UNIVERSIDAD DE SEVILLA

Departamento de Matemática Aplicada I

UNA GENERALIZACIÓN DE LAS ÁLGEBRAS DE LIE FILIFORMES

APÉNDICE

Jesús Mª Cabezas Martínez de Aragón

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Una generalización de las álgebras de Lie filiformes

Apéndice

Memoria presentada por Jesús Mª Cabezas Martínez de Aragón para optar al grado de Doctor en Matemáticas por la Universidad de Sevilla.

Vº Bº
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Fdo. José Ramón Gómez Martín, Catedrático de E.U. del Departamento de Matemática Aplicada I de la Universidad de Sevilla.

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Resumen

En este apéndice se desarrollan demostraciones que en la memoria: "Una generalización de las álgebras de Lie filiformes" están, bien incompletas, o bien se omiten, por ser casi análogas a otras especificadas en su totalidad.

Del capítulo 2: "Clasificación de las álgebras $(n-4)$-filiformes por extensiones centrales" se desarrollan en su totalidad:

- la demostración del teorema 2.5., en la cual aparecen las cinco familias de álgebras $(n-4)$-filiformes que sean extensiones centrales de primera generación del álgebra $\mathfrak{g}_{n-1}^{2}$.

- la demostración del teorema 2.6., en la cual aparecen las tres familias de álgebras $(n-4)$-filiformes que sean extensiones centrales de primera generación del álgebra $\mathfrak{g}_{n-1}^{n-3}$, con $\alpha \neq 0$.

Del capítulo 3: "Las álgebras de Lie p-filiformes como extensiones por derivaciones", una vez obtenida en la memoria la familia general de álgebras $(n-4)$-filiformes que son extensiones por derivaciones de las álgebras filiformes de dimensión 5 y siendo necesario analizar cuatro casos distintos, se expone en su totalidad la demostración de cada uno de ellos.

Del capítulo 4: "Aplicaciones geométricas" se completa

- la demostración del teorema 4.1., que da la dimensión del espacio de las derivaciones de $\mathfrak{g}_{n}^{2q-1}$, $1 \leq q \leq E(\frac{n-2}{2})$.

Se detalla cómo se obtienen las condiciones que resultan al exigir que cada d_i , $-2q \leq i \leq 2q$, sea una derivación.

- la demostración del teorema 4.2., que da la dimensión del espacio de las derivaciones de $\mathfrak{g}_n^{2s}$, $1 \leq s \leq E(\frac{n-3}{2})$.

Se especifican las condiciones iniciales que deben satisfacer las d_i , $-2s \leq i \leq 2s$, y que se deducen de $d_i(\mathfrak{g}_j) \subset \mathfrak{g}_{i+j}$, como también las posteriores que resultan al exigir que cada d_i sea, efectivamente, una derivación.

- la demostración del teorema 4.3., que da la dimensión del espacio de las derivaciones de $\mathfrak{g}_n^{n-2}$. Con las d_i , $-4 \leq i \leq 4$, que aparecen. Se siguen los pasos marcados en el punto anterior.

Indice

Clasificación de las álgebras (n-4)-filiformes por extensiones centrales

Corolario 2.2. *En dimensión n, la ley de un álgebra de Lie nilpotente compleja, de sucesión característica $(4,1,1,\ldots,1)$ y que sea una extensión central de primera generación, en una base $\{X_0, X_1, X_2, X_3, Y_1, Y_2, \ldots, Y_{n-5}, Z\}$, donde $Z \in \mathcal{Z}(\mathfrak{g})$, puede ser expresada mediante*

$$
\begin{array}{lcll}
[X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 2 \\
[X_0, X_3] & = & \alpha Z & \\
[X_0, Y_j] & = & d_j Z & 1 \leq j \leq n-5 \\
[X_1, X_2] & = & AY_{n-5} + aZ & \\
[X_1, X_3] & = & Ad_{n-5} Z & \\
[X_1, Y_j] & = & b_j Z & 1 \leq j \leq n-6 \\
[X_1, Y_{n-5}] & = & BX_3 + b_{n-5} Z & \\
[X_2, Y_{n-5}] & = & B\alpha Z & \\
[Y_i, Y_j] & = & c_{ij} Z & 1 \leq i < j \leq n-5,
\end{array}
$$

cumpliéndose

$$Ac_{j,n-5} = 0 \qquad 1 \leq j \leq n-6,$$

y donde el par (A, B), puede tomar únicamente los valores:

$$(0,0), \quad (0,1), \quad (1,0).$$

Extensiones centrales de primera generación de $\mathfrak{g}_{n-1}^2$

Corresponde al caso $(A, B) = (0, 1)$ del corolario 2.2.

Teorema 2.5. *Toda álgebra de Lie nilpotente compleja $(n-4)$-filiforme, de dimensión n y que sea extensión central de primera generación de $\mathfrak{g}_{n-1}^2$ es isomorfa a alguna de las álgebras de Lie de leyes*

$$\mathfrak{g}_n^{5,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-3}{2}) \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \end{array}$$

$$\mathfrak{g}_n^{6,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-5}{2}) \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-5}{2}) \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

$$\mathfrak{g}_n^{8,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-5}{2}) \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \end{array}$$

$$\mathfrak{g}_n^{9,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-6}{2}) \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

y cuando n es impar,

$$\mathfrak{g}_n^{6,E(\frac{n-5}{2})} \simeq \mathfrak{g}_n^{5,E(\frac{n-3}{2})}.$$

Demostración:

Según el corolario anterior, la ley de toda extensión central de primera generación de $\mathfrak{g}^2_{n-1}$, respecto de una cierta base $\{X_0, X_1, X_2, X_3, Y_1, Y_2, \ldots, Y_{n-5}, Z\}$, se puede expresar mediante

$$\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 2 \\ [X_0, X_3] & = & \alpha Z & \\ [X_0, Y_j] & = & d_j Z & 1 \leq j \leq n-5 \\ [X_1, X_2] & = & a Z & \\ [X_1, Y_j] & = & b_j Z & 1 \leq j \leq n-6 \\ [X_1, Y_{n-5}] & = & X_3 + b_{n-5} Z & \\ [X_2, Y_{n-5}] & = & \alpha Z & \\ [Y_i, Y_j] & = & c_{ij} Z & 1 \leq i < j \leq n-5. \end{array}$$

Se ha de verificar que $\alpha \neq 0$ porque, si $\alpha = 0$, al considerar

$$X_0^* = \sum_{i=0}^{3} A_i^0 X_i + \sum_{j=1}^{n-5} B_j^0 Y_j + C^0 Z$$

$$X_1^* = \sum_{i=0}^{3} A_i^1 X_i + \sum_{j=1}^{n-5} B_j^1 Y_j + C^1 Z,$$

se deduce, al ser Z un vector de $\mathcal{Z}(\mathfrak{g})$, que

$$\begin{array}{lcl} [X_0^*, X_1^*] & = & X_2^* \in < X_2, X_3, Z > \\ [X_0^*, X_2^*] & = & X_3^* \in < X_3, Z > \\ [X_0^*, X_3^*] & = & 0 \end{array}$$

y, en consecuencia, no aparecería ningún álgebra $(n-4)$ -filiforme.

Tras aplicar el cambio de base definido por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 3 \\ X_4^* & = & \alpha Z & \\ Y_j^* & = & Y_j & 1 \leq j \leq n-5, \end{array} \right.$$

la ley de $\mathfrak{g}$ se puede expresar como

$$\begin{array}{lcll}
[X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\
[X_0, Y_j] & = & d_j X_4 & 1 \le j \le n-5 \\
[X_1, X_2] & = & a X_4 & \\
[X_1, Y_j] & = & b_j X_4 & 1 \le j \le n-6 \\
[X_1, Y_{n-5}] & = & X_3 + b_{n-5} X_4 & \\
[X_2, Y_{n-5}] & = & X_4 & \\
[Y_i, Y_j] & = & c_{ij} X_4 & 1 \le i < j \le n-5.
\end{array}$$

Se puede suponer $d_j = 0 \quad 1 \le j \le n-5$, sin más que hacer el cambio:

$$\left\{ \begin{array}{lcll}
X_t^* & = & X_t & 0 \le t \le 4 \\
Y_j^* & = & Y_j - d_j X_3 & 1 \le j \le n-5.
\end{array} \right.$$

Se cumple, también, que $a \in \{0, 1\}$, porque, si $a \neq 0$, con el cambio de base definido por las relaciones

$$\left\{ \begin{array}{lcll}
X_0^* & = & \sqrt{a} X_0 & \\
X_1^* & = & X_1 & \\
X_2^* & = & \sqrt{a} X_2 & \\
X_3^* & = & a X_3 & \\
X_4^* & = & \sqrt{a^3} X_4 & \\
Y_k^* & = & Y_k & 1 \le k \le n-6 \\
Y_{n-5}^* & = & a Y_{n-5} &
\end{array} \right.$$

se obtiene que

$$\begin{array}{lcll}
[X_0^*, X_1^*] & = & [\sqrt{a} X_0, X_1] = \sqrt{a} X_2 = X_2^* & \\
[X_0^*, X_2^*] & = & [\sqrt{a} X_0, \sqrt{a} X_2] = a X_3 = X_3^* & \\
[X_0^*, X_3^*] & = & [\sqrt{a} X_0, a X_3] = \sqrt{a^3} X_4 = X_4^* & \\
[X_1^*, X_2^*] & = & [X_1, \sqrt{a} X_2] = \sqrt{a^3} X_4 = X_4^* & \\
[X_1^*, Y_j^*] & = & [X_1, Y_j] = b_j X_4 = \frac{b_j}{\sqrt{a^3}} X_4^* = b_j^* X_4^* & 1 \le j \le n-6 \\
[X_1^*, Y_{n-5}^*] & = & [X_1, a Y_{n-5}] = a X_3 + a b_{n-5} X_4 = X_3^* + b_{n-5}^* X_4^* & \\
[X_2^*, Y_{n-5}^*] & = & [\sqrt{a} X_2, a Y_{n-5}] = \sqrt{a^3} X_4 = X_4^* & \\
[Y_i^*, Y_j^*] & = & [Y_i, Y_j] = \frac{c_{ij}}{\sqrt{a^3}} X_4^* = c_{ij}^* X_4^* & 1 \le i < j \le n-6 \\
[Y_i^*, Y_{n-5}^*] & = & [Y_i, a Y_{n-5}] = \frac{c_{i,n-5}}{\sqrt{a}} X_4^* = c_{i,n-5}^* X_4^* & 1 \le i \le n-6
\end{array}$$

Por tanto, la ley de $\mathfrak{g}$ se expresa mediante

$$\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \qquad\qquad (\epsilon = a) \\ [X_1, Y_j] & = & b_j X_4 & 1 \leq j \leq n-6 \\ [X_1, Y_{n-5}] & = & X_3 + b_{n-5} X_4 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-5. \end{array}$$

