DEA model to compute the radial efficiency scores $DF_{t1}^{I}\left(x_0^{t2},y_0^{t2}\right)$ and $DF_{t1}^{I,VRS}\left(x_0^{t2},y_0^{t2}\right)$ is presented. This formulation is an extension of the relational Network DEA model proposed by Kao (2009) to general networks of processes. The notation used is the one proposed in Lozano (2011).

The main difference between Network DEA and conventional DEA is that while the latter considers a single process that consumes all the inputs and produces all the outputs, the former considers the existence of several processes each of which consumes its own set of inputs and produces its own set of outputs, in addition to consuming and producing intermediate products that are internal to the system under study.

For each process p of DMU j, denote x_{ij}^{p} as the observed amount of input i consumed and let y_{kj}^{p} be the observed amount of output j produced. Let $z_{rj}^{in,p}$ be the observed amount of intermediate product r consumed by process p of DMU j and $z_{rj}^{out,p}$ denote the observed amount of intermediate product r generated by process p of DMU j.

Let $P_I\left(i\right)$ be the set of processes that consume the input i and $P_O\left(k\right)$ the set of processes that generate the output o. In order to model the composition of intermediate flows inside the network, let $P^{out}\left(r\right)$ be the set of stages that produce the intermediate product r and $P^{in}\left(r\right)$ the set of processes that consume the intermediate product r.

In addition, λ_j^p stands for the set of multipliers that define the production possibility set of the process p, while θ symbolizes the proportional reduction of inputs of the DMU under assessment. Hence the input-oriented, Network DEA model to compute the maximal feasible radial reduction of inputs can be formulated (see Lozano, 2011) as

$$DF_0^I = \text{Min } \theta \qquad (9)$$

s.t.

$$\sum_{p \in P_I(i)} \sum_j \lambda_j^p x_{ij}^p \leq \theta \cdot x_{i0} \quad \forall i \qquad (10)$$

$$\sum_{p \in P_O(k)} \sum_j \lambda_j^p y_{kj}^p \geq y_{k0} \quad \forall k \qquad (11)$$

$$\sum_{p \in P^{out}(r)} \sum_j \lambda_j^p z_{rj}^{out,p} - \sum_{p \in P^{in}(r)} \sum_j \lambda_j^p z_{rj}^{in,p} \geq 0 \quad \forall r \qquad (12)$$

$$\lambda_j^p \geq 0 \quad \forall j \; \forall p \quad \theta \text{ free} \qquad (13)$$

The above model corresponds to assuming Constant Returns to Scale (CRS). In the VRS case the following constraints should be added.

$$\sum_j \lambda_j^p = 1 \quad \forall p \qquad (14)$$

Note that a characteristic feature of Network DEA models is that each process has its set of variables λ_j^p and the reason is that each process has its own technology. This leads to a larger overall production possibility set which increases the discriminatory power of the DEA model, so much so that it is very common in Network DEA to find that none of the DMUs is found to be efficient. That is so because in order for a DMU to be efficient, all its processes must be efficient – something which does not occur easily.

Another feature of Network DEA models is the intermediate products balance constraints (12). They guarantee that the amounts of intermediate products internally generated by the system are enough to satisfy the consumption of these intermediate products by those processes that require them.

3. Network DEA model for NBA teams

As stated previously, Network DEA has been developed to deal with the existence of multiple, linked processes inside a DMU. The network of processes used in this work is shown in Figure 1 and consists of four processes or stages. Process PERF (team-work performance) can be interpreted as an acquisition process, where the teams use the budget spent to sign up players. In an intuitive way, the more salary a player is paid, the better he should perform during matches. Therefore, the input of this first stage will be the total payroll of the team, while the number of attacking and defensive (against the opposing team) moves of the team are the corresponding outputs. The outputs of process PERF are actually intermediate products that are inputs for the two following stages, representing the offensive (OFF) and defensive (DEF) subsystems. Each of these processes generates one additional intermediate product which represents the number of points scored by the team and the inverse of the number of points scored by the opposing team, respectively.

The final stage (Wins Generation, WG) transforms the points scored by the team and by the opponent team into victories, which is the final output of the DMU. The choice of team payroll and points in the league as an input and output, respectively, can be regarded as a constant feature in works related to sports efficiency (e.g. Barros et al., 2010, in their estimation of efficiency scores for Brazilian soccer teams). Table 1 shows the definition and label assigned to each of the variables. These labels are used in Figure 1 and in the mathematical model below.

========================== Figure 1 ===========================

=========================== Table 1 ==============================

There are several points to be clarified. First of all, the number of moves is measured in absolute figures, i.e. the sum of the moves by all players in all matches of the regular season. Furthermore, the turnovers made by a team are an intermediate product that involves worse performance when it takes higher numerical values. Although traditionally these kinds of variables have coped with dummy variables or been treated as reverse products (Lewis and Sexton, 2004a), the easiest way to handle them is to work with the inverse of the quantity, in the same way as other authors have done previously (e.g. Cooper et al., 2009).

The offensive subsystem (OFF) evaluates the efficiency of the team in transforming the available offensive resources on the field into points while the defensive subsystem (DEF) evaluates how the team manages its defensive resources to minimize the points received. Robst et al. (2011) found no evidence that sports teams benefit from focusing on offense or defense, so both subsystems have being considered to be equally important in this paper.

Offensive and defensive subsystems are associated with the decisions of the coach, who has to plan proper strategies and tactics in order to maximize the number of points scored and minimize the number of points received, taking advantage of the skills and production abilities of his own players. The role of head coaches in team performance has been discussed in previous studies (e.g. Berri et al., 2009). As in the case of turnovers, and for the points made by the opponent, the inverse is taken as output, since a greater number of points received means worse performance.

With respect to the win generation stage (process WG), this assesses the competence of the team to administer the differences in points (points scored minus points received), so that the team would win the largest possible number of matches. The number of victories has many additional benefits, like a significant increase in attendance (Morse et al., 2008).

The proposed Network DEA model is the particularization of the model formulated in section 2.2 to the network shown in Figure 1. Note that this formulation corresponds to the CRS case, while in the VRS case the corresponding constraints (14) are considered.

$$DF_{t_1}^{I}\left(x_0^{t_2}, y_0^{t_2}\right) = \text{Min} \quad \theta \tag{15}$$

s.t.

$$\sum_j \lambda_j^{PERF} \cdot Budget_j^{t_1} \leq \theta \cdot Budget_0^{t_2} \tag{16}$$

$$\sum_j \lambda_j^{PERF} \cdot 2PA_j^{t_1} - \sum_j \lambda_j^{OFF} \cdot 2PA_j^{t_1} \geq 0 \tag{17}$$

$$\sum_j \lambda_j^{PERF} \cdot 3PA_j^{t_1} - \sum_j \lambda_j^{OFF} \cdot 3PA_j^{t_1} \geq 0 \tag{18}$$

$$\sum_j \lambda_j^{PERF} \cdot FTA_j^{t_1} - \sum_j \lambda_j^{OFF} \cdot FTA_j^{t_1} \geq 0 \tag{19}$$

$$\sum_j \lambda_j^{PERF} \cdot OffReb_j^{t_1} - \sum_j \lambda_j^{OFF} \cdot OffReb_j^{t_1} \geq 0 \tag{20}$$

$$\sum_j \lambda_j^{PERF} \cdot Assists_j^{t_1} - \sum_j \lambda_j^{OFF} \cdot Assists_j^{t_1} \geq 0 \tag{21}$$

$$\sum_j \lambda_j^{PERF} \cdot InvTO_j^{t_1} - \sum_j \lambda_j^{OFF} \cdot InvTO_j^{t_1} \geq 0 \tag{22}$$

$$\sum_j \lambda_j^{PERF} \cdot DefReb_j^{t_1} - \sum_j \lambda_j^{DEF} \cdot DefReb_j^{t_1} \geq 0 \tag{23}$$

$$\sum_j \lambda_j^{PERF} \cdot Steals_j^{t_1} - \sum_j \lambda_j^{DEF} \cdot Steals_j^{t_1} \geq 0 \tag{24}$$

$$\sum_j \lambda_j^{PERF} \cdot Blocks_j^{t_1} - \sum_j \lambda_j^{DEF} \cdot Blocks_j^{t_1} \geq 0 \tag{25}$$

$$\sum_j \lambda_j^{OFF} \cdot Points_j^{t_1} - \sum_j \lambda_j^{WG} \cdot Points_j^{t_1} \geq 0 \qquad (26)$$

$$\sum_j \lambda_j^{DEF} \cdot InvOppPoints_j^{t_1} - \sum_j \lambda_j^{WG} \cdot InvOppPoints_j^{t_1} \geq 0 \qquad (27)$$

$$\sum_j \lambda_j^{WG} \cdot Wins_j^{t_1} \geq Wins_0^{t_2} \qquad (28)$$

$$\lambda_j^{PERF}, \lambda_j^{OFF}, \lambda_j^{DEF}, \lambda_j^{WG} \geq 0 \qquad (29)$$

4. Results and discussion

The approach described in the previous sections has been applied to all 30 NBA teams using data corresponding to the regular seasons 2006/2007 to 2012/13. Each regular season consists of 82 matches, except for season 2011/12, when only 66 matches were played due to the lockout. Teams are grouped in two conferences, each conference consisting of three divisions. If the team performs well during the regular season, not only can it gain access to playoffs for the title, but it can also achieve a good ranking in the team's conference and thus have home ground advantage and play against less competitive teams in the first rounds of playoffs.

The data about the intermediate products and the output for all teams were taken from official statistics of NBA, available from their official website www.nba.com. Data corresponding to teams' budgets were extracted from http://www.storytellerscontracts.com, which is considered to be the most reliable website about NBA players' contracts. For the seven seasons included in this work, the input (Budget) and output (Wins) data are shown in Table 2. The budget data have been deflated, so budget data shown in Table 2 are in millions of constant 2009 dollars. Moreover, budget data have been normalized by computing the relative measure to the average budget from the corresponding season. Note that each season is identified by the year when the season finished. Thus, for example, the season 2008/2009 is referred to as season 2009 in the tables and figures. In addition, although the team OKC (Oklahoma City Thunder) was previously located in Seattle (and was known as the

Seattle Supersonics) before season 2008/2009 and the team NJ (New Jersey Nets) has been moved to New York (and is now known as the Brooklyn Nets) during season 2012/13, we refer to them as OKC and NJ respectively, during all seven seasons, to keep homogeneity within the tables and figures.

========================== Table 2 ===========================

Regarding the results, first of all, let us take a look at the efficiency scores of the 30 NBA teams in each season, computed using the network DEA approach proposed in section 3. These efficiency values are included in Table 3. When CRS scores differ from VRS ones, there is scale inefficiency and this means that the team is operating away from the Most Productive Scale Size (MPSS) (Banker, 1984). Thus, for instance, Memphis Grizzlies (MEM) is VRS efficient in 2008 but has a CRS efficiency score of just 0.335.

========================== Table 3 ===========================

Figure 2 shows, for each season, the average of the relative target budgets of all 30 teams computed using the proposed Network DEA approach. The relative target budget for every team is the relative budget of the corresponding projected point on the efficient frontier. In other words, it represents the normalized budget that the team would have consumed, had it been efficient. Note that the average normalized observed budget is constant and equal to unity. Moreover, it is larger than the average normalized VRS target budget which is, in its turn, larger than the average normalized CRS target budget. Note also that both relative target values decreased in the season prior to the lockout.

========================== Figure 2 ===========================

Note further that because this application considers just one exogenous input and one final output, the overall efficient frontier is a straight line (passing through the origin) in the case of CRS technology. Figure 3 shows the efficient projections of every team, for both Network DEA and conventional (i.e. single-process) DEA, thus revealing the corresponding efficient frontiers for every season under CRS. Since the frontier is a straight line, it means that the ratio of the target budget to target wins is the same for every team and represents the optimal (i.e. minimum) "effective cost" of a victory in each regular season. That means the optimal effective cost of a victory is defined as the proportional amount of non-normalized target budget consumed for achieving a single victory. Such a ratio corresponds to the inverse of the slope of the frontiers shown in Figure 3 times the average observed budget of that season. These optimal effective costs are shown in Figure 4. Note that the optimal effective costs estimated with Network DEA are lower than by conventional DEA. This results from the fact that the network DEA efficient frontier has a larger slope than the conventional DEA. Analogous to the evolution of the average normalized target budget, the "effective cost" increased in the 2009 and 2010 seasons, but decreased in the 2011 and 2013 seasons, i.e. the slope of the efficient frontiers in Figure 2 decreased for the 2009 and 2010 seasons, whereas it increased in 2011 and 2013.

========================== Figure 3 ===========================

========================== Figure 4 ===========================

Concerning the MPI results, Figure 5 includes the evolution of the (geometric) mean of the MPI of the 30 teams for the different periods computed using Network and single-process DEA. Furthermore, the evolution of the mean MPI components (as per the FGNZ decomposition) is also shown. Note that the mean MPI takes the same value for both Network and single-process DEA approaches. This is no coincidence. Actually, the MPI computed by both Network DEA and conventional DEA coincide for all teams. The reason must be that this application considers a single input and an input-orientation. Looking at the mean MPI, an slightly increasing pattern is evident prior to the lockout, with values less than unity in periods 2007-2008 and 2008-2009, and greater than unity in 2009-2010 and 2010-2011. Right after the lockout there was a dramatic decrease in productivity, due to the fewer number of games played in season 2011-12. Productivity recovered during the last season.

========================== Figure 5 ============================

Although the MPI computed by Network and single-process DEA are the same, the MPI components differ. Thus, for instance, according to Network DEA results, the slight productivity growth (on average) in the period 2009-2010 is explained by a positive scale efficiency change (i.e. PURESCACH>1) and a positive VRS efficiency change, hindered by small a technological regress. The conventional DEA approach, on the other hand, does not indicate any scale efficiency change (on average) in that period and attributes the productivity growth to a technological progress hindered by a worsening VRS efficiency change. This clearly shows that the results from the two approaches are dissimilar. Our claim is that those obtained by the Network DEA approach are more reliable than those of conventional DEA because Network DEA is a more fine-grained analysis that uses more information and therefore its results should be more informative and valid.

Looking again at the mean values of the MPI components (in Figure 5), the two periods in which there have been a large technological progress (positive frontier shift) have been the last one and the one previous to the lockout (explained by a significant drop in target budgets, as commented above). In other words, the best performing teams are increasing their efficiency by achieving better results while spending less money. Hence the fall in the season where the lockout occurred can be explained by the fewer number of matches played.

Not only has the technical change improved, but also there has been a steady increase in the VRS efficiency change during the last few seasons (Figure 5). Hence most teams are trying to catch up with the best practices, i.e. managing their economic resources in a more efficient way.

=========================== Figure 6 ==============================

With respect to the variation for individual teams, Figure 6 shows the evolution of the MPI and its components for the NBA champions for the last six seasons (Boston Celtics in 2008, Los Angeles Lakers in 2009 and 2010, Dallas Mavericks in 2011 and Miami Heats in 2012 and 2013). Note the high MPI value for Boston Celtics (BOS) in 2007-2008 due to the significant positive scale change that took place because the team achieved a greater number of wins (66) in 2008 without increasing its budget proportionally. Regarding the high MPI for Miami Heat (MIA) in 2008-2009, the team went through a significant development from season 2008 to 2009, becoming a top team and shaping one of the best rosters in the NBA without increasing its investment.

=========================== Table 4 ==============================

The specific MPI for each team in each period are shown in Table 4 while the MPI components estimated for each team are shown in Table 5. The (geometric) mean of the different divisions and of the whole league are also shown. Note that the technical change component takes the same value for each team in a given period. This is due to the fact that there is only one input and one output, and the CRS efficient frontiers in both periods (t and t+1) are a straight line.

=========================== Table 5 ==============================

Concerning the period 2010-2011 (right before the 2011 CBA), let us emphasize the fact that most teams underwent a worsening in their technical efficiency (VRS EFFCH less than unity) due in part to a significant technological progress (TECHCH=1.22). First, this finding shows that the production possibility set allowed the teams to reduce their payrolls, thus being able to cope with the economics losses – results that were in line with the negotiation of the 2011 CBA. Second, it can be deducted from the decrease in efficiency change that most teams were far away from the best practice frontier. However, according to the results from the following two periods, i.e. 2011-12 and

2012-13, after the lockout teams' managers must have worked hard to make up a roster with payroll and results able to catch up with the new frontier, since there is an increasing efficiency change pattern. To sum up, the 2011 CBA has set up a proper environment which has lead teams to control their budgets in a much more efficient way.

A least squares linear regression analysis has been performed in order to establish the effects of the different MPI components and of the normalized budget change on the wins change between two seasons. The standardized regression coefficients presented in Table 6 correspond to setting the change in wins as the independent variable while taking the normalized budget change, the VRS efficiency change and the scale change as the dependent variables. Since technology change does not vary across teams, its effect on win change is included in the estimated intercept of the regression.

========================== Table 6 ============================

Note that the influence of all three variables is significant and that the estimated coefficients are rather similar for both Network and single-process DEA. Moreover, it can be concluded from Table 6 that a budget change has little importance in the number of wins, i.e. a team would hardly get more wins by just spending more money on more expensive contracts. In the same way, improving efficiency to catch up with the frontier also has a relative effect on wins. However, the scale change is very relevant to achieve a greater number of wins, i.e., teams should aim to operate in the Most Productive Scale Size (MPSS) by managing their current resources to increase the number of victories in the regular season.

Finally, for comparison, as suggested by one of the reviewers, the network DEA approach proposed in Lewis et al. (2009) has been also applied to this dataset. The main advantage of their approach is that it allows computing explicitly the efficiency of every process (sub-DMU in their terminology). The adaptation of their approach to the present application is described in the Appendix. Table 7 shows the differences between the full efficiency computed by the Lewis-Lock-Sexton approach and the approach proposed in this paper.

========================== Table 7 ===========================

Note that the efficiencies of both approaches differ, with the proposed approach computing stricter efficiency scores. However, the Pearson´s correlation coefficient between the results of both approaches ranges from 0.878 to 0.965, which implies a rather high positive correlation between both Network DEA results. Due to lack of space, the efficiencies of the sub-DMUs computed by the Lewis-Lock-Sexton approach are displayed only for the Pacific Division in Figure 7. The results from other divisions are similar and are available from the authors upon request.

========================== Figure 7 ===========================

OFF and DEF efficiencies are very close to 1 for all teams and seasons, which implies that transforming offensive resources into points and defensive resource in minimizing received points is relatively straightforward, revealing the lesser extent of the influence of the coach decisions. Although PERF sub-system also exhibit an efficiency close to one, Los Angeles Lakers (LAL) have undergone a fall in PERF efficiency during the last few years, because of the relative poor performance of highly-paid players. In contrast, WG sub-system seems to be decisive to the overall efficiency, which makes a lot of sense, since the best teams master how to administer the differences in points in order to win the largest possible number of matches.

5. Summary and conclusions

In this paper an analysis of productivity change of NBA teams during the last seven years has been carried out. The results have shed light on the path taken by each team (and the NBA in general) in terms of the efficient use of its economic resources, specifically as regards the players' payroll. The research uses an innovative Network

DEA approach to assess the efficiency of teams and measure the distance to the corresponding efficient frontier. In general, although Network DEA models require much more data (e.g. about internal links and intermediate products) than the conventional DEA approach, the results obtained are more accurate and valid. In particular, the network of process considered consists of four stages: team performance (that uses the budget and produces offensive and defensive actions), offensive and defensive subsystems (that transform the offensive and defensive actions into points scored and points received, respectively) and a final wins generation stage (that produces the victories from the points scored and received). In total, a number of eleven intermediate products are considered thus increasing the complexity, but also the power, of the analysis with respect to the conventional DEA approach that, in this case, would involve a simple single-input, single-output problem.

It can be concluded from the study that during the last seasons there has been a technological progress consisting of a reduction in the budgets of the efficient teams. Although before the lockout there were teams that did not act accordingly and experienced an efficiency worsening, after the 2011 CBA was signed most teams have caught up with the best practices the most efficient teams have established, slashing their budgets without a significant drop in performance. Hence, the course of action towards efficiency is clear: budget reductions while maintaining (or improving) performance.

These conclusions also match up with regression results, that is, change in wins between seasons is mainly affected by the shift in scale efficiency, and thus managers should adjust their resources properly in order to operate in their MPSS. Concerning the 2011 CBA, this information supports the team owners' claims when negotiations took place and encourage players to adapt to the new realities of a changing world.

Appendix

In this appendix, the approach in Lewis et al. (2009) is adapted to the network DEA model shown in Figure 1. For a certain sub-DMU, an input-oriented CCR model is applied taking into account only the inputs and outputs of the sub-DMU under assessment. Note that the inputs and outputs of a sub-DMU may be intermediate products in the network DEA approach, e.g. the variables "Points" and "InvOppPoints" are intermediate products within the DMU but inputs for the process "Wins generation" (WG).

The input-oriented CCR model to compute the maximal feasible radial reduction of inputs for process WG in time period t1 can be formulated as:

$$\mathrm{WG\_Eff}^{t1} = \mathrm{Min} \quad \theta \tag{30}$$

s.t.

$$\sum_j \lambda_j^{WG} \cdot \mathrm{Points}_j^{t1} \leq \theta \cdot \mathrm{Points}_0^{t1} \tag{31}$$

$$\sum_j \lambda_j^{WG} \cdot \mathrm{InvOppPoints}_j^{t1} \leq \theta \cdot \mathrm{InvOppPoints}_0^{t1} \tag{32}$$

$$\sum_j \lambda_j^{WG} \cdot \mathrm{Wins}_j^{t1} \geq \mathrm{Wins}_0^{t1} \tag{33}$$

$$\lambda_j^{WG} \geq 0 \tag{34}$$

The input-oriented CCR model to compute the maximal feasible radial reduction of inputs for process OFF in time period t1 can be formulated as:

$$\mathrm{OFF\_Eff}^{t1} = \mathrm{Min} \quad \theta \tag{35}$$

s.t.

$$\sum_{j} \lambda_j^{WG} \cdot 2PA_j^{t_1} \leq \theta \cdot 2PA_0^{t_1} \tag{36}$$

$$\sum_{j} \lambda_j^{WG} \cdot 3PA_j^{t_1} \leq \theta \cdot 3PA_0^{t_1} \tag{37}$$

$$\sum_{j} \lambda_j^{WG} \cdot FTA_j^{t_1} \leq \theta \cdot FTA_0^{t_1} \tag{38}$$

$$\sum_{j} \lambda_j^{WG} \cdot OffReb_j^{t_1} \leq \theta \cdot OffReb_0^{t_1} \tag{39}$$

$$\sum_{j} \lambda_j^{WG} \cdot Assists_j^{t_1} \leq \theta \cdot Assists_0^{t_1} \tag{40}$$

$$\sum_{j} \lambda_j^{WG} \cdot InvTO_j^{t_1} \leq \theta \cdot InvTO_0^{t_1} \tag{41}$$

$$\sum_{j} \lambda_j^{WG} \cdot Points_j^{t_1} \geq Points_0^{t_1} \tag{42}$$

$$\lambda_j^{OFF} \geq 0 \tag{43}$$

The input-oriented CCR model to compute the maximal feasible radial reduction of inputs for process DEF in time period t1 can be formulated as:

$$DEF\_Eff^{t_1} = Min \quad \theta \tag{44}$$

s.t.

$$\sum_{j} \lambda_j^{WG} \cdot DefReb_j^{t_1} \leq \theta \cdot DefReb_0^{t_1} \tag{45}$$

$$\sum_{j} \lambda_j^{WG} \cdot Steals_j^{t_1} \leq \theta \cdot Steals_0^{t_1} \tag{46}$$

$$\sum_{j} \lambda_j^{WG} \cdot Blocks_j^{t_1} \leq \theta \cdot Blocks_0^{t_1} \tag{47}$$

$$\sum_j \lambda_j^{WG} \cdot InvOppPoints_j^{t1} \geq InvOppPoints_0^{t1} \tag{48}$$

$$\lambda_j^{DEF} \geq 0 \tag{49}$$

The input-oriented CCR model to compute the maximal feasible radial reduction of input for process PERF in time period t1 can be formulated as:

$$PERF\_Eff^{t1} = Min \quad \theta \tag{50}$$

s.t.

$$\sum_j \lambda_j^{WG} \cdot Budget_j^{t1} \leq \theta \cdot Budget_0^{t1} \tag{51}$$

$$\sum_j \lambda_j^{WG} \cdot 2PA_j^{t1} \geq 2PA_0^{t1} \tag{52}$$

$$\sum_j \lambda_j^{WG} \cdot 3PA_j^{t1} \geq 3PA_0^{t1} \tag{53}$$

$$\sum_j \lambda_j^{WG} \cdot FTA_j^{t1} \geq FTA_0^{t1} \tag{54}$$

$$\sum_j \lambda_j^{WG} \cdot OffReb_j^{t1} \geq OffReb_0^{t1} \tag{55}$$

$$\sum_j \lambda_j^{WG} \cdot Assists_j^{t1} \geq Assists_0^{t1} \tag{56}$$

$$\sum_j \lambda_j^{WG} \cdot InvTO_j^{t1} \geq InvTO_0^{t1} \tag{57}$$

$$\sum_j \lambda_j^{WG} \cdot DefReb_j^{t1} \geq DefReb_0^{t1} \tag{58}$$

$$\lambda_j^{PERF} \geq 0 \tag{59}$$

The efficiency for the entire DMU is computed in three steps according to the methodology proposed by Lewis et al. (2009). First, the optimal values for the inputs consumed by the WG process are computed using model (30)-(34). Second, using those optimal values in the models (35)-(43) and (44)-(49), instead of the corresponding observed values, the OFF and DEF efficiencies, respectively, are computed. Finally, the optimal values for the inputs consumed by the OFF and DEF processes are used in the model (50)-(59), instead of the observed values, and the efficiency score of process PERF, which will be the efficiency for the entire DMU, is computed. Note that the described method is just the opposite of the one described in Lewis et al. (2009), since the model used in that paper was output-oriented.

Acknowledgements.- The authors would like to thank the reviewers for their constructive and helpful remarks and suggestions.

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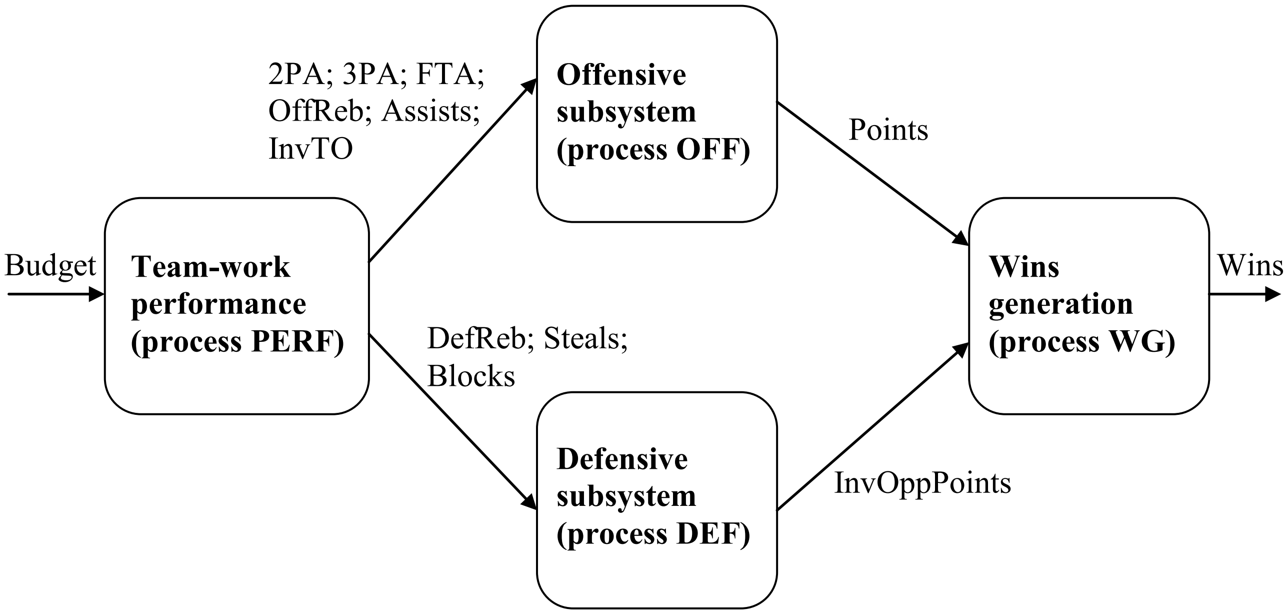
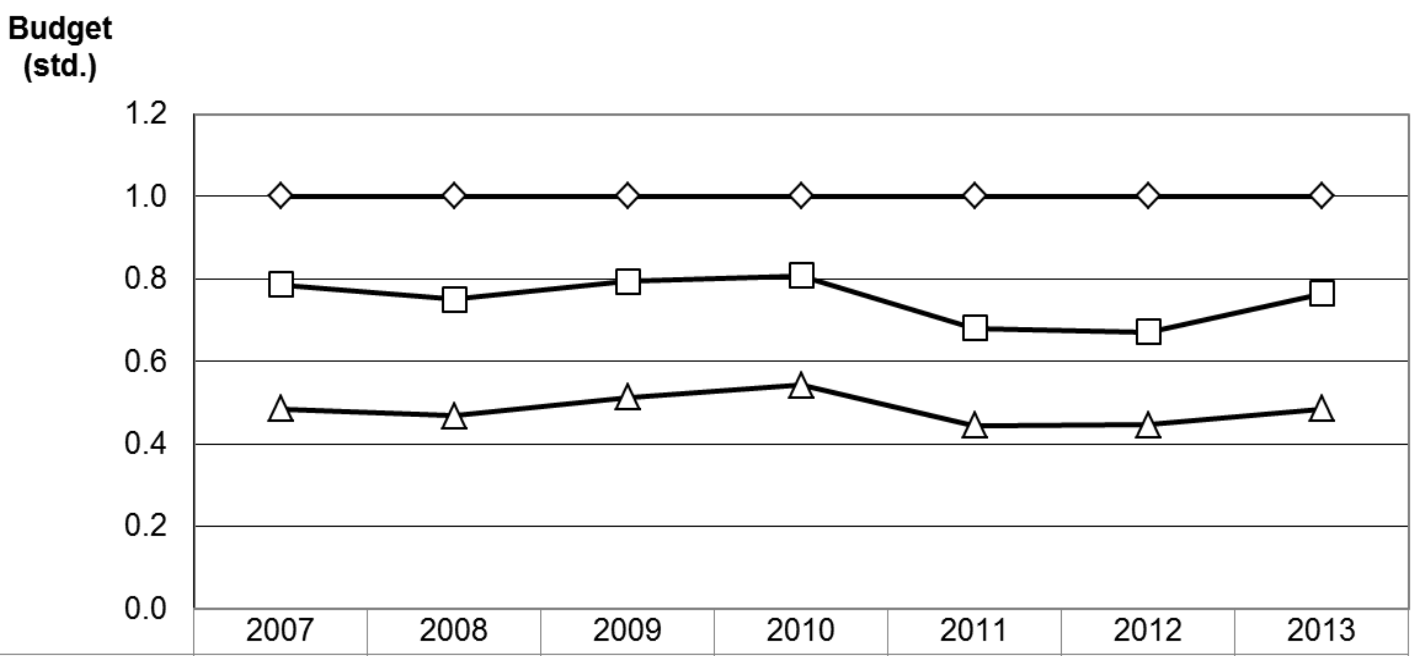


