# Effective Modulation Algorithm for Threelevel Converters

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**Abstract:** This paper presents an effective modulation algorithm (EMA) for high power voltage source three-level converter. This approach drastically reduces the computational load maintained, being substituted by decision making. This method permits the on-line computation of the switching sequence and the on-state durations of the respective switching state vectors corresponding to the modulation of a three-level inverter. Therefore a stored information from a memory or a table is not necessary in EMA. The modulation method used in this work presents the advantage of eliminating the angle from the calculations. This modulation technique permits an economic and simply electronic implementation.

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Keywords: modulation algorithm, three-level inverter, high power voltage source, switching state vectors, neutral-point voltage balancing.

## 1. Introduction

Although the advantages of multilevel converters were known since Akira Nabae proposed the topology NPC (Neutral Point Clamped) inverter in 1981 [1], implementation was limited due to the complexity of the switching control. In recent years multilevel voltage source inverter have been used in medium and high power applications [2]. They present the capability of increasing the output voltage magnitude and reducing the output voltage and current harmonic content, the switching frequency and the voltage supported by each power semiconductors. Three-level inverters permit to use the double voltage under the same type of switches. By synthesising the AC output voltage from several levels of voltages, staircase waveforms are produced, which approach the sinusoidal waveform with low harmonic distortion. Multilevel converter enables the ac voltage to be increased without transformer. The cancellation of low frequency harmonics from the ac voltages at the different levels means that the size of the ac inductances can be reduced. Due to these attractive characteristics several control algorithms of multilevel converter have been proposed. [3][4][5][6][7]

In this work, an effective approach that drastically reduces the computational load using a decision making is presented. This method is based on the decision based pulse width modulation developed by Holtz [8] for

conventional two-level converter. The modulation method used in this work presents the advantage of eliminating the angle from the calculations. The proposed effective modulation algorithm permits an simply implementation and reduces the load and the time computational.

# 2. Modulation Technique description

Three-phase quantities are usually transformed into the phasor representation since it simplifies the analysis of the modelled system. Since the switching of any power topology stay at discrete states, space vector modulation is used to approximate a reference voltage vector u\* calculating the time to its surrounding state vectors.

 $V_a$ ,  $V_b$  and  $V_c$  are the three-phase quantities usually transformed into the phasor representation. They are the amplitudes of the three 120° displaced phase voltages.

Three vectors  $u_1$ ,  $u_2$  and  $u_3$  are used to approximate the desired voltage vector  $u^*$  in polar coordinates in a control cycle  $T_m$ . The modulation law requires the actual voltage vector u to equal its reference value  $u^*$ . This signal is generated by a microcontroller.  $u^*$  is represented in the stationary reference frame.

$$u = u^* = V_a + V_b e^{\frac{j2\pi}{3}} + V_c e^{\frac{j4\pi}{3}}$$

$$= \text{Re} \{u^*\} + j \text{Img} \{u^*\}$$
(1)

During each modulation subcycle of duration  $T_m$  a switching sequence is generated. It is composed of three switching state vector  $u_1(t_1)$ ,  $u_2(t_2)$  and  $u_3(t_3)$ , where  $t_1$ ,  $t_2$  and  $t_3$  are the on-state durations of the active switching state vectors.

Referring to first sextant of the regular hexagon, the voltage space vector averaged over one sub cycle  $T_m$  is:

$$u = \frac{\left(t_1 u_1 + t_2 u_2\right)}{T_m}$$

In this paper, the problem is solved for voltage vector in the first sextant. However, this reference vector can be located in any of the six sectors of the regular hexagon which contains the switching state vectors. This problem is easily solved rotating  $u^*$  counter clock wise by an angle  $(n-1)\frac{\pi}{3}$ , where n is

the sextant number, n = 1,...,6. This rotation displaces any reference vector to the first  $60^{\circ}$  to be study there. The switching state vectors for the three-level inverter control are determined by reverse rotation.

The input to the modulation algorithm of the three-level converter is the normalized reference voltage vector. Thus, the first step of the method consist on localizing the sextant n = 1,..., 6 where is located the reference voltage vector  $u^*$ .

The voltage vector u\* is transformed into u\* $_{\text{flat}}$ . This transformation consist on scaling imaginary part multiplying it by  $\frac{1}{\sqrt{3}}$ . The

hexagon is flattened. Figure 1 shows the regular hexagon defined by the switching state vectors before and after of the transformation in the complex plane.

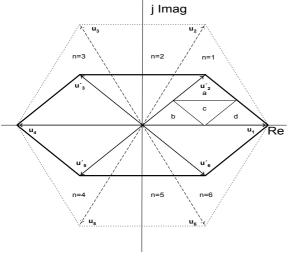


Figure 1. Switching state vectors before and after the transformation in the complex plane

The transformation of  $u^*$  into  $u^*_{\it flat}$  makes it possible to avoid on-line computations. These computations are substituted by decision making.