Al aplicar el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_0^* & = & X_0 - \frac{b_{n-5}}{2} Y_{n-5} & \\ X_1^* & = & X_1 & \\ X_2^* & = & X_2 + \frac{b_{n-5}}{2} X_3 + \frac{b_{n-5}^2}{2} X_4 & \\ X_3^* & = & X_3 + b_{n-5} X_4 & \\ X_4^* & = & X_4 & \\ Y_j^* & = & Y_j - \frac{b_{n-5}}{2} c_{j,n-5} X_3 & 1 \leq j \leq n-6 \\ Y_{n-5}^* & = & Y_{n-5} & \end{array} \right.$$

se obtiene que

$$\begin{array}{lcl} [X_0^*, X_1^*] & = & [X_0 - \frac{b_{n-5}}{2} Y_{n-5}, X_1] = X_2 + \frac{b_{n-5}}{2} X_3 + \frac{b_{n-5}^2}{2} X_4 = X_2^* \\ [X_0^*, X_2^*] & = & [X_0 - \frac{b_{n-5}}{2} Y_{n-5}, X_2 + \frac{b_{n-5}}{2} X_3 + \frac{b_{n-5}^2}{2} X_4] = X_3 + b_{n-5} X_4 = X_3^* \\ [X_0^*, X_3^*] & = & [X_0 - \frac{b_{n-5}}{2} Y_{n-5}, X_3 + b_{n-5} X_4] = X_4 = X_4^* \\ [X_0^*, Y_j^*] & = & [X_0 - \frac{b_{n-5}}{2} Y_{n-5}, Y_j - \frac{b_{n-5}}{2} c_{j,n-5} X_3] = \\ & = & (-\frac{b_{n-5}}{2} c_{j,n-5} + \frac{b_{n-5}}{2} c_{j,n-5}) X_4 = 0 \qquad 1 \leq j \leq n-6 \\ [X_1^*, X_2^*] & = & [X_1, X_2 + \frac{b_{n-5}}{2} X_3 + \frac{b_{n-5}^2}{2} X_4] = \epsilon X_4 = \epsilon X_4^* \\ [X_1^*, Y_j^*] & = & [X_1, Y_j - \frac{b_{n-5}}{2} c_{j,n-5} X_3] = [X_1, Y_j] = b_j X_4 = b_j X_4^* \qquad 1 \leq j \leq n-6 \\ [X_1^*, Y_{n-5}^*] & = & [X_1, Y_{n-5}] = X_3 + b_{n-5} X_4 = X_3^* \end{array}$$

$$\begin{array}{lcl}
[X_2^*, Y_{n-5}^*] & = & [X_2 + \frac{b_{n-5}}{2}X_3 + \frac{b_{n-5}^2}{2}X_4, Y_{n-5}] = X_4 = X_4^* \\
[Y_i^*, Y_j^*] & = & [Y_i - \frac{b_{n-5}}{2}c_{i,n-5}X_3, Y_j - \frac{b_{n-5}}{2}c_{j,n-5}X_3] = [Y_i, Y_j] = c_{ij}X_4 = \\
& = & c_{ij}X_4^* \qquad 1 \le i < j \le n-6 \\
[Y_i^*, Y_{n-5}^*] & = & [Y_i - \frac{b_{n-5}}{2}c_{i,n-5}X_3, Y_{n-5}] = [Y_i, Y_{n-5}] = c_{i,n-5}X_4 = \\
& = & c_{i,n-5}X_4^* \qquad 1 \le i \le n-6
\end{array}$$

y, entonces, $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$\begin{cases}
[X_0, X_i] = X_{i+1} & 1 \le i \le 3 \\
[X_1, X_2] = \epsilon X_4 & \epsilon \in \{0,1\} \\
[X_1, Y_j] = b_j X_4 & 1 \le j \le n-6 \\
[X_1, Y_{n-5}] = X_3 & \\
[X_2, Y_{n-5}] = X_4 & \\
[Y_i, Y_j] = c_{ij}X_4 & 1 \le i < j \le n-5.
\end{cases}$$

Se van a distinguir ahora dos casos, según sean todos los b_j , $1 \le j \le n-6$, nulos o no.

Caso: $b_j = 0 \qquad 1 \le j \le n-6$

El álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \le i < j \le n-5. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \le i < j \le n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le n-5 \end{cases}$$

se obtiene el álgebra

$$\mathfrak{g}_n^{5,1}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \le i < j \le n-6$, y existe algún $c_{i,n-5} \neq 0$ $1 \le i \le n-6$, se puede suponer $c_{1,n-5} \neq 0$. Basta con aplicar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 & \\ Y_1^* &= Y_i & & \\ Y_i^* &= Y_1 & & \\ Y_k^* &= Y_k & 1 \le k \le n-6 & k \notin \{1,i\} \\ Y_{n-5}^* &= Y_{n-5} & & \end{cases}$$

Se puede, además, suponer que $c_{1,n-5} = 1$ y $c_{k,n-5} = 0 \quad 2 \le k \le n-6$, sin más que efectuar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_1^* &= \frac{1}{c_{1,n-5}} Y_1 & \\ Y_k^* &= c_{1,n-5} Y_k - c_{k,n-5} Y_1 & 2 \le k \le n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtienen las álgebras siguientes, dependiendo del valor de ϵ:

$$\mathfrak{g}_n^{6,1}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_{n-5}] & = & X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,1}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_{n-5}] & = & X_4 & \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-6$, se puede suponer $c_{12} \neq 0$. Basta con efectuar el cambio de base definido por

$$\left\{ \begin{array}{lclll} X_t^* & = & X_t & 0 \leq t \leq 4 & \\ Y_1^* & = & Y_i & & \\ Y_2^* & = & Y_j & & \\ Y_i^* & = & Y_1 & & \\ Y_j^* & = & Y_2 & & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-6 & k \notin \{1,2,i,j\} \\ Y_{n-5}^* & = & Y_{n-5}. & & \end{array} \right.$$

Se puede, además, suponer $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \qquad 3 \leq k \leq n-5$, sin más que aplicar el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_1^* & = & \frac{1}{c_{12}} Y_1 & \\ Y_2^* & = & Y_2 & \\ Y_k^* & = & Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-5, \end{array} \right.$$

y se obtiene el álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 3 \leq i < j \leq n-5. \end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 3 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{array}{llcll} \mathfrak{g}_n^{5,2}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_1, Y_2] & = & X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 3 \leq i < j \leq n-6$, y existe algún $c_{i,n-5} \neq 0$ $3 \leq i \leq n-6$, se puede suponer $c_{3,n-5} \neq 0$.

Sin más que considerar cambios de base análogos a algunos anteriores se puede suponer: $c_{3,n-5} = 1$ y $c_{k,n-5} = 0 \quad 4 \leq k \leq n-6$ y se obtienen las álgebras

$$\begin{array}{llcll} \mathfrak{g}_n^{6,2}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_1, Y_2] & = & X_4 & \\ & [Y_3, Y_{n-5}] & = & X_4 & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{7,2}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, X_2] & = & X_4 & \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_1, Y_2] & = & X_4 & \\ & [Y_3, Y_{n-5}] & = & X_4 & \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-6$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-5$, y la ley de $\mathfrak{g}$ viene expresada por

$$\left\{ \begin{array}{lll} [X_0, X_i] & = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = \epsilon X_4 & \\ [X_1, Y_{n-5}] & = X_3 & \\ [X_2, Y_{n-5}] & = X_4 & \\ [Y_1, Y_2] & = X_4 & \\ [Y_3, Y_4] & = X_4 & \\ [Y_i, Y_j] & = c_{ij} X_4 & 5 \leq i < j \leq n-5. \end{array} \right.$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{5,k}$, $\mathfrak{g}_n^{6,k}$, $\mathfrak{g}_n^{7,k}$ $\quad 1 \leq k \leq r-1$ ó a una que tenga por ley:

$$\left\{ \begin{array}{lll} [X_0, X_i] & = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] & = X_3 & \\ [X_2, Y_{n-5}] & = X_4 & \\ [Y_{2k-1}, Y_{2k}] & = X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] & = c_{ij} X_4 & 2r-1 \leq i < j \leq n-5. \end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2r-1 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\left\{ \begin{array}{lll} X_0^* & = X_0 & \\ X_1^* & = X_1 + \epsilon Y_{n-5} & \\ X_t^* & = X_t & 2 \leq t \leq 4 \\ Y_k^* & = Y_k & 1 \leq k \leq n-5 \end{array} \right.$$

se obtiene el álgebra

$$\mathfrak{g}_n^{5,r} : \begin{array}{lll} [X_0, X_i] & = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] & = X_3 & \\ [X_2, Y_{n-5}] & = X_4 & \\ [Y_{2k-1}, Y_{2k}] & = X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \le i < j \le n-6$, y existe algún $c_{i,n-5} \ne 0$ $2r-1 \le i \le n-6$, se puede suponer $c_{2r-1,n-5} \ne 0$. Basta con hacer el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_i^* &= Y_{2r-1} & \\ Y_k^* &= Y_k & 1 \le k \le n-6 \qquad k \notin \{2r-1, i\} \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Se puede, además, suponer que $c_{2r-1,n-5} = 1$ y $c_{k,n-5} = 0 \quad 2r \le k \le n-6$, sin más que considerar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,n-5}} Y_{2r-1} & \\ Y_j^* &= c_{2r-1,n-5}Y_j - c_{j,n-5}Y_{2r-1} & 2r \le j \le n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtienen las álgebras siguientes

$$\mathfrak{g}_n^{6,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] &=& X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &=& X_4 & \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] &=& X_4 & \end{array}$$

* Si existe algún $c_{ij} \ne 0 \quad 2r-1 \le i < j \le n-6$, se puede suponer $c_{2r-1,2r} \ne 0$. Basta con aplicar el siguiente cambio de base:

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_{2r-1}^* &= Y_i \\ Y_{2r}^* &= Y_j \\ Y_i^* &= Y_{2r-1} \\ Y_j^* &= Y_{2r} \\ Y_k^* &= Y_k & 1 \le k \le n-6 \qquad k \notin \{2r-1, 2r, i, j\} \\ Y_{n-5}^* &= Y_{n-5}. \end{cases}$$

Se puede, además, suponer

$$c_{2r-1,2r} = 1 \text{ y } c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \le k \le n-5,$$

sin más que efectuar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} \\ Y_{2r}^* &= Y_{2r} \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \le k \le n-5, \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \le k \le r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \le i < j \le n-5, \end{cases}$$

llegándose a una situación parecida a las ya analizadas.

En consecuencia, aparecen las álgebras

$$\begin{array}{llll} \mathfrak{g}_n^{5,r}: & [X_0, X_i] = X_{i+1} & 1 \le i \le 3 & 1 \le r \le E(\frac{n-7}{2}) \\ & [X_1, Y_{n-5}] = X_3 \\ & [X_2, Y_{n-5}] = X_4 \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \le k \le r-1 \end{array}$$

$$\mathfrak{g}_n^{6,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-7}{2}) \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-7}{2}) \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

y justo antes del último paso del proceso, se obtiene que $\mathfrak{g}$ puede ser isomorfa a un álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le E(\frac{n-5}{2}) - 1 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 2E(\frac{n-5}{2}) - 1 \le i < j \le n-5. \end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-5}{2}) - 1 \le i < j \le n-5$, se obtiene el álgebra

$$\mathfrak{g}_n^{5,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le E(\frac{n-5}{2}) - 1. \end{array}$$

* Si $2E(\frac{n-5}{2}) - 1 \le n-7$ y $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-5}{2}) - 1 \le i < j \le n-6$, y

existe algún $c_{i,n-5} \neq 0 \quad 2E(\frac{n-5}{2}) - 1 \leq i \leq n-6$, se obtienen las álgebras

$$\mathfrak{g}_n^{6,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1 \\ [Y_{2E(\frac{n-5}{2})-1}, Y_{n-5}] & = & X_4 & \end{array}$$

y

$$\mathfrak{g}_n^{7,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1 \\ [Y_{2E(\frac{n-5}{2})-1}, Y_{n-5}] & = & X_4 & \end{array}$$

Y a continuación, hay que diferenciar dos posibilidades dependiendo de la paridad de n.