Figure 1. DMU as a network of processes



	2007	2008	2009	2010	2011	2012	2013
Observed	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Target VRS	0.79	0.75	0.79	0.81	0.68	0.67	0.76
Target CRS	0.48	0.47	0.51	0.54	0.44	0.45	0.48

Figure 2. Average normalized budgets (observed and target)

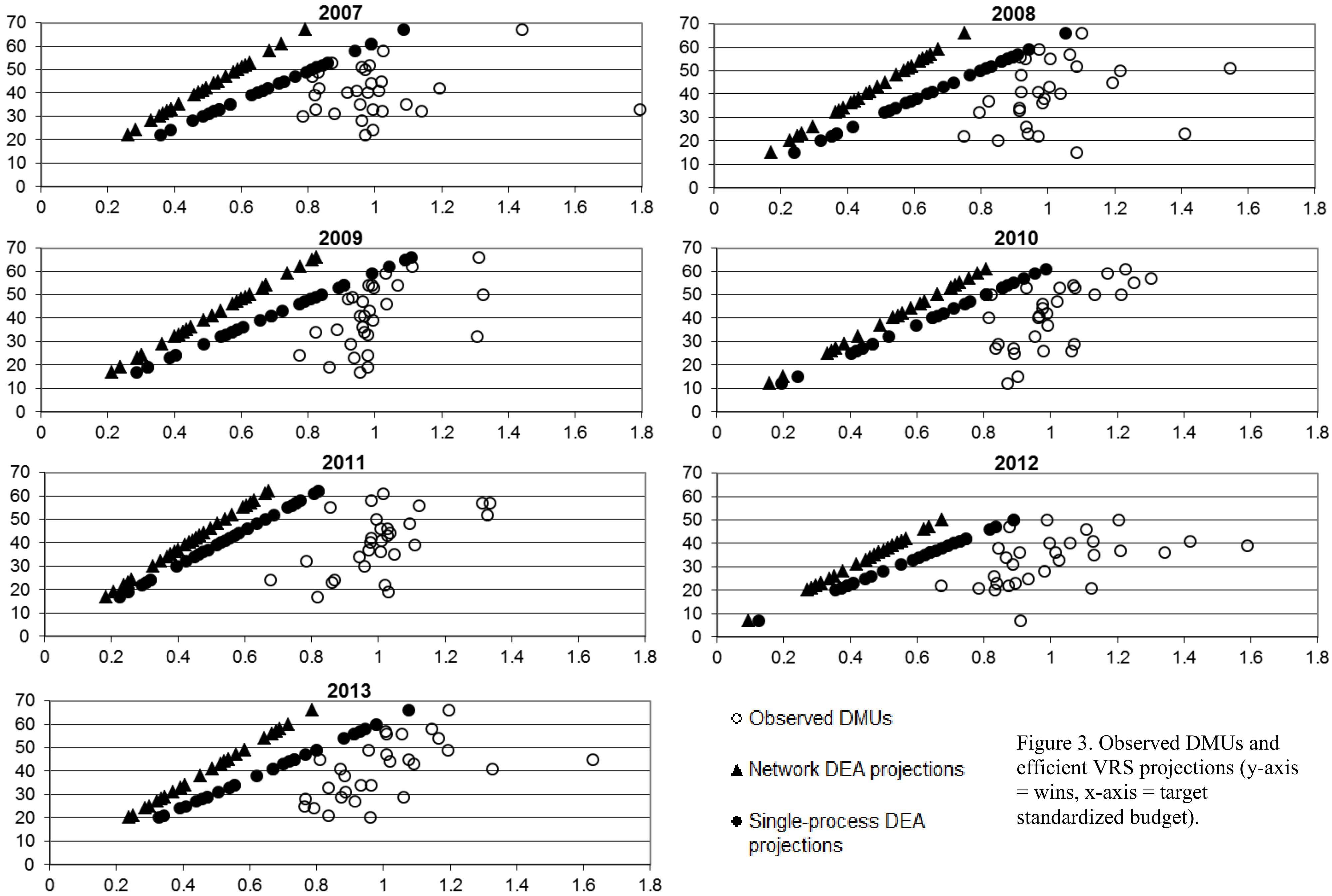
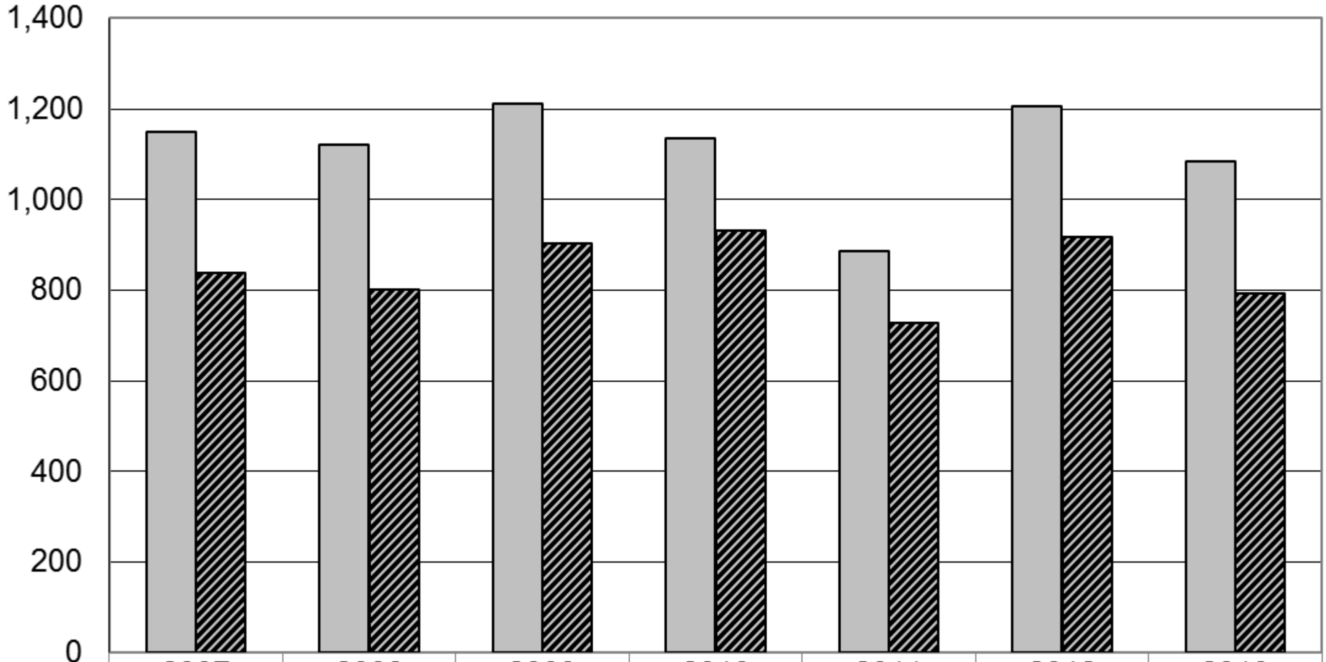


Figure 3. Observed DMUs and efficient VRS projections (y-axis = wins, x-axis = target standardized budget).



	2007	2008	2009	2010	2011	2012	2013
Single Process	1147.84	1120.30	1212.65	1133.67	886.27	1204.80	1083.43
Network DEA	836.65	799.76	902.92	929.41	725.49	915.96	791.63

Figure 4. CRS effective cost per victory in each regular season (in thousands of dollars).

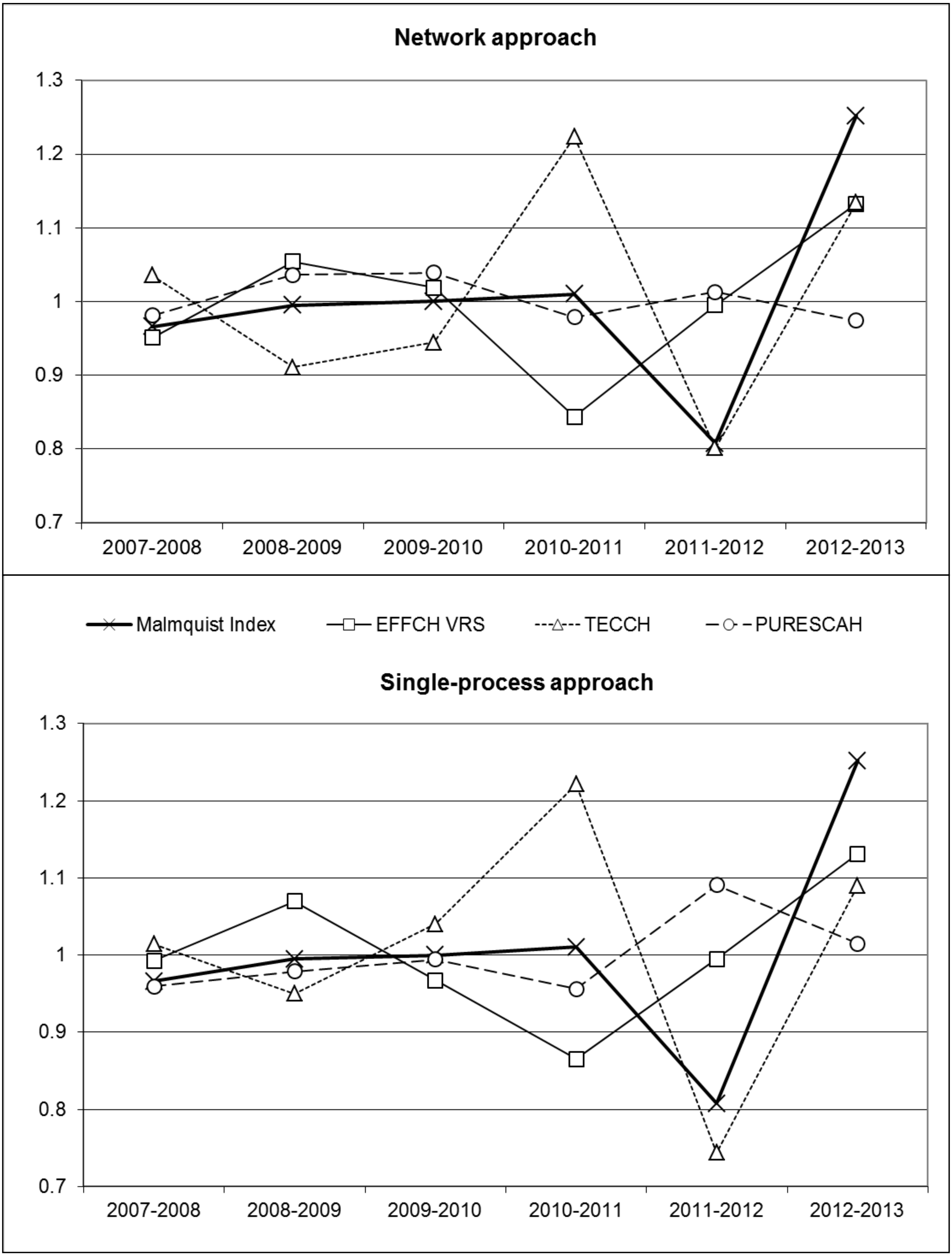


Figure 5. Evolution of mean MPI and its FGNZ components

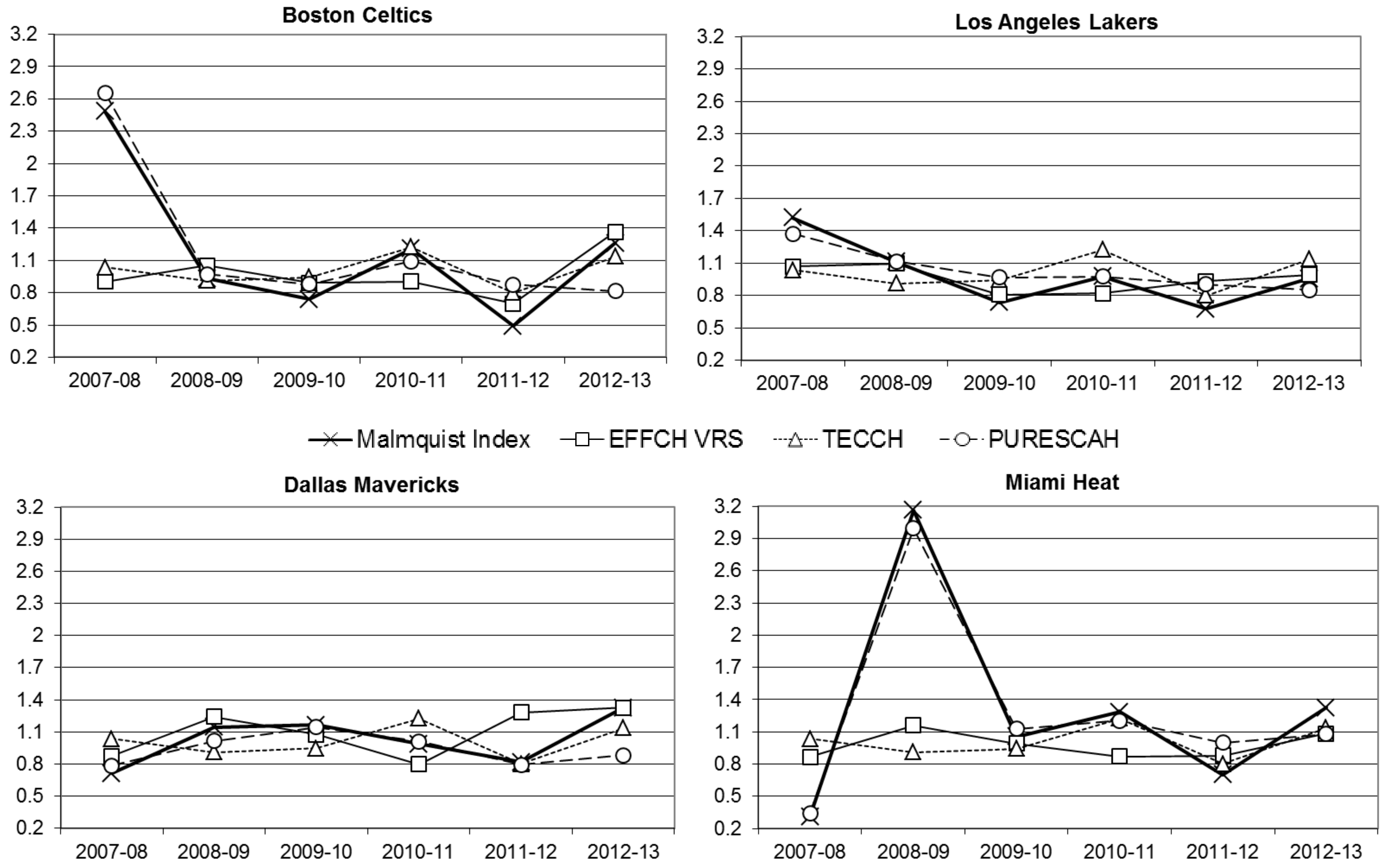


Figure 6. Evolution of MPI and its FGNZ components for the last four NBA champions.

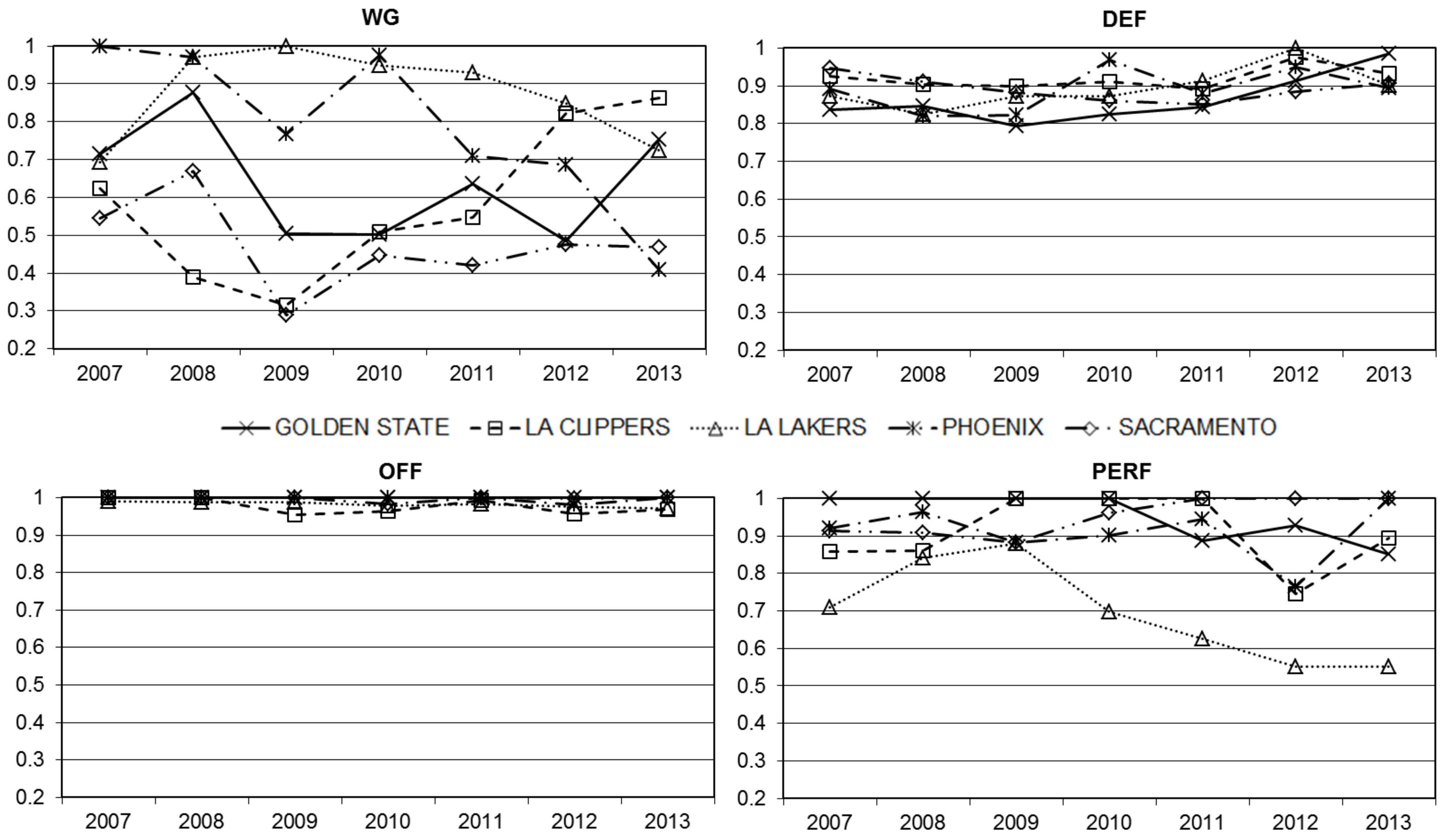


Figure 7. Efficiency of the four sub-DMUs for the Pacific Division according to the Lewis-Lock-Sexton methodology.

Name	Label	Type of variable
Total salaries of all players in the team	Budget	Input
Number of team victories	Wins	Output
2-Point shots attempted	2PA	Intermediate product
3-Point shots attempted	3PA	Intermediate product
Free throws attempted	FTA	Intermediate product
Offensive Rebounds	OffReb	Intermediate product
Number of Assists	Assists	Intermediate product
Inverse of Turnovers	InvTO	Intermediate product
Defensive rebounds	DefReb	Intermediate product
Number of Steals	Steals	Intermediate product
Blocked Shots	Blocks	Intermediate product
Points by the team	Points	Intermediate product
Inverse of Points by opponents	InvOppPoints	Intermediate product

Table 1. Model variables

		Budget														Wins						
		2007		2008		2009		2010		2011		2012		2013		2007	2008	2009	2010	2011	2012	2013
Division	**Teams**	Abs	Norm	Abs	Norm	Abs	Norm	Abs	Norm	Abs	Norm	Abs	Norm	Abs	Norm	Abs	Abs	Abs	Abs	Abs	Abs	Abs
Atlantic	BOS	70.4	0.99	77.3	1.10	80.1	1.11	85.1	1.21	75.2	1.12	107.8	1.59	88.1	1.33	24	66	62	50	56	39	41
	NJ	71.7	1.01	64.1	0.91	59.3	0.82	61.1	0.87	58.2	0.87	59.2	0.87	79.2	1.19	41	34	34	12	24	22	49
	NYK	127.1	1.79	98.9	1.41	94.2	1.31	75.1	1.07	65.6	0.98	61.4	0.91	77.5	1.17	33	23	32	29	42	36	54
	PHI	77.5	1.09	72.8	1.04	68.9	0.95	62.3	0.89	67.6	1.01	76.6	1.13	62.1	0.93	35	40	41	27	41	35	34
	TOR	57.5	0.81	68.1	0.97	70.7	0.98	67.6	0.96	68.3	1.02	60.6	0.89	64.0	0.96	47	41	33	40	22	23	34
Central	CHI	58.8	0.83	64.1	0.91	69.8	0.97	67.8	0.97	54.9	0.82	67.1	0.99	71.5	1.08	49	33	41	41	62	50	45
	CLE	68.9	0.97	83.7	1.19	94.6	1.31	85.9	1.22	69.0	1.03	53.1	0.78	52.7	0.79	50	45	66	61	19	21	24
	DET	61.7	0.87	68.2	0.97	71.8	0.99	58.6	0.83	64.2	0.96	63.2	0.93	70.6	1.06	53	59	39	27	30	25	29
	IND	67.7	0.95	69.0	0.98	69.6	0.96	66.9	0.95	65.1	0.97	50.6	0.75	63.6	0.96	35	36	36	32	37	42	49
	MIL	68.1	0.96	65.7	0.94	69.9	0.97	68.5	0.98	70.2	1.05	60.1	0.89	58.7	0.88	28	26	34	46	35	31	38
South-east	ATL	55.6	0.78	57.6	0.82	69.5	0.96	65.1	0.93	69.4	1.04	71.7	1.06	67.7	1.02	30	37	47	53	44	40	44
	CHA	58.2	0.82	55.7	0.79	64.1	0.89	68.6	0.98	63.3	0.94	61.7	0.91	55.4	0.83	33	32	35	44	34	7	21
	MIA	70.1	0.99	76.3	1.09	71.0	0.98	71.5	1.02	65.6	0.98	75.0	1.11	79.5	1.20	44	15	43	47	58	46	66
	ORL	65.1	0.92	76.3	1.09	74.5	1.03	82.1	1.17	88.8	1.33	82.0	1.21	63.9	0.96	40	52	59	59	52	37	20
	WAS	66.9	0.94	70.4	1.00	70.5	0.98	74.5	1.06	57.7	0.86	56.5	0.83	58.0	0.87	41	43	19	26	23	20	29
South-west	DAL	102.2	1.44	108.4	1.54	95.5	1.32	87.8	1.25	87.9	1.31	68.8	1.01	57.8	0.87	67	51	50	55	57	36	41
	HOU	69.7	0.98	70.6	1.01	71.9	1.00	69.3	0.99	68.8	1.03	58.7	0.87	53.8	0.81	52	55	53	42	43	34	45
	MEM	68.8	0.97	52.5	0.75	55.8	0.77	57.1	0.81	68.9	1.03	76.3	1.13	70.2	1.06	22	22	24	40	46	41	56
	NO	58.1	0.82	64.4	0.92	67.4	0.93	69.6	0.99	67.4	1.01	76.0	1.12	60.8	0.91	39	56	49	37	46	21	27
	SAS	72.6	1.02	62.6	0.89	71.5	0.99	79.4	1.13	67.9	1.01	81.6	1.20	76.1	1.14	58	56	54	50	61	50	58
North-west	DEN	72.3	1.02	85.4	1.22	70.9	0.98	75.3	1.07	66.6	0.99	57.1	0.84	66.9	1.01	45	50	54	53	50	38	57
	MIN	72.5	1.02	68.1	0.97	70.6	0.98	63.2	0.90	54.8	0.82	56.3	0.83	58.9	0.89	32	22	24	15	17	26	31
	OKC	62.2	0.88	59.7	0.85	67.7	0.94	56.7	0.81	57.4	0.86	59.4	0.88	65.0	0.98	31	20	23	50	55	47	60
	POR	80.7	1.14	64.6	0.92	77.1	1.07	57.8	0.82	73.3	1.09	66.5	0.98	55.6	0.84	32	41	54	50	48	28	33
	UT	68.1	0.96	60.5	0.86	66.3	0.92	72.0	1.03	74.4	1.11	90.9	1.34	72.5	1.09	51	54	48	53	39	36	43
Pacific	GSW	59.0	0.83	64.6	0.92	66.8	0.93	68.6	0.98	67.5	1.01	56.8	0.84	67.1	1.01	42	48	29	26	36	23	47
	LAC	69.3	0.98	66.0	0.94	62.4	0.86	59.2	0.84	52.6	0.78	67.5	1.00	67.1	1.01	40	23	19	29	32	40	56
	LAL	84.6	1.19	74.8	1.07	78.8	1.09	91.3	1.30	89.3	1.33	96.2	1.42	108.1	1.63	42	57	65	57	57	41	45
	PHO	70.0	0.99	65.4	0.93	74.8	1.04	75.0	1.07	65.3	0.97	69.5	1.02	50.7	0.76	61	55	46	54	40	33	25
	SAC	70.5	0.99	69.4	0.99	69.0	0.96	62.6	0.89	45.3	0.68	45.5	0.67	51.0	0.77	33	38	17	25	24	22	28
Average		70.9	1.00	70.2	1.00	72.2	1.00	70.2	1.00	67.0	1.00	67.8	1.00	66.5	1.00	41	41	41	41	41	33	41

Note: Abs. budget data in millions of US$

Table 2. Input-output data for the NBA regular seasons from 2006/2007 to 2012/2013.

		2007		2008		2009		2010		2011		2012		2013	
Division	**Teams**	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS
Atlantic	BOS	0.29	0.79	0.68	0.71	0.70	0.75	0.55	0.67	0.54	0.60	0.33	0.42	0.37	0.58
	NJ	0.48	0.77	0.42	0.82	0.52	0.95	0.18	0.93	0.30	0.78	0.34	0.77	0.49	0.64
	NYK	0.22	0.44	0.19	0.53	0.31	0.60	0.36	0.75	0.46	0.69	0.54	0.74	0.55	0.65
	PHI	0.38	0.72	0.42	0.69	0.54	0.82	0.40	0.91	0.44	0.67	0.42	0.59	0.43	0.82
	TOR	0.68	0.97	0.48	0.77	0.42	0.80	0.55	0.84	0.23	0.66	0.35	0.75	0.42	0.79
Central	CHI	0.70	0.95	0.41	0.82	0.53	0.81	0.56	0.84	0.82	0.98	0.68	0.69	0.50	0.71
	CLE	0.61	0.81	0.43	0.63	0.63	0.70	0.66	0.66	0.20	0.66	0.36	0.86	0.36	0.96
	DET	0.72	0.91	0.69	0.77	0.49	0.79	0.43	0.97	0.34	0.71	0.36	0.72	0.33	0.72
	IND	0.43	0.82	0.42	0.76	0.47	0.81	0.44	0.85	0.41	0.70	0.76	0.90	0.61	0.80
	MIL	0.34	0.82	0.32	0.80	0.44	0.81	0.62	0.83	0.36	0.65	0.47	0.76	0.51	0.86
South-east	ATL	0.45	1.00	0.51	0.91	0.61	0.82	0.76	0.87	0.46	0.65	0.51	0.63	0.51	0.75
	CHA	0.47	0.95	0.46	0.94	0.49	0.88	0.60	0.83	0.39	0.72	0.10	0.74	0.30	0.91
	MIA	0.52	0.79	0.16	0.69	0.55	0.80	0.61	0.79	0.64	0.69	0.56	0.61	0.66	0.66
	ORL	0.51	0.85	0.70	0.88	0.71	0.80	0.67	0.69	0.42	0.51	0.41	0.55	0.25	0.79
	WAS	0.51	0.83	0.49	0.75	0.24	0.79	0.32	0.76	0.29	0.79	0.32	0.80	0.40	0.87
South-west	DAL	0.55	0.56	0.38	0.48	0.47	0.60	0.58	0.65	0.47	0.52	0.48	0.66	0.56	0.88
	HOU	0.62	0.80	0.62	0.74	0.67	0.81	0.56	0.82	0.45	0.66	0.53	0.77	0.66	0.94
	MEM	0.27	0.81	0.33	1.00	0.39	1.00	0.65	0.99	0.48	0.66	0.49	0.60	0.63	0.72
	NO	0.56	0.96	0.70	0.82	0.66	0.85	0.49	0.81	0.50	0.67	0.25	0.60	0.35	0.83
	SAS	0.67	0.77	0.65	0.76	0.68	0.81	0.59	0.71	0.65	0.67	0.56	0.57	0.60	0.67
North-west	DEN	0.52	0.77	0.47	0.61	0.69	0.82	0.65	0.75	0.54	0.68	0.61	0.80	0.67	0.76
	MIN	0.37	0.77	0.26	0.77	0.31	0.79	0.22	0.90	0.22	0.83	0.42	0.81	0.42	0.86
	OKC	0.42	0.89	0.26	0.84	0.31	0.82	0.82	1.00	0.70	0.79	0.72	0.77	0.73	0.79
	POR	0.33	0.69	0.50	0.80	0.63	0.75	0.80	0.98	0.47	0.62	0.39	0.68	0.47	0.91
	UT	0.63	0.82	0.71	0.87	0.65	0.86	0.68	0.79	0.38	0.61	0.36	0.50	0.47	0.70
Pacific	GSW	0.60	0.94	0.59	0.81	0.39	0.84	0.35	0.83	0.39	0.67	0.37	0.80	0.55	0.76
	LAC	0.48	0.80	0.28	0.80	0.28	0.90	0.46	0.96	0.44	0.86	0.54	0.67	0.66	0.76
	LAL	0.42	0.66	0.61	0.70	0.74	0.77	0.58	0.62	0.46	0.51	0.39	0.47	0.33	0.47
	PHO	0.73	0.81	0.60	0.72	0.56	0.76	0.67	0.76	0.44	0.69	0.44	0.65	0.39	1.00
	SAC	0.39	0.79	0.47	0.81	0.22	0.81	0.37	0.91	0.38	1.00	0.44	1.00	0.43	0.99

