Why are neutral elements immutable?

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Abstract

It is not exaggerated to say that all of the people, included the youngest, know the important role played by the elements 0 and 1 in Mathematics. One of the main reasons for these elements (normally named *units*, although its right name is *neutral elements)* to be fundamental is their uniqueness within any mathematical structure. However, the main goal of this paper is to show that these roles could be discussed by redefining the usual mathematical laws, quite more in agreement with some aspects of the reality, particularly in the physical world.

Keywords: Units in Mathematics and Physics, New Aritmethic, Isonumbers.

Resumen

No es exagerado decir que todas las personas, incluidas las más jóvenes, conocen la importancia de los elementos 0 y 1 en Matemáticas. Una de las razones por las que estos elementos (normalmente llamados *unidades*, aunque su nombre correcto sea el de *elementos neutros)* son fundamentales es por su unicidad dentro de cualquier estructura matemática. Sin embargo, el objetivo principal de este artículo es mostrar que esta importancia pudiera ser discutida si se redefinen las leyes matemáticas usuales, lo cual estaría más de acuerdo con algunos aspectos actuales de la realidad, particularmente en el mundo físico.

Palabras clave: Unidades en Matemáticas y Física, Nueva Aritmética, Isonúmeros.

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I. INTRODUCTION

The objective of this paper is to raise the question of the immutability of the *neutral elements*, and take advantage of it to also give a very short introduction to a new Mathematics which is at present appearing: the *Isomathematics*.

Since our more youthful years, we all have been aware of the existence of certain elements, colloquially named *units*, although its right name is *neutral elements*.

Indeed, in the first place, we are taught to count one-to-one: 1, 2, 3, 4,... Secondly, we are told that these last objects, named *numbers,* are related between themselves by different laws. So, we learn to add, firstly from one to one: 1+1=2, 2+1=3,... Teachers tell us that the symbol + represents the *addition* and that such a law has a neutral element: the zero (0), which means that $a + 0 = 0 + a = a$, whichever the number a is.

Later, we are taught to multiply and, in a similar way as before, we are told that the symbol × represents the *multiplication or product a*nd that the neutral element of this law is 1, that is: $a\times 1=1\times a$, and we are also taught that + and × are related between themselves by the *distributive property.*

These two neutral elements and such a property allow to form in our mind the idea of a *mathematical structure. A*t the same time, we easily understand that several types of numbers have to be consecutively appearing: *Z-numbers,* which get round the problem of computing the *opposite* of a *N-numbers*; *Q-numbers*, which do the same with respect to the *inverse* of a *Z-numbers*; later R and C, and so on, in the way that each type of them extends the previous one. all these sets have something in common: elements 0 and 1 are the neutral elements in all of them, with respect to the two laws previously indicated.

Here, we do not pretend to discuss the importance of such neutral elements, that is to say, we do not try to put in doubt the transcendence or their uniqueness, but, why have these elements to be always precisely the 0 and the 1 and not others different?, that is, why do not we let them to have numerical values depending on the mathematical structure under consideration? For example, why not consider $\sqrt{2}$, instead of 1? or why not to define: $a\times\sqrt{2}=\sqrt{2}\times a=a$? Note that the choice of $\sqrt{2}$ is anecdotal, because it could be replaced in fact with any real number, although we have taken this choice to do more amazing this subject.

II. DO OTHER POSSIBILITIES REALLY EXIST?

It is reasonable to think that 0 and 1 are the neutral elements for different reasons: by agreement, by notation or, even, by logic (please, if $\sqrt{2}$ is even an irrational number!). However, a most serious response would be to say that, in any case, the laws + and × should be redefined to accept other neutral elements. Well, let us redefine them. How long the equality 2×2=4 is going to be imposed to us? Naturally, someone will tell us that 2×2=1 (mod 3) but this response would not satisfy to us either. In fact, we do not want to reduce our sets of numbers. Why do we have to restrict the imagination of our students by obliging them to accept the unique possibility 2×2=4? Can it be thought that in our world this is truly so? Note that, if we think in Mathematics as a tool to understand the Universe, some physical theories, like Relativity, for instance, are already becoming phased out.

Indeed, it is sufficiently proved that $(R^3,+,\times)$ is not a real model for understanding the Universe. In fact, what we call *unit*:

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is not such an *unit* in the physical consideration of Universe.

To get a better understanding of these subjects, let us say that Mathematics generally used in quantitative sciences of the 20-th century were based on ordinary fields with characteristic zero, a trivial (left, right) unit I=+1 and on an ordinary associative product between generic quantities *a, b* of a given set, such as matrices, vector fields, etc. Such a Mathematics is known to be linear, local differential (beginning from its topology), and Hamiltonian, thus solely representing a finite number of isolated point-particles with action-at-a-distance forces derivable from a potential. Such a Mathematics was proved to provide an exact and invariant representation of planetary and atomic systems as well as, more generally, of all the so-called *exterior dynamical systems* in which all constituents can be well approximated as being point-like.

By contrast, the great majority of systems in the physical reality are nonlinear, nonlocal (of integral and other type) and not completely representable with a Hamiltonian in the coordinates of the experimenter. This is the case for all systems historically called *interior dynamical systems,* such as the structure of: planets, strongly interacting particles (such as protons and neutrons), nuclei, molecules, stars, and other systems. The latter systems cannot be consistently reduced to a finite number of isolated point-particles. Therefore, the mathematics so effective for exterior systems is only approximate at best for interior systems. For these reasons, the unit has not got the exclusive character from which it is always presupposed, but it depends on several external factors, like coordinates, speed, acceleration, temperature, density,...