Caso: n par ($E(\frac{n-5}{2}) = \frac{n-6}{2}$)

* Si existe algún $c_{ij} \neq 0 \quad 2E(\frac{n-5}{2}) - 1 = n-7 \leq i < j \leq n-6 \Leftrightarrow c_{n-7,n-6} \neq 0$, se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-6}{2}. \end{array} \right.$$

Al aplicar el cambio de base definido por

$$\left\{ \begin{array}{lcll} X_0^* & = & X_0 & \\ X_1^* & = & X_1 + \epsilon Y_{n-5} & \\ X_t^* & = & X_t & 2 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \end{array} \right.$$

se obtiene el álgebra

$$\mathfrak{g}_n^{5,E(\frac{n-3}{2})}: \begin{array}{lcll} [X_0,X_i] & = & X_{i+1} & 1\le i\le 3 \\ [X_1,Y_{n-5}] & = & X_3 & \\ [X_2,Y_{n-5}] & = & X_4 & \\ [Y_{2k-1},Y_{2k}] & = & X_4 & 1\le k\le E(\frac{n-3}{2})-1. \end{array}$$

Caso: n impar ($E(\frac{n-5}{2})=\frac{n-5}{2}$)

* Si existe algún $c_{ij}\neq 0$ $2E(\frac{n-5}{2})-1=n-6\le i<j\le n-5$ $\Leftrightarrow$ $c_{n-6,n-5}\neq 0$, y aplicando el cambio de base:

$$\left\{\begin{array}{lcll} X_t^* & = & X_t & 0\le t\le 4 \\ Y_k^* & = & Y_k & 1\le k\le n-5 \quad k\neq n-6 \\ Y_{n-6}^* & = & \frac{1}{c_{n-6,n-5}}Y_{n-6} & \end{array}\right.$$

se obtiene la ley:

$$\left\{\begin{array}{lcll} [X_0,X_i] & = & X_{i+1} & 1\le i\le 3 \\ [X_1,X_2] & = & \epsilon X_4 & \epsilon\in\{0,1\} \\ [X_1,Y_{n-5}] & = & X_3 & \\ [X_2,Y_{n-5}] & = & X_4 & \\ [Y_{2k-1},Y_{2k}] & = & X_4 & 1\le k\le \frac{n-5}{2}. \end{array}\right.$$

* Si $\epsilon=0$, dicha ley corresponde a

$$\mathfrak{g}_n^{5,E(\frac{n-3}{2})}: \begin{array}{lcll} [X_0,X_i] & = & X_{i+1} & 1\le i\le 3 \\ [X_1,Y_{n-5}] & = & X_3 & \\ [X_2,Y_{n-5}] & = & X_4 & \\ [Y_{2k-1},Y_{2k}] & = & X_4 & 1\le k\le E(\frac{n-3}{2})-1, \end{array}$$

álgebra que también aparece cuando n es par.

Se observa que en este caso (n impar), dicho álgebra $\mathfrak{g}_n^{5,E(\frac{n-3}{2})}$ coincide con $\mathfrak{g}_n^{6,E(\frac{n-5}{2})}$.

* Si $\epsilon=1$, la ley corresponde a $\mathfrak{g}_n^{7,E(\frac{n-5}{2})}$, álgebra ya obtenida.

Entonces, se concluye que en el caso: $b_j = 0 \quad 1 \leq j \leq n-6$, surgen las familias

$$\begin{array}{llll} \mathfrak{g}_n^{5,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [X_1, Y_{n-5}] = X_3 & & \\ & [X_2, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llll} \mathfrak{g}_n^{6,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-5}{2}) \\ & [X_1, Y_{n-5}] = X_3 & & \\ & [X_2, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & \\ & [Y_{2r-1}, Y_{n-5}] = X_4 & & \end{array}$$

$$\begin{array}{llll} \mathfrak{g}_n^{7,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-5}{2}) \\ & [X_1, X_2] = X_4 & & \\ & [X_1, Y_{n-5}] = X_3 & & \\ & [X_2, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & \\ & [Y_{2r-1}, Y_{n-5}] = X_4 & & \end{array}$$

y cuando n es impar, $\mathfrak{g}_n^{5,E(\frac{n-3}{2})} \simeq \mathfrak{g}_n^{6,E(\frac{n-5}{2})}$.

Caso: $\exists j \in \{1, 2, \ldots, n-6\} : b_j \neq 0$

El álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_j] &= b_j X_4 & 1 \leq j \leq n-6 \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5. \end{cases}$$

Los cambios de base definidos por las relaciones

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{n-6}^* &= Y_j & \\ Y_j^* &= Y_{n-6} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{j, n-6\} \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_j^* &= b_{n-6} Y_j - b_j Y_{n-6} & 1 \leq j \leq n-7 \\ Y_{n-6}^* &= \frac{1}{b_{n-6}} Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

permiten, el primero suponer $b_{n-6} \neq 0$ y el segundo, $b_j = 0 \quad 1 \leq j \leq n-7$ y $b_{n-6} = 1$. Entonces, la ley de $\mathfrak{g}$ está determinada por:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5. \end{cases}$$

*** Si $c_{ij} = 0 \quad \forall i, j \quad 1 \le i < j \le n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{array}{llll} \mathfrak{g}_n^{8,1}: & [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ & [X_1, Y_{n-6}] &= X_4 & \\ & [X_1, Y_{n-5}] &= X_3 & \\ & [X_2, Y_{n-5}] &= X_4 & \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i, j \quad 1 \le i < j \le n-7$, y existe algún $c_{i,n-6} \neq 0$ $1 \le i \le n-7$, se puede suponer $c_{1,n-6} \neq 0$. Basta con efectuar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 & \\ Y_1^* &= Y_i & & \\ Y_i^* &= Y_1 & & \\ Y_k^* &= Y_k & 1 \le k \le n-7 & k \notin \{1, i\} \\ Y_{n-6}^* &= Y_{n-6} & & \\ Y_{n-5}^* &= Y_{n-5} & & \end{cases}$$

Se puede, además, suponer que $c_{1,n-6} = 1$ y $c_{k,n-6} = 0 \quad 2 \le k \le n-7$, sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_1^* &= \frac{1}{c_{1,n-6}} Y_1 & \\ Y_k^* &= c_{1,n-6} Y_k - c_{k,n-6} Y_1 & 2 \le k \le n-7 \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene la ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_{n-6}] &= X_4 & \\ [Y_i, Y_{n-5}] &= c_{i,n-5} X_4 & 1 \leq i \leq n-6. \end{cases}$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 1 \leq i \leq n-6$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_{n-6}] &= X_4 & \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \quad\quad t \neq 1 \\ X_1^* &= X_1 - Y_1 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2, n-6\} \\ Y_2^* &= Y_{n-6} & & \\ Y_{n-6}^* &= Y_2 & & \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a

$$\mathfrak{g}_n^{5,2}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_1, Y_2] &=& X_4 & \end{array}$$

ya obtenida anteriormente.

* Si existe algún $c_{i,n-5} \neq 0 \quad 2 \leq i \leq n-7$, se puede suponer $c_{2,n-5} \neq 0$. Basta con considerar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= Y_1 \\ Y_2^* &= Y_i \\ Y_i^* &= Y_2 \\ Y_k^* &= Y_k & 2 \leq k \leq n-7 \qquad k \notin \{2, i\} \\ Y_{n-6}^* &= Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

Se puede, además, suponer que

$$c_{1,n-5} = 0, \ c_{2,n-5} = 1, \ c_{k,n-5} = 0 \qquad 3 \leq k \leq n-7,$$

sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_0^* &= \sqrt{c_{2,n-5}} X_0 \\ X_1^* &= c_{2,n-5} X_1 \\ X_2^* &= \sqrt{c_{2,n-5}^3} X_2 \\ X_3^* &= c_{2,n-5}^2 X_3 \\ X_4^* &= \sqrt{c_{2,n-5}^5} X_4 \\ Y_k^* &= c_{2,n-5} Y_k - c_{k,n-5} Y_2 & 1 \leq k \leq n-7 \qquad k \neq 2 \\ Y_2^* &= \sqrt{c_{2,n-5}} Y_2 \\ Y_{n-6}^* &= \sqrt{c_{2,n-5}^3} Y_{n-6} \\ Y_{n-5}^* &= c_{2,n-5} Y_{n-5} \end{cases}$$

En efecto, se obtiene que

$$
\begin{aligned}
[X_0^*, X_1^*] &= [\sqrt{c_{2,n-5}}X_0, c_{2,n-5}X_1] = \sqrt{c_{2,n-5}^3}X_2 = X_2^* \\
[X_0^*, X_2^*] &= [\sqrt{c_{2,n-5}}X_0, \sqrt{c_{2,n-5}^3}X_2] = c_{2,n-5}^2 X_3 = X_3^* \\
[X_0^*, X_3^*] &= [\sqrt{c_{2,n-5}}X_0, c_{2,n-5}^2 X_3] = \sqrt{c_{2,n-5}^5}X_4 = X_4^* \\
[X_1^*, X_2^*] &= [c_{2,n-5}X_1, \sqrt{c_{2,n-5}^3}X_2] = \epsilon\sqrt{c_{2,n-5}^5}X_4 = \epsilon X_4^* \\
[X_1^*, Y_{n-6}^*] &= [c_{2,n-5}X_1, \sqrt{c_{2,n-5}^3}Y_{n-6}] = \sqrt{c_{2,n-5}^5}X_4 = X_4^* \\
[X_1^*, Y_{n-5}^*] &= [c_{2,n-5}X_1, c_{2,n-5}Y_{n-5}] = c_{2,n-5}^2 X_3 = X_3^* \\
[X_2^*, Y_{n-5}^*] &= [\sqrt{c_{2,n-5}^3}X_2, c_{2,n-5}Y_{n-5}] = \sqrt{c_{2,n-5}^5}X_4 = X_4^* \\
[Y_1^*, Y_{n-6}^*] &= [c_{2,n-5}Y_1 - c_{1,n-5}Y_2, \sqrt{c_{2,n-5}^3}Y_{n-6}] = \sqrt{c_{2n-5}^5}X_4 = X_4^* \\
[Y_2^*, Y_{n-5}^*] &= [\sqrt{c_{2,n-5}}Y_2, c_{2,n-5}Y_{n-5}] = \sqrt{c_{2,n-5}^5}X_4 = X_4^* \\
[Y_k^*, Y_{n-5}^*] &= [c_{2,n-5}Y_k - c_{k,n-5}Y_2, c_{2,n-5}Y_{n-5}] = (c_{2,n-5}^2 c_{k,n-5} - c_{k,n-5}c_{2,n-5}^2)X_4 = \\
&= 0.X_4 = 0 \qquad 1 \leq k \leq n-7 \qquad k \neq 2 \\
[Y_{n-6}^*, Y_{n-5}^*] &= [\sqrt{c_{2,n-5}^3}Y_{n-6}, c_{2,n-5}Y_{n-5}] = \sqrt{c_{2,n-5}^5}.c_{n-6,n-5}.X_4 = \beta X_4,
\end{aligned}
$$

y la ley de $\mathfrak{g}$ es:

$$
\left\{
\begin{array}{lcll}
[X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] &=& \epsilon X_4 & \epsilon \in \{0,1\} \\
[X_1, Y_{n-6}] &=& X_4 & \\
[X_1, Y_{n-5}] &=& X_3 & \\
[X_2, Y_{n-5}] &=& X_4 & \\
[Y_1, Y_{n-6}] &=& X_4 & \\
[Y_2, Y_{n-5}] &=& X_4 & \\
[Y_{n-6}, Y_{n-5}] &=& \beta X_4 &
\end{array}
\right.
$$

Al aplicar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} - \beta Y_2 & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

se transforma en

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_{n-6}] &= X_4 & \\ [Y_2, Y_{n-5}] &= X_4 & \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* &= X_1 - Y_1 & & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5, & \end{cases}$$

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2, n-6\} \\ Y_2^* &= Y_{n-6} & & \\ Y_{n-6}^* &= Y_2 & & \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{3, n-6\} \\ Y_3^* &= Y_{n-6} & & \\ Y_{n-6}^* &= Y_3 & & \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_3, Y_{n-5}] &= X_4 & \end{cases}$$

que corresponde a $\mathfrak{g}_n^{6,2}$ si $\epsilon = 0$ y a $\mathfrak{g}_n^{7,2}$ si $\epsilon = 1$, ambas ya obtenidas.