Table 3. Efficiency scores for CRS and VRS Network DEA approaches

		Network Malmquist Index					
Division	**Teams**	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013
Atlantic	BOS	2.483	0.932	0.739	1.209	0.492	1.262
	NJ	0.919	1.111	0.333	2.005	0.912	1.632
	NYK	0.887	1.502	1.105	1.585	0.925	1.166
	PHI	1.151	1.167	0.708	1.336	0.762	1.176
	TOR	0.729	0.797	1.233	0.520	1.192	1.373
	Mean	1.112	1.077	0.750	1.217	0.823	1.311
Central	CHI	0.612	1.174	1.002	1.781	0.668	0.827
	CLE	0.733	1.334	0.991	0.370	1.452	1.129
	DET	0.996	0.646	0.824	0.968	0.857	1.019
	IND	1.000	1.020	0.898	1.135	1.478	0.911
	MIL	0.953	1.265	1.341	0.709	1.047	1.230
	Mean	0.843	1.055	0.997	0.875	1.052	1.013
Southeast	ATL	1.177	1.083	1.172	0.743	0.891	1.142
	CHA	1.004	0.978	1.142	0.800	0.213	3.274
	MIA	0.310	3.166	1.056	1.283	0.702	1.327
	ORL	1.402	0.935	0.883	0.778	0.780	0.680
	WAS	0.987	0.454	1.260	1.092	0.898	1.384
	Mean	0.873	1.073	1.095	0.917	0.622	1.361
Southwest	DAL	0.711	1.145	1.164	0.988	0.816	1.328
	HOU	1.034	0.974	0.799	0.984	0.937	1.417
	MEM	1.298	1.055	1.584	0.911	0.814	1.457
	NO	1.283	0.861	0.711	1.226	0.409	1.577
	SAS	1.000	0.962	0.811	1.362	0.690	1.220
	Mean	1.041	0.995	0.968	1.081	0.706	1.394
Northwest	DEN	0.931	1.339	0.898	1.019	0.896	1.255
	MIN	0.724	1.082	0.679	1.248	1.506	1.118
	OKC	0.636	1.092	2.526	1.038	0.834	1.144
	POR	1.567	1.148	1.202	0.722	0.650	1.383
	UT	1.180	0.834	0.989	0.680	0.764	1.469
	Mean	0.955	1.087	1.129	0.917	0.890	1.267
Pacific	GSW	1.034	0.601	0.849	1.345	0.768	1.694
	LAC	0.598	0.899	1.564	1.186	0.984	1.381
	LAL	1.520	1.113	0.736	0.977	0.675	0.958
	PHO	0.859	0.837	1.138	0.812	0.785	1.018
	SAC	1.244	0.431	1.577	1.265	0.924	1.112
	Mean	1.001	0.736	1.119	1.099	0.820	1.205
Mean		0.9663	0.995	1.0002	1.0104	0.8077	1.2517

Table 4. MPI computed using the proposed Network DEA approach

		2007-08			2008-09			2009-10			2010-11			2011-12			2012-13		
Division	Teams	EFFCH VRS	TECCH	PURE SCACH	EFFCH VRS	TECCH	PURE SCACH	EFFCH VRS	TECCH	PURE SCACH	EFFCH VRS	TECCH	PURE SCACH	EFFCH VRS	TECCH	PURE SCACH	EFFCH VRS	TECCH	PURE SCACH
Atlantic	BOS	0.90	1.04	2.66	1.05	0.91	0.97	0.89	0.94	0.88	0.90	1.22	1.09	0.70	0.80	0.88	1.36	1.13	0.81
	NJ	1.06	1.04	0.84	1.16	0.91	1.05	0.98	0.94	0.36	0.84	1.22	1.95	0.99	0.80	1.15	0.83	1.13	1.73
	NYK	1.21	1.04	0.70	1.12	0.91	1.47	1.26	0.94	0.93	0.92	1.22	1.41	1.07	0.80	1.08	0.88	1.13	1.16
	PHI	0.96	1.04	1.16	1.19	0.91	1.07	1.11	0.94	0.68	0.74	1.22	1.48	0.89	0.80	1.07	1.38	1.13	0.75
	TOR	0.80	1.04	0.88	1.03	0.91	0.85	1.05	0.94	1.24	0.79	1.22	0.54	1.13	0.80	1.32	1.06	1.13	1.15
Central	CHI	0.86	1.04	0.68	0.99	0.91	1.30	1.03	0.94	1.03	1.17	1.22	1.25	0.71	0.80	1.18	1.03	1.13	0.71
	CLE	0.77	1.04	0.91	1.12	0.91	1.31	0.94	0.94	1.11	0.99	1.22	0.31	1.30	0.80	1.39	1.12	1.13	0.89
	DET	0.85	1.04	1.13	1.02	0.91	0.69	1.23	0.94	0.71	0.73	1.22	1.08	1.02	0.80	1.05	1.00	1.13	0.90
	IND	0.93	1.04	1.04	1.07	0.91	1.05	1.04	0.94	0.91	0.82	1.22	1.13	1.29	0.80	1.43	0.89	1.13	0.90
	MIL	0.98	1.04	0.94	1.01	0.91	1.38	1.03	0.94	1.38	0.78	1.22	0.74	1.17	0.80	1.11	1.14	1.13	0.95
South-east	ATL	0.91	1.04	1.25	0.90	0.91	1.32	1.06	0.94	1.17	0.75	1.22	0.81	0.97	0.80	1.14	1.18	1.13	0.85
	CHA	0.99	1.04	0.98	0.93	0.91	1.15	0.94	0.94	1.29	0.87	1.22	0.75	1.03	0.80	0.26	1.24	1.13	2.33
	MIA	0.87	1.04	0.35	1.16	0.91	2.99	0.99	0.94	1.13	0.87	1.22	1.20	0.88	0.80	1.00	1.09	1.13	1.08
	ORL	1.03	1.04	1.31	0.90	0.91	1.13	0.87	0.94	1.08	0.74	1.22	0.86	1.09	0.80	0.90	1.43	1.13	0.42
	WAS	0.90	1.04	1.06	1.06	0.91	0.47	0.96	0.94	1.39	1.03	1.22	0.86	1.02	0.80	1.09	1.09	1.13	1.12
South-west	DAL	0.87	1.04	0.79	1.24	0.91	1.01	1.07	0.94	1.15	0.80	1.22	1.01	1.28	0.80	0.80	1.33	1.13	0.88
	HOU	0.93	1.04	1.08	1.08	0.91	0.99	1.01	0.94	0.83	0.81	1.22	1.00	1.18	0.80	1.00	1.22	1.13	1.03
	MEM	1.24	1.04	1.01	1.00	0.91	1.16	0.99	0.94	1.69	0.66	1.22	1.12	0.91	0.80	1.12	1.21	1.13	1.06
	NO	0.85	1.04	1.45	1.04	0.91	0.91	0.96	0.94	0.78	0.83	1.22	1.21	0.89	0.80	0.57	1.39	1.13	1.00
	SAS	0.98	1.04	0.99	1.07	0.91	0.98	0.88	0.94	0.98	0.93	1.22	1.19	0.85	0.80	1.01	1.18	1.13	0.91
North-west	DEN	0.80	1.04	1.13	1.34	0.91	1.10	0.92	0.94	1.04	0.91	1.22	0.92	1.17	0.80	0.96	0.95	1.13	1.16
	MIN	1.01	1.04	0.70	1.03	0.91	1.16	1.13	0.94	0.63	0.92	1.22	1.11	0.98	0.80	1.93	1.07	1.13	0.92
	OKC	0.94	1.04	0.65	0.98	0.91	1.22	1.21	0.94	2.20	0.79	1.22	1.07	0.97	0.80	1.08	1.03	1.13	0.98
	POR	1.17	1.04	1.30	0.94	0.91	1.34	1.30	0.94	0.98	0.63	1.22	0.94	1.10	0.80	0.73	1.33	1.13	0.91
	UT	1.06	1.04	1.08	0.99	0.91	0.92	0.91	0.94	1.15	0.77	1.22	0.72	0.82	0.80	1.16	1.40	1.13	0.93
Pacific	GSW	0.86	1.04	1.16	1.03	0.91	0.64	0.98	0.94	0.91	0.81	1.22	1.35	1.19	0.80	0.80	0.94	1.13	1.58
	LAC	0.99	1.04	0.58	1.12	0.91	0.88	1.07	0.94	1.55	0.90	1.22	1.08	0.78	0.80	1.57	1.12	1.13	1.09
	LAL	1.07	1.04	1.37	1.10	0.91	1.11	0.81	0.94	0.97	0.82	1.22	0.98	0.93	0.80	0.91	0.99	1.13	0.85
	PHO	0.90	1.04	0.92	1.05	0.91	0.87	0.99	0.94	1.21	0.92	1.22	0.72	0.94	0.80	1.04	1.53	1.13	0.59
	SAC	1.03	1.04	1.16	1.00	0.91	0.48	1.12	0.94	1.49	1.10	1.22	0.94	1.00	0.80	1.15	0.99	1.13	0.99

Table 5. Components from MPI decomposition obtained by Network DEA

	2007-08		2008-09		2009-10	
Component	Network	Single	Network	Single	Network	Single
Budget Change	0.079	0.444*	0.236*	0.212*	0.275*	0.300*
EFFCH VRS	0.077	0.575*	0.229*	0.293*	0.302*	0.393*
PURESCACH	0.995*	0.715*	0.979*	0.883*	0.983*	0.915*
	2010-11		**2011-12**		**2012-13**	
Component	Network	Single	Network	Single	Network	Single
Budget Change	0.297*	0.289*	0.238	0.427*	0.029	0.460*
EFFCH VRS	0.348*	0.425*	0.265	0.403*	0.028	0.428*
PURESCACH	0.986*	0.861*	0.994*	0.927*	0.999*	0.915*

* p-value ≤ 0.001

Table 6. Standardized regression coefficients for the change in the variable Wins between seasons.

Division	Teams	2007	2008	2009	2010	2011	2012	2013
Atlantic	BOS	0.12	0.23	0.29	0.24	0.29	0.30	0.23
	NJ	0.36	0.18	0.19	0.05	0.13	0.21	0.26
	NYK	0.20	0.20	0.41	0.31	0.51	0.41	0.45
	PHI	0.37	0.30	0.38	0.14	0.23	0.31	0.42
	TOR	0.32	0.26	0.20	0.21	0.16	0.16	0.20
Central	CHI	0.30	0.22	0.27	0.21	0.18	0.31	0.32
	CLE	0.34	0.24	0.24	0.15	0.08	0.16	0.18
	DET	0.28	0.31	0.35	0.20	0.18	0.15	0.17
	IND	0.20	0.23	0.24	0.22	0.27	0.24	0.39
	MIL	0.38	0.18	0.24	0.34	0.17	0.25	0.28
Southeast	ATL	0.28	0.31	0.24	0.22	0.19	0.23	0.27
	CHA	0.23	0.18	0.21	0.21	0.14	0.05	0.17
	MIA	0.18	0.06	0.29	0.20	0.26	0.27	0.25
	ORL	0.35	0.30	0.29	0.33	0.42	0.41	0.21
	WAS	0.30	0.26	0.12	0.08	0.19	0.15	0.24
Southwest	DAL	0.16	0.21	0.20	0.19	0.29	0.28	0.35
	HOU	0.37	0.28	0.29	0.29	0.34	0.27	0.34
	MEM	0.18	0.22	0.20	0.35	0.42	0.35	0.37
	NO	0.29	0.30	0.34	0.29	0.24	0.18	0.22
	SAS	0.22	0.30	0.32	0.16	0.34	0.28	0.28
Northwest	DEN	0.37	0.35	0.31	0.25	0.39	0.39	0.33
	MIN	0.23	0.15	0.15	0.11	0.20	0.22	0.20
	OKC	0.30	0.22	0.27	0.18	0.30	0.28	0.27
	POR	0.13	0.27	0.28	0.20	0.30	0.21	0.26
	UT	0.37	0.29	0.35	0.32	0.17	0.28	0.21
Pacific	GSW	0.40	0.41	0.27	0.26	0.36	0.24	0.36
	LAC	0.37	0.13	0.15	0.12	0.27	0.31	0.34
	LAL	0.22	0.28	0.26	0.16	0.17	0.22	0.19
	PHO	0.27	0.40	0.28	0.33	0.40	0.20	0.21
	SAC	0.41	0.28	0.11	0.11	0.26	0.24	0.24
Mean		0.28	0.25	0.26	0.21	0.26	0.25	0.27
Min		0.12	0.06	0.11	0.05	0.08	0.05	0.17
Max		0.41	0.41	0.41	0.35	0.51	0.41	0.45
Spearman coef.		0.878*	0.965*	0.946*	0.933*	0.889*	0.941*	0.958*

* p-value ≤ 0.01

Table 7. Differences between Lewis-Lock-Sexton approach and the proposed approach from 2007 to 2013 seasons.

Applied Mathematical Modelling 37 (2013) 1665–1676

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Network DEA approach to airports performance assessment considering undesirable outputs

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ARTICLE INFO

Article history:
Received 30 June 2010
Received in revised form 3 April 2012
Accepted 21 April 2012
Available online 30 April 2012

Keywords:
Network DEA
Intermediate products
Undesirable outputs
Directional distance function
Airports efficiency

ABSTRACT

In this paper, a directional distance approach is proposed to deal with network DEA problems in which the processes may generate not only desirable final outputs but also undesirable outputs. The proposed approach is applied to the problem of modelling and benchmarking airport operations. The corresponding network DEA model considers two process (Aircraft Movement and Aircraft Loading) with two final outputs (Annual Passenger Movement and Annual Cargo handled), one intermediate product (Aircraft Traffic Movements) and two undesirable outputs (Number of Delayed Flights and Accumulated Flight Delays). The proposed approach has been applied to Spanish airports data for year 2008 comparing the computed directional distance efficiency scores with those obtained using a conventional, single-process directional distance function approach. From this comparison, it can be concluded that the proposed network DEA approach has more discriminatory power than its single-process counterpart, uncovering more inefficiencies and providing more valid results.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric tool for assessing the relative efficiency of homogeneous Decision Making Units (DMU). A DMU is commonly modelled as a single process that transforms inputs into outputs. The mathematical function for this transformation is unknown and the relative efficiency of the DMUs is assessed based only on the input and output data of the observed DMUs. From this, assuming certain technology assumptions, a Production Possibility Set (PPS) is inferred which contains all feasible input–output combinations. A DMU is labelled relative efficient if it cannot be found a feasible operation point (i.e. within the PPS) that produces more output without consuming more inputs or that consumes less input without producing less output. On the contrary, if the amounts of inputs consumed to produce the current outputs can be reduced or if the amount of outputs produced with the current inputs can be increased then the DMU is relative inefficient and an efficiency score based on the estimated potential improvements is computed.

It occurs often that, apart from consuming inputs and producing desirable outputs, the DMUs also generate undesirable outputs. That is rather common in many production settings in which pollution, noise, etc., are unwillingly but inevitably generated. There are many DEA approaches that can handle this situation basically through the assumption of an appropriate technology. The basic assumption in these cases is the joint weak disposability of desirable and undesirable outputs [1]. This implies that a reduction of the amount of undesirable outputs reduces also the desirable outputs because of the required diversion of productive resources to that end. A common way of handling efficiency assessment when there are undesirable

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0307-904X/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2012.04.041

outputs is through the Directional Distance Function (DDF) [2]. Such approach is a generalization of the common radial input or output-oriented efficiency measures and allows a simultaneous reduction both of inputs and of undesirable outputs as well as an increase in the desirable outputs. This type of DEA approaches has been widely used in environmental efficiency assessment (e.g. [3,4]).

Also, conventional DEA approaches consider the production process of a DMU as a black box. There are, however, other DEA applications in which instead of considering just a single process, a finer analysis is done which considers different interrelated processes each one with its own exogenous inputs and final outputs and also with intermediate products that are generated and consumed within the system. This type of DEA approaches are generally known as network DEA (e.g. [5]).

In this paper we present a Directional Distance Function (DDF) network DEA approach that takes into account undesirable outputs, applying it to assess the efficiency of airports. The structure of the paper is the following. Section briefly reviews the related literature on network DEA. Section 3 introduces the notation and defines the network technology concept. In section 4 the proposed directional distance network DEA approach is presented. Section 5 presents the application of the proposed approach to the relative efficiency assessment of airports, using data from the 39 Spanish airports for year 2008. Finally, Section 6 summarizes and concludes.

2. Brief network DEA literature review

As it has been mentioned above, network DEA approaches take into consideration the internal configuration of the DMUs instead of considering them as black boxes. Thus, while conventional DEA considers a single process that consumes all the inputs and produces all the outputs, network DEA considers the existence of several processes each of which consumes its owns set of inputs and produce its own set of outputs, in addition to consuming and producing intermediate products. These intermediate products are considered as inputs for some stages are outputs for others.

Most network DEA papers deal with series-of-processes systems (e.g. [6,7]), although parallel-processes (e.g. [8]) and general networks of processes have also been studied (e.g. [9,10]). Also, although most approaches use a radial input or output orientation (e.g. [6,7], etc.), other approaches such as Slacks-Based Measure (SBM) (e.g. [11,12]), Slacks-Based Inefficiency (SBI) [13] and cost efficiency (e.g. [14,15]) have also been proposed. The number of applications of network DEA has also grown and spans transportation (e.g. [16,17]), banking (e.g. [13,14]), utilities (e.g. [11,18]) and sports (e.g. [19,20]), among others.

In this paper, we present a novel network DEA approach that can handle the case in which some processes may generate undesirable outputs. To the best of our knowledge, there are just a few network DEA approaches that consider undesirable factors. Thus, Kordrostami and Amirteimoori [21] consider a multistage system and take into account the undesirable factors (which can also be intermediate products) with a minus sign in the computation of the virtual inputs and virtual outputs of a multiplier formulation. Hua and Bian [22] extend the approach to a more general network of processes, not necessarily in series. In both papers, a multiplier DEA form is used. Fukuyama and Weber [13] consider a two-stage series system and presents an application to assess the efficiency of banks.

Therefore, the network DEA approach proposed in this paper makes two main methodological contributions. One is to define formally the PPS of a general network of processes with undesirable outputs. The other is the use of a DDF efficiency measure computed on an envelopment form DEA model. In addition, the proposed approach has been applied to an important sector allowing more valid airports efficiency estimates than those computed by conventional, single-process DEA approaches.

3. Network technology including undesirable outputs

Network DEA considers that there are n DMUs which are structurally homogeneous, i.e. they consist of the same types of processes with the same interrelationships among them. Let P be the number of processes. Let $I(p)$ the set of exogenous inputs used in process p and, for each $i \in I(p)$, let x_{ij}^p denote the observed amount of exogenous input i consumed by process p of DMU j. Let $P_I(i)$ the set of processes that consume the exogenous input i and $x_{ij} = \sum_{p \in P_I(i)} x_{ij}^p$ the total amount of exogenous input i consumed by all processes of DMU j. Similarly, let $O(p)$ the set of final (hence desirable) outputs of process p and, for each $k \in O(p)$, let y_{kj}^p denote the observed amount of final output k produced by process p of DMU j. Let $P_O(k)$ the set of processes that produce the final output k and $y_{kj} = \sum_{p \in P_O(k)} y_{kj}^p$ the total amount of final output k produced by all processes of DMU j. Also, let $U(p)$ the set of undesirable outputs of process p and, for each $b \in U(p)$, let u_{bj}^p denote the observed amount of undesirable output b produced by process p of DMU j. Let $P_U(b)$ the set of processes that produce the undesirable output b and $u_{bj} = \sum_{p \in P_U(b)} u_{bj}^p$ the total amount of undesirable output k produced by all processes of DMU j. Let also $P_U = \cup_b P_U(b)$ the set processes that generate undesirable outputs of any kind.

In addition to these exogenous inputs, final outputs and undesirable outputs, there exist R intermediate products internally generated and consumed. Thus, let $P^{\text{out}}(r)$ the set of processes that generate the intermediate product r and for each $p \in P^{\text{out}}(r)$ let z_{rj}^p the observed amount of intermediate product r generated by process p of DMU j. Also, let $P^{\text{in}}(r)$ the set of processes that consume the intermediate product r and for each $p \in P^{\text{in}}(r)$ let z_{rj}^p the observed amount of intermediate product r consumed by process p of DMU j. Finally, let $R^{\text{out}}(p)$ and $R^{\text{in}}(p)$ be the sets of the intermediate products produced and consumed, respectively, by a certain process p. Without loss of generality we may assume that

$$\sum_{p \in P^{\text{out}}(r)} z_{rj}^p = \sum_{p \in P^{\text{in}}(r)} z_{rj}^p \quad \forall r \ \forall j \tag{1}$$

i.e. the intermediate products consumed by a DMU are completely generated within the system.

In order to be able to formulate a network DEA model it is convenient to first establish the corresponding network technology, i.e. the network PPS, with all the feasible, i.e. attainable, (inputs, final outputs, undesirable outputs) combinations. To that end let us first consider the PPS of an individual process p. We should distinguish between processes which do not generate undesirable outputs and those that do. In the former case the PPS is given by

$$T_p = \left\{ (x_i^p, y_k^p, z_r^p) : \begin{array}{ll} \exists \lambda_j^p \in \Lambda_p \ \forall j \quad x_i^p \geqslant \sum_j \lambda_j^p x_{ij}^p \ \forall i \in I(p) & y_k^p \leqslant \sum_j \lambda_j^p y_{kj}^p \ \forall k \in O(p) \\ z_r^p \geqslant \sum_j \lambda_j^p z_{rj}^p \ \forall r \in R^{\text{in}}(p) & z_r^p \leqslant \sum_j \lambda_j^p z_{rj}^p \ \forall r \in R^{\text{out}}(p) \end{array} \right\}$$

where, as in conventional DEA, the set Λ_p represent the Returns to Scale (RTS) assumption for process p.

In the case of processes that generate undesirable outputs, i.e. $p \in P_U$, the corresponding PPS should incorporate the joint weak disposability of desirable (both final and intermediate products) and undesirable outputs, thus leading to

$$\widehat{T}_p = \left\{ (x_i^p, y_k^p, u_b^p, z_r^p) : \begin{array}{ll} \exists 0 \leqslant \theta_p \leqslant 1 \quad \exists \lambda_j^p \in \Lambda_p \ \forall j & x_i^p \geqslant \sum_j \lambda_j^p x_{ij}^p \ \forall i \in I(p) \\ y_k^p \leqslant \theta_p \cdot \sum_j \lambda_j^p y_{kj}^p \ \forall k \in O(p) & u_b^p = \theta_p \cdot \sum_j \lambda_j^p y_{kj}^p \ \forall b \in U(p) \\ z_r^p \geqslant \sum_j \lambda_j^p z_{rj}^p \ \forall r \in R^{\text{in}}(p) & z_r^p \leqslant \theta_p \cdot \sum_j \lambda_j^p z_{rj}^p \ \forall r \in R^{\text{out}}(p) \end{array} \right\}$$

Note that when there are undesirable outputs the variable θ_p is not needed (or equivalently it takes its maximum allowed value, i.e. $\theta_p = 1$) in the case of a Constant Returns to Scale (CRS) process (see e.g. [23,1]).

Using the PPS of the individual processes, the corresponding network PPS can be defined as

$$T = \left\{ (x_i, y_k, u_b) : \begin{array}{ll} \exists (x_i^p, y_k^p, u_b^p, z_r^p) \in \widehat{T}_p \ \forall p \in P_U & \exists (x_i^p, y_k^p, z_r^p) \in T_p \ \forall p \notin P_U \\ x_i \geqslant \sum_{p \in P_I(i)} x_{ij}^p \ \forall i & y_k \leqslant \sum_{p \in P_O(k)} y_{kj}^p \ \forall k \\ u_b = \sum_{p \in P_U(b)} u_{bj}^p \ \forall b & \\ \sum_{p \in P^{\text{out}}(r)} z_{rj}^p - \sum_{p \in P^{\text{in}}(r)} z_{rj}^p \geqslant 0 & \forall r \end{array} \right\}$$

4. Proposed directional distance approach

Once the network technology has been appropriately defined, in this section a directional distance network DEA approach is proposed. The Directional Distance Function (DDF) measures the distance from a certain operation point (e.g. DMU 0) to the weakly efficient subset of the network PPS along a given direction vector $\mathbf{g} = (g_i^x, g_k^y, g_b^u)$ [2]. It is, therefore, the largest step size that can be given along that direction from that operation point without abandoning the network PPS. The DDF approach is a common approach when undesirable outputs are present (e.g. [2,1,24–27], etc.) and it includes as particular cases the pure input and pure output orientations.

Because the resulting model is a Linear Program (LP) we will first formulate the proposed approach for the case that all processes exhibit CRS. Let β the DDF of DMU 0 along direction vector $\mathbf{g} = (g_i^x, g_k^y, g_b^u)$ which can be computed as

Max β

subject to

$$\sum_{p \in P_I(i)} \sum_j \lambda_j^p x_{\text{ij}}^p \leqslant x_{i0} - \beta \cdot g_i^x \quad \forall i \tag{3}$$

$$\begin{gathered} \sum_{p \in P_O(k)} \sum_j \lambda_j^p y_{\text{kj}}^p \geqslant y_{k0} + \beta \cdot g_k^y \quad \forall k \\ \sum_{p \in P_U(b)} \sum_j \lambda_j^p u_{\text{bj}}^p = u_{b0} - \beta \cdot g_b^u \quad \forall b \end{gathered} \tag{4}$$

$$\sum_{p \in P^{\text{out}}(r)} \sum_j \lambda_j^p z_{\text{rj}}^p - \sum_{p \in P^{\text{in}}(r)} \sum_j \lambda_j^p z_{\text{rj}}^p \geqslant 0 \quad \forall r \tag{5}$$

$$\lambda_j^p \geqslant 0 \quad \forall j \ \forall p \quad \beta \text{ free} \tag{6}$$

Constraints (3) and (4) respectively impose the corresponding exogenous input reductions and final outputs increases. For each input i, the left hand size of the corresponding constraint (3) computes the sum, for all the processes that consume that input, of the target input of the operation points of these processes. The corresponding right hand size relates the target total input consumption to the current input consumption thus bounding from below the maximum step size β that can be achieved along the direction given by vector $\mathbf{g} = (g_i^x, g_k^y, g_b^u)$. Similarly, for each output k, the left hand size of the corresponding constraint (4) computes the sum, for all the processes that produce that output, of the target output of the operation points of these processes. The corresponding right hand size relates the target total production of output k to the current production of output k, also bounding from below the maximum step size β that can be achieved along $\mathbf{g} = (g_i^x, g_k^y, g_b^u)$.

Constraints (5) impose, for each undesirable output, the possible reduction that can be obtained using that reduction, as before, to bound the maximum step size along direction $\mathbf{g} = (g_i^x, g_k^y, g_b^u)$. An important difference with the previous constraints is that, due to the weak disposability of undesirable outputs assumption, these constraints are equalities.

Finally, constraints (6) are global balance constraints imposing that the amount of each intermediate product produced in the system is sufficient to satisfy the amount of that intermediate product that is consumed. Thus, for each intermediate product, the first term in the constraint represents the sum of the target production of that intermediate product by the processes that produce it while the second terms computes the sum of the target consumption of that intermediate product by the processes that consume it.

The optimal solution to the above model, denoted with an asterisk superscript, gives the target operation point for each process p

$$\hat{x}_i^p = \sum_j \left(\lambda_j^p\right)^* x_{\mathrm{ij}}^p \quad \forall i \in I(p) \tag{7}$$

$$\hat{y}_k^p = \sum_j \left(\lambda_j^p\right)^* y_{\mathrm{kj}}^p \quad \forall k \in O(p) \tag{8}$$

$$\hat{u}_b^p = \sum_j \left(\lambda_j^p\right)^* u_{\mathrm{bj}}^p \quad \forall b \in U(p) \tag{9}$$

$$\hat{z}_r^p = \sum_j \left(\lambda_j^p\right)^* z_{\mathrm{rj}}^p \quad \forall r \in R^{\mathrm{in}}(p) \cup R^{\mathrm{out}}(p) \tag{10}$$

From the targets of the different processes, the targets for the system total exogenous inputs, final outputs and undesirable outputs can then be computed as

$$\hat{x}_i = \sum_{p \in P_I(i)} \hat{x}_i^p \leqslant x_{i0} = \sum_{p \in P_I(i)} x_{i0}^p \quad \forall i \tag{11}$$

$$\hat{y}_k = \sum_{p \in P_O(k)} \hat{y}_k^p \geqslant y_{k0} = \sum_{p \in P_O(k)} y_{k0}^p \quad \forall k \tag{12}$$

$$\hat{u}_b = \sum_{p \in P_U(b)} \hat{u}_{\prime b}^p \leqslant u_{b0} = \sum_{p \in P_U(b)} u_{b0}^p \quad \forall b \tag{13}$$

The above model assumes that CRS prevails for all processes. If that is not the case, the resulting model is generally a Non-Linear Program (NLP). In order to formulate this non-CRS case, let P_{CRS}, P_{VRS} and P_{NIRS} be the sets of processes with CRS, Variable Returns to Scale (VRS) and Non-Increasing Returns to Scale (NIRS) PPS respectively. The corresponding DDF network DEA model is

$$\mathrm{Max} \quad \beta \tag{14}$$

subject to

$$\sum_{p \in P_I(i)} \sum_j \lambda_j^p x_{\mathrm{ij}}^p \leqslant x_{i0} - \beta \cdot g_i^x \quad \forall i \tag{15}$$

$$\sum_{p \in P_O(k)} \theta_p \cdot \sum_j \lambda_j^p y_{\mathrm{kj}}^p \geqslant y_{k0} + \beta \cdot g_k^y \quad \forall k \tag{16}$$

$$\sum_{p \in P_U(b)} \theta_p \cdot \sum_j \lambda_j^p u_{\mathrm{bj}}^p = u_{b0} - \beta \cdot g_b^u \quad \forall b \tag{17}$$

$$\sum_{p \in P^{\mathrm{out}}(r)} \theta_p \cdot \sum_j \lambda_j^p z_{\mathrm{rj}}^p - \sum_{p \in P^{\mathrm{in}}(r)} \sum_j \lambda_j^p z_{\mathrm{rj}}^p \geqslant 0 \quad \forall r \tag{18}$$

$$\sum_j \lambda_j^p = 1 \quad \forall p \in P_{\mathrm{VRS}} \tag{19}$$

$$\sum_j \lambda_j^p \leqslant 1 \quad \forall p \in P_{\mathrm{NIRS}} \tag{20}$$

$$\theta_p = 1 \quad \forall p \notin P_U \tag{21}$$

$$\theta_p = 1 \quad \forall p \in (P_U \cap P_{\mathrm{CRS}}) \tag{22}$$

$$0 \leqslant \theta_p \leqslant 1 \quad \forall p \in (P_U \cap P_{\mathrm{VRS}}) \cup (P_U \cap P_{\mathrm{NIRS}}) \tag{23}$$

$$\lambda_j^p \geqslant 0 \quad \forall j \; \forall p \quad \beta \quad \text{free} \tag{24}$$

The interpretation of constraints (15)–(18) is similar to that of constraints (3)–(5) with the added feature of the variable θ_p that is required to account for the weak disposability of undesirable outputs. Although such variable is only required for those processes that generate undesirable outputs (i.e. $p \in P_U$) and only in the case that the Returns to Scale (RTS) of the process is not CRS, in order to simplify the formulation, a variable θ_p has been assumed for each process p, although, through constraints (21) and (22) the variable is set to unity for those processes for which it is not required. In other words, constraints (21) and (22) indicate that variable θ_p does not intervene for those processes that do not generate undesirable outputs or generate undesirable outputs but exhibit CRS. For the processes p that generate undesirable outputs and do not exhibit CRS, constraints (3) bound the variable θ_p below unity, consistent with the definition of the corresponding PPS $\widehat{T}_p$.

Constraints (19) and (20) impose the RTS constraints on the sum of the intensity variables of the different processes. Note that the above model reduces to model (2)–(6) in the case that all processes belong to P_{CRS} (i.e. $P_{\text{VRS}} = P_{\text{NIRS}} = \emptyset$).