Once the sextant n has been localized into the regular hexagon, the identification of the sector a, b, c or d into the sextant, the switching sequence to approximate a reference voltage vector and the on-state durations are calculated by rotating u\* to the first sextant. The rotated reference voltage vector is:

$$u_{g} = u_{ga} + j u_{gb} = u \cdot exp(-j(n-1)\frac{\pi}{3});$$

$$n=1,...,6$$
(3)

This vector  $\mathbf{u}_g$  is transformed in another with identical real part and reduced imaginary part.

$$u_{gf} = u_{ga} + j \frac{\sqrt{3}}{3} u_{gb} = u_{gfa} + j u_{gfb}$$
 (4)

This simplifies the sector identification. In general, a vector  $\mathbf{u}^*_{gf}$  can be expressed in function of the time of its surrounding state vectors.

$$u^*_{gf} = t_1 u^1_{gf} + t_2 u^2_{gf} + t_3 u^3_{gf}$$
 (5)

In the algorithm the subcycle  $T_{nm}$  is normalized to the unit:

$$T_{nm} = 1 = t_1 + t_2 + t_3 \tag{6}$$

Therefore, the problem of expressing  $u^*_{gf}$  as a function of its surrounding state vectors can be stated in a new illuminating manner:  $u^*_{gf} - u^2_{gf}$  can be regarded as the new vector to approximate the active switching state vectors  $(u^1_{gf} - u^2_{gf})$  and  $(u^3_{gf} - u^2_{gf})$ , and  $t_1$  and  $t_3$  being the corresponding on-state durations.  $u^*_{gf}$  can be studied from the centre of the regular hexagon which correspond to  $u^2_{gf}$ . It is shown in Figure 2. In this case  $t_2 = 1 - t_1 - t_3$  and  $u^*_{gf}$  can be expressed:

$$u^*_{gf} = t_1 (u^1_{gf} - u^2_{gf}) + t_3 (u^3_{gf} - u^2_{gf}) + u^2_{gf}$$
(7)

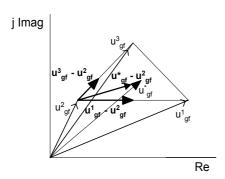


Figure 2. Example for studying  $u^*_{gf}$  from the centre of the regular subhexagon corresponding to  $u^2_{af}$ 

It is necessary to locate the triangular sector and the subhexagon where  $u^*_{gf}$  is found. For determinate the sector where  $u^*_{gf}$  is, a displacement to the centre node of the each regular subhexagon is done. Localization of the vertical and horizontal site of the centre node is necessary.

Once the sector where is located u\*<sub>qf</sub> is known the expressions to calculate the onstate durations given by Holtz conventional inverters [8] are allowed. The locations of the switching state vectors in the complex plane results very easy. Since the transformed sectors are separated by 45° lines, the sector can be readily identified by comparing the real and the imaginary parts of a complex number. Thus, the method used in this work presents the advantage of eliminating the angle from the calculations. Moreover, the numeric evaluation of the onstate durations are reduced to a simple addition. This efficient algorithm uses the minimum number of possible comparators.

It is necessary a reverse rotation and transformation to obtain the definitive state vectors and on-state durations which generate the original normalized reference voltage vector. For recuperate the original sectors is necessary to distinguish between two cases. The first one consist on to pass from the first sextant to the next odd sextant.

 $u_{gf}^*$  is multiplied by  $e^{\frac{j2\pi s}{3}}$ , s = 1,2

$$u^* = u^*_{gf} e^{\frac{j2\pi s}{3}}$$
(8)

In general, s = 1 corresponds to the pass from the sextant n = 1 to n = 3 and s = 2 to the pass from n = 1 to n = 5.

The following equation shows the general expression for a 120° rotation.

$$u^* = V_a e^{\frac{js2\pi}{3}} + V_b e^{\frac{j(s+1)2\pi}{3}} + V_c e^{\frac{j(s+2)2\pi}{3}}$$
(9)

The output of the algorithm at this point is a (3 x 3) matrix containing Va, Vb and Vc for the three switching vectors in the first sextant and a vector containing their corresponding on-state durations.

S matrix shows the switching states vectors in the first sextant given by the modulation algorithm. For 120° rotation the following transformation of the S matrix is obtained:

$$S = \begin{pmatrix} V_{a}^{1} & V_{b}^{1} & V_{c}^{1} \\ V_{a}^{2} & V_{b}^{2} & V_{c}^{2} \\ V_{a}^{3} & V_{b}^{3} & V_{c}^{3} \end{pmatrix} \rightarrow \begin{pmatrix} V_{c}^{1} & V_{a}^{1} & V_{b}^{1} \\ V_{c}^{2} & V_{a}^{2} & V_{b}^{2} \\ V_{c}^{3} & V_{a}^{3} & V_{b}^{3} \end{pmatrix}$$

$$T = \begin{bmatrix} t_{1} & t_{2} & t_{3} \end{bmatrix}$$
(10)

The state vectors and the on-state durations of a three level inverter in sextant n = 3 and n = 5 from n = 1 are calculated.