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2 \leq i \leq n-7$, pero $c_{1,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se puede suponer $c_{n-6,n-5} = 0$, sin más que aplicar el cambio de base definido por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq n-6 \\ Y_{n-5}^* & = & Y_{n-5} + c_{n-6,n-5} Y_1 & \end{array} \right.$$

y se obtiene la ley determinada por

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_{n-6}] & = & X_4 & \\ [Y_1, Y_{n-5}] & = & c_{1,n-5} X_4 & \end{array} \right.$$

Al hacer el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_0^* & = & X_0 + \frac{c_{1,n-5}}{2} Y_{n-5} & \\ X_1^* & = & X_1 & \\ X_2^* & = & X_2 - \frac{c_{1,n-5}}{2} X_3 & \\ X_3^* & = & X_3 - c_{1,n-5} X_4 & \\ X_4^* & = & X_4 & \\ Y_1^* & = & Y_1 + \frac{c_{1,n-5}^2}{2} X_3 & \\ Y_k^* & = & Y_k & 2 \leq k \leq n-7 \\ Y_{n-6}^* & = & Y_{n-6} & \\ Y_{n-5}^* & = & -c_{1,n-5} Y_{n-6} + Y_{n-5} & \end{array} \right.$$

se obtiene que

$$
\begin{aligned}
[X_0^*, X_1^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, X_1] = X_2 - \tfrac{c_{1,n-5}}{2} X_3 = X_2^* \\
[X_0^*, X_2^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, X_2 - \tfrac{c_{1,n-5}}{2} X_3] = X_3 - c_{1,n-5} X_4 = X_3^* \\
[X_0^*, X_3^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, X_3 - c_{1,n-5} X_4] = X_4 = X_4^* \\
[X_0^*, Y_1^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, Y_1 + \tfrac{c_{1,n-5}^2}{2} X_3] = 0 \\
[X_1^*, Y_{n-6}^*] &= [X_1, Y_{n-6}] = X_4 = X_4^* \\
[X_1^*, Y_{n-5}^*] &= [X_1, -c_{1,n-5} Y_{n-6} + Y_{n-5}] = X_3 - c_{1,n-5} X_4 = X_3^* \\
[X_2^*, Y_{n-5}^*] &= [X_2 - \tfrac{c_{1,n-5}}{2} X_3, -c_{1,n-5} Y_{n-6} + Y_{n-5}] = X_4 = X_4^* \\
[Y_1^*, Y_{n-6}^*] &= [Y_1 + \tfrac{c_{1,n-5}^2}{2} X_3, Y_{n-6}] = X_4 = X_4^* \\
[Y_1^*, Y_{n-5}^*] &= [Y_1 + \tfrac{c_{1,n-5}^2}{2} X_3, -c_{1,n-5} Y_{n-6} + Y_{n-5}] = 0.
\end{aligned}
$$

Y la ley de $\mathfrak{g}$ se convierte en

$$
\left\{
\begin{array}{lcll}
[X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] &=& \epsilon X_4 & \epsilon \in \{0, 1\} \\
[X_1, Y_{n-6}] &=& X_4 & \\
[X_1, Y_{n-5}] &=& X_3 & \\
[X_2, Y_{n-5}] &=& X_4 & \\
[Y_1, Y_{n-6}] &=& X_4 &
\end{array}
\right.
$$

Esta situación ya ha sido analizada y resulta $\mathfrak{g} \simeq \mathfrak{g}_n^{5,2}$.

*** Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $1 \leq i \leq n-7$, se puede suponer $c_{1,n-5} \neq 0$. Basta con efectuar el cambio de base siguiente:

$$\begin{cases} X_t^* & = X_t \qquad 0 \leq t \leq 4 \\ Y_1^* & = Y_i \\ Y_i^* & = Y_1 \\ Y_k^* & = Y_k \qquad 1 \leq k \leq n-7 \qquad k \notin \{1,i\} \\ Y_{n-6}^* & = Y_{n-6} \\ Y_{n-5}^* & = Y_{n-5} \end{cases}$$

Al considerar

$$\begin{cases} X_0^* & = \sqrt[3]{c_{1,n-5}}X_0 \\ X_1^* & = \sqrt[3]{c_{1,n-5}^2}X_1 \\ X_2^* & = c_{1,n-5}X_2 \\ X_3^* & = \sqrt[3]{c_{1,n-5}^4}X_3 \\ X_4^* & = \sqrt[3]{c_{1,n-5}^5}X_4 \\ Y_1^* & = \sqrt[3]{c_{1,n-5}^2}Y_1 \\ Y_k^* & = c_{1,n-5}Y_k - c_{k,n-5}Y_1 \qquad 2 \leq k \leq n-6 \\ Y_{n-5}^* & = \sqrt[3]{c_{1,n-5}^2}Y_{n-5} \end{cases}$$

se obtiene que

$$[X_0^*, X_1^*] = [\sqrt[3]{c_{1,n-5}}X_0, \sqrt[3]{c_{1,n-5}^2}X_1] = c_{1,n-5}X_2 = X_2^*$$

$$[X_0^*, X_2^*] = [\sqrt[3]{c_{1,n-5}}X_0, c_{1,n-5}X_2] = \sqrt[3]{c_{1,n-5}^4}X_3 = X_3^*$$

$$[X_0^*, X_3^*] = [\sqrt[3]{c_{1,n-5}}X_0, \sqrt[3]{c_{1,n-5}^4}X_3] = \sqrt[3]{c_{1,n-5}^5}X_4 = X_4^*$$

$$[X_1^*, X_2^*] = [\sqrt[3]{c_{1,n-5}^2}X_1, c_{1,n-5}X_2] = \epsilon\sqrt[3]{c_{1,n-5}^5}X_4 = \epsilon X_4^*$$

$$[X_1^*, Y_{n-6}^*] = [\sqrt[3]{c_{1,n-5}^2}X_1, c_{1,n-5}Y_{n-6} - c_{n-6,n-5}Y_1] = \sqrt[3]{c_{1,n-5}^5}X_4 = X_4^*$$

$$[X_1^*, Y_{n-5}^*] = [\sqrt[3]{c_{1,n-5}^2}X_1, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \sqrt[3]{c_{1,n-5}^4}X_3 = X_3^*$$

$$\begin{aligned}
[X_2^*, Y_{n-5}^*] &= [c_{1,n-5}X_2, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \sqrt[3]{c_{1,n-5}^5}X_4 = X_4^* \\
[Y_1^*, Y_{n-5}^*] &= [\sqrt[3]{c_{1,n-5}^2}Y_1, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \sqrt[3]{c_{1,n-5}^7}X_4 = \beta X_4^* \\
[Y_k^*, Y_{n-5}^*] &= [c_{1,n-5}Y_k - c_{k,n-5}Y_1, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \\
&= (\sqrt[3]{c_{1,n-5}^5}c_{k,n-5} - c_{k,n-5}\sqrt[3]{c_{1,n-5}^5})X_4 = 0.X_4 = 0 \quad 2 \leq k \leq n-6.
\end{aligned}$$

La ley de $\mathfrak{g}$ es:

$$\begin{cases}
[X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] = \epsilon X_4 & \epsilon \in \{0,1\} \\
[X_1, Y_{n-6}] = X_4 & \\
[X_1, Y_{n-5}] = X_3 & \\
[X_2, Y_{n-5}] = X_4 & \\
[Y_1, Y_{n-5}] = \beta X_4 & \beta \neq 0.
\end{cases}$$

Se puede suponer $\beta = 1$, sin más que hacer el cambio de base:

$$\begin{cases}
X_0^* = \sqrt[5]{\beta}X_0 & \\
X_1^* = \sqrt[5]{\beta^2}X_1 & \\
X_2^* = \sqrt[5]{\beta^3}X_2 & \\
X_3^* = \sqrt[5]{\beta^4}X_3 & \\
X_4^* = \beta X_4 & \\
Y_1^* = \frac{1}{\sqrt[5]{\beta^2}}Y_1 & \\
Y_k^* = Y_k & 2 \leq k \leq n-7 \\
Y_{n-6}^* = \sqrt[5]{\beta^3}Y_{n-6} & \\
Y_{n-5}^* = \sqrt[5]{\beta^2}Y_{n-5} &
\end{cases}$$

En efecto:

$$\begin{aligned}
[X_0^*, X_1^*] &= [\sqrt[5]{\beta}X_0, \sqrt[5]{\beta^2}X_1] = \sqrt[5]{\beta^3}X_2 = X_2^* \\
[X_0^*, X_2^*] &= [\sqrt[5]{\beta}X_0, \sqrt[5]{\beta^3}X_2] = \sqrt[5]{\beta^4}X_3 = X_3^* \\
[X_0^*, X_3^*] &= [\sqrt[5]{\beta}X_0, \sqrt[5]{\beta^4}X_3] = \beta X_4 = X_4^* \\
[X_1^*, X_2^*] &= [\sqrt[5]{\beta^2}X_1, \sqrt[5]{\beta^3}X_2] = \epsilon\beta X_4 = \epsilon X_4^* \\
[X_1^*, Y_{n-6}^*] &= [\sqrt[5]{\beta^2}X_1, \sqrt[5]{\beta^3}Y_{n-6}] = \beta X_4 = X_4^*
\end{aligned}$$

$$[X_1^*, Y_{n-5}^*] = [\sqrt[5]{\beta^2}X_1, \sqrt[5]{\beta^2}Y_{n-5}] = \sqrt[5]{\beta^4}X_3 = X_3^*$$

$$[X_2^*, Y_{n-5}^*] = [\sqrt[5]{\beta^3}X_2, \sqrt[5]{\beta^2}Y_{n-5}] = \beta X_4 = X_4^*$$

$$[Y_1^*, Y_{n-5}^*] = [\frac{1}{\sqrt[5]{\beta^2}}Y_1, \sqrt[5]{\beta^2}Y_{n-5}] = \beta X_4 = X_4^*.$$

Por tanto, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] = \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] = X_4 & \\ [X_1, Y_{n-5}] = X_3 & \\ [X_2, Y_{n-5}] = X_4 & \\ [Y_1, Y_{n-5}] = X_4 & \end{cases}$$

Se puede suponer $\epsilon = 0$, sin más que aplicar el cambio de base:

$$\begin{cases} X_t^* = X_t & 0 \le t \le 4 \quad t \ne 1 \\ X_1^* = X_1 + \epsilon Y_{n-5} & \\ Y_1^* = Y_1 + \epsilon Y_{n-6} & \\ Y_k^* = Y_k & 2 \le k \le n-5, \end{cases}$$

y se obtiene el álgebra

$$\mathfrak{g}_n^{9,1}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_{n-5}] & = & X_4 & \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-6$ y $c_{i,n-5} = 0 \quad \forall i \quad 1 \leq i \leq n-7$, pero $c_{n-6,n-5} \neq 0$ se distinguen dos casos, dependiendo del valor de ϵ.