After solving the above NLP, the target operation point for each process can be computed as

$$\hat{x}_i^p = \sum_j \left(\lambda_j^p\right)^* x_{\text{ij}}^p \quad \forall i \in I(p) \tag{25}$$

$$\hat{y}_k^p = \theta_p^* \cdot \sum_j \left(\lambda_j^p\right)^* y_{\text{kj}}^p \quad \forall k \in O(p) \tag{26}$$

$$\hat{u}_b^p = \theta_p^* \cdot \sum_j \left(\lambda_j^p\right)^* u_{\text{bj}}^p \quad \forall b \in U(p) \tag{27}$$

$$\hat{z}_r^p = \theta_p^* \cdot \sum_j \left(\lambda_j^p\right)^* z_{\text{rj}}^p \quad \forall r \in R^{\text{out}}(p) \tag{28}$$

$$\hat{z}_r^p = \sum_j \left(\lambda_j^p\right)^* z_{\text{rj}}^p \quad \forall r \in R^{\text{in}}(p) \tag{29}$$

leading to the corresponding system targets as per (11)–(13).

5. Application to airports efficiency assessment

The motivation for this paper is the application of network DEA to modelling airport operations where in addition to the desirable outputs (Annual Passenger Movement and Total Cargo handled) there are also some undesirable outputs (Number of Delayed Flights and Accumulated Flight Delays). There are many airports benchmarking studies that have used DEA (e.g. [28–34]). However almost all these DEA studies consider a DMU as a single process. An exception is Yu [16] which presents a SBM network DEA approach to airports operations, although it does not consider undesirable outputs. The undesirable outputs considered in DEA applications to airports are aircraft noise [24,27] and airplanes delays [26,35]. Taking into account the undesirable effects of airport operations not only increases the realism of the analysis but also contributes to a fairer performance assessment. Thus, when undesirable outputs are ignored, DEA models tend to label as efficient those airports with a higher activity level, some of which may be oversaturated and causing excessive pollution, noise and inconveniences to passengers. When such saturated airports are considered as efficient then all airports are projected using them as benchmarks which means that the targets thus computed would also suffer from those drawbacks. It makes more sense to include undesirable outputs in the DEA model, provided the data are available, in which case the DEA model will not only reduce the inputs and increase the desirable outputs but also reduce the undesirable outputs thus detecting inefficiencies with respect to the latter variables. Thus, for an airport to be considered efficient it must happen that its undesirable outputs level cannot

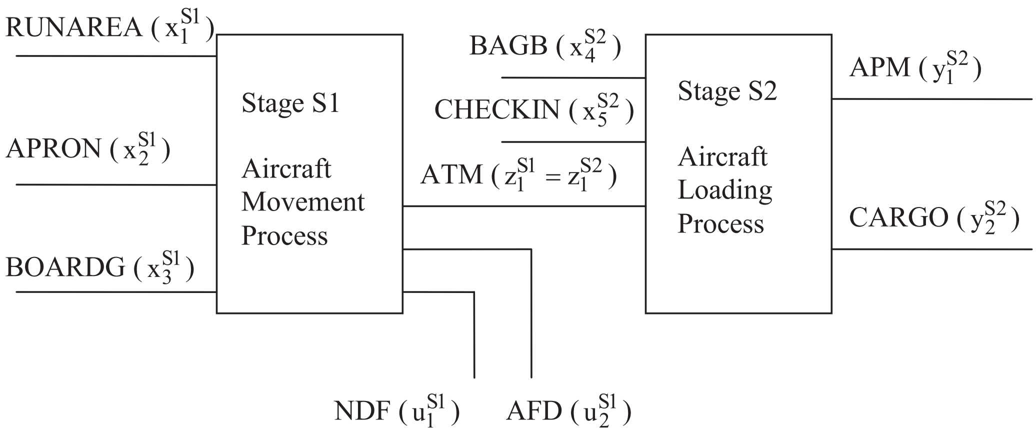

Fig. 1. DMU as two-stage system.

Table 1
Inputs and outputs (desirable and undesirable) with their abbreviations.

	Variable	Units	Label
Inputs	Total runway area	Square meters	RUNAREA
	Apron capacity	Number of stands	APRON
	Number of boarding gates	Number of gates	BOARDG
	Number of baggage belts	Number of belts	BAGB
	Number of check-in counters	Number of counters	CHECKIN
Intermediate product	Aircraft Traffic Movements	Thousand operations	ATM
Outputs (desirable)	Annual Passenger Movements	Thousand passengers	APM
	Cargo handled	Tonnes	CARGO
Outputs (undesirable)	Number of Delayed Flights	Number of flights	NDF
	Accumulated Flight Delays	Min	AFD

Table 2
Observed data for 39 Spanish airports for year 2008.

Airport	IATA	RUNAREA	APRON	BOARDG	ATM	NDF	AFD	CHECKIN	BAGB	APM	CARGO
A Coruña	LCG	87,300	5	4	17.719	1218	23783.4	10	3	1174.970	283.571
Albacete	ABC	162,000	2	2	2.113	58	1376.5	4	1	19.254	8.924
Alicante	ALC	135,000	31	16	81.097	7642	142445.8	42	9	9578.304	5982.313
Almeria	LEI	144,000	15	5	18.280	1114	20149.1	17	4	1024.303	21.322
Asturias	OVD	99,000	7	9	18.371	1310	23893.5	11	3	1530.245	139.465
Badajoz	BJZ	171,000	1	2	4.033	137	2365.4	4	1	81.010	0.000
Barcelona	BCN	475,020	121	65	321.693	33,036	645924.6	143	19	30272.084	103996.489
Bilbao	BIO	207,000	21	12	61.682	4592	80848.2	36	7	4172.903	3178.758
Cordoba	ODB	62,100	23	1	9.604	14	254.4	1	0	22.230	0.000
El Hierro	VDE	37,500	3	2	4.775	27	641.6	5	1	195.425	171.717
Fuerteventura	FUE	153,000	34	10	44.552	3920	72179.7	34	8	4492.003	2722.661
Girona-Costa Brava	GRO	108,000	17	7	49.927	4992	100305.6	18	3	5510.970	184.127
Gran Canaria	LPA	139,500	55	38	116.252	7463	136380.7	86	19	10212.123	33695.248
Granada-Jaen	GRX	134,550	11	3	19.279	951	17868.8	12	3	1422.014	66.889
Ibiza	IBZ	126,000	25	12	57.233	6193	152840.1	48	8	4647.360	3928.387
Jerez	XRY	103,500	9	5	50.551	1174	19292.2	13	3	1303.817	90.428
La Gomera	GMZ	45,000	3	2	3.393	17	420.7	5	1	41.890	7.863
La Palma	SPC	99,000	5	5	20.109	423	8286.0	13	2	1151.357	1277.264
Lanzarote	ACE	108,000	24	16	53.375	5104	101685.6	49	8	5438.178	5429.589
Leon	LEN	94,500	5	2	5.705	442	7191.5	3	1	123.183	15.979
Madrid Barajas	MAD	927,000	263	230	469.746	52,526	908360.0	484	53	50846.494	329186.631
Malaga	AGP	144,000	43	30	119.821	15,548	277663.8	85	16	12813.472	4800.271
Melilla	MLN	64,260	5	2	10.959	218	2979.6	4	1	314.643	386.340
Murcia	MJV	138,000	5	5	19.339	1344	24103.1	18	4	1876.255	2.730
Palma de Mallorca	PMI	295,650	86	68	193.379	26,038	501486.0	204	16	22832.857	21395.791
Pamplona	PNA	99,315	7	2	12.971	666	11691.8	4	1	434.477	52.942
Reus	REU	110,475	5	5	26.676	943	18240.8	8	3	1278.074	119.848
Salamanca	SLM	150,000	6	2	12.450	427	6626.1	4	2	60.103	0.000
San Sebastian	EAS	78,930	6	3	12.282	713	11184.0	6	2	403.191	63.791
Santander	SDR	104,400	8	5	19.198	1004	17842.0	8	2	856.606	37.482
Santiago	SCQ	144,000	16	12	21.945	2007	34322.3	19	5	1917.466	2418.798
Saragossa	ZAZ	302,310	12	3	14.584	1095	19547.6	6	2	594.952	21438.894
Seville	SVQ	151,200	23	10	65.067	2567	51084.9	42	6	4392.148	6102.264
Tenerife North	TFN	153,000	16	16	67.800	1783	32637.0	37	5	4236.615	20781.674
Tenerife South	TFS	144,000	44	22	60.779	5254	110818.9	87	14	8251.989	8567.093
Valencia	VLC	144,000	35	18	96.795	4998	102719.2	42	8	5779.343	13325.799
Valladolid	VLL	180,000	7	5	13.002	843	14760.6	8	2	479.689	34.650
Vigo	VGO	108,000	8	6	17.934	1535	25593.6	12	3	1278.762	1481.939
Vitoria	VIT	157,500	18	3	12.225	669	11585.8	7	2	67.818	34989.727

be reduced without sacrificing output. Otherwise, if its undesirable outputs level is excessive the airport will be labeled as inefficient and potential improvements will be computed.

In order to carry out a network DEA approach to airports two processes can be distinguished: one related to the movement of the aircrafts (and therefore aimed at maximizing its throughput given the available resources) and a second one related to the airplanes load factors (both in terms of passengers and cargo). The distinction between these two processes has already been considered in the literature. Thus, Gillen and Lall [28,36] and Pels et al. [29,30] present the results of two separate DEA models each. Gillen and Lall [28,36] call them Movements (or Airside) and Terminal services (passengers and cargo) and do not establish any links between both models except their sharing some inputs. Pels et al. [30] refer to the two DEA models as an airport model (oriented to aircraft movement) and an airlines model (oriented towards selling aircraft

Table 3
Results of proposed DDF network DEA approach.

Airport	β^*	θ^*_{S1}	ATM^*_0	NDF^*_0	AFD^*_0	APM^*_0	$CARGO^*_0$	$\Delta\ ATM^*_0$	$\Delta\ NDF^*_0$	$\Delta\ AFD^*_0$	$\Delta\ APM^*_0$	$\Delta\ CARGO^*_0$
A Coruña	0.648	1.000	18.5	429	8382.1	1935.8	533.3	0.7	−789	−15401.3	760.9	249.8
Albacete	0.000	1.000	2.1	58	1376.5	19.3	8.9	0.0	0	0.0	0.0	0.0
Alicante	0.014	1.000	85.1	7537	140497.8	9709.3	8677.9	4.0	−105	−1948.0	131.0	2695.6
Almeria	0.786	1.000	16.0	238	4310.9	1829.5	1534.2	−2.3	−876	−15838.2	805.2	1512.9
Asturias	0.626	1.000	23.4	490	8940.0	2487.9	381.5	5.0	−820	−14953.5	957.7	242.1
Badajoz	0.197	0.803	3.2	110	1898.7	97.0	11.6	−0.8	−27	−466.7	16.0	11.6
Barcelona	0.000	1.000	321.7	33,036	645924.6	30272.1	103996.5	0.0	0	0.0	0.0	0.0
Bilbao	0.676	1.000	59.5	1490	26231.5	6991.9	5326.2	−2.2	−3102	−54616.7	2819.0	2147.4
Cordoba	0.000	1.000	9.6	14	254.4	22.2	0.0	0.0	0	0.0	0.0	0.0
El Hierro	0.081	0.919	4.4	25	589.7	211.2	185.6	−0.4	−2	−51.9	15.8	13.9
Fuerteventura	0.565	1.000	59.6	1706	31409.9	7029.3	4534.3	15.1	−2214	−40769.8	2537.3	1811.6
Girona-Costa Brava	0.000	1.000	49.9	4992	100305.6	5511.0	184.1	0.0	0	0.0	0.0	0.0
Gran Canaria	0.171	1.000	104.7	6186	113043.8	11959.6	39461.1	−11.6	−1277	−23336.9	1747.5	5765.8
Granada-Jaen	0.529	1.000	20.1	448	8424.6	2173.6	1120.6	0.8	-503	−9444.2	751.6	1053.7
Ibiza	0.223	1.000	45.2	4813	118783.7	5682.9	4803.7	−12.0	−1380	−34056.4	1035.5	875.3
Jerez	0.673	1.000	19.9	384	6310.9	2181.1	1,387.4	−30.7	−790	−12981.3	877.3	1297.0
La Gomera	0.264	0.736	2.5	13	309.7	52.9	9.9	−0.9	−4	−110.9	11.1	2.1
La Palma	0.329	1.000	14.9	284	5563.9	1529.6	1696.9	−5.2	−139	−2722.1	378.2	419.6
Lanzarote	0.409	1.000	64.3	3014	60047.7	7665.0	7652.9	10.9	−2090	−41637.9	2226.8	2223.3
Leon	0.799	1.000	7.3	89	1446.3	221.6	28.7	1.6	−353	−5745.2	98.4	12.8
Madrid Barajas	0.000	1.000	469.7	52,526	908360.0	50846.5	329186.6	0.0	0	0.0	0.0	0.0
Malaga	0.060	1.000	117.5	14,621	261116.1	13577.1	13491.8	−2.3	−927	−16547.7	763.6	8691.5
Melilla	0.281	0.719	7.9	157	2141.2	403.2	495.0	−3.1	−61	−838.4	88.5	108.7
Murcia	0.522	1.000	25.0	643	11529.9	2855.0	1966.1	5.7	−701	−12573.3	978.7	1963.4
Palma de Mallorca	0.000	1.000	193.4	26,038	501486.0	22832.9	21395.8	0.0	0	0.0	0.0	0.0
Pamplona	0.450	1.000	11.0	366	6430.7	630.0	76.8	−2.0	−300	−5261.1	195.5	23.8
Reus	0.515	1.000	20.8	458	8851.9	1935.9	181.5	−5.9	−485	−9389.0	657.9	61.7
Salamanca	0.910	0.569	3.5	38	596.6	114.8	12.7	−9.0	−389	−6029.5	54.7	12.7
San Sebastian	0.840	1.000	8.4	114	1784.1	742.1	117.4	−3.9	−599	−9399.9	338.9	53.6
Santander	0.744	1.000	14.9	257	4564.7	1494.1	155.9	−4.3	−747	−13277.4	637.5	118.4
Santiago	0.707	1.000	29.2	588	10059.7	3272.9	4128.7	7.2	−1419	−24262.7	1355.5	1709.9
Saragossa	0.000	1.000	14.6	1095	19547.6	595.0	21438.9	0.0	0	0.0	0.0	0.0
Seville	0.358	1.000	51.5	1647	32781.1	5965.9	8288.7	−13.6	−920	−18303.7	1573.7	2186.4
Tenerife North	0.262	1.000	52.2	1315	24076.6	5347.8	26232.6	−15.6	−468	−8560.5	1111.2	5450.9
Tenerife South	0.234	1.000	80.8	4025	84890.9	10182.7	10571.5	20.0	−1229	−25928.1	1930.7	2004.4
Valencia	0.404	1.000	72.4	2979	61218.3	8114.3	18709.7	−24.4	−2019	−41500.9	2335.0	5383.9
Valladolid	0.862	1.000	9.2	117	2042.1	893.0	590.7	−3.8	−726	−12718.5	413.3	556.0
Vigo	0.731	1.000	20.9	412	6876.5	2213.9	2565.7	3.0	−1123	−18717.1	935.2	1083.8
Vitoria	0.000	1.000	12.2	669	11585.8	67.8	34989.7	0.0	0	0.0	0.0	0.0

seats). They go as far as stating Aircraft Traffic Movements can be considered as an intermediate good that is 'produced' and then 'consumed' in the production of Annual Passenger Movements. That is exactly what we have done in this paper.

Hence, the intermediate product linking the two processes considered in this paper is the variable ATM that represents the aircraft movements in and out of the airport. That the movement of aircraft is considered an intermediate product instead of a final output makes sense since the movement of aircraft (in civilian airports of course) is not an end in itself but a means of providing the service of air transporting people and goods. This intermediate product is therefore produced in stage S1 and "consumed" in stage S2. As a consequence of the aircraft traffic generating unwanted flight delays, stage S1 also generates two undesirable outputs. The final outputs are produced in stage S2. Thus, we propose the two-stage network shown in Fig. 1. Table 1 lists the definition and units of measurement of the different variables. Note that a specific feature of this DEA model is that all the inputs are non-discretionary. This means that the DEA model used should not aim at reducing them below their current, observed levels (see [37]).

Table 2 shows the dataset for 39 Spanish airports for year 2008. The data have been obtained from the Spanish Airport and Air Navigation Authority (AENA, http://www.aena.es/csee/Satellite?pagename=Estadisticas/Home, last accessed 2012–03-30) except the flights delays data which have been kindly provided by the CODA (Central Office for Delay Analysis) service of Eurocontrol (http://www.eurocontrol.int/eatm/public/standard_page/coda.html, last accessed 2012–03-30). As the sample includes airports of very different sizes and since the airports do not operate in a perfectly competitive market (something that could guarantee that they have optimal scale sizes), it is safer to assume that the two processes have VRS.

Note that, although in general, as discussed in the previous section, when the processes have RTS different to CRS, the proposed approach leads to an NLP model, in this specific application the model can be linearized as shown in the Appendix. This is due to two concurring facts. One is that there is only one process that generates undesirable outputs (i.e. $P_U = \{S1\}$) and the other is that the exogenous inputs consumed in that process are not considered as inputs to the other process. Note also that since all the inputs are non-discretionary the corresponding components of the direction vector $\mathbf{g} = (g_i^x, g_k^y, g_b^u)$ should be zero, i.e. $\mathbf{g} = (0, g_k^y, g_b^u)$. As for the components corresponding to the final and the undesirable outputs, we have

Table 4
Results of DDF network DEA approach with non-worsening-ATM constraint.

Airport	β^*	θ^*_{S1}	ATM^*_0	NDF^*_0	AFD^*_0	APM^*_0	$CARGO^*_0$	$\Delta\ ATM^*_0$	$\Delta\ NDF^*_0$	$\Delta\ AFD^*_0$	$\Delta\ APM^*_0$	$\Delta\ CARGO^*_0$
A Coruña	0.648	1.000	18.5	429	8382.1	1935.8	533.3	0.7	−789	−15401.3	760.9	249.8
Albacete	0.000	1.000	2.1	58	1376.5	19.3	8.9	0.0	0	0.0	0.0	0.0
Alicante	0.014	1.000	85.1	7537	140497.8	9709.3	8677.9	4.0	−105	−1948.0	131.0	2695.6
Almeria	0.731	1.000	18.3	299	5417.0	1773.2	76.2	0.0	−815	−14732.1	748.9	54.9
Asturias	0.626	1.000	23.4	490	8940.0	2487.9	381.5	5.0	−820	−14953.5	957.7	242.1
Badajoz	0.000	1.000	4.0	137	2365.4	81.0	79.2	0.0	0	0.0	0.0	0.0
Barcelona	0.000	1.000	321.7	33,036	645924.6	30272.1	103996.5	0.0	0	0.0	0.0	0.0
Bilbao	0.627	1.000	61.7	1713	30160.0	6789.1	5171.7	0.0	−2879	−50688.2	2616.2	1992.9
Cordoba	0.000	1.000	9.6	14	254.4	22.2	0.0	0.0	0	0.0	0.0	0.0
El Hierro	0.000	1.000	4.8	27	641.6	195.4	171.7	0.0	0	0.0	0.0	0.0
Fuerteventura	0.565	1.000	59.6	1706	31409.9	7029.3	4534.3	15.1	−2214	−40769.8	2537.3	1811.6
Girona-Costa Brava	0.000	1.000	49.9	4992	100305.6	5511.0	184.1	0.0	0	0.0	0.0	0.0
Gran Canaria	0.000	1.000	116.3	7463	136380.7	10212.1	33695.3	0.0	0	0.0	0.0	0.0
Granada-Jaen	0.529	1.000	20.1	448	8424.6	2173.6	1120.6	0.8	−503	−9444.2	751.6	1053.7
Ibiza	0.000	1.000	57.2	6193	152840.1	4647.4	3928.4	0.0	0	0.0	0.0	0.0
Jerez	0.000	1.000	50.6	1174	19292.2	1303.8	90.4	0.0	0	0.0	0.0	0.0
La Gomera	0.000	1.000	3.4	17	420.7	41.9	7.9	0.0	0	0.0	0.0	0.0
La Palma	0.000	1.000	20.1	423	8,286.0	1151.4	1277.3	0.0	0	0.0	0.0	0.0
Lanzarote	0.409	1.000	64.3	3014	60047.7	7665.0	7652.9	10.9	−2,090	−41637.9	2226.8	2223.3
Leon	0.799	1.000	7.3	89	1446.3	221.6	28.7	1.6	−353	−5745.2	98.4	12.8
Madrid Barajas	0.000	1.000	469.7	52,526	908360.0	50846.5	329186.6	0.0	0	0.0	0.0	0.0
Malaga	0.000	1.000	119.8	15,548	277663.8	12813.5	4800.3	0.0	0	0.0	0.0	0.0
Melilla	0.000	1.000	11.0	218	2979.6	314.6	386.3	0.0	0	0.0	0.0	0.0
Murcia	0.522	1.000	25.0	643	11529.9	2855.0	1966.1	5.7	−701	−12573.3	978.7	1,963.4
Palma de Mallorca	0.000	1.000	193.4	26,038	501486.0	22832.9	21395.8	0.0	0	0.0	0.0	0.0
Pamplona	0.000	1.000	13.0	666	11691.8	434.5	52.9	0.0	0	0.0	0.0	0.0
Reus	0.000	1.000	26.7	943	18240.8	1278.1	119.8	0.0	0	0.0	0.0	0.0
Salamanca	0.000	1.000	12.5	427	6626.1	111.3	6.3	0.0	0	0.0	0.0	0.0
San Sebastian	0.692	1.000	12.3	219	3439.9	682.4	108.0	0.0	−494	−7744.1	279.2	44.2
Santander	0.631	1.000	19.2	371	6586.4	1397.0	61.1	0.0	−633	−11255.7	540.4	23.6
Santiago	0.707	1.000	29.2	588	10059.7	3272.9	4128.7	7.2	−1419	−24262.7	1355.5	1709.9
Saragossa	0.000	1.000	14.6	1095	19547.6	595.0	21438.9	0.0	0	0.0	0.0	0.0
Seville	0.004	1.000	65.1	2556	50859.1	4411.6	6129.2	0.0	−11	−225.8	19.4	27.0
Tenerife North	0.000	1.000	67.8	1783	32637.0	4236.6	20781.7	0.0	0	0.0	0.0	0.0
Tenerife South	0.234	1.000	80.8	4025	84890.9	10182.7	10571.5	20.0	−1229	−25928.1	1930.7	2004.4
Valencia	0.000	1.000	96.8	4998	102719.2	5779.3	13325.8	0.0	0	0.0	0.0	0.0
Valladolid	0.746	1.000	13.0	214	3743.8	837.7	60.5	0.0	−629	−11016.8	358.0	25.9
Vigo	0.731	1.000	20.9	412	6876.5	2213.9	2565.7	3.0	−1123	−18717.1	935.2	1083.8
Vitoria	0.000	1.000	13.5	669	11585.8	67.8	34989.7	1.3	0	0.0	0.0	0.0

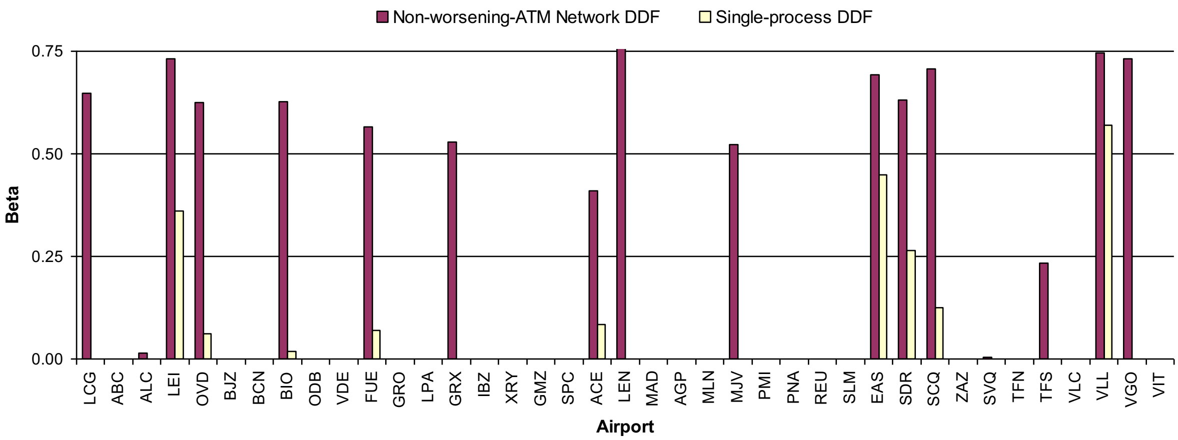

Fig. 2. Proposed network DDF approach versus single-process DDF approach.

adopted a proportional directional distance vector $\mathbf{g} = (0, y_{k0}, u_{b0})$. This means that the computed optimal step size β^* can be interpreted as the percentage that all output variables can be simultaneously improved, where improvement means reduction in the case of undesirable outputs and increase in the case of desirable outputs.

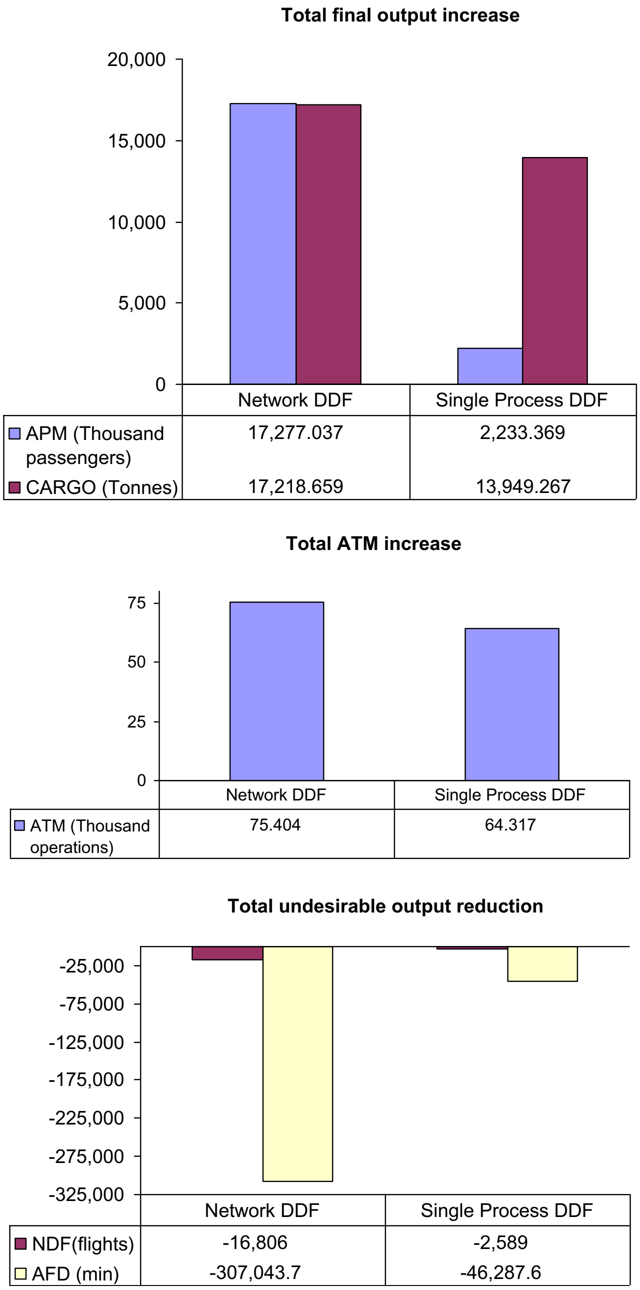

Fig. 3. Overall results.

Table 3 shows the optimal step size β^* computed by the proposed approach together with the optimal value of θ^*_{S1} and the output values for the computed targets and the change those targets represent from the corresponding observed values. Note that there are seven airports for which $\beta^* = 0$. Note also that since the variable ATM is considered in principle as an intermediate output the only requirement that the model imposes on the corresponding target value is just that enough of this intermediate product is produced in stage S1 to satisfy the amount consumed in stage S2. This means that it is possible for some DMUs that the target for this variable be lower that the observed value, i.e. $\mathrm{ATM}^*_0 < \mathrm{ATM}_0$. This means that the optimal value of that variable, i.e. the one that leads to the largest proportional increase in the final outputs and simultaneous proportional decrease of the undesirable outputs requires sometimes that the level of the aircraft movement operations be downscaled (thus reducing congestion), downscaling that is more than compensated with an improvement in the efficiency of the aircraft loading operations, so that in the end APM and CARGO increase and NDF and AFD decrease.

Since voluntarily reducing the level of aircraft operations is a recommendation that, even when optimal, may not be agreeable to many airport managers and as an example of the flexibility of the proposed approach we have also solved the model imposing the additional constraint that the target of variable ATM cannot be lower than the observed value, i.e. $\mathrm{ATM}_0^* \geqslant \mathrm{ATM}_0$. In this way, the target values not only of APM and CARGO but also of ATM are guaranteed to be higher than the observed values (or at least stay the same) while the target values for NDF and AFD are guaranteed to be lower than the observed values (or at least stay the same, if). Note that, because of a reduction in the feasibility region, imposing the above constraint always leads to a smaller value, which means that smaller improvements are achieved for the final and the undesirable outputs. In particular, the number of airports for which $\beta^* = 0$ has increased to 22, which is more than half of the airports. For those airports for which improvements are feasible, the magnitude of the estimated improvements is significant, as indicated by the relatively large β^* values.

In order to compare the proposed network DEA approach with the conventional (i.e. single-stage) DDF approach and to be consistent with the assumptions of Table 4 the direction vector that has been considered has a zero value for the component corresponding to ATM output, i.e. $g^y_{\mathrm{ATM}} = 0$. Fig. 2 shows the corresponding values of the two approaches. It can be seen that the number of weakly efficient DMUs increases (from 22 to 30) when DMUs are modelled as single processes and that the improvement possibilities are much reduced. Thus, $\beta^*_{\mathrm{NETWORK}} \geqslant \beta^*_{\mathrm{SINGLE\ PROCESS}}$ for all DMUs with average β^* decreasing from 0.236 to 0.051 and the average $(\beta^*|\beta^* > 0)$ decreasing from 0.542 to 0.223. The most important conclusion that can be extracted from these results is that the single-process conventional DEA approach has less discriminatory power that the network DEA approach.

Finally, the overall potential improvements (i.e. aggregated for the whole Spanish airport system) uncovered by the proposed network DDF approach and by the single-process DDF approach are presented in Fig. 3 which shows the total improvements in the final outputs (APM and CARGO) and in the intermediate product (ATM) as well as the reductions in the two undesirable outputs (NFD and AFD). These aggregated improvements are of interest because the 39 airports are under the responsibility of AENA, which in addition to holding the managers of the different airports accountable of its performance can also look for system-wide improvements. It can be noted that both approaches estimate almost equivalent aggregated APM increases but the network DEA approach leads to a much bigger increase in CARGO and much bigger reductions in the flight delays than the single-process DEA approach, and this without increasing ATM much more.