Pass  $n = 1 \rightarrow n = 3$ :

The state vectors in n = 1 and sector 3:

$$\begin{pmatrix} 210\\110\\100 \end{pmatrix} \text{ are transformed into: } \begin{pmatrix} 021\\011\\010 \end{pmatrix} \text{ in } n = 3$$

and sector 3

Pass  $n = 3 \rightarrow n = 5$ :

The state vectors in n = 3 and sector 3:

$$\begin{pmatrix} 021\\011\\010 \end{pmatrix} \text{ are transformed into: } \begin{pmatrix} 102\\101\\001 \end{pmatrix} \text{ in n = 5}$$

and sector 3

The component  $V_c$  in n = 1 and n = 2 is cero.

If the original reference voltage vector  $u^*$  was in a even sextant an counter clock wise  $60^\circ$  rotation will be necessary from n = 1 to n = 2. Then a  $120^\circ$  rotation from n = 2 to n = 4 or 6 will be carry out if it is needed.

The pass from n = 1 to n= 2 can be performed with a  $60^{\circ}$  reverse rotation  $j\pi$ 

multiplying by e  $^3$  the equation (1) and projecting the vector in the second sextant over  $V_a$  and  $V_b$  exes. Equalling real and imaginary parts the following transformation of the S matrix is obtained:

$$S = \begin{pmatrix} V_{a}^{1} & V_{b}^{1} & V_{c}^{1} \\ V_{a}^{2} & V_{b}^{2} & V_{c}^{2} \\ V_{a}^{3} & V_{b}^{3} & V_{c}^{3} \end{pmatrix} \rightarrow \begin{pmatrix} V_{a}^{1} - V_{b}^{1} & V_{a}^{1} & V_{c}^{1} \\ V_{a}^{2} - V_{b}^{2} & V_{a}^{2} & V_{c}^{2} \\ V_{a}^{3} - V_{b}^{3} & V_{a}^{3} & V_{c}^{3} \end{pmatrix}$$

$$T = \begin{bmatrix} t_{1} & t_{2} & t_{3} \end{bmatrix}$$
(11)

Following with the previous example for a three level converter:

Pass  $n = 1 \rightarrow n = 2$ :

The state vectors in n = 1 and sector 3:

$$\begin{pmatrix} 210\\110\\100 \end{pmatrix} \text{ are transformed into: } \begin{pmatrix} 120\\010\\110 \end{pmatrix} \text{ in } n = 2$$

and sector 3

The signal flow graph in figure 3. shows the decision based modulation algorithm. The voltage reference vector u\* is transformed to u\*<sub>gf</sub> which is sampled at a clock and fed to a decision tree. Before converter block the sequence of switching states and their respective durations are generated in the decision tree algorithm. Finally a load is connected to the converter.

## 3. Simulation results

EMA has been implemented in a simulated diode clamped multilevel converter.

The following figures show the simulation results. The considered operation conditions of the modelled three-level converter are: switching frequency f = 1 kHz, voltage DC-link 1600 V and R-L load. Where R = 4  $\Omega$  and L = 1mH.

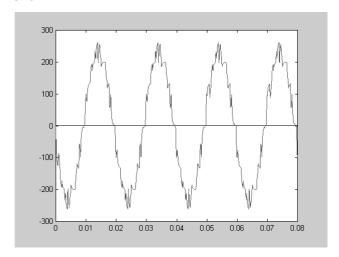


Figure 4. Simulated current curve of the three-level converter

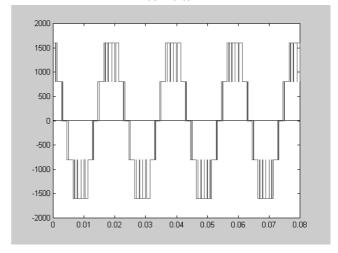


Figure 5. Simulated phase-phase voltage curve of the three-level converter

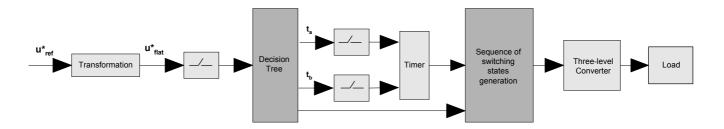


Figure 3. Decision based modulation algorithm

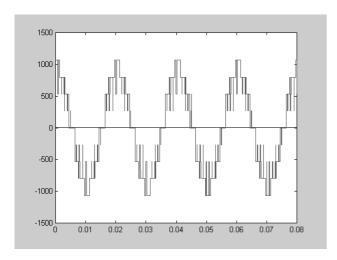


Figure 6. Simulated phase-neutral voltage curve of the three-level converter

## 4. Summary

The efficient modulation algorithm presented in this work is very useful to readily calculate the on-line computation of the switching sequence and the on-state durations of the respective switching state vectors corresponding to the modulation of a threelevel inverter without physically affect to the load connected to the multilevel converter. The vector selection is adjusted according to the input reference to improve the voltage generation being balanced the DC-link capacitor voltage. The simulation results show an excellent performance of the system using decision based modulation algorithm. The expected results provided by other modulation techniques have been obtained using EMA. However, the computational effort using EMA is drastically reduced compared with other conventional Space Vector Pulse Width Modulation (SVPWM) techniques. This modulation technique permits an economic and simply electronic implementation. The effectiveness of the suggested EMA for three-level converter is being verified by experimental results using a DSP.

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