Caso: $\epsilon = 0$

Se puede suponer $c_{n-6,n-5} = 1$, sin más que hacer el cambio de base:

$$\begin{cases} X_0^* &= X_0 & \\ X_t^* &= c_{n-6,n-5} X_t & 1 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5, \end{cases}$$

y la ley de $\mathfrak{g}$ viene determinada por

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{n-6}, Y_{n-5}] &= X_4 & \end{cases}$$

Esta ley se consigue al aplicar a la ley

$$\mathfrak{g}_n^{7,1}: \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_{n-5}] &= X_4 & \end{array}$$

los siguientes cambios de base sucesivamente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & \\ Y_1^* &= Y_{n-6} & & \\ Y_{n-6}^* &= Y_1 & & \\ Y_k^* &= Y_k & 2 \leq k \leq n-5 & k \neq n-6 \end{cases} \text{ y}$$

$$\begin{cases} X_0^* &= X_0 & & \\ X_1^* &= -X_1 - Y_{n-5} & & \\ X_t^* &= -X_t & 2 \leq t \leq 4 & \\ Y_k^* &= -Y_k & 1 \leq k \leq n-7 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq n-7 & k = \dot{2} \\ Y_{n-6}^* &= -Y_{n-6} & & \\ Y_{n-5}^* &= Y_{n-5} & & \end{cases}$$

En consecuencia, $\mathfrak{g} \simeq \mathfrak{g}_n^{7,1}$.

Caso: $\epsilon = 1 \quad c_{n-6,n-5} = 1$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \quad t \neq 1 \\ X_1^* = X_1 + Y_{n-5} & \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \end{cases}$$

y

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ Y_1^* = Y_{n-6} & \\ Y_{n-6}^* = Y_1 & \\ Y_k^* = Y_k & 2 \leq k \leq n-5 \quad k \neq n-6 \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{6,1}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_{n-5}] & = & X_4 & \end{array}$$

Caso: $\epsilon = 1 \quad c_{n-6,n-5} \neq 1$

Al aplicar el cambio de base definido por

$$\left\{\begin{array}{lcll} X_0^* & = & X_0 & \\ X_1^* & = & -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5} & \\ X_t^* & = & -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_t & 2 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \qquad k \neq n-6 \\ Y_{n-6}^* & = & \frac{1}{c_{n-6,n-5}-1}Y_{n-6} & \end{array}\right.$$

se obtienen que

$$[X_0^*, X_1^*] = [X_0, -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_2 = X_2^*$$

$$[X_0^*, X_t^*] = [X_0, -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_t] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_{t+1} = X_{t+1}^* \qquad 2 \leq t \leq 3$$

$$\begin{array}{lcl} [X_1^*, X_2^*] & = & [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}, -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_2] = \\ & = & \frac{c_{n-6,n-5}^2}{4(1-c_{n-6,n-5})^2}X_4 + \frac{c_{n-6,n-5}^2-2c_{n-6,n-5}}{4(1-c_{n-6,n-5})^2}X_4 = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4 = X_4^* \end{array}$$

$$\begin{array}{lcl} [X_1^*, Y_{n-6}^*] & = & [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}, \frac{1}{c_{n-6,n-5}-1}Y_{n-6}] = \\ & = & \frac{-c_{n-6,n-5}}{2(1-c_{n-6,n-5})(c_{n-6,n-5}-1)}X_4 - \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})(c_{n-6,n-5}-1)}X_4 = \\ & = & -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4 = X_4^* \end{array}$$

$$[X_1^*, Y_{n-5}^*] = [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}, Y_{n-5}] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_3 = X_3^*$$

$$[X_2^*, Y_{n-5}^*] = [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_2, Y_{n-5}] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4 = X_4^*$$

$$[Y_{n-6}^*, Y_{n-5}^*] = [\frac{1}{c_{n-6,n-5}-1}Y_{n-6}, Y_{n-5}] = \frac{c_{n-6,n-5}}{c_{n-6,n-5}-1}X_4 = 2(-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4) = 2X_4^*$$

Por tanto, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\left\{\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{n-6}, Y_{n-5}] & = & 2X_4 & \end{array}\right.$$

Los cambios de base sucesivos:

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= 2X_1 + Y_{n-5} \\ X_t^* &= 2X_t & 2 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le n-5 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_1^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_1 \\ Y_k^* &= Y_k & 2 \le k \le n-5 \qquad k \ne n-6 \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\begin{aligned} \mathfrak{g}_n^{7,1}: \; [X_0, X_i] &= X_{i+1} \qquad 1 \le i \le 3 \\ [X_1, X_2] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_1, Y_{n-5}] &= X_4 \end{aligned}$$

ya obtenida.

*** Si existe algún $c_{ij} \ne 0 \quad 1 \le i < j \le n-7$, se puede suponer $c_{12} \ne 0$. Basta con efectuar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_1^* &= Y_i \\ Y_2^* &= Y_j \\ Y_i^* &= Y_1 \\ Y_j^* &= Y_2 \\ Y_k^* &= Y_k & 1 \le k \le n-7 \qquad k \notin \{1,2,i,j\} \\ Y_{n-6}^* &= Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \qquad 3 \le k \le n-5$, sin más que hacer el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 & \\ Y_2^* &= Y_2 & \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-5, \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \leq i < j \leq n-5. \end{cases}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{array}{llll} \mathfrak{g}_n^{8,2}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, Y_{n-6}] = X_4 & \\ & [X_1, Y_{n-5}] = X_3 & \\ & [X_2, Y_{n-5}] = X_4 & \\ & [Y_1, Y_2] = X_4 & \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-7$, y existe algún $c_{i,n-6} \neq 0$ $3 \leq i \leq n-7$, se puede suponer $c_{3,n-6} \neq 0$.

Sin más que considerar cambios de base análogos a algunos anteriores se puede suponer: $c_{3,n-6} = 1$ y $c_{k,n-6} = 0 \quad 4 \leq k \leq n-7$ y se obtiene la ley determinada por

$$\begin{cases} [X_0, X_i] & = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = X_4 & \\ [X_1, Y_{n-5}] & = X_3 & \\ [X_2, Y_{n-5}] & = X_4 & \\ [Y_1, Y_2] & = X_4 & \\ [Y_3, Y_{n-6}] & = X_4 & \\ [Y_i, Y_{n-5}] & = c_{i,n-5} X_4 & 3 \leq i \leq n-6. \end{cases}$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 3 \leq i \leq n-6$, se cumple que $\mathfrak{g}$ es isomorfa a $\mathfrak{g}_n^{5,3}$, ya obtenida.

* Si existe algún $c_{i,n-5} \neq 0 \quad 4 \leq i \leq n-7$, se cumple $\mathfrak{g} \simeq \mathfrak{g}_n^{6,3}$ si $\epsilon = 0$ y $\mathfrak{g} \simeq \mathfrak{g}_n^{7,3}$ si $\epsilon = 1$, ambas ya obtenidas.

* Si $c_{i,n-5} = 0 \quad \forall i \quad 4 \leq i \leq n-7$, pero $c_{3,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se cumple $\mathfrak{g} \simeq \mathfrak{g}_n^{5,3}$, ya obtenida.

*** Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0 \quad 3 \leq i \leq n-7$, se obtiene el álgebra

$$\mathfrak{g}_n^{9,2}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \\ [Y_3, Y_{n-5}] & = & X_4 & \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-6$ y $c_{i,n-5} = 0 \quad \forall i \quad 3 \leq i \leq n-7$, pero $c_{n-6,n-5} \neq 0$, se cumple:

$$\mathfrak{g} \simeq \mathfrak{g}_n^{7,2} \text{ si } \epsilon = 0$$

$$\mathfrak{g} \simeq \mathfrak{g}_n^{6,2} \text{ si } \epsilon = 1 \text{ y } c_{n-6,n-5} = 1$$

$$\mathfrak{g} \simeq \mathfrak{g}_n^{7,2} \text{ si } \epsilon = 1 \text{ y } c_{n-6,n-5} \neq 1.$$

*** Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-7$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-5$. La ley de $\mathfrak{g}$ viene expresada por

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_3, Y_4] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 5 \leq i < j \leq n-5. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{8,k}$, $\mathfrak{g}_n^{9,k}$, $\mathfrak{g}_n^{5,k^*}$, $\mathfrak{g}_n^{6,k^*}$, $\mathfrak{g}_n^{7,k^*}$ $\quad 1 \leq k \leq r-1 \quad 1 \leq k^* \leq r$, ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r-1 \leq i < j \leq n-5. \end{cases}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\mathfrak{g}_n^{8,r} : \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &=& X_4 & \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1. \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-7$ y existe algún $c_{i,n-6} \neq 0$ $2r-1 \leq i \leq n-7$, se puede suponer $c_{2r-1,n-6} \neq 0$. Basta con efectuar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_i^* &= Y_{2r-1} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-7 \quad k \notin \{2r-1, i\} \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Se puede, además, suponer que $c_{2r-1,n-6} = 1$ y $c_{k,n-6} = 0 \quad 2r \leq k \leq n-7$, sin más que considerar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,n-6}} Y_{2r-1} & \\ Y_k^* &= c_{2r-1,n-6} Y_k - c_{k,n-6} Y_{2r-1} & 2r \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene la ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 & \\ [Y_i, Y_{n-5}] &= c_{i,n-5} X_4 & 2r-1 \leq i \leq n-6. \end{cases}$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2r-1 \leq i \leq n-6$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 & \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \quad t \neq 1 \\ X_1^* &= X_1 - Y_{2r-1} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2r, n-6\} \\ Y_{2r}^* &= Y_{n-6} & & \\ Y_{n-6}^* &= Y_{2r} & & \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a

$$\mathfrak{g}_n^{5,r+1}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r. \end{array}$$

* Si existe algún $c_{i,n-5} \neq 0 \quad 2r \leq i \leq n-7$, se puede suponer $c_{2r,n-5} \neq 0$. Basta con hacer el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-1 \\ Y_{2r}^* &= Y_i & \\ Y_i^* &= Y_{2r} & \\ Y_k^* &= Y_k & 2r \leq k \leq n-7 \quad k \notin \{2r, i\} \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Se puede, además, suponer que

$$c_{2r-1,n-5} = 0, \; c_{2r,n-5} = 1 \text{ y } c_{k,n-5} = 0 \qquad 2r+1 \leq k \leq n-7,$$

sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_0^* &= \sqrt{c_{2r,n-5}} X_0 & & \\ X_1^* &= c_{2r,n-5} X_1 & & \\ X_2^* &= \sqrt{c_{2r,n-5}^3} X_2 & & \\ X_3^* &= c_{2r,n-5}^2 X_3 & & \\ X_4^* &= \sqrt{c_{2r,n-5}^5} X_4 & & \\ Y_k^* &= \sqrt{c_{2r,n-5}^5} Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_k^* &= c_{2r,n-5} Y_k - c_{k,n-5} Y_{2r} & 2r-1 \leq k \leq n-7 & k \neq 2r \\ Y_{2r}^* &= \sqrt{c_{2r,n-5}} Y_{2r} & & \\ Y_{n-6}^* &= \sqrt{c_{2r,n-5}^3} Y_{n-6} & & \\ Y_{n-5}^* &= c_{2r,n-5} Y_{n-5} & & \end{cases}$$

Y, en consecuencia, la ley de $\mathfrak{g}$ es:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 & \\ [Y_{2r}, Y_{n-5}] &= X_4 & \\ [Y_{n-6}, Y_{n-5}] &= \beta X_4 & \end{cases}$$

Aplicando el cambio:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} - \beta Y_{2r} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

se transforma en

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 \\ [Y_{2r}, Y_{n-5}] &= X_4 \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* &= X_1 - Y_{2r-1} \\ Y_k^* &= Y_k & 1 \leq k \leq n-5, \end{cases}$$

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2r, n-6\} \\ Y_{2r}^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_{2r} \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2r+1, n-6\} \\ Y_{2r+1}^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_{2r+1} \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_{2r+1}, Y_{n-5}] &= X_4 \end{cases}$$

que corresponde a $\mathfrak{g}_n^{6,r+1}$ si $\epsilon = 0$ y a $\mathfrak{g}_n^{7,r+1}$ si $\epsilon = 1$, ambas ya obtenidas.