6. Conclusions

In this paper a network DEA approach considering that some or all of the processes generate undesirable outputs has been proposed. The proposed approach is based on the definition of the network technology resulting from the composition of the PPS of the individual process, some of which are assumed to produce undesirable outputs. The proposed approach has been applied to the benchmarking of airports operations, modelled as two serial stages corresponding to aircrafts movements and aircrafts loading respectively. The aircrafts movements stage is the one that generates the undesirable outputs related to flights delays.

Using a dataset of 39 Spanish airports for year 2008 the corresponding DDF efficiency scores have been computed and compared with those of the corresponding single-process DDF approach leading to the important conclusion that the single-process conventional DEA approach has less discriminatory power that the network DEA approach. In other words, the network DEA approach uncovers much larger inefficiencies in the current operation points than does the single-process approach. This translates in that the network DEA approach is able to compute much larger improvements (especially for CARGO and for the two undesirable outputs) than the single-process approach. In addition, our claim is that the network DEA analysis is more valid because it represents a more fine-grained approach than just considering all inputs and outputs ascribed to a single, aggregated process.

About possible continuations of this research we may mention that since the DDF approach only guarantees weakly efficient targets, an SBM-like approach (like that of [11] or [12]) could be used to handle the undesirable outputs considered in this paper albeit possibly leading to a slightly more complex model formulation. Another interesting line of research is that of using the DDF approach proposed in this paper with panel data to compute the equivalent to Malmquist–Luenberger Productivity indicators for network DEA systems.

Acknowledgments

This research was carried out with the financial support of the Spanish Ministry of Science grant DPI2010–16201, and FEDER. The authors would also like to thank Eurocontrol for providing the data on flights delays.

Appendix A

The particular DDF network DEA model for the airports benchmarking application presented in Section 5 is:

$$\text{Max} \quad \beta \tag{30}$$

subject to

$$\sum_j \lambda_j^{S1} \cdot \text{RUNAREA}_j \leqslant \text{RUNAREA}_0 \tag{31}$$

$$\sum_j \lambda_j^{S1} \cdot \text{APRON}_j \leqslant \text{APRON}_0 \tag{32}$$

$$\sum_j \lambda_j^{S1} \cdot \text{BOARDG}_j \leqslant \text{BOARDG}_0 \tag{33}$$

$$\theta_{S1} \cdot \sum_j \lambda_j^{S1} \cdot \text{ATM}_j - \sum_j \lambda_j^{S2} \cdot \text{ATM}_j \geqslant 0 \tag{34}$$

$$\theta_{S1} \cdot \sum_j \lambda_j^{S1} \cdot \text{NDF}_j = \text{NDF}_0 \cdot (1-\beta) \tag{35}$$

$$\theta_{S1} \cdot \sum_j \lambda_j^{S1} \cdot \text{AFD}_j = \text{AFD}_0 \cdot (1-\beta) \tag{36}$$

$$\sum_j \lambda_j^{S2} \cdot \text{CHECKIN}_j \leqslant \text{CHECKIN}_0 \tag{37}$$

$$\sum_j \lambda_j^{S2} \cdot \text{BAGB}_j \leqslant \text{BAGB}_0 \tag{38}$$

$$\sum_j \lambda_j^{S2} \cdot \text{APM}_j \geqslant \text{APM}_0 \cdot (1+\beta) \tag{39}$$

$$\sum_j \lambda_j^{S2} \cdot \text{CARGO}_j \geqslant \text{CARGO}_0 \cdot (1+\beta) \tag{40}$$

$$\sum_j \lambda_j^{S1} = 1 \tag{41}$$

$$\sum_j \lambda_j^{S2} = 1 \tag{42}$$

$$0 \leqslant \theta_{S1} \leqslant 1 \tag{43}$$

$$\lambda_j^{S1} \geqslant 0 \quad \lambda_j^{S2} \geqslant 0 \quad \beta \text{ free} \tag{44}$$

This model takes into account that only the process S1 generates undesirable outputs, that only process S2 generates desirable outputs and that both processes are VRS. Note also that, as commented in section 5, a direction vector $\mathbf{g} = (0, g_k^y, g_b^u)$ has been considered. Hence, the step size β is bounded from above only by the desirable and undesirable outputs potential improvements.

The above NLP can be linearized defining new variables $\hat{\lambda}_j^{S1} = \theta_I \cdot \lambda_j^{S1}$. This allows substituting the corresponding constraints involving λ_j^I by these new all-linear ones:

$$\sum_j \hat{\lambda}_j^{S1} \cdot \text{RUNAREA}_j \leqslant \theta_{S1} \cdot \text{RUNAREA}_0 \tag{45}$$

$$\sum_j \hat{\lambda}_j^{S1} \cdot \text{APRON}_j \leqslant \theta_{S1} \cdot \text{APRON}_0 \tag{46}$$

$$\sum_j \hat{\lambda}_j^{S1} \cdot \text{BOARDG}_j \leqslant \theta_{S1} \cdot \text{BOARDG}_0 \tag{47}$$

$$\sum_j \hat{\lambda}_j^{S1} \cdot \text{ATM}_j - \sum_j \lambda_j^{S2} \cdot \text{ATM}_j \geqslant 0 \tag{48}$$

$$\sum_j \hat{\lambda}_j^{S1} \cdot \text{NDF}_j = \text{NDF}_0 \cdot (1-\beta) \tag{49}$$

$$\sum_j \hat{\lambda}_j^{S1} \cdot \text{AFD}_j = \text{AFD}_0 \cdot (1-\beta) \tag{50}$$

$$\sum_j \hat{\lambda}_j^{S1} = \theta_{S1} \tag{51}$$

$$\hat{\lambda}_j^{S1} \geqslant 0 \quad \lambda_j^{S2} \geqslant 0 \quad \beta \text{ free} \tag{52}$$

The corresponding final and undesirable outputs system targets can be computed as

$$\mathrm{NDF}_0^* = \sum_j \left(\hat{\lambda}_j^{S1}\right)^* \cdot \mathrm{NDF}_j = \mathrm{NDF}_0 \cdot (1 - \beta^*) \tag{53}$$

$$\mathrm{AFD}_0^* = \sum_j \left(\hat{\lambda}_j^{S1}\right)^* \cdot \mathrm{AFD}_j = \mathrm{AFD}_0 \cdot (1 - \beta^*) \tag{54}$$

$$\mathrm{APM}_0^* = \sum_j \left(\lambda_j^{S2}\right)^* \cdot \mathrm{APM}_j \geqslant \mathrm{APM}_0 \cdot (1 + \beta^*) \tag{55}$$

$$\mathrm{CARGO}_0^* = \sum_j \left(\lambda_j^{S2}\right)^* \cdot \mathrm{CARGO}_j \geqslant \mathrm{CARGO}_0 \cdot (1 + \beta^*) \tag{56}$$

and for the key intermediate product the computed target is

$$\mathrm{ATM}_0^* = \sum_j \left(\hat{\lambda}_j^{S1}\right)^* \cdot \mathrm{ATM}_j = \theta_{S1}^* \cdot \sum_j \left(\lambda_j^{S1}\right)^* \cdot \mathrm{ATM}_j \tag{57}$$

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Chapter 10
Network Fuzzy Data Envelopment Analysis

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Abstract In this chapter a general approach to handle fuzzy data when the units under analysis are formed by a network of processes is presented. Conventional DEA assumes a single-process that consumes all the different inputs and produces all the different outputs. Network DEA, on the contrary, considers different interrelated processes, each one with its own inputs, its own outputs and, very important, its own technology. This allows a more fine-grained analysis although at the expense of requiring more data. Conventional Network DEA approaches assume crisp data although recently two proposals have been made that can process fuzzy data in the special cases of a serial two-stage system and of parallel production processes. There is, however, a need to deal with general networks of processes which can have fuzzy input or output data. In this chapter, several Fuzzy DEA approaches are extended to Network DEA. The resulting models are illustrated on a dataset from the literature.

Keywords Efficiency assessment · Network DEA · Fuzzy data

1 Introduction

Network DEA refers to a growing number of DEA approaches that instead of assuming that all inputs and outputs are consumed and produced, respectively, by a single process, the system can be modelled as formed by distinct sub-processes. Each sub-process is in itself a process that consumes a subset of the inputs and

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A. Emrouznejad and M. Tavana (eds.), *Performance Measurement with Fuzzy Data Envelopment Analysis*, Studies in Fuzziness and Soft Computing 309,
DOI: 10.1007/978-3-642-41372-8_10,

produces a subset of the outputs. A feature of Network DEA is the possible existence of intermediate products that are produced by a process and consumed by another. These endogenously generated-and-consumed products represent the links between the different stages or processes. Another key feature of Network DEA is that the technology, i.e. the production possibility set, is modelled at the process level. Thus, each process will have its own technology, with, for example, its own returns to scale assumption.

The number of Network DEA approaches has grown in the last decade, both at the theory and at the application level. Seminal works were those of Färe and Grosskopf [3, 4], Sexton and Lewis [24] and Lewis and Sexton [17, 18]. The field, however, did not gain momentum until the relational network DEA approach of Kao and Hwang [12, 13] and Kao [10, 11] as well as the Network SBM approach of Tone and Tsutsui [25]. There are other Network DEA approaches like the weighted additive efficiency decomposition approach of Chen et al. [5] and Cook et al. [6] or the Network Slacks-Based Inefficiency (NSBI) approach of Fukuyama and Weber [7] as well as many applications in different sectors, such as manufacturing (e.g. [19]), supply chain management [1], transportation (e.g. [27]), tourism (e.g. [28]), finance (e.g. [2]), management (e.g. [8]), education (e.g. 22) and sports (e.g. [21]).

Although, in principle, different technologies, metrics and orientations can be used in Network DEA, for the sake of simplicity we will assume a Variable Returns to Scale (VRS) radial, input-oriented approach. First we will present the crisp data model formulation but, before, it is important to introduce appropriate notation (see [20]). Thus, let us assume that there exist n DMUs all of which are structurally homogeneous, i.e. all of them have the same number and type of processes. Each process consumes a different subset of inputs and produces a different subset of outputs. Let $I(p)$ be the set of exogenous inputs used in process p and, for each $i \in I(p)$, let x_{ij}^{p} denote the observed amount of exogenous input i consumed by process p of DMU j. Similarly, let $O(p)$ the set of final outputs of process p and, for each $k \in O(p)$, let y_{kj}^{p} denote the observed amount of final output k produced by process p of DMU j. Let $P_I(i)$ be the set of processes that consume the exogenous input i and $x_{ij} = \sum_{p \in P_I(i)} x_{ij}^{p}$ the total amount of exogenous input i consumed by all processes of DMU j. Let $P_O(k)$ be the set of processes that produce the final output k and $y_{kj} = \sum_{p \in P_O(k)} y_{kj}^{p}$ the total amount of final output k produced by all processes of DMU j.

In addition to exogenous inputs and outputs, there exist R intermediate products. Let $P^{out}(r)$ be the set of processes that generate the intermediate product r so that for each $p \in P^{out}(r)$ let z_{rj}^{p} the observed amount of intermediate product r generated by process p of DMU j. Analogously, let $P^{in}(r)$ be the set of processes that consume the intermediate product r and for each $p \in P^{in}(r)$ let z_{rj}^{p} the observed amount of intermediate product r consumed by process p of DMU j. Let us assume that an intermediate product r cannot be consumed and produced simultaneously by a process, i.e. $P^{out}(r) \cap P^{in}(r) = \emptyset \quad \forall r$. Also, without loss of generality, let us assume that

$$\sum_{p \in P^{out}(r)} z_{rj}^{p} = \sum_{p \in P^{in}(r)} z_{rj}^{p} \quad \forall r \, \forall j$$

i.e. the intermediate products are completely generated and consumed within the own DMU. Finally, to facilitate the model formulation it is convenient to define the sets $R^{out}(p)$ and $R^{in}(p)$ corresponding to the intermediate products produced and consumed, respectively, by a certain process p.

Note that the sets $P^{out}(r)$ and $P^{in}(r)$ (or, equivalently, $R^{out}(p)$ and $R^{in}(p)$) jointly determine all the structure of intermediate flows within the system. Thus, for example, a system consisting of just parallel process with no intermediate flows ($R = 0$) would have $R^{out}(p) = R^{in}(p) = \emptyset \quad \forall p$. On the contrary, a typical multi-stage series system would have $R^{out}(p) = R^{in}(p+1) \quad 1 < p < P$ and $R^{in}(1) = R^{out}(P) = \emptyset$.

To formulate the multiplier VRS radial input-oriented multiplier formulation of relational Network DEA model, let

J index of specific DMU being assessed
u_i weight of exogenous input i
v_k weight of final output k
w_r weight of intermediate product r
η_p VRS free intercept variable
E_J Efficiency of DMU J

1.1 Model I: Multiplier Form of Crisp Network DEA

$$E_J = \text{Max} \sum_{k} \sum_{p \in P_O(k)} v_k \, y_{kJ}^{p} + \sum_{p} \eta_p$$

$$s.t. \sum_{i} \sum_{p \in P_I(i)} u_i \, x_{iJ}^{p} = 1$$

$$\sum_{k \in O(p)} v_k y_{kj}^{p} + \sum_{r \in R^{out}(p)} w_r z_{rj}^{p} + \eta_p - \sum_{i \in I(p)} u_i x_{ij}^{p} - \sum_{r \in R^{in}(p)} w_r z_{rj}^{p} \le 0 \quad \forall j \, \forall p$$

$$u_i, v_k, w_r \ge 0 \quad \forall i \, \forall k \, \forall r \quad \eta_p \text{ free } \forall p$$

This model looks for the values of the weights of the inputs, outputs and intermediate products that maximize the virtual output of the DMU J under assessment. A basic feature of the relational network DEA approach is that all processes that consume an input or intermediate product or produce an output or intermediate product use the same weight for that input, output or intermediate product. As for the constraints, there are two types. One is that the virtual input of the DMU J is set to unity, as it is common in input-oriented DEA models. The second set of constraints guarantees that the efficiency of all processes is bounded by unity. Adding these constraints for the different processes of each DMU leads to a set of constraints (not included in the model because they are redundant) which guarantee that, with the weights chosen by DMU J, the efficiency of every DMU is not greater than unity. Finally, note that, for each process p only its own subsets of inputs, output and intermediate products are taken into account. Analogously, to compute the virtual input and output only the processes that consume an input or produce an output are taken into account.

To formulate the dual of model I, let

θ Uniform reduction factor of the inputs consumption of DMU J

λ_j^p Intensity variable of process p of DMU j

1.2 Model II: Envelope Form of Crisp Network DEA

$$
\begin{aligned}
& E_J = \operatorname{Min}\ \theta \\
& s.t. \sum_{p \in P_I(i)} \sum_j \lambda_j^p x_{ij}^p \le \theta \sum_{p \in P_I(i)} x_{iJ}^p \quad \forall i \\
& \sum_{p \in P_O(k)} \sum_j \lambda_j^p y_{kj}^p \ge \sum_{p \in P_O(k)} y_{kJ}^p \quad \forall k \\
& \sum_{p \in P^{out}(r)} \sum_j \lambda_j^p z_{rj}^p - \sum_{p \in P^{in}(r)} \sum_j \lambda_j^p z_{rj}^p \ge 0 \quad \forall r \\
& \sum_j \lambda_j^p = 1 \quad \forall p \\
& \lambda_j^p \ge 0 \quad \forall j\, \forall p \quad \theta \text{ free}
\end{aligned}
$$

This envelopment form Network DEA model finds the maximum radial contraction of the inputs consumed by DMU J by looking for an appropriate point within the overall Production Possibility Set (PPS) [20]. Thus, the projected operation point must maintain the total amount of outputs produced by DMU J and be such that for each intermediate product the total amount produced by the different processes must be sufficient to satisfy the amounts needed by the different processes that consume it. A basic feature of this type of Network DEA models is that each process p has its own set of intensity variables λ_j^p. In other words, each process has its own process PPS with its own Returns To Scale assumption. In the above model it has been assumed that all processes operate under VRS.

2 Extension of Kao and Liu Approach to Network Fuzzy DEA

Kao and Liu [16] and Kao and Lin [14] have extended Kao and Liu [15] Fuzzy DEA approach to two-stage serial systems and to parallel production systems. In this section, the approach is extended to general networks of processes such as those described in the previous section. The difference with the crisp Network DEA models I and II will lay on the consideration of fuzzy data for the inputs, outputs and intermediate products. Thus, let us assume that the inputs, outputs and intermediate products consumed or produced by each process are given as LR-type Fuzzy Numbers (LRFN)

$$\widetilde{X}_{ij}^{p} = \left\{ \left(x_{ij}^{p}\right)^{L}, \left(x_{ij}^{p}\right)^{R}, \left(\beta_{ij}^{p}\right)^{L}, \left(\beta_{ij}^{p}\right)^{R} \right\}_{L_i^p, R_i^p}$$

$$\widetilde{Y}_{kj}^{p} = \left\{ \left(y_{kj}^{p}\right)^{L}, \left(y_{kj}^{p}\right)^{R}, \left(\widehat{\beta}_{kj}^{p}\right)^{L}, \left(\widehat{\beta}_{kj}^{p}\right)^{R} \right\}_{\widehat{L}_k^p, \widehat{R}_k^p}$$

$$\widetilde{Z}_{rj}^{p} = \left\{ \left(z_{rj}^{p}\right)^{L}, \left(z_{rj}^{p}\right)^{R}, \left(\hat{\hat{\beta}}_{rj}^{p}\right)^{L}, \left(\hat{\hat{\beta}}_{rj}^{p}\right)^{R} \right\}_{\hat{\hat{L}}_r^p, \hat{\hat{R}}_r^p}$$

where $L_i^p, R_i^p, \widehat{L}_k^p, \widehat{R}_k^p, \hat{\hat{L}}_r^p, \hat{\hat{R}}_r^p : [0,1] \rightarrow [0,1]$ are non-increasing, continuous shape functions and

$$L_i^p(0) = R_i^p(0) = \widehat{L}_k^p(0) = \widehat{R}_k^p(0) = \hat{\hat{L}}_r^p(0) = \hat{\hat{R}}_r^p(0) = 1 \quad \forall i \forall k \forall r \forall p$$

$$L_i^p(1) = R_i^p(1) = \widehat{L}_k^p(1) = \widehat{R}_k^p(1) = \hat{\hat{L}}_r^p(1) = \hat{\hat{R}}_r^p(1) = 0 \quad \forall i \forall k \forall r \forall p$$

The corresponding membership functions are of the type

$$\mu_{\widetilde{X}}(x) = \begin{cases} 1 & \text{if } (x)^L \le x \le (x)^R \\ L\left(\frac{(x)^L - x}{(\beta)^L}\right) & \text{if } (x)^L - (\beta)^L \le x \le (x)^L \\ R\left(\frac{x - (x)^R}{(\beta)^R}\right) & \text{if } (x)^R \le x \le (x)^R + (\beta)^R \\ 0 & \text{otherwise} \end{cases}$$

The α-cuts of $\widetilde{X}_{ij}^{p}$, $\widetilde{Y}_{kj}^{p}$ and $\widetilde{Z}_{rj}^{p}$ are the intervals

$$\left(\widetilde{X}_{ij}^{p}\right)_{\alpha} = \left[\left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{L}, \left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{U}\right]$$

$$\left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{L} = \left(x_{ij}^{p}\right)^{L} - L_i^{p^*}(\alpha) \cdot \left(\beta_{ij}^{p}\right)^{L}$$

$$\left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{U} = \left(x_{ij}^{p}\right)^{R} + R_i^{p^*}(\alpha) \cdot \left(\beta_{ij}^{p}\right)^{R}$$

$$\left(\widetilde{Y}^p_{kj}\right)_\alpha = \left[\left(\widetilde{Y}^p_{kj}\right)^L_\alpha, \left(\widetilde{Y}^p_{kj}\right)^U_\alpha\right]$$
$$\left(\widetilde{Y}^p_{kj}\right)^L_\alpha = \left(y^p_{kj}\right)^L - \widehat{L}^{p^*}_k(\alpha) \cdot \left(\widehat{\beta}^p_{kj}\right)^L$$
$$\left(\widetilde{Y}^p_{kj}\right)^U_\alpha = \left(y^p_{kj}\right)^R + \widehat{R}^{p^*}_k(\alpha) \cdot \left(\widehat{\beta}^p_{kj}\right)^R$$

$$\left(\widetilde{Z}^p_{rj}\right)_\alpha = \left[\left(\widetilde{Z}^p_{rj}\right)^L_\alpha, \left(\widetilde{Z}^p_{rj}\right)^U_\alpha\right]$$
$$\left(\widetilde{Z}^p_{rj}\right)^L_\alpha = \left(z^p_{rj}\right)^L - \hat{\hat{L}}^{p^*}_r(\alpha) \cdot \left(\hat{\hat{\beta}}^p_{rj}\right)^L$$
$$\left(\widetilde{Z}^p_{rj}\right)^U_\alpha = \left(z^p_{rj}\right)^R + \hat{\hat{R}}^{p^*}_r(\alpha) \cdot \left(\hat{\hat{\beta}}^p_{rj}\right)^R$$

where the inverse shape functions are defined as

$$L^*(\alpha) = \sup\{h : L(h) \geq \alpha\}$$

$$R^*(\alpha) = \sup\{h : R(h) \geq \alpha\}$$

For example, for trapezoidal and triangular fuzzy numbers

$$L(h) = R(h) = 1 - h$$
$$L^*(\alpha) = R^*(\alpha) = 1 - \alpha$$

The Kao and Liu [15, 16] approach is α-level based, according to the classification of Fuzzy DEA approaches given by Hatami-Marbini et al. [9]. For each $\alpha \in [0, 1]$ the corresponding α-cut of the efficiency of DMU J $\widetilde{E}_J$ can be expressed as

$$\left(\widetilde{E}_J\right)_\alpha = \left[\left(\widetilde{E}_J\right)^L_\alpha, \left(\widetilde{E}_J\right)^U_\alpha\right]$$

where the upper and lower limits can be computed using the following pair of models.

2.1 Model III: Kao and Liu Upper Limit

$$\left(\widetilde{E}_J\right)^U_\alpha = \max \sum_k \sum_{p \in P_O(k)} v_k \cdot \left(\widetilde{Y}^p_{kJ}\right)^U_\alpha + \sum_p \eta_p$$

s.t.

$$\sum_i \sum_{p \in P_I(i)} u_i \cdot \left(\widetilde{X}^p_{iJ}\right)^L_\alpha = 1$$

$$\sum_{k\in O(p)} v_k \cdot \left(\widetilde{Y}_{kJ}^{p}\right)_{\alpha}^{U} + \sum_{r\in R^{out}(p)} w_r \cdot z_{rJ}^{p} + \eta_p$$
$$- \sum_{i\in I(p)} u_i \cdot \left(\widetilde{X}_{iJ}^{p}\right)_{\alpha}^{L} - \sum_{r\in R^{in}(p)} w_r \cdot z_{rJ}^{p} \le 0 \quad \forall p$$

$$\sum_{k\in O(p)} v_k \cdot \left(\widetilde{Y}_{kj}^{p}\right)_{\alpha}^{L} + \sum_{r\in R^{out}(p)} w_r \cdot z_{rj}^{p} + \eta_p$$
$$- \sum_{i\in I(p)} u_i \cdot \left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{U} - \sum_{r\in R^{in}(p)} w_r \cdot z_{rj}^{p} \le 0 \quad \forall j \ne J\, \forall p$$

$$\left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{L} \le z_{rj}^{p} \le \left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{U} \quad \forall j \forall p \forall r \in R^{in}(p) \cup R^{out}(p)$$

$$u_i, v_k, w_r \ge 0 \quad \forall i\, \forall k\, \forall r$$

The above model corresponds to

$$\left(\widetilde{E}_J\right)_{\alpha}^{U} = \max_{\substack{\left(x_{ij}^{p}\right)_{\alpha}^{L} \le x_{ij}^{p} \le \left(x_{ij}^{p}\right)_{\alpha}^{U} \\ \left(y_{kj}^{p}\right)_{\alpha}^{L} \le y_{kj}^{p} \le \left(y_{kj}^{p}\right)_{\alpha}^{U} \\ \left(z_{rj}^{p}\right)_{\alpha}^{L} \le z_{rj}^{p} \le \left(z_{rj}^{p}\right)_{\alpha}^{U}}} E_J$$

where the efficiency E_J is computed as per model I. The model takes into account that the maximum efficiency for DMU occurs when

$$\begin{aligned}
x_{iJ}^{p} &= \left(x_{iJ}^{p}\right)_{\alpha}^{L} && \forall p \forall i \in I(p) \\
x_{ij}^{p} &= \left(x_{ij}^{p}\right)_{\alpha}^{U} && \forall j \ne J \forall p \forall i \in I(p) \\
y_{kJ}^{p} &= \left(y_{kJ}^{p}\right)_{\alpha}^{U} && \forall p \forall k \in O(p) \\
y_{kj}^{p} &= \left(y_{kj}^{p}\right)_{\alpha}^{L} && \forall j \ne J \forall p \forall k \in O(p)
\end{aligned}$$

Since for the intermediate products it is not known in advance which value, between the corresponding lower and upper limits, leads to the maximum efficiency the corresponding amounts are left as variables in the model. Unfortunately, this makes model III non-linear although it can be easily linearised introducing the new variables

$$\widehat{z}_{rj}^{p} = w_r \cdot z_{rj}^{p} \quad \forall j\, \forall p\, \forall r \in R^{in}(p) \cup R^{out}(p)$$

and reformulating the corresponding constraints as

$$\sum_{k\in O(p)} v_k \cdot \left(\widetilde{Y}^p_{kJ}\right)^U_\alpha + \sum_{r\in R^{out}(p)} \widehat{z}^p_{rJ} + \eta_p - \sum_{i\in I(p)} u_i \cdot \left(\widetilde{X}^p_{iJ}\right)^L_\alpha - \sum_{r\in R(p)} \widehat{z}^p_{rJ} \le 0 \quad \forall p$$

$$\sum_{k\in O(p)} v_k \cdot \left(\widetilde{Y}^p_{kj}\right)^L_\alpha + \sum_{r\in R^{out}(p)} \widehat{z}^p_{rj} + \eta_p - \sum_{i\in I(p)} u_i \cdot \left(\widetilde{X}^p_{ij}\right)^U_\alpha - \sum_{r\in R^{in}(p)} \widehat{z}^p_{rj} \le 0 \quad \forall j \ne J\, \forall p$$

$$w_r \cdot \left(\widetilde{Z}^p_{rj}\right)^L_\alpha \le \widehat{z}^p_{rj} \le w_r \cdot \left(\widetilde{Z}^p_{rj}\right)^U_\alpha \quad \forall j\, \forall p\, \forall r \in R^{in}(p) \cup R^{out}(p)$$

2.2 Model IV: Kao and Liu Lower Limit

$$\left(\widetilde{E_J}\right)^L_\alpha = \min\ \theta$$

$s.t.$

$$\sum_{p\in P_I(i)} \lambda^p_J \cdot \left(\widetilde{X}^p_{iJ}\right)^U_\alpha + \sum_{p\in P_I(i)} \sum_{j\ne J} \lambda^p_j \cdot \left(\widetilde{X}^p_{ij}\right)^L_\alpha \le \theta \sum_{p\in P_I(i)} \left(\widetilde{X}^p_{iJ}\right)^U_\alpha \quad \forall i$$

$$\sum_{p\in P_O(k)} \lambda^p_J \cdot \left(\widetilde{Y}^p_{kJ}\right)^L_\alpha + \sum_{p\in P_O(k)} \sum_{j\ne J} \lambda^p_j \cdot \left(\widetilde{Y}^p_{kj}\right)^U_\alpha \ge \sum_{p\in P_O(k)} \left(\widetilde{Y}^p_{kJ}\right)^L_\alpha \quad \forall k$$

$$\sum_{p\in P^{out}(r)} \sum_j \lambda^p_j \cdot z^p_{rj} - \sum_{p\in P^{in}(r)} \sum_j \lambda^p_j \cdot z^p_{rj} \ge 0 \quad \forall r$$

$$\left(\widetilde{Z}^p_{rj}\right)^L_\alpha \le z^p_{rj} \le \left(\widetilde{Z}^p_{rj}\right)^U_\alpha \quad \forall j\, \forall p\, \forall r \in R^{in}(p) \cup R^{out}(p)$$

$$\sum_j \lambda^p_j = 1 \quad \forall p$$

$$\lambda^p_j \ge 0 \quad \forall j\, \forall p \qquad \theta \text{ free}$$

The above model corresponds to

$$\left(\widetilde{E}_J\right)^L_\alpha = \min_{\substack{\left(x^p_{ij}\right)^L_\alpha \le x^p_{ij} \le \left(x^p_{ij}\right)^U_\alpha \\ \left(y^p_{kj}\right)^L_\alpha \le y^p_{kj} \le \left(y^p_{kj}\right)^U_\alpha \\ \left(z^p_{rj}\right)^L_\alpha \le z^p_{rj} \le \left(z^p_{rj}\right)^U_\alpha}} E_J$$

where the efficiency E_J is computed as per model II. In model IV it has been taken into account that the minimum efficiency for DMU J occurs when

$$\begin{aligned} x_{iJ}^{p} &= \left(x_{iJ}^{p}\right)_{\alpha}^{U} \quad \forall p \forall i \in I(p) \\ x_{ij}^{p} &= \left(x_{ij}^{p}\right)_{\alpha}^{L} \quad \forall j \neq J \forall p \forall i \in I(p) \\ y_{kJ}^{p} &= \left(y_{kJ}^{p}\right)_{\alpha}^{L} \quad \forall p \forall k \in O(p) \\ y_{kj}^{p} &= \left(y_{kj}^{p}\right)_{\alpha}^{U} \quad \forall j \neq J \forall p \forall k \in O(p) \end{aligned}$$

Again, since for the intermediate products it is not known in advance which value, between the corresponding lower and upper limits, leads to the minimum efficiency the corresponding amounts are left as variables in the model. Unfortunately, this makes model III non-linear although it can be easily linearised introducing the new variables

$$\widehat{z}_{rj}^{p} = \lambda_j^p \cdot z_{rj}^{p} \quad \forall r \forall j \ \forall p$$

and reformulating the corresponding constraints as

$$\sum_{p \in P^{out}(r)} \sum_{j} \widehat{z}_{rj}^{p} - \sum_{p \in P^{in}(r)} \sum_{j} \widehat{z}_{rj}^{p} \geq 0 \quad \forall r$$

$$\lambda_j^p \cdot \left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{L} \leq \widehat{z}_{rj}^{p} \leq \lambda_j^p \cdot \left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{U} \quad \forall j \ \forall p \ \forall r \in R^{in}(p) \cup R^{out}(p)$$

Kao and Liu [15, 16], in addition to estimating the overall efficiency of each DMU, indicate how to estimate also the efficiency of the different processes. However, since one of the characteristics of multiplier-form DEA models (like Model I or Model III) is that there can be alternative optimal solutions then it is not clear how to compute the efficiency of individual processes. Since this issue is open and requires further research (even in the crisp data case) we will not try to compute process efficiencies.