That is, the unit matrix above indicated is not constant, but we have to consider it variable: $I_3 = I_3(x,\dot{x},\ddot{x},t,m,...)$. The question is how can we vary a mathematical unit, which is, by agreement, unique. A possible response to this question appeared in 1980, when a group of theoretical physicists began to construct a new mathematical foundation, which is, at present, known as *Isomathematics* (see [2]). In this way, let us suppose that we wish to construct the set of *real isonumbers* having a given real number I as isounit. First, we define the set $\hat{R} = \{\hat{a} = a \times I : a \in R\}$. Next, we redefine the usual product in the following way: $\hat{a} \hat{\times} \hat{b} = a \times b$, where $\hat{\times}$ is now named *isoproduct.* So, let us observe that $\hat{a} \times 1 = \hat{1} \times \hat{a} = \hat{a}$, for all $\hat{a} \in \hat{R}$. Therefore, $\hat{1} = I$ is the unit of $\hat{\times}$. Specifically, the answer to the question: "*can* $I = \sqrt{2}$ *be the unit element of the real numbers with respect to the product?*" is affirmative.

So, under such a choice, the following computations are obtained:

$$\hat{2} = 2\sqrt{2} = 2\sqrt{2} = \hat{2} \hat{\times} \hat{1}; \qquad \hat{3} = 3\sqrt{2} = \hat{3} \hat{\times} \hat{1};$$
$$\hat{6} = 6\sqrt{2} = \hat{2} \hat{\times} \hat{3}.$$

In this way, a new mathematical structure has been reached. It is formed by the same set of elements as the initial one, which constitutes a difference with the case of Z_3 Many other examples can be shown. If we think in an equality already considered in this paper, 2×2=4 we would have now, under the new conditions, that $\hat{2} \hat{\times} \hat{2} = 4\sqrt{2}$. Moreover, it is not difficult to prove that $\hat{2} \hat{\times} \hat{2} = 1$ when I=1/4 is the isounit, without working in Z_3.

So, although it does not constitute a demonstration of the immutability of isounits by itself, previous reasonings show that there is an infinite number of isoproducts and associated isounits and the neutral elements are therefore dependent of the considered isoproducts.

III. AN EXAMPLE: LORENTZ ISOSYMMETRY

The changes previously described have involved, among many other subjects, the appearance of a new cleaning energy, named *hadronic energy*, a possible generalization of Einstein's Relativity Theory, the prediction of the existence of the antimatter and, as a consequence, of the antigravity, the modification of the Einstein-Podolski-Rosen's Theory. For a more complete information about all of the subjects here commented the reader can consult both the text and the references of [5], for instance.

As a possible example, let us see the case of the Lorentz isosymmetry. So, let $M(x,\ \eta,\ R)$ be the Minkowskian *R-space*, with local chart $x=(x^k)=(r,\ x^4)$ (being $r \in R^3$ and $x^4=c_0 t$) and 4-dimensional metric $\eta = diag_n(1,1,1,-1)$. It will be $x^2 = x^{1^2} + x^{2^2} + x^{3^2} - x^{4^2}$. In the absence of gravity, space-time is determined as a smooth flat manifold endowed with the Minkowsky metric

η. Santilli proposes in [4] the *Minkowskian* $\hat{R}-space\ \hat{M}(\hat{x},\hat{\eta},\hat{R})$, by defining the *isometric* $\hat{\eta}=diag(\hat{\eta}_{11},\hat{\eta}_{22},\hat{\eta}_{33},\hat{\eta}_{44})$ and so, $\hat{x}^2=\sum_{x=1}^{4}x^i\hat{\eta}_{ii}x^i$.

Transformation group of the space-time living the space-time interval, $ds^2=dx^i\eta_{ij}dx^j$, invariant is called *Lorentz group*. Besides, the Lorentz symmetry is one of the fundamental symmetries of physical theories. It is correct into ordinary conditions, but not with extended particles, high energies or unusual physical conditions [1], such as non-linear dependence. To solve it, is possible to use the transformation groups corresponding to the Minkowsky isometric (*Lorentz isogroup* [4]). These transformations are formally linear and local on the Minkowskian isospace $\hat{M}$, but generally non-linear and non-local on the conventional space M. They provide methods for the explicit construction of (generally nonlinear but local) symmetries of conventional gravitational metrics, such as Schwarzschild's metric. Besides, Einstein's gravitation or any other gravitational theory (not necessarily Riemannian) with metric $\hat{\eta}=T\eta(\det(T)>0)$ admits the conventional Lorentz symmetry as a global isotopic symmetry.

IV. FINAL CONCLUSIONS

Nowadays, mathematicians and physicist throughout the world are trying to endow this new theory with a right mathematical foundation which makes it serious and consistent. It requires a first step by step construction of each usual mathematical structures (see [3], for instance).

So, according with the previous exposition, it is not hazardous to affirm that we are dived in the beginnings of a new arithmetic which could supply non imaginable facts in a non distant future, in the physical world, above all. We think that this impression should be transmitted to young students to extend their minds and to allow them to elaborate their own conjectures about this new situation.

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