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2r \leq i \leq n-7$, pero $c_{2r-1,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se puede suponer $c_{n-6,n-5} = 0$, sin más que efectuar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} + c_{n-6,n-5}Y_{2r-1} & \end{cases}$$

y se obtiene la ley determinada por

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 & \\ [Y_{2r-1}, Y_{n-5}] &= c_{2r-1,n-5}X_4 & \end{cases}$$

Se puede suponer $c_{2r-1,n-5} = 0$, sin más que aplicar el cambio de base:

$$\begin{cases} X_0^* &= X_0 + \frac{c_{2r-1,n-5}}{2}Y_{n-5} & \\ X_1^* &= X_1 & \\ X_2^* &= X_2 - \frac{c_{2r-1,n-5}}{2}X_3 & \\ X_3^* &= X_3 - c_{2r-1,n-5}X_4 & \\ X_4^* &= X_4 & \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= Y_{2r-1} + \frac{c_{2r-1,n-5}^2}{2}X_3 & \\ Y_k^* &= Y_k & 2r \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= -c_{2r-1,n-5}Y_{n-6} + Y_{n-5} & \end{cases}$$

La situación que resulta ya ha sido analizada y resulta ser $\mathfrak{g}$ isomorfa a

$$\mathfrak{g}_n^{5,r+1}: \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r. \end{array}$$

*** Si $c_{ij} = 0$ $\forall i, j$ $2r-1 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $2r-1 \leq i \leq n-7$, se puede suponer $c_{2r-1,n-5} \neq 0$. Basta con considerar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= Y_i & \\ Y_i^* &= Y_{2r-1} & \\ Y_k^* &= Y_k & 2r-1 \leq k \leq n-7 \quad k \notin \{2r-1, i\} \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Al considerar

$$\begin{cases} X_0^* &= \sqrt[3]{c_{2r-1,n-5}}X_0 & & \\ X_1^* &= \sqrt[3]{c_{2r-1,n-5}^2}X_1 & & \\ X_2^* &= c_{2r-1,n-5}X_2 & & \\ X_3^* &= \sqrt[3]{c_{2r-1,n-5}^4}X_3 & & \\ X_4^* &= \sqrt[3]{c_{2r-1,n-5}^5}X_4 & & \\ Y_k^* &= \sqrt[3]{c_{2r-1,n-5}^5}Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_{2r-1}^* &= \sqrt[3]{c_{2r-1,n-5}^2}Y_{2r-1} & & \\ Y_k^* &= c_{2r-1,n-5}Y_k - c_{k,n-5}Y_{2r-1} & 2r \leq k \leq n-6 & \\ Y_{n-5}^* &= \sqrt[3]{c_{2r-1,n-5}^2}Y_{n-5} & & \end{cases}$$

se obtiene la ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] &= \beta X_4 & \beta \neq 0. \end{cases}$$

Se puede suponer $\beta = 1$, sin más que hacer el cambio de base:

$$\begin{cases} X_0^* &= \sqrt[5]{\beta} X_0 \\ X_1^* &= \sqrt[5]{\beta^2} X_1 \\ X_2^* &= \sqrt[5]{\beta^3} X_2 \\ X_3^* &= \sqrt[5]{\beta^4} X_3 \\ X_4^* &= \beta X_4 \\ Y_k^* &= \beta Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_{2r-1}^* &= \frac{1}{\sqrt[5]{\beta^2}} Y_{2r-1} \\ Y_k^* &= Y_k & 2r \leq k \leq n-7 \\ Y_{n-6}^* &= \sqrt[5]{\beta^3} Y_{n-6} \\ Y_{n-5}^* &= \sqrt[5]{\beta^2} Y_{n-5} \end{cases}$$

En consecuencia, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] &= X_4 \end{cases}$$

Se puede suponer $\epsilon = 0$, sin más que aplicar el cambio de base:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* &= X_1 + \epsilon Y_{n-5} \\ Y_{2r-1}^* &= Y_{2r-1} + \epsilon Y_{n-6} \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \neq 2r-1, \end{cases}$$

y se obtiene el álgebra

$$\mathfrak{g}_n^{9,r} : \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] &= X_4 \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \;\; 2r-1 \leq i < j \leq n-6$ y $c_{i,n-5} = 0 \;\; \forall i$ $2r-1 \leq i \leq n-7$, pero $c_{n-6,n-5} \neq 0$ se distinguen dos casos, dependiendo del valor de ϵ.

Caso: $\epsilon = 0$

Se puede suponer $c_{n-6,n-5} = 1$, sin más que aplicar el cambio de base:

$$\left\{\begin{array}{lcllll} X_0^* & = & X_0 & & \\ X_t^* & = & c_{n-6,n-5}X_t & 1 \leq t \leq 4 & \\ Y_k^* & = & c_{n-6,n-5}Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_k^* & = & Y_k & 2r-1 \leq k \leq n-5, & \end{array}\right.$$

y la ley de $\mathfrak{g}$ viene determinada por

$$\left\{\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{n-6}, Y_{n-5}] & = & X_4 & \end{array}\right.$$

Esta ley se consigue al aplicar a

$$\begin{array}{llcll} \mathfrak{g}_n^{7,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, X_2] & = & X_4 & \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ & [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

los siguientes cambios de base sucesivamente:

$$\left\{\begin{array}{lclll} X_t^* & = & X_t & 0 \leq t \leq 4 & \\ Y_{2r-1}^* & = & Y_{n-6} & & \\ Y_{n-6}^* & = & Y_{2r-1} & & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 & k \notin \{2r-1, n-6\} \end{array}\right.$$

y

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= -X_1 - Y_{n-5} \\ X_t^* &= -X_t & 2 \le t \le 4 \\ Y_k^* &= -Y_k & 1 \le k \le n-7 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \le k \le n-7 & k = \dot{2} \\ Y_{n-6}^* &= -Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

En consecuencia, $\mathfrak{g} \simeq \mathfrak{g}_n^{7,r}$.

Caso: $\epsilon = 1 \quad c_{n-6,n-5} = 1$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 & t \ne 1 \\ X_1^* &= X_1 + Y_{n-5} \\ Y_k^* &= Y_k & 1 \le k \le n-5 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_{2r-1}^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_{2r-1} \\ Y_k^* &= Y_k & 1 \le k \le n-5 & k \notin \{2r-1, n-6\} \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{6,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] &=& X_3 \\ [X_2, Y_{n-5}] &=& X_4 \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] &=& X_4 \end{array}$$

Caso: $\epsilon = 1 \quad c_{n-6,n-5} \neq 1$

Al aplicar el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5} \\ X_t^* &= -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_t & 2 \leq t \leq 4 \\ Y_k^* &= -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_k^* &= Y_k & 2r-1 \leq k \leq n-5 & k \neq n-6 \\ Y_{n-6}^* &= \frac{1}{c_{n-6,n-5}-1}Y_{n-6} \end{cases}$$

Se obtiene que $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= X_4 \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{n-6}, Y_{n-5}] &= 2X_4 \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= 2X_1 + Y_{n-5} \\ X_t^* &= 2X_t & 2 \leq t \leq 4 \\ Y_k^* &= 2Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_k^* &= Y_k & 2r-1 \leq k \leq n-5 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_{2r-1} \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2r-1, n-6\} \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{7,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

*** Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-7$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con considerar el cambio de base expresado por

$$\left\{ \begin{array}{lclll} X_t^* & = & X_t & 0 \leq t \leq 4 & \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 & \\ Y_{2r-1}^* & = & Y_i & & \\ Y_{2r}^* & = & Y_j & & \\ Y_i^* & = & Y_{2r-1} & & \\ Y_j^* & = & Y_{2r} & & \\ Y_k^* & = & Y_k & 2r-1 \leq k \leq n-7 & k \notin \{2r-1, 2r, i, j\} \\ Y_{n-6}^* & = & Y_{n-6} & & \\ Y_{n-5}^* & = & Y_{n-5} & & \end{array} \right.$$

Se puede, además, suponer que $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k}$ $2r+1 \leq k \leq n-5$, sin más que hacer el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* & = & \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* & = & Y_{2r} & \\ Y_k^* & = & Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-5, \end{array} \right.$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \leq i < j \leq n-5. \end{cases}$$

Se llega a una situación análoga a las ya consideradas.

En consecuencia, aparecen las álgebras

$$\begin{array}{llllll} \mathfrak{g}_n^{8,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-7}{2}) \\ & [X_1, Y_{n-6}] & = & X_4 & & \\ & [X_1, Y_{n-5}] & = & X_3 & & \\ & [X_2, Y_{n-5}] & = & X_4 & & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llllll} \mathfrak{g}_n^{9,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-7}{2}) \\ & [X_1, Y_{n-6}] & = & X_4 & & \\ & [X_1, Y_{n-5}] & = & X_3 & & \\ & [X_2, Y_{n-5}] & = & X_4 & & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 & \\ & [Y_{2r-1}, Y_{n-5}] & = & X_4 & & \end{array}$$

y justo antes del último paso del proceso, se obtiene que $\mathfrak{g}$ puede ser isomorfa a un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1 \\ [Y_i, Y_j] &= c_{ij} X_4 & 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-5. \end{cases}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-5$, se obtiene el álgebra

$$\mathfrak{g}_n^{8,E(\frac{n-5}{2})} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1. \end{array}$$

*** Si existe algún $c_{i,n-6} \neq 0 \quad 2E(\frac{n-5}{2}) - 1 \leq i \leq n-7$, se cumple que la ley de $\mathfrak{g}$ está determinada por

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2E(\frac{n-5}{2})-1}, Y_{n-6}] & = & X_4 & \\ [Y_i, Y_{n-5}] & = & c_{in-5} X_4 & 2E(\frac{n-5}{2}) - 1 \leq i \leq n-6. \end{array} \right.$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2E(\frac{n-5}{2}) - 1 \leq i \leq n-6$, se obtiene $\mathfrak{g} \simeq \mathfrak{g}_n^{5,E(\frac{n-3}{2})}$.