3 Extension of Saati et al. Approach to Network Fuzzy DEA

Saati et al. [23] propose a Fuzzy DEA approach that considers Triangular Fuzzy Numbers (TFN). Therefore, let us assume that the inputs, outputs and intermediate products consumed or produced by each process are given as

$$\widetilde{X}_{ij}^{p} = \left\{ \left(x_{ij}^{p}\right)^{-}, \left(x_{ij}^{p}\right)^{0}, \left(x_{ij}^{p}\right)^{+} \right\}$$

$$\widetilde{Y}_{kj}^{p} = \left\{ \left(y_{kj}^{p}\right)^{-}, \left(y_{kj}^{p}\right)^{0}, \left(y_{kj}^{p}\right)^{+} \right\}$$

$$\widetilde{Z}_{rj}^{p} = \left\{ \left(z_{rj}^{p}\right)^{-}, \left(z_{rj}^{p}\right)^{0}, \left(z_{rj}^{p}\right)^{+} \right\}$$

whose corresponding α-cuts are

$$\left(\widetilde{X}_{ij}^{p}\right)_{\alpha} = \left[\left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{L}, \left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{U} \right]$$
$$\left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{L} = \alpha \cdot \left(x_{ij}^{p}\right)^{0} + (1-\alpha) \cdot \left(x_{ij}^{p}\right)^{-}$$
$$\widetilde{X}_{\alpha}^{U} = \alpha \cdot \left(x_{ij}^{p}\right)^{0} + (1-\alpha) \cdot \left(x_{ij}^{p}\right)^{+}$$

$$\left(\widetilde{Y}_{kj}^{p}\right)_{\alpha} = \left[\left(\widetilde{Y}_{kj}^{p}\right)_{\alpha}^{L}, \left(\widetilde{Y}_{kj}^{p}\right)_{\alpha}^{U} \right]$$
$$\left(\widetilde{Y}_{kj}^{p}\right)_{\alpha}^{L} = \alpha \cdot \left(y_{kj}^{p}\right)^{0} + (1-\alpha) \cdot \left(y_{kj}^{p}\right)^{-}$$
$$\left(\widetilde{Y}_{kj}^{p}\right)_{\alpha}^{U} = \alpha \cdot \left(y_{kj}^{p}\right)^{0} + (1-\alpha) \cdot \left(y_{kj}^{p}\right)^{+}$$

$$\left(\widetilde{Z}_{rj}^{p}\right)_{\alpha} = \left[\left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{L}, \left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{U} \right]$$
$$\left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{L} = \alpha \cdot \left(z_{rj}^{p}\right)^{0} + (1-\alpha) \cdot \left(z_{rj}^{p}\right)^{-}$$
$$\left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{U} = \alpha \cdot \left(z_{rj}^{p}\right)^{0} + (1-\alpha) \cdot \left(z_{rj}^{p}\right)^{+}$$

Of course, TFNs are just a special case of LRFNs where

$$\left(x_{ij}^{p}\right)^{0} = \left(x_{ij}^{p}\right)^{L} = \left(x_{ij}^{p}\right)^{R}$$
$$\left(x_{ij}^{p}\right)^{-} = \left(x_{ij}^{p}\right)^{0} - \left(\beta_{ij}^{p}\right)^{L}$$
$$\left(x_{ij}^{p}\right)^{+} = \left(x_{ij}^{p}\right)^{0} + \left(\beta_{ij}^{p}\right)^{R}$$

and

$$L(h) = R(h) = 1 - h$$
$$L^{*}(\alpha) = R^{*}(\alpha) = 1 - \alpha$$

This approach is also α-level based. For each $\alpha \in [0,1]$ an efficiency score $E_J(\alpha)$ can be computed using the following model

3.1 Model V: Saati et al.

$$E_J(\alpha) = \max \sum_{k} \sum_{p \in P_O(k)} v_k\, y_{kJ}^p + \sum_p \eta_p$$

s.t.

$$\sum_i \sum_{p \in P_I(i)} u_i\, x_{iJ}^p = 1$$

$$\sum_{k \in O(p)} v_k\, y_{kj}^p + \sum_{r \in R^{out}(p)} w_r \cdot z_{rj}^p + \eta_p - \sum_{i \in I(p)} u_i\, x_{ij}^p - \sum_{r \in R^{in}(p)} w_r \cdot z_{rj}^p \le 0 \ \forall j \forall p$$

$$\left(\widetilde{X}_{ij}^p\right)_\alpha^L \le x_{ij}^p \le \left(\widetilde{X}_{ij}^p\right)_\alpha^U \quad \forall j\, \forall p \forall i \in I(p)$$

$$\left(\widetilde{Y}_{kj}^p\right)_\alpha^L \le y_{kj}^p \le \left(\widetilde{Y}_{kj}^p\right)_\alpha^U \quad \forall j\, \forall p \forall k \in O(p)$$

$$\left(\widetilde{Z}_{rj}^p\right)_\alpha^L \le z_{rj}^p \le \left(\widetilde{Z}_{rj}^p\right)_\alpha^U \quad \forall j\, \forall p\, \forall r \in R^{in}(p) \cup R^{out}(p)$$

$$u_i, v_k, w_r \ge 0 \quad \forall i\, \forall k\, \forall r$$

Same as with the Kao and Liu upper limit model III, this corresponds to

$$\left(\widetilde{E}_J\right)_\alpha^U = \max_{\substack{\left(x_{ij}^p\right)_\alpha^L \le x_{ij}^p \le \left(x_{ij}^p\right)_\alpha^U \\ \left(y_{kj}^p\right)_\alpha^L \le y_{kj}^p \le \left(y_{kj}^p\right)_\alpha^U \\ \left(z_{rj}^p\right)_\alpha^L \le z_{rj}^p \le \left(z_{rj}^p\right)_\alpha^U}} E_J$$

but maintaining x_{ij}^p and y_{kj}^p as variables also. Introducing new variables

$$\widehat{x}_{ij}^p = u_i\, x_{ij}^p \quad \forall j\, \forall p\, \forall i \in I(p)$$

$$\widehat{y}_{kj}^p = v_k\, y_{kj}^p \quad \forall j\, \forall p\, \forall k \in O(p)$$

$$\widehat{z}_{rj}^p = w_r\, z_{rj}^p \quad \forall j\, \forall p\, \forall r \in R^{in}(p) \cup R^{out}(p)$$

leads to the following Linear Program (LP)

$$E_J(\alpha) = \max \sum_k \sum_{p \in P_O(k)} \widehat{y}_{kJ}^p + \sum_p \eta_p$$

s.t.

$$\sum_{i} \sum_{p \in P_I(i)} \widehat{x}_{iJ}^{p} = 1$$

$$\sum_{k \in O(p)} \widehat{y}_{kj}^{p} + \sum_{r \in R^{out}(p)} \widehat{z}_{rj}^{p} + \eta_p - \sum_{i \in I(p)} \widehat{x}_{ij}^{p} - \sum_{r \in R^{in}(p)} \widehat{z}_{rj}^{p} \leq 0 \quad \forall j \, \forall p$$

$$u_i \cdot \left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{L} \leq \widehat{x}_{ij}^{p} \leq u_i \cdot \left(\widetilde{X}_{ij}^{p}\right)_{\alpha}^{U} \quad \forall j \, \forall p \, \forall i \in I(p)$$

$$v_k \cdot \left(\widetilde{Y}_{kj}^{p}\right)_{\alpha}^{L} \leq \widehat{y}_{kj}^{p} \leq v_k \cdot \left(\widetilde{Y}_{kj}^{p}\right)_{\alpha}^{U} \quad \forall j \, \forall p \, \forall k \in O(p)$$

$$w_r \cdot \left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{L} \leq \widehat{z}_{rj}^{p} \leq w_r \cdot \left(\widetilde{Z}_{rj}^{p}\right)_{\alpha}^{U} \quad \forall j \, \forall p \, \forall r \in R^{in}(p) \cup R^{out}(p)$$

Summarising, the efficiency score computed by the Saati et al. approach coincides with the Kao and Liu upper limit of model III. This is something that will be confirmed in the numerical results section.

4 Extension of Wang et al. Approach to Network Fuzzy DEA

Wang et al. [26] proposed an α-level set approach which, contrary to the Kao and Liu [15] approach, uses the same crisp production frontier as reference for all DMUs and for all values of α. Such fixed production frontier is inferred from the best performing values of the DMUs, which correspond to the $\alpha = 0.0$ upper limit for the outputs and the $\alpha = 0.0$ lower limit for the inputs. For the intermediate outputs the data are not fixed but they can vary between their $\alpha = 0.0$ lower and upper limits. For each $\alpha \in [0, 1]$ the corresponding α-cut of the efficiency of DMU J $\widetilde{E}_J$ can be expressed as

$$\left(\widetilde{E}_J\right)_{\alpha} = \left[\left(\widetilde{E}_J\right)_{\alpha}^{L}, \left(\widetilde{E}_J\right)_{\alpha}^{U}\right]$$

where the upper and lower limits can be computed using the following pair of models.

4.1 Model VI: Wang et al. Upper Limit

$$\left(\widetilde{E}_J\right)_\alpha^U = \max \sum_k \sum_{p \in P_O(k)} v_k \cdot \left(\widetilde{Y}_{kJ}^p\right)_\alpha^U + \sum_p \eta_p$$

$$s.t.$$

$$\sum_i \sum_{p \in P_I(i)} u_i \cdot \left(\widetilde{X}_{iJ}^p\right)_\alpha^L = 1$$

$$\sum_{k \in O(p)} v_k \cdot \left(\widetilde{Y}_{kj}^p\right)_{0.0}^U + \sum_{r \in R^{out}(p)} w_r \cdot z_{rj}^p + \eta_p -$$

$$- \sum_{i \in I(p)} u_i \cdot \left(\widetilde{X}_{ij}^p\right)_{0.0}^L - \sum_{r \in R^{in}(p)} w_r \cdot z_{rj}^p \le 0 \quad \forall j \forall p$$

$$\left(\widetilde{Z}_{rj}^p\right)_{0.0}^L \le z_{rj}^p \le \left(\widetilde{Z}_{rj}^p\right)_{0.0}^U \quad \forall j\, \forall p\, \forall r \in R^{in}(p)p \cup R^{out}(p)$$

$$u_i, v_k, w_r \ge 0 \quad \forall i\, \forall k\, \forall r$$

4.2 Model VII: Wang et al. Lower Limit

$$\left(\widetilde{E}_J\right)_\alpha^L = \min \ \theta$$

$$s.t.$$

$$\sum_{p \in P_I(i)} \sum_j \lambda_j^p \cdot \left(\widetilde{X}_{ij}^p\right)_{0.0}^L \le \theta \sum_{p \in P_I(i)} \left(\widetilde{X}_{ij}^p\right)_\alpha^U \quad \forall i$$

$$\sum_{p \in P_O(k)} \sum_j \lambda_j^p \cdot \left(\widetilde{Y}_{kj}^p\right)_{0.0}^U \ge \sum_{p \in P_O(k)} \left(\widetilde{Y}_{kj}^p\right)_\alpha^L \quad \forall k$$

$$\sum_{p \in P^{out}(r)} \sum_j \lambda_j^p \cdot z_{rj}^p - \sum_{p \in P^{in}(r)} \sum_j \lambda_j^p \cdot z_{rj}^p \ge 0 \quad \forall r$$

$$\left(\widetilde{Z}_{rj}^p\right)_{0.0}^L \le z_{rj}^p \le \left(\widetilde{Z}_{rj}^p\right)_{0.0}^U \quad \forall j\, \forall p\, \forall r \in R^{in}(p) \cup R^{out}(p)$$

$$\sum_j \lambda_j^p = 1 \quad \forall p$$

$$\lambda_j^p \ge 0 \quad \forall j\, \forall p \qquad \theta \text{ free}$$

The above two models can be linearised exactly the same as Kao and Liu models III and IV, respectively.

5 Numerical Experiments

In this section the proposed models will be applied to a dataset from the literature. Although this chapter deals with general networks of processes, the only published Network Fuzzy DEA datasets are those in Kao and Liu [16] and Kao and Lin [14]. However, although Kao and Lin [14] considers a parallel production system problem with fuzzy data, the specific dataset used to illustrate their approach is of a shared-inputs type, i.e. the actual amounts of an input consumed by each process is not known but it is left to the DEA model the task of determining, within certain bounds, the share of the inputs that is supposedly consumed by each process. Since that type of Network DEA models is different to the one considered here the corresponding dataset cannot be used for our purpose.

The dataset in Kao and Liu [16] corresponds to a simple two-stage system with two inputs, two intermediate products and two outputs, as shown in Fig. 1. Although the models formulated in this chapter can deal with VRS, for comparison with Kao and Liu [16] Constant Returns to Scale (CRS) will be assumed, which means that variables η_p should be dropped from multiplier formulations and accordingly the convexity constraints on the intensity variables λ_j^p should also be dropped from the envelope formulations.

Tables A.1, A.2, and A.3, in the appendix, show the TFN corresponding to the inputs, intermediate products and outputs, respectively, of the 24 DMUs in Kao and Liu [16]. Tables 1 and 2 show, for different values of $\alpha \in [0, 1]$ the lower and upper limits of the corresponding α-cuts computed by Kao and Liu models III and IV, respectively. There are some minor differences with respect to the results reported by Kao and Liu [16]. The results obtained have been calculated with the dataset shown in Tables A.1, A.2 and A.3, which corresponds exactly to the dataset that appears in Kao and Liu [16]. It seems that the results reported in that paper were obtained using a dataset with more precision, as in Kao and Hwang [12].

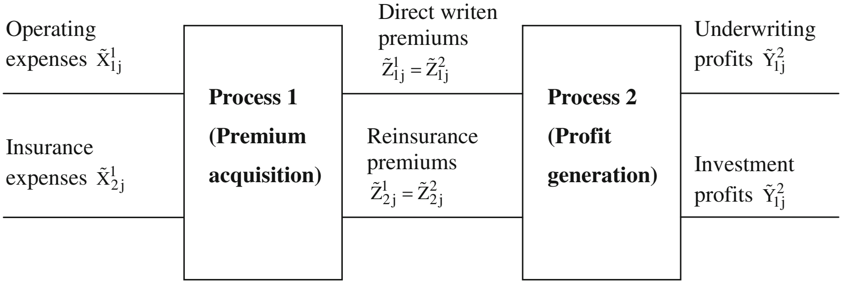

Fig. 1 Two-stage system from Kao and Liu [16]

Table 1 Upper limit of α-cuts of efficiency as per model III Kao and Liu

DMU	α = 0.0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1.0
1	0.904	0.862	0.820	0.779	0.741	0.700
2	0.795	0.759	0.724	0.691	0.659	0.626
3	0.861	0.824	0.788	0.754	0.721	0.690
4	0.426	0.399	0.373	0.348	0.326	0.304
5	1.000	1.000	0.968	0.905	0.847	0.792
6	0.510	0.487	0.465	0.444	0.416	0.389
7	0.375	0.353	0.332	0.313	0.295	0.277
8	0.371	0.350	0.329	0.310	0.292	0.275
9	0.294	0.278	0.263	0.249	0.236	0.224
10	0.636	0.598	0.562	0.529	0.497	0.468
11	0.217	0.204	0.192	0.181	0.170	0.159
12	0.942	0.902	0.864	0.828	0.793	0.760
13	0.276	0.260	0.246	0.232	0.219	0.207
14	0.392	0.369	0.347	0.327	0.307	0.289
15	0.794	0.753	0.715	0.678	0.644	0.612
16	0.433	0.408	0.383	0.361	0.339	0.319
17	0.484	0.456	0.430	0.406	0.383	0.361
18	0.350	0.330	0.310	0.292	0.275	0.259
19	0.497	0.480	0.464	0.448	0.433	0.413
20	0.725	0.685	0.647	0.609	0.573	0.539
21	0.247	0.235	0.223	0.212	0.201	0.191
22	0.783	0.743	0.706	0.671	0.638	0.606
23	0.556	0.522	0.489	0.459	0.431	0.404
24	0.176	0.166	0.156	0.147	0.139	0.131

Table 2 Lower limit of α-cuts of efficiency as per model IV Kao and Liu

DMU	$\alpha = 0.0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	0.500	0.535	0.572	0.612	0.654	0.700
2	0.447	0.478	0.511	0.547	0.585	0.626
3	0.549	0.575	0.603	0.632	0.661	0.690
4	0.213	0.229	0.246	0.265	0.284	0.304
5	0.563	0.603	0.646	0.692	0.741	0.792
6	0.279	0.298	0.319	0.341	0.364	0.389
7	0.203	0.216	0.230	0.245	0.261	0.277
8	0.203	0.215	0.229	0.243	0.259	0.275
9	0.163	0.174	0.185	0.198	0.211	0.224
10	0.337	0.360	0.385	0.411	0.439	0.468
11	0.116	0.124	0.132	0.141	0.150	0.159
12	0.559	0.595	0.634	0.676	0.720	0.760
13	0.153	0.163	0.173	0.184	0.195	0.207
14	0.212	0.225	0.240	0.256	0.272	0.289
15	0.460	0.489	0.520	0.551	0.580	0.612
16	0.233	0.248	0.264	0.281	0.300	0.319
17	0.266	0.283	0.301	0.320	0.340	0.361
18	0.189	0.202	0.215	0.229	0.243	0.259
19	0.309	0.328	0.348	0.368	0.391	0.413
20	0.395	0.421	0.448	0.476	0.507	0.539
21	0.145	0.154	0.162	0.171	0.181	0.191
22	0.510	0.528	0.547	0.566	0.586	0.606
23	0.291	0.311	0.332	0.355	0.379	0.404
24	0.095	0.101	0.108	0.115	0.123	0.131

Table 3 Upper limit of α-cuts of efficiency as per model VI Wang et al

DMU	$\alpha = 0.0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 0.10$
1	0.792	0.773	0.754	0.736	0.719	0.702
2	0.707	0.691	0.674	0.658	0.642	0.627
3	0.779	0.761	0.743	0.725	0.708	0.691
4	0.344	0.336	0.328	0.320	0.312	0.305
5	0.897	0.876	0.855	0.835	0.815	0.795
6	0.441	0.431	0.421	0.411	0.401	0.391
7	0.309	0.303	0.297	0.291	0.286	0.280
8	0.308	0.302	0.296	0.290	0.284	0.278
9	0.249	0.244	0.239	0.235	0.230	0.225
10	0.519	0.509	0.499	0.489	0.479	0.469
11	0.179	0.175	0.172	0.168	0.165	0.162
12	0.841	0.825	0.809	0.793	0.777	0.761
13	0.233	0.228	0.224	0.220	0.215	0.211
14	0.322	0.316	0.309	0.303	0.297	0.292
15	0.681	0.667	0.654	0.641	0.629	0.616
16	0.354	0.347	0.340	0.333	0.327	0.320
17	0.404	0.396	0.388	0.380	0.373	0.366
18	0.288	0.282	0.276	0.271	0.266	0.261
19	0.460	0.451	0.443	0.434	0.426	0.418
20	0.609	0.597	0.585	0.573	0.561	0.550
21	0.205	0.203	0.200	0.198	0.196	0.193
22	0.709	0.690	0.672	0.654	0.637	0.621
23	0.446	0.438	0.429	0.421	0.413	0.405
24	0.147	0.144	0.141	0.138	0.135	0.132

Table 4 Lower limit of α-cuts of efficiency as per model VII Wang et al

DMU	$\alpha = 0.0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	0.494	0.506	0.519	0.531	0.544	0.557
2	0.440	0.450	0.461	0.473	0.484	0.496
3	0.487	0.499	0.511	0.523	0.536	0.549
4	0.213	0.219	0.224	0.230	0.235	0.241
5	0.563	0.577	0.591	0.605	0.620	0.635
6	0.279	0.286	0.293	0.300	0.307	0.315
7	0.203	0.207	0.212	0.216	0.220	0.225
8	0.203	0.207	0.211	0.215	0.219	0.224
9	0.163	0.166	0.169	0.173	0.176	0.180
10	0.337	0.344	0.351	0.358	0.365	0.372
11	0.116	0.119	0.121	0.124	0.126	0.129
12	0.554	0.565	0.576	0.588	0.600	0.612
13	0.153	0.156	0.160	0.163	0.166	0.169
14	0.212	0.216	0.220	0.224	0.229	0.233
15	0.449	0.458	0.467	0.476	0.486	0.496
16	0.233	0.238	0.243	0.248	0.253	0.258
17	0.266	0.271	0.276	0.282	0.287	0.293
18	0.189	0.193	0.197	0.201	0.205	0.209
19	0.304	0.310	0.316	0.322	0.329	0.335
20	0.395	0.403	0.412	0.420	0.429	0.438
21	0.145	0.147	0.149	0.151	0.153	0.155
22	0.478	0.482	0.486	0.490	0.494	0.498
23	0.291	0.297	0.303	0.309	0.315	0.321
24	0.095	0.097	0.099	0.101	0.103	0.105

As advanced in Sect. 3, the efficiency scores computed by the Saati et al. model V coincide with the upper limits of Kao and Liu shown in Table 1.

Tables 3 and 4 show the upper and lower limits, respectively, of the efficiency scores, for the different possibility values $\alpha \in [0, 1]$, computed by the Wang et al. models VI and VII. Note that for $\alpha = 1$, the upper and lower limits in these tables do not coincide between themselves.

Let us make one final remark about a fact that may have been noticed by the reader and it is that, for almost none of the different approaches tried, no DMU is assessed as efficient, i.e. almost all efficiency scores are below unity, even for $\alpha = 0.0$ upper limits. This is not surprising and, actually, it is quite common in Network DEA because when each DMU consists of several processes then all of them must be efficient for the overall DMU to be efficient. That is a tall order that does occur often. It occurs more often that one process of a DMU may be efficient but not all the others.

6 Conclusions

This chapter has shown how to extend several Fuzzy DEA approaches to the Network DEA context. It has helped greatly the fact that the notation used for the crisp Network DEA approach allows for a rather simple formulation of the model for general networks. Extending those formulations to handle fuzzy data is not straightforward but not difficult, as the approaches shown in this chapter show.

Since Network DEA represents, in general, a more fine-grained level of analysis which can lead to more valid results (although at the expense of requiring more detailed data), the possibility of applying a Network Fuzzy DEA approach for those problems in which the data are uncertain contributes to enhance the usefulness of the approach.

Acknowledgments This research was carried out with the financial support of the Spanish Ministry of Science grant DPI2010-16201, and FEDER.

Appendix

See Tables A.1, A.2, and A.3.

Table A.1 Inputs data of Kao and Liu [16] problem

DMU	$\left(x^1_{1j}\right)^-$	$\left(x^1_{1j}\right)^0$	$\left(x^1_{1j}\right)^+$	$\left(x^1_{2j}\right)^-$	$\left(x^1_{2j}\right)^0$	$\left(x^1_{2j}\right)^+$
1	1113	1178	1256	636	673	717
2	1305	1381	1472	1278	1352	1441
3	1112	1177	1255	559	592	631
4	568	601	641	561	594	633
5	6331	6699	7141	3167	3351	3572
6	2483	2627	2800	631	668	712
7	1853	1942	2047	1377	1443	1521
8	3615	3789	3994	1787	1873	1974
9	1495	1567	1652	906	950	1001
10	1243	1303	1373	1238	1298	1368
11	1872	1962	2068	641	672	708
12	2473	2592	2732	620	650	685
13	2481	2609	2739	1301	1368	1436
14	1328	1396	1466	940	988	1037
15	2077	2184	2293	619	651	684
16	1152	1211	1272	395	415	436
17	1382	1453	1526	1032	1085	1139
18	720	757	795	520	547	574
19	151	159	167	173	182	191
20	138	145	152	50	53	56
21	80	84	88	25	26	27
22	14	15	16	9	10	10
23	51	54	57	27	28	29
24	155	163	171	223	235	246

Table A.2 Intermediate products data of Kao and Liu [16] problem

DMU	$\left(z^1_{1j}\right)^-$	$\left(z^1_{1j}\right)^0$	$\left(z^1_{1j}\right)^+$	$\left(z^1_{2j}\right)^-$	$\left(z^1_{2j}\right)^0$	$\left(z^1_{2j}\right)^+$
1	7041	7451	7943	809	856	912
2	9469	10020	10681	1712	1812	1932
3	4513	4776	5091	529	560	597
4	2999	3174	3383	351	371	395
5	35335	37362	39680	1657	1753	1869
6	9211	9747	10390	900	952	1015
7	10193	10685	11262	613	643	678
8	16473	17267	18199	1082	1134	1195
9	10945	11473	12093	521	546	575
10	7832	8210	8653	481	504	531
11	6890	7222	7612	613	643	678
12	9000	9434	9943	1067	1118	1178
13	13239	13921	14617	771	811	852
14	7034	7396	7766	442	465	488
15	9911	10422	10943	712	749	786
16	5331	5606	5886	382	402	422
17	7318	7695	8080	325	342	359
18	3453	3631	3813	946	995	1045
19	1083	1141	1196	458	483	506
20	300	316	331	124	131	137
21	214	225	236	38	40	42
22	49	52	54	13	14	15
23	233	245	257	47	49	51
24	452	476	499	611	644	675

Table A.3 Outputs data of Kao and Liu [16] problem

DMU	$\left(y^1_{1j}\right)^-$	$\left(y^1_{1j}\right)^0$	$\left(y^1_{1j}\right)^+$	$\left(y^1_{2j}\right)^-$	$\left(y^1_{2j}\right)^0$	$\left(y^1_{2j}\right)^+$
1	930	984	1049	644	681	726
2	1160	1228	1309	788	834	889
3	277	293	312	622	658	701
4	234	248	264	167	177	189
5	7419	7851	8369	3709	3925	4184
6	1619	1713	1826	392	415	442
7	2136	2239	2360	419	439	463
8	3720	3899	4110	593	622	656
9	995	1043	1099	252	264	278
10	1619	1697	1789	529	554	584
11	1418	1486	1566	17	18	19
12	1502	1574	1659	867	909	958
13	3432	3609	3789	212	223	234
14	1332	1401	1471	316	332	349
15	3191	3355	3523	528	555	583
16	812	854	897	187	197	207
17	2990	3144	3301	353	371	390
18	658	692	727	155	163	171
19	493	519	544	44	46	48
20	337	355	372	25	26	27
21	48	51	53	6	6	6
22	78	82	86	4	4	4
23	1	1	1	17	18	19
24	135	142	149	15	16	17

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CHAPTER TWENTY

EFFICIENCY ASSESSMENT OF US STATES USING NETWORK DEA ANALYSIS

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Abstract

The current financial and economic crisis has brought into focus the sustainability of public finances and the need for efficiency in the provision of public services both at the national and state level. Thus, the aim of this this chapter is to assess the relative efficiency of the 50 U.S. states, as well as estimating for each of them feasible reductions in taxes, debt and public expenditures, by applying a two-stage network data envelopment analysis (DEA) approach. The more recently available data, namely those of fiscal years 2007-2011, have been used. The results show that almost half of the states were relatively efficient and were consistently so during this period. Other states, however, such as New Jersey, Pennsylvania, Massachusetts, Michigan and Maryland, not only had more debt than necessary but also had higher taxes to finance their excessive expenditures. It also seems from the results that, on average, states governed by the Democratic Party, showed greater inefficiencies relative to GDP than those governed by the Republican Party.

20.1 Introduction

The current economic and financial crisis has hit a number of countries (mainly the U.S. and Europe) in a way that has not been witnessed since the Great Depression. The first response of most countries was aimed at maintaining liquidity and sustaining economic activity through government spending. The sharp decrease in tax revenues has meant that some governments have had to run large deficits and increase their debt to unsustainable levels. Worried about the risks of sovereign debt defaults, the financial markets have reacted in some cases by raising debt yields and credit default swap prices and forcing many countries onto a deficit-reduction path that jeopardizes the possibility of an economic recovery.

It is against this background of watchful financial markets overseeing the sustainability of public finances and indebted governments forced to raise taxes and enact cutbacks in many programmes (including education, health care and welfare) that the present study can be framed. In order to keep governments providing essential public services it is necessary that they do so in a sustainable way, which means that inefficiencies, in their many forms (i.e. overspending, higher than necessary taxes, excessive borrowing), must be removed from the system.

The relevance of improving public sector efficiency has long been emphasized by Organisation for Economic Co-operation and Development (OECD) policy analysts (e.g. Curristine, Lonti, & Joumard, 2007), who pointed out that governments should increase the use of performance information in budget processes, focusing on results instead of each government department trying to obtain as much money as possible. Many studies have been published assessing the efficiency of governments managing the public expenditure. Afonso, Schuknecht, and Tanzi (2005) computed (i) public sector performance (PSP), defined as the outcome of public sector activities (i.e., several economic indicators as outputs) and (ii) public sector efficiency (PSE), defined as the outcomes relative to the resources employed. As well as PSP and PSE indicators, the non-parametric technique Free Disposal Hull (FDH) was applied to assess the public sector efficiency of 23 industrialized countries, concluding that countries with small public sectors report higher efficiency. Angelopoulos, Philippopoulos, and Tsionas (2008) also used PSE indicators and Stochastic Frontier Analysis (SFA), along with an econometric model, to study the relationship between growth and the size of the government for 52 countries. This relationship depends on the technical efficiency of the public spending of each country.

Newer studies are moving towards the using of DEA for the assessment of efficiency via appropriate cost and outcome measures of public policies (Afonso, Schuknecht, & Tanzi, 2010; Hauner & Kyobe, 2010). Adam, Delis, and Kammas (2011) proposed computing measures of efficiency using SFA and DEA that are able to remove socio-economic and exogenous factors across different countries.

However, these previous papers failed to take into account that not only public expenditure but also tax affects the efficiency of public finances. Therefore, the composition of tax revenue should not be ignored, as stated by Afonso et al. (2005) and Bierbrauer and Sahm (2010). The latter actually studied the interdependence of optimal tax and expenditure policies in a democratic mechanism framework.

Although there have been many studies about optimal taxation, such as Golosov, Kocherlakota, and Tsyvinski (2003), where the capital and commodity taxations are examined, or Bierbrauer (2009), who proposed an incentive to eliminate biases which arise when tax revenues are devoted to public goods, there appears to be a paucity of literature heeding both tax revenues and public expenditure to evaluate the sustainability of public finances. Within this context, the proposed network DEA methodology arises as a way to take into account the full process for the provision of public services.

Thus the main objective of the chapter is to compute feasible reductions in tax revenues and debt incurred, and to identify overspending in the provision of public services by U.S. states. Although the methodology can be also apply to other governments, at regional or national levels, the U.S. states have been selected for reasons of data standardization and availability.

The structure of the paper is the following. In section 2 the methodology, i.e. network DEA, is introduced, together with the specific model implemented and the data used. In section 3, the results of the efficiency assessment are presented and discussed. Finally, section 4 summarizes and concludes.

20.2 Methodology

This section presents the mathematical tool used to measure the states' inefficiency, namely Network DEA, which can be used to gauge how the states' efficiencies evolve and how they impact on their finances. First of all, an introduction reviewing some of the literature on Network DEA and its foundations is presented. Second, the proposed approach is formulated, explaining all the variables included in the model. Lastly, there is an overview of the specific data used.

20.2.1 Introduction to network DEA

DEA is a non-parametric mathematical tool commonly used to assess the relative efficiency of a number of similar (i.e. homogeneous) Decision Making Units (DMUs). Among many other applications, DEA has been applied extensively to assess the efficiency of public services and administrations such as, for example, education (e.g. Avkiran, 2001), hospitals (e.g. Caballer-Tarazona et al., 2010), police stations (e.g. García-Sánchez, Rodríguez-Domínguez, & Parra-Domínguez, 2013), justice courts (e.g. Pedraja-Chaparro & Salinas-Jimenez, 1996), public transport (e.g. Hilmola, 2011), municipal services (e.g. Benito-López, Moreno-Enguix, & Solana-Ibañez, 2011), national governments (e.g. Adam et al., 2011), etc.