* Si $c_{2E(\frac{n-5}{2})-1,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se obtiene $\mathfrak{g} \simeq \mathfrak{g}_n^{5,E(\frac{n-3}{2})}$.

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $2E(\frac{n-5}{2}) - 1 \leq i \leq n-7$ (esta situación solamente ocurre cuando $E(\frac{n-5}{2}) = \frac{n-6}{2}$ $\Leftrightarrow$ *n par* $\Leftrightarrow \frac{n-6}{2} = E(\frac{n-6}{2})$), se obtiene el álgebra:

$$\mathfrak{g}_n^{9,E(\frac{n-6}{2})} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-6}{2}) - 1 \\ [Y_{2E(\frac{n-6}{2})-1}, Y_{n-5}] & = & X_4 & \end{array}$$

Cuando n es impar, la última álgebra de dicha familia es $\mathfrak{g}_n^{9,E(\frac{n-7}{2})} = \mathfrak{g}_n^{9,E(\frac{n-6}{2})}$.

* Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-6$ y $c_{i,n-5} = 0 \quad \forall i$ $2E(\frac{n-5}{2}) - 1 \leq i \leq n-7$, pero $c_{n-6,n-5} \neq 0$ se distinguen dos casos, dependiendo del valor de ϵ.

$\mathfrak{g} \simeq \mathfrak{g}_n^{7,E(\frac{n-5}{2})}$ si $\epsilon = 0$

$\mathfrak{g} \simeq \mathfrak{g}_n^{6,E(\frac{n-5}{2})}$ si $\epsilon = 0 \quad c_{n-6,n-5} = 1$

$\mathfrak{g} \simeq \mathfrak{g}_n^{7,E(\frac{n-5}{2})}$ si $\epsilon = 0 \quad c_{n-6,n-5} \neq 1$.

En consecuencia, se concluye que en el caso: $\exists j \in \{1,2,\ldots,n-6\} : b_j \neq 0$, surgen las familias

$$\begin{array}{llllll}
\mathfrak{g}_n^{8,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-5}{2}) \\
& [X_1, Y_{n-6}] & = & X_4 & & \\
& [X_1, Y_{n-5}] & = & X_3 & & \\
& [X_2, Y_{n-5}] & = & X_4 & & \\
& [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 &
\end{array}$$

$$\begin{array}{llllll}
\mathfrak{g}_n^{9,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-6}{2}) \\
& [X_1, Y_{n-6}] & = & X_4 & & \\
& [X_1, Y_{n-5}] & = & X_3 & & \\
& [X_2, Y_{n-5}] & = & X_4 & & \\
& [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 & \\
& [Y_{2r-1}, Y_{n-5}] & = & X_4 & &
\end{array}$$

□

Extensiones centrales de primera generación de $\mathfrak{g}_{n-1}^{n-3}$

Corresponde al caso $(A, B) = (1, 0)$ del corolario 2.2.

Extensiones centrales de $\mathfrak{g}_{n-1}^{n-3}$, caso $\alpha \neq 0$

Teorema 2.6. *Toda álgebra de Lie nilpotente compleja $(n-4)$-filiforme, de dimensión n y que sea extensión central de primera generación de $\mathfrak{g}_{n-1}^{n-3}$ con $\alpha \neq 0$ (en el sentido explicado anteriormente) es isomorfa a alguna de las álgebras de Lie de leyes*

$$\begin{array}{lllll}
\mathfrak{g}_n^{10,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & & 1 \leq r \leq E(\frac{n-4}{2}) \\
 & [X_1, X_2] = Y_{n-5} & & & \\
 & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & &
\end{array}$$

$$\begin{array}{lllll}
\mathfrak{g}_n^{11,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & & 1 \leq r \leq E(\frac{n-5}{2}) \\
 & [X_1, X_2] = Y_{n-5} & & & \\
 & [X_1, Y_{n-6}] = X_4 & & & \\
 & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & &
\end{array}$$

$$\begin{array}{lllll}
\mathfrak{g}_n^{12,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & & 1 \leq r \leq E(\frac{n-4}{2}) \\
 & [X_1, X_2] = Y_{n-5} & & & \\
 & [X_1, Y_{n-5}] = X_4 & & & \\
 & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1. & &
\end{array}$$

Demostración: Toda álgebra de Lie de las consideradas en este teorema es isomorfa a una de ley

$$\begin{array}{lll}
[X_0, X_i] & = X_{i+1} & 1 \leq i \leq 2 \\
[X_0, X_3] & = \alpha Z & \\
[X_0, Y_j] & = d_j Z & 1 \leq j \leq n-5 \\
[X_1, X_2] & = Y_{n-5} + a Z & \\
[X_1, X_3] & = d_{n-5} Z & \\
[X_1, Y_j] & = b_j Z & 1 \leq j \leq n-5 \\
[Y_i, Y_j] & = c_{ij} Z & 1 \leq i < j \leq n-6.
\end{array}$$

Además, se puede suponer que

$$\alpha = 1 \quad d_j = 0 \quad 1 \leq j \leq n-6 \quad \text{y} \quad a = 0,$$

sin más que aplicar sucesivamente los cambios de base siguientes:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 3 \\ X_4^* &= \alpha Z & \\ Y_j^* &= Y_j & 1 \leq j \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} + aZ & \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_j^* &= Y_j - d_j X_3 & 1 \leq j \leq n-6 \\ Y_{n-5}^* &= Y_{n-5}\,. & \end{cases}$$

Entonces, la ley de $\mathfrak{g}$ se convierte en

$$\begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_0, Y_{n-5}] &=& d_{n-5} X_4 & \\ [X_1, X_2] &=& Y_{n-5} & \\ [X_1, X_3] &=& d_{n-5} X_4 & \\ [X_1, Y_j] &=& b_j X_4 & 1 \leq j \leq n-5 \\ [Y_i, Y_j] &=& c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{array}$$

Además, se puede suponer $d_{n-5} = 0$, sin más que hacer seguidamente los siguientes cambios de base:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{n-5}^* &= Y_{n-5} - d_{n-5} X_3 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* &= X_1 - d_{n-5} X_0 & & \\ Y_j^* &= Y_j & 1 \leq j \leq n-5. & \end{cases}$$

En consecuencia, los productos corchete no nulos de $\mathfrak{g}$, salvo antisimetría, son:

$$\begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &=& Y_{n-5} & \\ [X_1, Y_j] &=& b_j X_4 & 1 \leq j \leq n-5 \\ [Y_i, Y_j] &=& c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{array}$$

Se deben distinguir ahora los casos $b_{n-5} = 0$ y $b_{n-5} \neq 0$.

Cuando $b_{n-5} = 0$ se deben estudiar por separado los casos en que sean $b_j = 0$, $1 \leq j \leq n-6$, o bien exista algún $b_j \neq 0$.

Cuando $b_{n-5} \neq 0$ se puede probar que se pueden suponer nulos todos los restantes b_j. En todos los casos se va a distinguir la nulidad o no de los c_{ij}.

Caso 1: $b_{n-5} = 0$

El álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_j] & = & b_j X_4 & 1 \leq j \leq n-6 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{array}$$

Caso 1.1: $b_j = 0 \quad 1 \leq j \leq n-6$

La ley de $\mathfrak{g}$ viene determinada por

$$\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{10,1} : \left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \end{array} \right.$$

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-6$, se puede suponer $c_{12} \neq 0$. Basta con efectuar el cambio de base expresado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_1^* & = & Y_i & \\ Y_2^* & = & Y_j & \\ Y_i^* & = & Y_1 & \\ Y_j^* & = & Y_2 & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \quad k \notin \{1, 2, i, j\}. \end{array} \right.$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \qquad 3 \leq k \leq n-6$, y para demostrarlo es suficiente considerar el cambio de base siguiente:

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_1^* & = & \frac{1}{c_{12}} Y_1 & \\ Y_2^* & = & Y_2 & \\ Y_k^* & = & Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-6 \\ Y_{n-5}^* & = & Y_{n-5} & \end{array} \right.$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 3 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{10,2} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_1, Y_2] &= X_4 & \end{cases}$$

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-6$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-6$, sin más que hacer cambios de base análogos a los anteriores. Se obtiene un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_1, Y_2] &= X_4 & \\ [Y_3, Y_4] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 5 \leq i < j \leq n-6. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{10,k}$, $\quad 1 \leq k \leq r-1$ ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r-1 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2r-1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{10,r} : \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-6$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con aplicar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_{2r}^* &= Y_j & \\ Y_i^* &= Y_{2r-1} & \\ Y_j^* &= Y_{2r} & \\ Y_k^* &= Y_k & 1 \le k \le n-5 \quad k \notin \{2r-1, 2r, i, j\}. \end{cases}$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \le k \le n-6$. Esto se consigue con el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* &= Y_{2r} & \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \le k \le n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \le k \le r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \le i < j \le n-6 \end{cases}$$

llegándose a una situación análoga a las ya consideradas.

Es evidente que el proceso finaliza cuando $r = E(\frac{n-6}{2}) + 1 = E(\frac{n-4}{2})$, por lo que cuando $b_{n-5} = 0$ y $b_j = 0 \quad 1 \le j \le n-6$, surge la familia

$$\begin{array}{llll} \mathfrak{g}_n^{10,r}: & [X_0, X_i] = X_{i+1} & 1 \le i \le 3 & 1 \le r \le E(\frac{n-4}{2}) \\ & [X_1, X_2] = Y_{n-5} & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \le k \le r-1. & \end{array}$$

Caso 1.2: $\exists j \in \{1,2,\dots,n-6\} : b_j \neq 0$

Como $b_{n-5} = 0$, se sabe que la ley de $\mathfrak{g}$ viene determinada por

$$\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_j] & = & b_j X_4 & 1 \leq j \leq n-6 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{array}$$

Los cambios de base definidos por las relaciones

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{n-6}^* & = & Y_j & \\ Y_j^* & = & Y_{n-6} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \quad k \notin \{j, n-6\} \end{array} \right.$$

y

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_j^* & = & b_{n-6} Y_j - b_j Y_{n-6} & 1 \leq j \leq n-7 \\ Y_{n-6}^* & = & \frac{1}{b_{n-6}} Y_{n-6} & \\ Y_{n-5}^* & = & Y_{n-5} & \end{array} \right.$$

permiten, el primero suponer $b_{n-6} \neq 0$ y el segundo, $b_j = 0 \quad 1 \leq j \leq n-7$ y $b_{n-6} = 1$. Entonces, la ley de $\mathfrak{g}$ está determinada por

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,1} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-7$ y existe algún $c_{i,n-6} \neq 0$ $1 \leq i \leq n-7$, se puede suponer $c_{1,n-6} \neq 0$. Basta con aplicar el cambio de

base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= Y_i & \\ Y_i^* &= Y_1 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \qquad k \notin \{1, i\}. \end{cases}$$

Al efectuar el cambio de base expresado por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= c_{1,n-6}X_1 - Y_1 & \\ X_t^* &= c_{1,n-6}X_t & 2 \leq t \leq 4 \\ Y_1^* &= Y_1 & \\ Y_k^* &= c_{1,n-6}Y_k - c_{k,n-6}Y_1 & 2 \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= c_{1,n-6}^2 Y_{n-5} & \end{cases}$$

se obtiene que

$$\begin{array}{lcll} [X_0^*, X_1^*] &=& [X_0, c_{1,n-6}X_1 - Y_1] = c_{1,n-6}X_2 = X_2^* & \\ [X_0^*, X_t^*] &=& [X_0, c_{1,n-6}X_t] = c_{1,n-6}X_{t+1} = X_{t+1}^* & 2 \leq t \leq 3 \\ [X_1^*, X_2^*] &=& [c_{1,n-6}X_1 - Y_1, c_{1,n-6}X_2] = c_{1,n-6}^2 Y_{n-5} = Y_{n-5}^* & \\ [X_1, Y_{n-6}^*] &=& [c_{1,n-6}X_1 - Y_1, Y_{n-6}] = c_{1,n-6}X_4 - c_{1,n-6}X_4 = 0 & \\ [Y_k^*, Y_{n-6}] &=& [c_{1,n-6}Y_k - c_{k,n-6}Y_1, Y_{n-6}] = & \\ &=& (c_{1,n-6}c_{k,n-6} - c_{k,n-6}c_{1,n-6}).X_4 = 0.X_4 = 0 & 2 \leq k \leq n-7 \\ [Y_1^*, Y_{n-6}] &=& [Y_1, Y_{n-6}] = c_{1,n-6}X_4 = X_4^* & \end{array}$$

En consecuencia, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_1, Y_{n-6}] &= X_4 & \end{cases}$$

y al hacer el cambio siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_2^* &= Y_{n-6} & \\ Y_{n-6}^* &= Y_2 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \qquad k \notin \{2, n-6\}, \end{cases}$$

se transforma en

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [Y_1, Y_2] &= X_4 \end{cases}$$

que corresponde a $\mathfrak{g}_n^{10,2}$, ya obtenida anteriormente.