Traditionally DEA has considered the units under assessment as black boxes, consisting of a single process that uses inputs to produce outputs. There exist, however, situations in which multiple, interrelated stages can be distinguished. This has given rise to the emergence of network DEA approaches (e.g. Färe & Grosskopf, 1996, 2000; Löthgren & Tambour 1999; Färe, Grosskopf, & Whittaker, 2007). Since the literature on network DEA has increased substantially in recent years, a review of that literature is outside the scope of this paper. Suffice it to say that most approaches and applications correspond to two-stage or multi-stage serial systems (e.g. Liang, Cook, & Zhu, 2008; Cook et al., 2007; Chen et al., 2009), although parallel and general networks of processes have also been considered (Kao, 2009a, 2009b; Kao & Hwang, 2010). Not only have radial models been used but also non-radial (e.g. Tone & Tsutsui, 2009; Fukuyama & Weber, 2010), dynamic (e.g. Chen, 2009; Tone & Tsutsui, 2010) and cost efficiency models (e.g. Fukuyama & Matousek, 2011; Lozano, 2011) have been developed.

As for applications, network DEA has been mainly used in transportation (e.g. Yu, 2010; Bichou, 2011; Lozano, Gutiérrez, & Moreno, 2013), sport (Lewis, Lock, & Sexton, 2009; Moreno & Lozano, 2012) and finance (e.g. Kao & Hwang, 2008; Avkiran, 2009). Particularly interesting, however, in our case, is the application of network DEA to the assessment of the macroeconomic efficiency of OECD countries using the input-output data of the different sectors of the economy (Prieto & Zofío, 2007).

20.2.2 Proposed network DEA approach

Figure 20-1 shows the two stages which have been considered. The first is labelled *Public Finance* and corresponds to state revenue collection, basically from tax receipts. Different taxes have been included, although for most states the biggest shares correspond to income tax (personal and corporate) and sales taxes. Seven states (namely, Alaska, Florida, South Dakota, Nevada, Texas, Washington and Wyoming) do not have personal income tax with the last four also lacking corporate income tax; five states (namely Alaska, Delaware, Montana, New Hampshire and Oregon) do not have sales tax. Another important fraction, about 25% on average, of state funds comes from the federal government. The basic outputs of this stage are the different state expenditures that are financed with these tax receipts. Again, six expenditure concepts have been included with, approximately, about half of the total expenditures going to education (K-12 and college) and health care. What is more important is that all these expenditures are inputs to the second stage, which has the state population and GDP as non-discretionary outputs, and has been labelled *Public Services Provision*. Thus, all expenditures are treated as intermediate products.

For the sake of completeness, it has to be said that the input Other Revenue groups together the inter-governmental revenue, current charges, miscellaneous general revenue, utility revenue, liquor stores revenue and insurance trust revenue. Similarly while the intermediate product Other Expenditure groups together other and unallocable expenditures, utility expenditure, liquor store expenditure and insurance trust expenditure.

Note that, apart from the revenue inputs, the Public Finance stage has two additional inputs which correspond to the Cash and holdings from the previous year on the one hand and, on the other, the Debt from the previous year. Although both are non-discretionary they differ in that Debt is considered a normal (i.e. undesirable) input while Cash and holdings is considered a desirable input (i.e. the larger the better). Moreover, three final outputs of that stage have been considered: one of them desirable, namely Cash and holdings at the end of the year, and the other two undesirable, namely Interest on general debt and Debt at the end of the year. Table 20-1 list and summarizes the inputs and outputs of each stage.

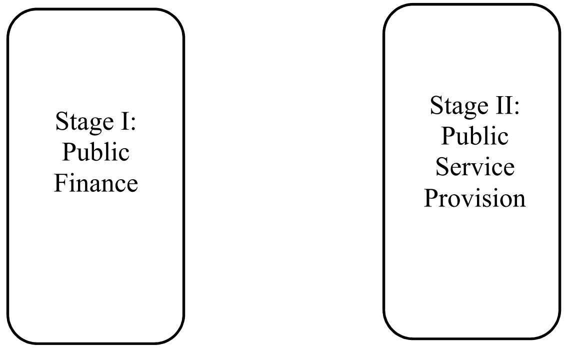


Figure 20-1 Proposed two-stage approach

<u>Inputs</u>:

- x_1 General sales taxes (GenSalTax)
- x_2 Selective sales taxes (SelSalTax)
- x_3 Licence taxes (LicenTax)
- x_4 Individual income tax (IndInTax)
- x_5 Corporate income tax (CorInTax)
- x_6 Other taxes (OthTax)
- x_7 Other revenue (OthRev)
- x_8 Debt previous year (DebtPrev)
- d Cash and holdings previous year (CashPrev)

<u>Intermediate products</u>:

- z_1 Education expenditure (EduExp)
- z_2 Public welfare expenditure (WelExp)
- z_3 Health care expenditure (HealthExp)
- z_4 Public safety expenditure (SafeExp)
- z_5 Governmental administration expenditure (GovExp)
- z_6 Other expenditure (OthExp)

<u>Outputs:</u>

- y Cash and holdings at the end of the year (Cash)
- u_1 Interests on general debt (IntDebt)
- u_2 Debt at the end of the year (Debt)
- w_1 State population (Pop)
- w_2 State Gross Domestic Product (GDP)

Table 20–1 Inputs and outputs of each stage

Stage I (Public Finance)		
Input	**Non-discretionary**	**Desirable**
GenSalTax		
SelSalTax		
LicenTax		
IndInTax		
CorInTax		
OthTax		
OthRev	Yes	
DebtPrev	Yes	
CashPrev	Yes	Yes
Output	**Non-discretionary**	**Undesirable**
Cash		
IntDebt		Yes
Debt		Yes
Stage II (Public Service Provision)		
Input	**Non-discretionary**	**Desirable**
EduExp		
WelExp		
HealthExp		
SafeExp		
GovExp		
OthExp		
Output	**Non-discretionary**	**Undesirable**
Pop	Yes	
GDP	Yes	

With respect to the network DEA model to be used, there are several issues that have to be answered. One of these is the assumed technology. In this respect, since it seems reasonable to expect the existence of scale effects in the Public Service Provision stage Variable Returns to Scale (VRS) have been assumed in that stage while Constant Returns to Scale (CRS) have been assumed in the Public Finance stage. The two undesirable outputs of the Public Finance stage are assumed to be jointly and weakly disposable with the other (desirable) outputs. Also, there is the issue of the intermediate products between stages (i.e. the expenditures); they have been considered as free links (Tone & Tsutsui, 2009).

Another issue is the model orientation and distance function. In this paper, the Directional Distance Measure (DDM) of efficiency proposed in Färe and Grosskopf (2010) is used. This efficiency measure corresponds to the sum of multiple directional distance functions along the different inputs and outputs. Note that neither the non-discretionary inputs of stage I nor the population output of stage II is assigned a directional distance function. Also note that, for the two undesirable outputs of stage I, the directional distance function aims at reducing the amounts of those outputs that are produced. This DDM efficiency score is particularly suited to this application since all the inputs and outputs involved are measured in the same units (constant US$) which means that each directional distance corresponds to the absolute US$ amount that the corresponding input (i.e. tax receipt) or output (i.e. cash, debt or debt interest payment) can be feasibly improved. The resulting efficiency score represents the sum of all those amounts since all those input and output improvements are assumed to be made simultaneously. Thus, the computed DDM is a measure of the total relative inefficiency of the assessed DMU and the corresponding targets lie on the efficient frontier.

Let 0 be the index of the DMU being assessed and the multipliers corresponding to stages I and II respectively. The proposed network DEA model is:

$$
\begin{aligned}
& DDM_0 = Max \quad \beta^{GST} + \beta^{SST} + \beta^{LT} + \beta^{IIT} \\
& + \beta^{CIT} + \beta^{OT} + \gamma^{Cash} + \alpha^{Debt} + \alpha^{ID} \\
& \text{s.t.} \\
& \sum_j \lambda_j^I \cdot GenSalTax \le GenSalTax_0 - \beta^{GST} \cdot 1, \\
& \sum_j \lambda_j^I \cdot SelSalTax \le SelSalTax_0 - \beta^{SST} \cdot 1, \\
& \sum_j \lambda_j^I \cdot LicenTax \le LicenTax_0 - \beta^{LT} \cdot 1, \\
& \sum_j \lambda_j^I \cdot IndInTax \le IndInTax_0 - \beta^{IIT} \cdot 1, \\
& \sum_j \lambda_j^I \cdot CorInTax \le CorInTax_0 - \beta^{CIT} \cdot 1, \\
& \sum_j \lambda_j^I \cdot OthTax \le OthTax_0 - \beta^{OT} \cdot 1, \\
& \sum_j \lambda_j^I \cdot OthRev \le OthRev_0,
\end{aligned} \tag{20.1}
$$

$$\sum_j \lambda_j^I \cdot DebtPrev \leq DebtPrev_0,$$

$$\sum_j \lambda_j^I \cdot CashPrev \geq CashPrev_0,$$

$$\sum_j \lambda_j^I \cdot Cash_j \geq Cash_0 + \gamma^{Cash} \cdot 1,$$

$$\sum_j \lambda_j^I \cdot IntDebt_j = IntDebt_0 - \alpha^{ID} \cdot 1,$$

$$\sum_j \lambda_j^I \cdot Debt_j = Debt_0 - \alpha^{Debt} \cdot 1,$$

$$\sum_j \lambda_j^I \cdot z_{kj} - \sum_j \lambda_j^{II} \cdot z_{kj} = 0,\ k = 1,2,\ldots,6,$$

$$\sum_j \lambda^{II} \cdot Pop_j \geq Pop_0,$$

$$\sum_j \lambda^{II} \cdot GDP_j \geq GDP_0,$$

$$\sum_j \lambda_j^{II} = 1,$$

where all variables are non-negative and $z_{kj},\ k = 1,2,\ldots,6,$ refers to the expenditures in Figure 20-1. Note the different sign of the inequality for the case of the constraint corresponding to the desirable input CashPrev and also the equal sign of the constraints corresponding to the two undesirable outputs IntDebt and Debt. Finally, note the VRS constraint associated with stage II.

In summary, the proposed approach considers that an efficient state is one that, in order to provide its public services, spends what is necessary but without burdening taxpayers, either directly (through taxes) or indirectly (through debt), more than is required.

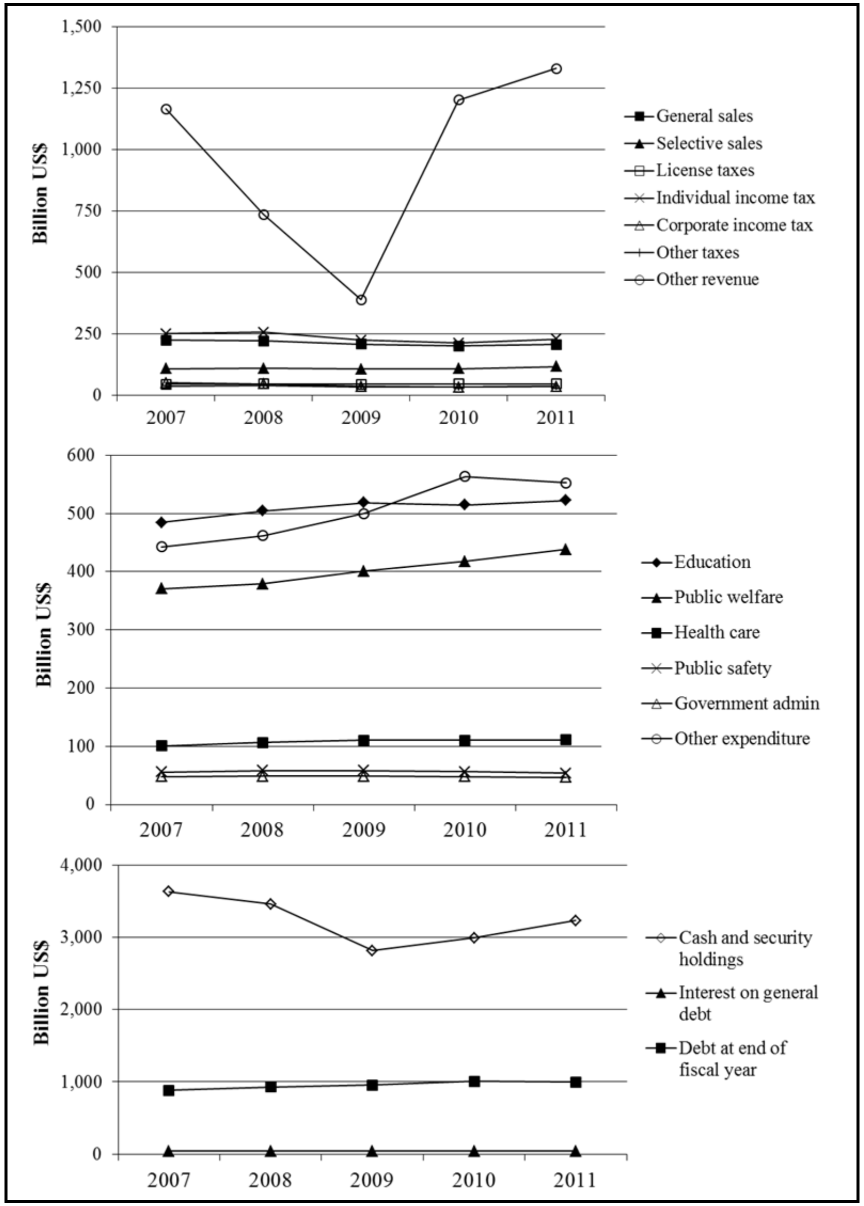
20.2.3 Data

Before analyzing the results obtained, let us comment first about the data. Note that in most states the fiscal year runs from July 1st to the following June 30th. The only exceptions are New York (whose fiscal year starts on April 1st), Texas (September 1st) and Alabama and Michigan (October 1st). Therefore, when this research was carried out, the states were in fiscal year 2013. However, the most recent data available correspond to fiscal year 2011 – a two-year lag. Those of fiscal year 2012 are scheduled to be published in winter 2014. Once they are available, the proposed approach can, of course, be applied to them. The data used in this study include the first years of the current economic crisis. Note, in this respect, that in those years, in order to close their budget shortfalls, many states have had to implement a mixed strategy that combined cutting expenditures, raising taxes, drawing down reserve funds and using federal aid from the American Recovery and Reinvestment Act of 2009 (ARRA). Although tax revenues have risen lately, the state budget problems are not yet solved, due to the continuing high unemployment and economic uncertainty and the end of the exceptional federal assistance previously provided by the ARRA.

About the source of the data: most of them come from the U.S. Census Bureau, which publishes an Annual Survey of State Government Finances (http://www.census.gov/govs/state/), accompanied by the corresponding Survey Methodology (http://www2.census.gov/govs/state/11_methodology.pdf) and Technical Documentation (http://www2.census.gov/govs/state/statetechdoc2011.pdf). Population data come from the Population Estimates Program of the U.S. Census Bureau (http://www.census.gov/popest/estimates.html). GDP data were obtained from the Regional Economic Accounts published by the Bureau of Economic Analysis (http://www.bea.gov/regional/index.htm). GDP deflator values (used to express all monetary amounts in constant 2005 US$) were obtained from the corresponding National Economic Accounts (http://www.bea.gov/national/index.htm#gdp).

Figure 20-2 shows the distribution of the revenues and expenditures in each fiscal year. It also shows the cash and holding balance, interest debt and outstanding debt. Note that, apart from federal transfers (included as other revenue), the major sources of revenue for state governments are individual income tax and general sales tax, both of which have declined in recent years. Note also the significant increase in the federal transfers to the states in fiscal year 2010 due to the ARRA. As regards expenditures, Education and welfare (which includes Medicaid expenditure) are the largest spending categories for state governments, with the latter having significantly increased in recent years. Finally, note the increase of debt in this period; however, compared with outstanding federal government debt, the states' debt is relatively modest.

Figure 20-2 Revenue and expenditure distribution and evolution



20.3 Results and discussion

Regarding the results, Figure 20-3, Figure 20-4 and Figure 20-5 show, in decreasing order, the DDM efficiency scores of the relatively inefficient states for each fiscal year. Note that a contemporaneous approach has been adopted, i.e. the states' efficiencies for each year are computed using only the observations for that year. Adopting an inter-temporal approach, i.e. jointly benchmarking all observations in all the periods, would result in a stricter efficiency assessment but one that may be unfair, since the economic and budgetary circumstances in each year are different which would render the comparisons meaningless.

Figure 20-3 DDM efficiency scores for fiscal year 2007

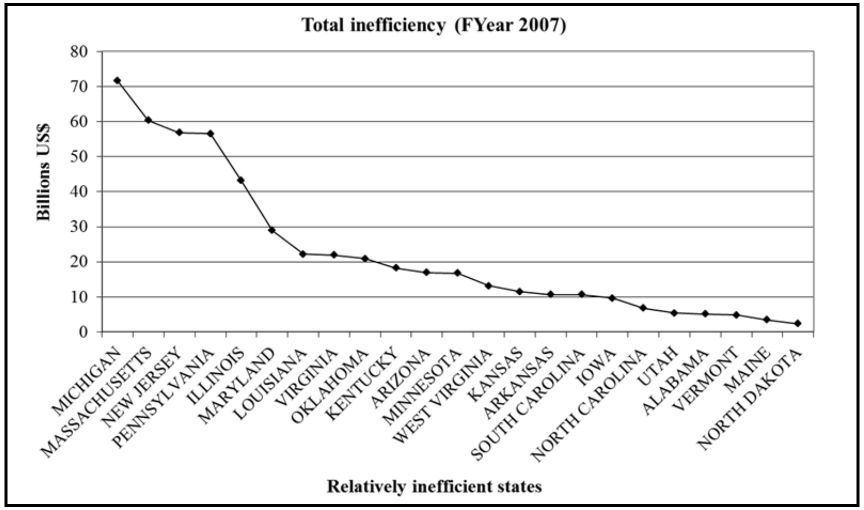


Figure 20-4 DDM efficiency scores for fiscal years 2009–2010

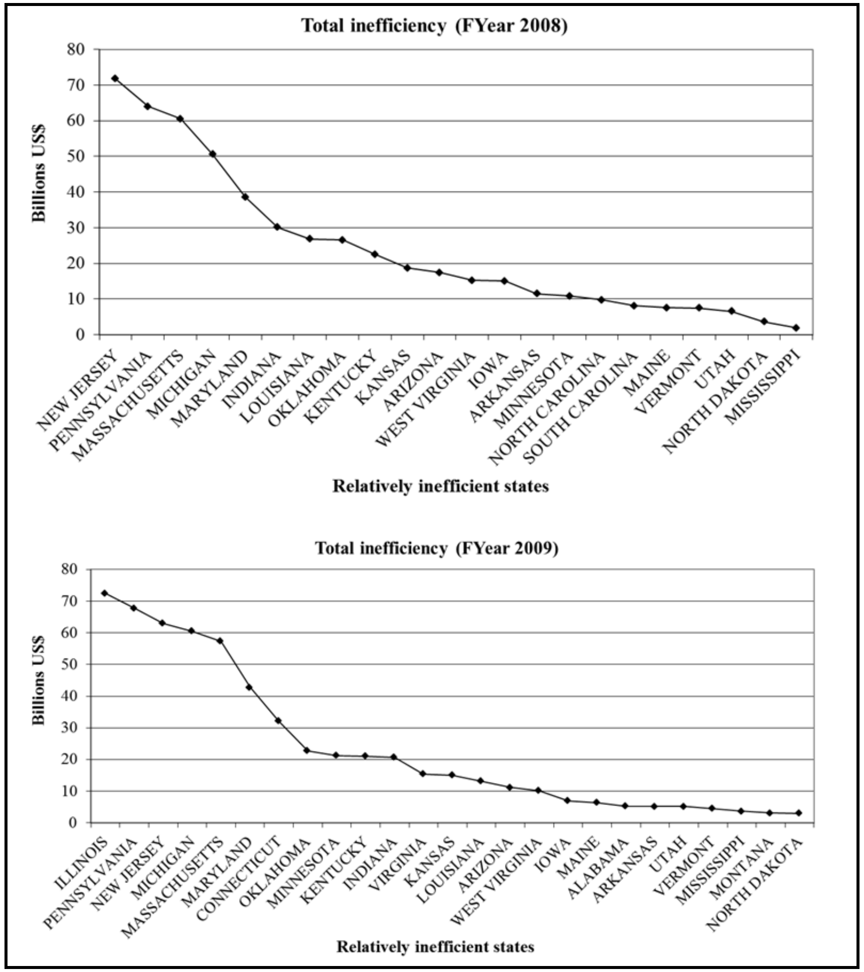
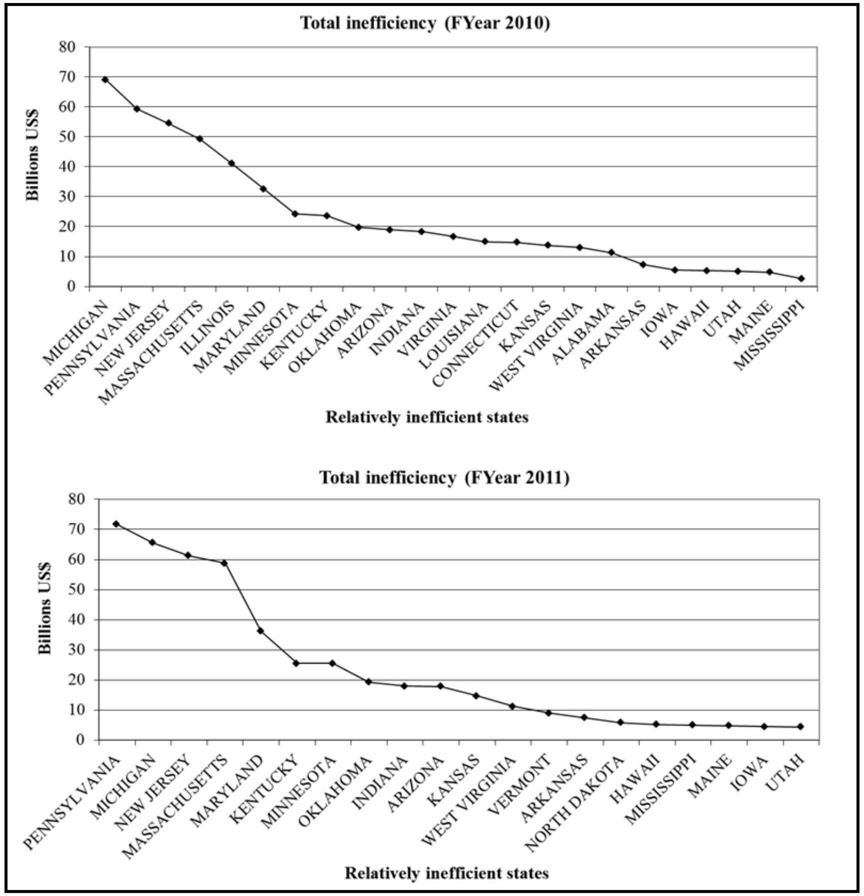


Figure 20-5 DDM efficiency scores for fiscal years 2010–2011



Note how certain states, such as New Jersey, Pennsylvania, Massachusetts, Michigan and Maryland, are highly inefficient year after year. Another group of states, such as Kentucky, Kansas, Minnesota and West Virginia, also have significant inefficiencies every year. Some states (such as Illinois, Connecticut, Louisiana, Oklahoma and Virginia) have been inefficient some years but seem to have improved lately. Also a number of states have occasional or relatively minor (i.e. below 10 billion US$) inefficiencies. Finally, there 22 states (namely Alaska, California, Colorado, Delaware, Florida, Georgia, Idaho, Missouri, Nebraska, Nevada, New Hampshire, New Mexico, New York, Ohio, Oregon, Rhode Island, South Dakota, Tennessee, Texas, Washington, Wisconsin and Wyoming) that have been relatively efficient all the years of the period under study. More detailed results showing the decomposition of the DDM efficiency score according to the different sources of inefficiency are shown in Table 20-2, Table 20-3 and Table 20-4. Note that the rows corresponding to those states that have been relatively efficient in all these years are not shown.

Figure 20-6 shows the states' total inefficiency as well as the decomposition of that inefficiency. It can be seen that the total states' inefficiency represents a huge amount (around 500 billion US$) and that it increased significantly in 2009 although it has decreased sharply in the last two years. More than half the total inefficiency is related to maintaining low, system-wide, liquidity levels. The second source of inefficiency is excessive borrowing, which is more than 100 billion US$ higher than necessary. Debt inefficiency (i.e. excess debt) increased in 2009 and 2010 but has decreased lately. The third source of inefficiency is the excessively high taxes, overall around 50 billion US$ each year. The last, and minor, source of inefficiency is excessive interest payments, related obviously to the excess debt held which was commented on above.

Table 20–2 Decomposition of DDM efficiency scores (billions US$) for year 2007

		Fyear 2007			
		$\sum\beta$	γ^{CASH}	α^{INT}	γ^{CASH}
Alabama	**AL**	1.014	3.004	0.000	1.073
Arizona	**AZ**	3.677	12.657	0.040	0.594
Arkansas	**AR**	2.365	7.556	0.000	0.763
Connecticut	**CT**	0.000	0.000	0.000	0.000
Hawaii	**HI**	0.000	0.000	0.000	0.000
Illinois	**IL**	6.175	5.060	1.688	30.216
Indiana	**IN**	0.000	0.000	0.000	0.000
Iowa	**IA**	1.207	7.483	0.065	0.837
Kansas	**KS**	1.652	9.735	0.047	0.000
Kentucky	**KY**	2.462	14.073	0.000	1.710
Louisiana	**LA**	0.871	20.380	0.210	0.737
Maine	**ME**	1.178	0.319	0.074	1.858
Maryland	**MD**	4.754	16.269	0.344	7.551
Massachusetts	**MA**	6.409	6.987	2.246	44.680
Michigan	**MI**	5.113	62.842	0.000	3.684
Minnesota	**MN**	7.428	8.256	0.090	0.973
Mississippi	**MS**	0.000	0.000	0.000	0.000
Montana	**MT**	0.000	0.000	0.000	0.000
New Jersey	**NJ**	10.983	13.553	1.008	31.275
North Carolina	**NC**	4.489	0.000	0.000	2.320
North Dakota	**ND**	0.138	2.102	0.054	0.000
Oklahoma	**OK**	1.774	17.535	0.094	1.474
Pennsylvania	**PA**	6.619	35.974	0.679	13.262
South Carolina	**SC**	0.914	5.930	0.000	3.802
Utah	**UT**	1.660	2.440	0.048	1.200
Vermont	**VT**	0.887	3.469	0.039	0.423
Virginia	**VA**	3.889	16.999	0.000	1.065
West Virginia	**WV**	1.375	9.969	0.000	1.772
	Total	77.034	282.592	6.726	151.271

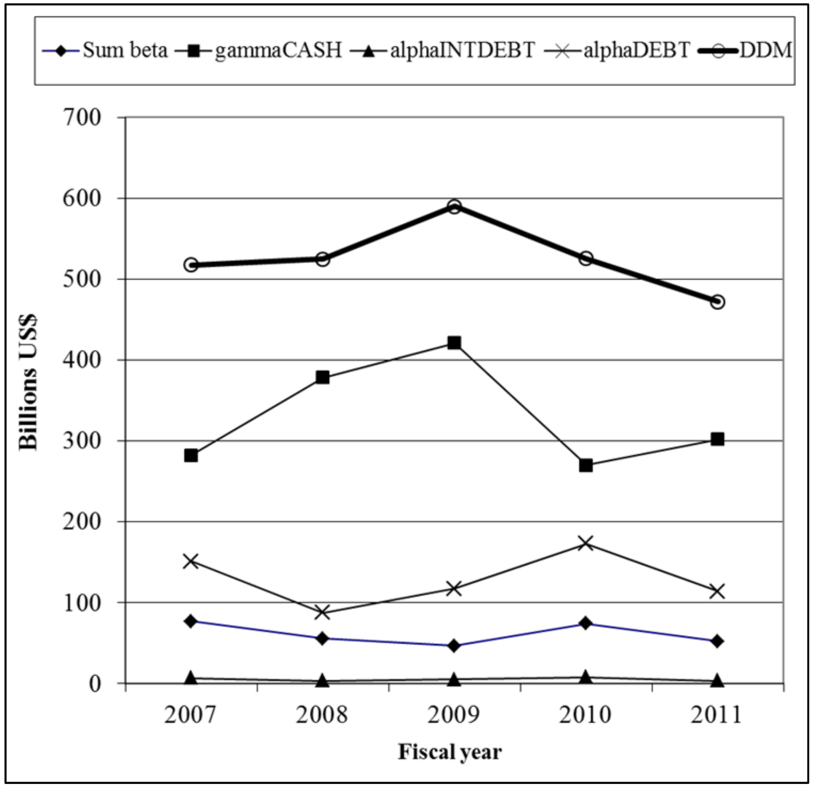
Table 20–3 Decomposition of DDM efficiency scores (billions US$) for years 2008 and 2009

	Fyear 2008				Fyear 2009			
	$\sum\beta$	γ^{CASH}	α^{INT}	γ^{CASH}	$\sum\beta$	γ^{CASH}	α^{INT}	γ^{CASH}
AL	0.000	0.000	0.000	0.000	1.224	4.037	0.009	0.000
AZ	2.140	15.105	0.000	0.203	1.516	8.594	0.028	1.057
AR	2.258	9.230	0.000	0.000	2.087	2.388	0.000	0.706
CT	0.000	0.000	0.000	0.000	3.204	18.088	0.596	10.303
HI	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IL	0.000	0.000	0.000	0.000	2.868	45.572	1.338	22.699
IN	3.466	24.320	0.123	2.171	4.339	8.759	0.161	7.421
IA	0.736	14.096	0.046	0.193	1.630	4.467	0.000	0.832
KS	2.366	16.314	0.048	0.000	0.685	14.286	0.063	0.000
KY	2.705	18.258	0.000	1.594	1.485	19.155	0.003	0.379
LA	0.929	24.390	0.135	1.396	1.161	9.842	0.261	1.918
ME	0.706	6.235	0.038	0.589	0.240	6.132	0.022	0.000
MD	3.766	28.724	0.217	5.854	0.950	41.788	0.053	0.000
MA	6.559	7.116	2.236	44.631	3.461	11.696	2.093	40.069
MI	3.375	45.519	0.000	1.657	2.799	53.146	0.000	4.563
MN	7.471	3.337	0.070	0.000	6.510	14.403	0.033	0.342
MS	1.252	0.374	0.000	0.305	1.768	1.474	0.000	0.408
MT	0.000	0.000	0.000	0.000	0.246	2.112	0.012	0.709
NJ	5.179	52.545	0.171	13.873	2.287	45.302	0.314	15.070
NC	6.331	0.000	0.000	3.400	0.000	0.000	0.000	0.000
ND	0.205	3.346	0.070	0.023	0.288	2.719	0.054	0.000
OK	0.837	25.657	0.075	0.000	1.345	21.307	0.012	0.126
PA	2.198	54.707	0.278	6.783	2.368	57.756	0.201	7.462
SC	0.388	4.907	0.000	2.796	0.000	0.000	0.000	0.000
UT	0.878	5.648	0.002	0.000	1.196	3.944	0.000	0.041
VT	0.732	5.732	0.068	0.914	0.696	3.753	0.016	0.098
VA	0.000	0.000	0.000	0.000	1.435	11.445	0.106	2.425
WV	1.247	12.865	0.000	1.105	0.853	8.751	0.000	0.533
Total	55.722	378.426	3.575	87.487	46.642	420.916	5.376	117.162

	Fyear 2010				Fyear 2011			
	$\sum\beta$	γ^{CASH}	α^{INT}	γ^{CASH}	$\sum\beta$	γ^{CASH}	α^{INT}	γ^{CASH}
AL	1.405	9.834	0.011	0.036	0.000	0.000	0.000	0.000
AZ	1.379	16.931	0.048	0.611	0.930	16.257	0.057	0.605
AR	1.623	5.258	0.000	0.421	1.925	5.230	0.000	0.328
CT	3.033	0.000	0.561	11.196	0.000	0.000	0.000	0.000
HI	2.034	0.194	0.129	2.919	2.009	0.000	0.144	3.108
IL	3.679	3.817	1.842	31.726	0.000	0.000	0.000	0.000
IN	3.999	4.597	0.408	9.260	3.400	8.139	0.243	6.171
IA	1.747	2.816	0.000	0.951	1.621	2.675	0.000	0.234
KS	2.167	11.280	0.078	0.204	1.879	12.509	0.000	0.329
KY	3.197	17.679	0.156	2.613	1.686	22.378	0.116	1.354
LA	1.028	10.656	0.441	2.821	0.000	0.000	0.000	0.000
ME	1.257	2.099	0.083	1.313	0.130	4.001	0.020	0.629
MD	4.819	20.235	0.269	7.298	3.520	26.103	0.238	6.377
MA	5.597	1.836	1.844	39.926	6.727	2.304	1.964	47.750
MI	5.342	59.432	0.140	4.130	5.847	52.934	0.097	6.670
MN	6.908	17.036	0.003	0.256	7.471	16.888	0.039	1.106
MS	1.365	0.475	0.000	0.756	1.299	3.672	0.000	0.089
MT	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
NJ	9.324	10.856	1.075	33.204	5.183	26.821	0.578	28.697
NC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ND	0.000	0.000	0.000	0.000	1.408	4.415	0.010	0.000
OK	0.886	18.050	0.040	0.750	0.870	16.819	0.046	1.581
PA	8.217	37.295	0.598	13.162	3.647	63.129	0.033	4.915
SC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UT	0.895	4.155	0.000	0.000	0.593	2.971	0.000	0.863
VT	0.000	0.000	0.000	0.000	0.575	7.428	0.031	1.003
VA	2.850	5.052	0.344	8.468	0.000	0.000	0.000	0.000
WV	1.600	10.273	0.033	1.083	1.408	7.670	0.023	2.124
Total	74.350	269.856	8.104	173.106	52.127	302.345	3.639	113.933

Table 20–4 Decomposition of DDM efficiency scores (billions US$) for years 2010 and 2011

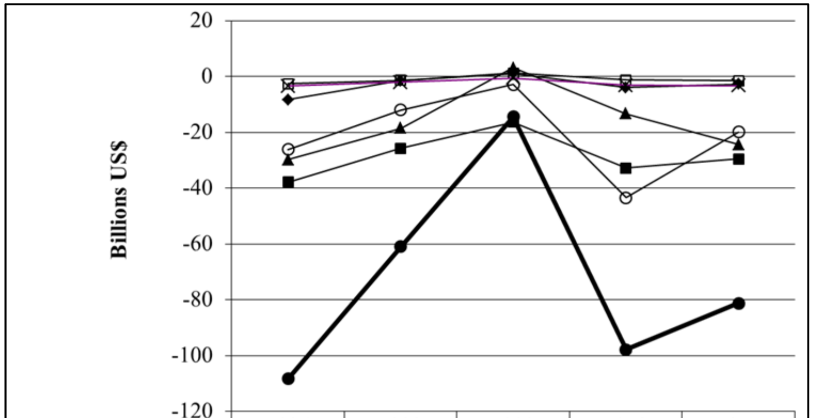
Figure 20-6 Total states' inefficiency and its decomposition



The proposed approach not only identifies the amount and sources of the states' financial inefficiencies but also quantifies the reductions (in some cases, increases) in expenditures that might have been made. Thus, Table 20-5, Table 20-6, Table 20-7, Table 20-8 and Table 20-9 in the appendix, show the reductions/increases in each of the six expenditure concepts for each of the inefficient states in each fiscal year. Note that, as in Table 20-2, Table 20-3 and Table 20-4, the states that have been consistently efficient during this period are not shown because their target values coincide with the observed values, and therefore no changes in expenditure are deemed necessary. Looking at the weighted average row, it can be seen that in the first year of the period under study all expenditures seem to have been in need of important reductions (between 5% and 10%). The need for expenditure reductions was smaller in 2008. In 2009, welfare and government expenditure reductions were still possible although at a lower rate while expenditure in education, health care and public safety should have slightly increased. In the last two years the expenditures were again excessive, with the largest potential reductions (around 7%) in welfare and government expenditures and smaller reductions (below 5%) in education and health care.

It is also interesting to analyze the sum of the absolute expenditure changes for all the states. The aggregate results are shown in Figure 20-7. It is noticeable how the aggregated observed expenditures were well above (of the order of 100 billion US$) those computed by the model for the first year of the crisis, were then significantly reduced in the following two years and have come back again to their usual values in 2010 and 2011. The reduction in the estimation of the aggregated overspending of the states during years 2008 and 2009 occurred for all expenditure categories. It is significant that the model estimates that the expenditures on education, health care and public safety have been cut excessively so that in 2009 3.1 billon US$ more, 1.1 billion US$ more and 1.2 billions US$ more, respectively, should have been spent on these three areas. Note also how in that year the overspending in welfare was still high (16.3 billion U$) but much lower than its usual level (of around 30 billion US$). In the last year, except for health care, relative overspending has almost returned to pre-recession levels. Note that we are not saying that health care spending is not high in absolute terms, but that since it has increased for all states, in relative terms the excess health care expenditures for the inefficient states are not as high as, for example, those on welfare.

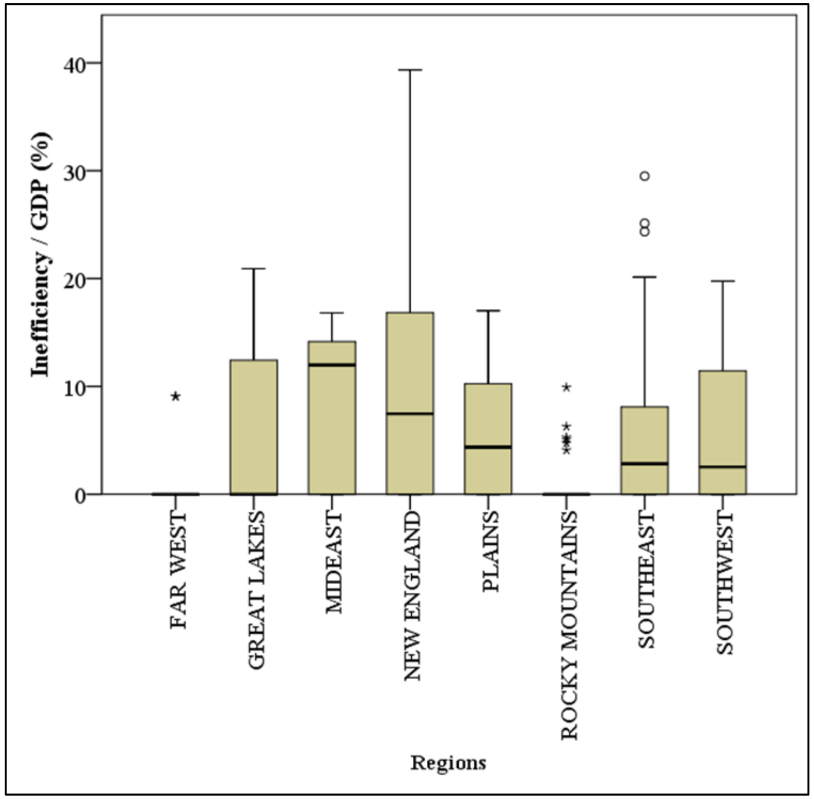
Figure 20-7 Projected aggregate states' expenditures changes



	2007	2008	2009	2010	2011
Education	-0,030	-0,018	0,003	-0,013	-0,024
Public welfare	-0,038	-0,026	-0,016	-0,033	-0,030
Health care	-0,008	-0,001	0,001	-0,004	-0,003
Public safety	-0,003	-0,001	0,001	-0,001	-0,002
Government admin.	-0,003	-0,002	-0,001	-0,003	-0,003
Other expenditure	-0,026	-0,012	-0,003	-0,043	-0,020
Total expenditure	-0,108	-0,061	-0,014	-0,098	-0,081

In order to study the distribution of total inefficiencies according to economic regions, Figure 20-8 shows a boxplot of the states' inefficiencies grouped according to the economic regions defined by the Bureau of Economic Analysis and corresponding to the five years covered. Note that total inefficiency has been normalized by GDP, to take into account differences across states and regions in terms of their economic size. Note also that whereas the Mideast and New England show the larger feasible reductions, fewer sources of inefficiency have been detected for the Far West and Rocky Mountains.

Figure 20-8 Total states' inefficiency for all years normalized by GDP and grouped into economic regions



Although some previous studies, such as Hauner and Kyobe (2010) and Afonso et al. (2010) have applied a regression with explanatory variables, identifying the determinants of inefficiencies is beyond the aim of this paper. However, it is interesting to plot the inefficiencies revealed by the model versus population or GDP so that any pattern of influence can be studied. Figure 20-9 shows the average aggregate inefficiencies (normalized by GDP) from 2007 to 2011 versus average population in that period. Similarly, Figure 20-10 shows the average aggregate inefficiency versus average GDP. It can be seen that the most inefficient states (relative to their GDP), such as West Virginia and Vermont, have a low population and that no feasible reductions are detected in large states, such as Texas and California. This may be an effect of the VRS assumption considered in the model, which tends to assess as relatively efficient those states having the minimum value of an input or the maximum value of an output. In general, except for the above effect, no clear relationship between GDP and inefficiency can be inferred.

Figure 20-9 Average states' inefficiency normalized by GDP versus average population

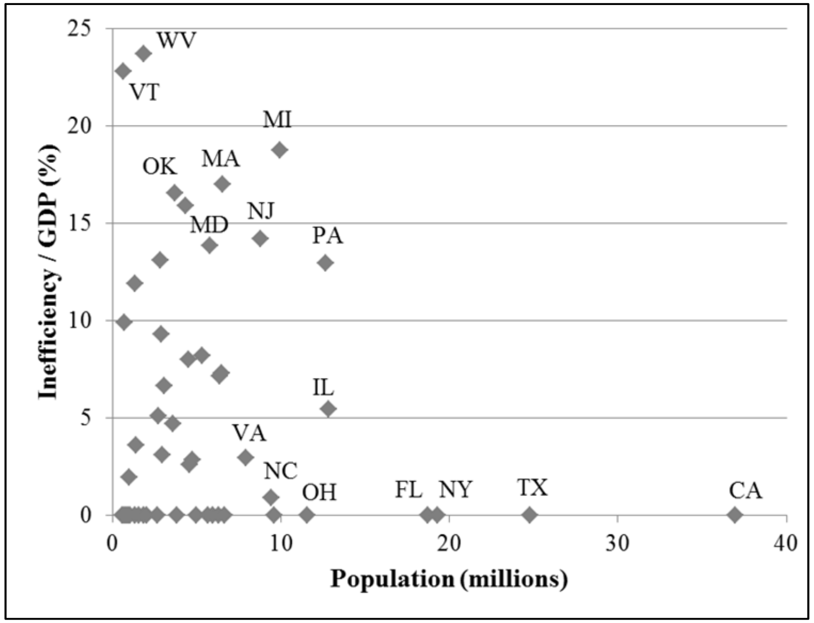
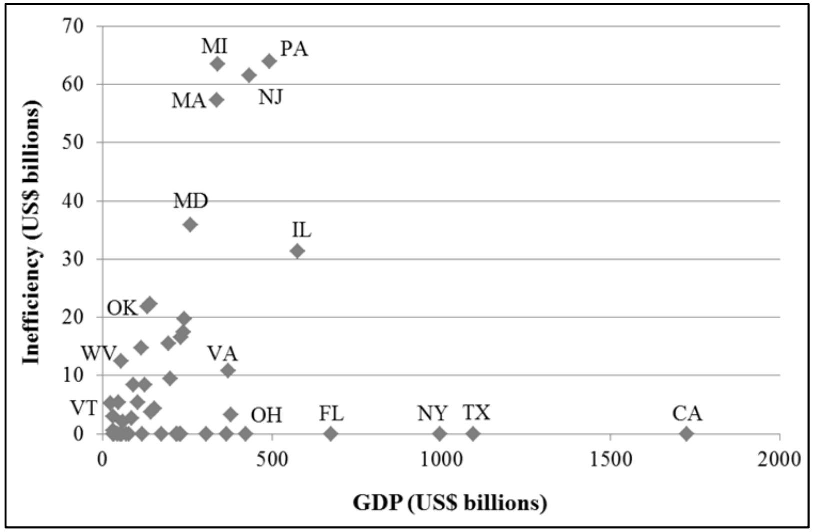


Figure 20-10 Average states' inefficiency versus average GDP



Finally, there is some room for discussion about the influence of political ideology on tax burdens, size of governments' budgets and overspending. Thus, Potrafke (2011) found, by means of a dynamic panel data model, that government ideology, namely left- or right-wing parties, hardly influenced the allocation of public expenditures in OECD countries. On the other hand, Adam et al. (2011) found that countries with right-wing governments have higher efficiency. In our case, a non-parametric t-test has been carried out to compare the average inefficiency of the states governed by the Democratic and Republican parties. The t-test points out that, on average, states governed by the Democratic Party, showed greater inefficiencies relative to GDP (mean = 6.85, standard error = 0.72) than states governed by the Republican Party (mean = 3.89, standard error = 0.55). This difference was significant at the $p = 0.001$ level; however, it did not represent a large effect ($r = 0.21$). These findings are more in line with Adam et al. (2011), implying that states governed by the Republican Party should better control public expenditure thus reducing the tax burden.

Some words of caution and caveats about the assumptions and limitations of the study are due. An important assumption in DEA is that the units under assessment are homogeneous. That can be argued to hold in the case of U.S. states but only up to a point. Thus, it has been commented above that states' fiscal years are not always coincident and that their legislatures have the freedom and power to tax people differently. They can also decide which specific programmes to finance, which additional funds to set up, etc. Moreover states are very diverse, in terms of population size, demographics, geographical extension, natural resources, etc. It is logical to expect that the wealth and needs of (and services demanded by) their citizens may be different. In summary, there is therefore much built-in heterogeneity in the system. Through aggregation into broad tax and expenditure classes, the effects of this heterogeneity are alleviated but may not disappear completely. That is why, for example, the efficient targets for the expenditures would have to be assessed in the light of the political process of each state. It would be unreasonable to expect from such a simple methodology as DEA, which uses just a table of tax receipts and expenditures figures, to reach target levels that are consistent with the priorities and specific circumstances of each state.

In addition to the above, it must be acknowledged that the proposed approach considers a simplified view of the state finances and budgeting processes, which are more complex in reality and involve multiple players, revenue projections, negotiations and compromises, mid-year revisions, etc. The proposed approach just represents an attempt to use the officially approved figures to compare, ex post, how efficient the states have been in managing the sources and uses of available funds and resources, and to what extent, in relative terms, the spending that has taken place relates to the population and GDP of each state.

20.4 Summary and conclusions

In this paper, a network DEA approach has been applied to assess the efficiency of U.S. states. The analysis is based on a two-stage network, with a first process labelled Public Finance (aimed at collecting tax receipts and other revenue in order to finance expenditures obtaining, as by-products, liquidity and debt levels) and a second process labelled Public Services Provision (that actually transforms the state budget expenditures into services to the people and to the economy). This viewpoint sees expenditures as intermediate products, i.e. as means to support the ends of the system, and taxes as the basic inputs that feed the overall system. As such, and consistent with the conventional DEA efficiency goal of reducing the amounts of inputs required to maintain output levels, higher than necessary taxes are considered as inefficiencies. Also considered to be inefficiencies are excessive borrowings and their corresponding interest payments. Finally, treated as a desirable variable (the larger the cash and liquid assets, the better) maintaining a low liquidity level is also considered to be a source of inefficiency.

The proposed approach has been applied to the period 2007-2011 and the results show that 22 states have managed their finances in a relatively efficient way during this period. However, unfortunately, not all states have performed so well. Some seem to have overspent, taxing the people excessively and/or financing expenditures through borrowing. Between the best and the worst performers there are degrees in the level of relative efficiency of the public finances of the states during this period. Also some states have been more efficient in some years and less so in others. In general, the estimated total inefficiency of the states, including excess debt (and debt service), excess taxes and insufficient cash holdings, is about 500 billion US$ and although it increased 20% in the first two years of the crisis it has come back to its "normal" level. The results also suggest that, on average, states governed by the Democratic Party, showed greater inefficiencies relative to GDP than those governed by the Republican Party.

An interesting feature of the proposed network DEA model is that it is able to compute target levels not only for the inputs and outputs but also for the intermediate products. That allows the setting of efficient targets for the expenditures of a state. All these targets take into account the population and GDP of a state, and the expenditure levels of their peer states. In this way, it can be said that the best state budgeting practices would be spread and the efficiency of the whole system improved. Analysis of the aggregated discrepancies between the target and observed expenditures sheds much light on the evolution of these expenditures over these years. It has been found that excess expenditures were reduced during the first two years of the crisis but have since then almost returned to their previous levels.

Although identification of the inefficient states may not be news, in the sense that the general public and the media already know about the budget problems of many states, the quantification of these states' inefficiencies through a benchmarking exercise such as the one carried out in this study may be relevant and contribute to the debate. Also, as a tool to analyze state budgets and help prevent an inefficient use of taxpayers' money, we believe that the proposed network DEA model and the DEA methodology in general, may represent a useful first step.

Finally, as mentioned in the results discussion, as a topic for further research, it may be interesting to study the determinants of the estimated inefficiencies. It is a common limitation of DEA that, although inefficiencies are uncovered, it does not provide many clues about the reasons that may explain those inefficiencies. Thus, a regression analysis could be carried out to identify some economic, institutional, demographic and geographical explanatory variables that may influence the performance of taxation and expenditure of the states (e.g. Hauner & Kyobe, 2010).

Appendix

Table 20–5 Reductions/increases in expenditures for year 2007 (%)

Fyear 2007

	EduExp	WelExp	HealthExp	SafeExp	GovExp	OthExp
Alabama	-30.9	-16.2	-52.7	9.2	-18.7	1.2
Arizona	-5.2	-24.6	-9.4	-19.4	-13.6	-15.4
Arkansas	-32.2	-36.6	-26.9	11.1	-38.2	13.0
Connecticut	0.0	0.0	0.0	0.0	0.0	0.0
Hawaii	0.0	0.0	0.0	0.0	0.0	0.0
Illinois	21.5	-24.4	-4.6	31.4	5.6	-26.1
Indiana	0.0	0.0	0.0	0.0	0.0	0.0
Iowa	-24.3	-22.8	-30.5	36.8	-15.8	-0.1
Kansas	-26.4	-18.9	-43.1	6.9	-31.9	-0.4
Kentucky	-30.3	-37.4	-39.5	1.9	-20.0	-13.5
Louisiana	11.3	15.0	-16.0	-9.2	6.4	-30.7
Maine	-7.8	-48.9	-39.7	7.0	-32.9	-10.5
Maryland	-19.1	-27.7	-43.1	-49.1	-41.7	-17.2
Massachusetts	3.2	-44.0	31.2	-23.4	-25.7	-25.5
Michigan	-38.6	-25.2	-18.3	-18.8	66.3	3.6
Minnesota	-33.7	-36.9	59.2	-12.4	-24.5	-13.8
Mississippi	0.0	0.0	0.0	0.0	0.0	0.0
Montana	0.0	0.0	0.0	0.0	0.0	0.0
New Jersey	-4.2	-22.4	-24.9	-14.6	-29.0	-35.9
North Carolina	-19.5	-17.7	-13.7	-15.9	-1.8	-5.3
North Dakota	-15.5	6.8	100.0	60.0	13.6	-10.4
Oklahoma	-23.5	-27.2	8.3	-19.2	-0.5	10.3
Pennsylvania	2.2	-37.0	-17.5	-6.4	-32.2	-16.4
South Carolina	-17.5	-23.1	-40.2	0.4	-48.5	-32.0
Utah	-33.4	10.4	-38.3	-4.1	-54.9	15.3
Vermont	-50.9	-37.0	-1.2	-34.5	-4.4	30.0
Virginia	-9.6	5.6	-39.9	-30.6	-5.4	10.7
West Virginia	-25.2	-31.1	34.6	8.6	-37.0	-10.6
Weighted aver.	*-6.2*	*-10.2*	*-8.2*	*-4.8*	*-7.2*	*-5.9*

Table 20–6 Reductions/increases in expenditures for year 2008 (%)

	Fyear 2008					
	EduExp	WelExp	HealthExp	SafeExp	GovExp	OthExp
Alabama	0.0	0.0	0.0	0.0	0.0	0.0
Arizona	0.0	-24.6	-4.6	-17.4	-6.1	-19.5
Arkansas	-34.1	-36.6	-33.0	12.7	-36.1	18.9
Connecticut	0.0	0.0	0.0	0.0	0.0	0.0
Hawaii	0.0	0.0	0.0	0.0	0.0	0.0
Illinois	0.0	0.0	0.0	0.0	0.0	0.0
Indiana	-15.8	-28.2	91.5	19.9	41.7	-0.3
Iowa	-15.7	-5.8	-16.9	32.2	-11.8	26.9
Kansas	-25.7	-10.3	-26.8	-2.7	-12.3	5.6
Kentucky	-19.5	-25.7	-6.0	1.4	-11.6	-8.3
Louisiana	7.5	10.1	-17.1	-10.1	5.1	-38.4
Maine	25.0	-36.7	-19.1	47.8	-19.0	11.6
Maryland	-10.7	-11.6	-18.2	-40.0	-29.3	2.0
Massachusetts	16.0	-40.2	47.4	-17.1	-29.7	-24.1
Michigan	-34.8	-24.3	-15.0	-19.0	54.5	10.5
Minnesota	-29.5	-44.5	39.8	7.4	-24.9	-27.1
Mississippi	-21.9	-26.6	-28.9	-2.0	16.3	-28.3
Montana	0.0	0.0	0.0	0.0	0.0	0.0
New Jersey	28.3	5.6	22.9	8.4	-18.8	-12.1
North Carolina	-20.9	-12.9	-18.6	-11.6	6.3	4.8
North Dakota	-0.5	-0.1	174.7	72.1	45.2	-0.9
Oklahoma	-11.8	-22.5	31.8	-2.7	37.1	38.2
Pennsylvania	13.5	-14.2	-1.5	-10.8	-22.2	3.6
South Carolina	-16.7	-16.1	-39.5	2.4	-51.6	-36.4
Utah	-24.8	33.1	-29.6	16.2	-42.4	17.1
Vermont	-39.8	-39.2	16.3	-22.0	14.8	42.7
Virginia	0.0	0.0	0.0	0.0	0.0	0.0
West Virginia	-5.3	-11.4	85.1	18.7	-15.2	1.9
Weighted aver.	*-3.7*	*-6.8*	*-1.4*	*-2.3*	*-4.2*	*-2.6*

	Fyear 2009					
	EduExp	WelExp	HealthExp	SafeExp	GovExp	OthExp
Alabama	-39.0	-8.2	-47.4	7.0	9.8	-3.1
Arizona	4.4	-22.4	-4.0	-26.1	8.7	-16.9
Arkansas	-31.2	-19.9	-34.7	34.6	-23.2	-2.3
Connecticut	41.0	-22.1	-35.5	27.8	-19.8	-7.4
Hawaii	0.0	0.0	0.0	0.0	0.0	0.0
Illinois	37.5	-15.7	24.0	45.7	35.8	-9.2
Indiana	-23.8	-17.8	110.9	25.0	35.9	2.4
Iowa	-21.5	-14.6	-33.5	23.0	-42.2	-11.7
Kansas	-3.4	12.4	-36.6	28.1	-23.1	11.1
Kentucky	-1.7	-14.0	-1.6	35.2	17.7	15.0
Louisiana	0.3	-0.7	-27.2	-10.3	3.1	-30.4
Maine	50.0	-29.6	1.9	65.5	-9.8	18.6
Maryland	31.6	18.4	17.0	-10.7	17.1	35.0
Massachusetts	33.2	-27.5	114.8	2.5	-16.2	-11.8
Michigan	-11.6	0.9	17.6	-5.9	103.9	27.1
Minnesota	-34.9	-31.1	66.3	-4.1	-16.7	-10.7
Mississippi	-25.1	-39.2	-41.5	2.3	24.2	-34.4
Montana	-6.7	-5.3	57.1	-4.2	-26.2	0.2
New Jersey	43.9	24.5	56.1	35.5	-0.4	-9.7
North Carolina	0.0	0.0	0.0	0.0	0.0	0.0
North Dakota	10.9	6.1	183.6	82.6	61.9	0.1
Oklahoma	-23.1	-18.4	17.7	-20.3	10.9	21.0
Pennsylvania	13.3	-15.6	-4.9	-4.8	-41.1	0.3
South Carolina	0.0	0.0	0.0	0.0	0.0	0.0
Utah	-36.7	13.9	-35.2	0.4	-45.3	8.8
Vermont	-37.9	-26.0	22.1	-19.8	28.8	31.0
Virginia	-1.3	15.8	-37.5	-22.5	-9.6	6.8
West Virginia	5.7	-5.5	107.4	24.1	-0.3	10.4
Weighted aver.	*0.6*	*-4.1*	*1.0*	*2.0*	*-1.1*	*-0.6*

Table 20–7 Reductions/increases in expenditures for year 2009 (%)

Table 20–8 Reductions/increases in expenditures for year 2010 (%)

	Fyear 2010					
	EduExp	WelExp	HealthExp	SafeExp	GovExp	OthExp
Alabama	-29.7	-14.4	-48.3	6.3	0.2	4.0
Arizona	11.6	-18.9	4.1	-19.3	28.0	-14.6
Arkansas	-30.7	-29.7	-11.8	3.4	-12.2	5.4
Connecticut	14.6	-12.2	-49.2	-9.0	-43.7	-19.8
Hawaii	-14.0	1.4	-48.3	11.2	-40.7	-24.9
Illinois	31.0	-19.9	5.7	34.8	-2.5	-35.6
Indiana	-27.9	-17.2	110.7	20.0	52.2	-6.7
Iowa	-17.7	-25.7	-25.9	27.2	-39.5	-12.1
Kansas	-25.4	-8.6	-40.3	4.2	-16.6	-5.3
Kentucky	-27.8	-27.3	-8.1	7.4	-4.1	-11.9
Louisiana	-7.7	1.6	-30.9	-21.5	-12.1	-28.2
Maine	4.5	-43.1	-7.9	12.5	0.9	-4.8
Maryland	-10.8	-26.5	-19.3	-35.7	-25.2	-5.6
Massachusetts	12.4	-35.9	76.0	-11.4	-26.3	-21.6
Michigan	-17.3	-5.0	15.6	1.5	121.0	-3.5
Minnesota	-21.3	-40.6	83.5	2.3	-28.7	-19.5
Mississippi	-13.0	-35.4	-45.5	6.4	-18.8	-32.6
Montana	0.0	0.0	0.0	0.0	0.0	0.0
New Jersey	7.3	-10.9	1.9	-5.2	-19.9	-38.4
North Carolina	0.0	0.0	0.0	0.0	0.0	0.0
North Dakota	0.0	0.0	0.0	0.0	0.0	0.0
Oklahoma	-26.8	-20.1	29.8	-22.9	-7.8	12.9
Pennsylvania	10.8	-26.7	-6.9	-11.3	-45.6	-29.0
South Carolina	0.0	0.0	0.0	0.0	0.0	0.0
Utah	-32.2	9.8	-32.6	7.3	-54.6	4.5
Vermont	0.0	0.0	0.0	0.0	0.0	0.0
Virginia	7.7	6.6	-43.4	-27.1	-28.4	5.7
West Virginia	-6.6	-4.2	132.6	11.4	9.2	-10.0
Weighted aver.	*-2.6*	*-7.9*	*-3.5*	*-2.2*	*-6.5*	*-7.7*

Table 20–9 Reductions/increases in expenditures for year 2011 (%)

	Fyear 2011					
	EduExp	WelExp	HealthExp	SafeExp	GovExp	OthExp
Alabama	0.0	0.0	0.0	0.0	0.0	0.0
Arizona	9.4	-22.4	3.0	-8.3	36.5	-10.8
Arkansas	-38.5	-28.7	-28.4	-3.4	-29.7	-0.7
Connecticut	0.0	0.0	0.0	0.0	0.0	0.0
Hawaii	-26.7	-19.6	-58.4	15.7	-49.1	-34.4
Illinois	0.0	0.0	0.0	0.0	0.0	0.0
Indiana	-26.7	-18.9	125.6	29.4	39.6	0.6
Iowa	-22.9	-25.7	-14.3	24.9	-25.7	-11.7
Kansas	-26.1	-8.7	-42.7	5.7	-21.6	0.8
Kentucky	-18.2	-16.1	-6.9	14.7	-12.9	-2.8
Louisiana	0.0	0.0	0.0	0.0	0.0	0.0
Maine	34.4	-30.9	1.0	38.2	-19.6	13.3
Maryland	-14.7	-18.2	-19.2	-43.7	-37.3	7.6
Massachusetts	9.8	-33.9	57.9	-17.9	-35.2	-21.9
Michigan	-34.2	-21.4	-13.6	-19.5	59.0	-6.2
Minnesota	-23.9	-35.5	92.4	-0.7	-31.0	-13.3
Mississippi	-15.9	-31.8	-37.8	4.6	-3.6	-24.6
Montana	0.0	0.0	0.0	0.0	0.0	0.0
New Jersey	3.7	-9.0	-10.9	-12.2	-31.4	-28.6
North Carolina	0.0	0.0	0.0	0.0	0.0	0.0
North Dakota	-17.5	-0.8	40.0	38.5	24.1	-14.7
Oklahoma	-23.9	-17.0	15.6	-24.8	-21.6	10.9
Pennsylvania	0.2	-25.3	-34.2	-11.7	-48.5	-14.1
South Carolina	0.0	0.0	0.0	0.0	0.0	0.0
Utah	-29.8	12.0	-38.4	12.4	-59.1	-3.6
Vermont	-39.6	-36.8	8.8	-27.8	37.8	30.4
Virginia	0.0	0.0	0.0	0.0	0.0	0.0
West Virginia	-10.7	-15.3	88.5	9.1	-8.6	-16.7
Weighted aver.	*-4.7*	*-6.7*	*-2.5*	*-2.8*	*-7.0*	*-3.6*

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