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-7$, se puede suponer $c_{12} \neq 0$. Basta con considerar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t \quad 0 \leq t \leq 4 \\ Y_1^* &= Y_i \\ Y_2^* &= Y_j \\ Y_i^* &= Y_1 \\ Y_j^* &= Y_2 \\ Y_k^* &= Y_k \quad 1 \leq k \leq n-5 \quad k \notin \{1, 2, i, j\}. \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \quad 3 \leq k \leq n-6$, sin más que aplicar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t \quad 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 \\ Y_2^* &= Y_2 \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 \quad 3 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [X_1, Y_{n-6}] &= X_4 \\ [Y_1, Y_2] &= X_4 \\ [Y_i, Y_j] &= c_{ij} X_4 \quad 3 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 3 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,2} : \begin{array}{lcl} [X_0, X_i] &=& X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &=& Y_{n-5} \\ [X_1, Y_{n-6}] &=& X_4 \\ [Y_1, Y_2] &=& X_4 \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-7$, y existe algún $c_{i,n-6} \neq 0$ $3 \leq i \leq n-7$,se obtiene el álgebra

$$\mathfrak{g}_n^{10,3}: \begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [Y_1, Y_2] &= X_4 \\ [Y_3, Y_4] &= X_4 \end{cases}$$

ya obtenida.

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-7$, se obtiene un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [X_1, Y_{n-6}] &= X_4 \\ [Y_1, Y_2] &= X_4 \\ [Y_3, Y_4] &= X_4 \\ [Y_i, Y_j] &= c_{ij}X_4 \quad 5 \leq i < j \leq n-6. \end{cases}$$

Tanto en el caso anterior como en éste, se consiguen las leyes sin más que considerar cambios de base análogos a algunos anteriores.

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{11,k} \quad \mathfrak{g}_n^{10,k+1} \quad 1 \leq k \leq r-1$, o a un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [X_1, Y_{n-6}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 \quad 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij}X_4 \quad 2r-1 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,r}: \begin{array}{lcl} [X_0, X_i] &=& X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &=& Y_{n-5} \\ [X_1, Y_{n-6}] &=& X_4 \\ [Y_{2k-1}, Y_{2k}] &=& X_4 \quad 1 \leq k \leq r-1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-7$ y existe algún $c_{i,n-6} \neq 0$ $2r-1 \leq i \leq n-7$, se puede suponer $c_{2r-1,n-6} \neq 0$. Basta con aplicar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_i^* &= Y_{2r-1} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \qquad k \notin \{2r-1, i\}. \end{cases}$$

Al efectuar el cambio:

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= c_{2r-1,n-6}X_1 - Y_{2r-1} & \\ X_t^* &= c_{2r-1,n-6}X_t & 2 \leq t \leq 4 \\ Y_k^* &= c_{2r-1,n-6}Y_k & 1 \leq k \leq 2r-2 \quad k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \quad k = \dot{2} \\ Y_{2r-1}^* &= Y_{2r-1} & \\ Y_k^* &= c_{2r-1,n-6}Y_k - c_{k,n-6}Y_{2r-1} & 2r \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= c_{2r-1,n-6}^2 Y_{n-5} & \end{cases}$$

se obtienen los siguientes productos corchete no nulos, salvo antisimetría:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 & \end{cases}$$

y al hacer

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r}^* &= Y_{n-6} & \\ Y_{n-6}^* &= Y_{2r} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \qquad k \notin \{2r, n-6\}, \end{cases}$$

la ley anterior se transforma en

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \end{cases}$$

que corresponde a $\mathfrak{g}_n^{10,r+1}$, ya obtenida anteriormente.

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-7$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con aplicar el siguiente cambio de base:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_{2r}^* &= Y_j & \\ Y_i^* &= Y_{2r-1} & \\ Y_j^* &= Y_{2r} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r-1, 2r, i, j\}. \end{cases}$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \leq k \leq n-5$. Se consigue con el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* &= Y_{2r} & \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-6}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \leq i < j \leq n-6. \end{cases}$$

Se llega a una situación parecida a las ya analizadas.

En consecuencia, van apareciendo las álgebras nuevas

$$\begin{array}{llll} \mathfrak{g}_n^{11,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-7}{2}) - 1 \\ & [X_1, X_2] = Y_{n-5} & & \\ & [X_1, Y_{n-6}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & \end{array}$$

y también las ya obtenidas anteriormente:

$$\mathfrak{g}_n^{10,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \qquad 2 \leq r \leq E(\frac{n-7}{2}) \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1, \end{array}$$

y justo antes del último paso del proceso, se obtiene la siguiente ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 2E(\frac{n-7}{2}) - 1 \leq i < j \leq n-6. \end{array} \right.$$

A continuación, hay que diferenciar dos casos, dependiendo de la paridad de la dimensión de $\mathfrak{g}$.

Caso n par $(E(\frac{n-7}{2}) = \frac{n-8}{2})$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-7}{2}) - 1 = n - 9 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11, E(\frac{n-7}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-7}{2}) - 1 = n - 9 \leq i < j \leq n-7$, y existe algún $c_{i,n-6} \neq 0 \quad n-9 \leq i \leq n-7$, se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-8}{2} = E(\frac{n-6}{2}) - 1, \end{array} \right.$$

que corresponde a $\mathfrak{g}_n^{10, E(\frac{n-6}{2})}$, ya conocida.

* Si existe algún $c_{ij} \neq 0 \quad 2E(\frac{n-7}{2}) - 1 = n - 9 \leq i < j \leq n-7$, se puede suponer $c_{n-9,n-8} \neq 0$, y se obtiene el álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-8}{2} = E(\frac{n-5}{2}) - 1 \\ [Y_{n-7}, Y_{n-6}] & = & c_{n-7,n-6} X_4 & \end{array} \right.$$

- Si $c_{n-7,n-6} = 0$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1. \end{array}$$

- Si $c_{n-7,n-6} \neq 0$, al aplicar el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \quad t \neq 1 \\ X_1^* & = & X_1 - \frac{1}{c_{n-7,n-6}} Y_{n-7} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \quad k \neq n-7 \\ Y_{n-7}^* & = & \frac{1}{c_{n-7,n-6}} Y_{n-7} & \end{array} \right.$$

se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-6}{2} = E(\frac{n-4}{2}) - 1, \end{array} \right.$$

que corresponde a $\mathfrak{g}_n^{10,E(\frac{n-4}{2})}$, ya conocida.

Caso n impar $(E(\frac{n-7}{2}) = \frac{n-7}{2})$

El álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 2E(\frac{n-7}{2}) - 1 \leq i < j \leq n-6. \end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-7}{2}) - 1 = n-8 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,E(\frac{n-7}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-7}{2}) - 1 = n - 8 \leq i < j \leq n - 7 \Leftrightarrow c_{n-8,n-7} = 0$, y existe algún $c_{i,n-6} \neq 0 \quad n - 8 \leq i \leq n - 7$, se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-7}{2} = E(\frac{n-4}{2}) - 1, \end{array} \right.$$

que corresponde a $\mathfrak{g}_n^{10,E(\frac{n-4}{2})}$, ya conocida.

* Si existe algún $c_{ij} \neq 0 \quad 2E(\frac{n-7}{2}) - 1 = n - 8 \leq i < j \leq n - 7 \Leftrightarrow c_{n-8,n-7} \neq 0$, se obtiene el álgebra:

$$\mathfrak{g}_n^{11,E(\frac{n-5}{2})} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1. \end{array}$$

Como conclusión del caso: $b_{n-5} = 0$ y $\exists j \in \{1, 2, \ldots, n-6\} : b_j \neq 0$, se observa que surge la familia nueva de álgebras siguiente:

$$\mathfrak{g}_n^{11,r} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-5}{2}) \\ [X_1, X_2] & = & Y_{n-6} & \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r - 1. \end{array}$$

Caso 2: $b_{n-5} \neq 0$

El álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_j] & = & b_j X_4 & 1 \leq j \leq n-5 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{array}$$

Al efectuar el siguiente cambio de base:

$$\left\{ \begin{array}{lcll} X_0^* & = & \sqrt[3]{b_{n-5}} X_0 & \\ X_1^* & = & \frac{1}{\sqrt[6]{b_{n-5}}} X_1 & \\ X_2^* & = & \sqrt[6]{b_{n-5}} X_2 & \\ X_3^* & = & \sqrt{b_{n-5}} X_3 & \\ X_4^* & = & \sqrt[6]{b_{n-5}^5} X_4 & \\ Y_j^* & = & b_{n-5} Y_j - b_j Y_{n-5} & 1 \leq j \leq n-6 \\ Y_{n-5}^* & = & Y_{n-5} & \end{array} \right.$$

se obtienen los siguientes productos corchete:

$$\begin{array}{lcl} [X_0^*, X_1^*] & = & [\sqrt[3]{b_{n-5}} X_0, \frac{1}{\sqrt[6]{b_{n-5}}} X_1] = \sqrt[6]{b_{n-5}} X_2 = X_2^* \\ [X_0^*, X_2^*] & = & [\sqrt[3]{b_{n-5}} X_0, \sqrt[6]{b_{n-5}} X_2] = \sqrt{b_{n-5}} X_3 = X_3^* \\ [X_0^*, X_3^*] & = & [\sqrt[3]{b_{n-5}} X_0, \sqrt{b_{n-5}} X_3] = \sqrt[6]{b_{n-5}^5} X_4 = X_4^* \\ [X_1^*, X_2^*] & = & [\frac{1}{\sqrt[6]{b_{n-5}}} X_1, \sqrt[6]{b_{n-5}} X_2] = [X_1, X_2] = Y_{n-5} = Y_{n-5}^* \\ [X_1^*, Y_{n-5}^*] & = & [\frac{1}{\sqrt[6]{b_{n-5}}} X_1, Y_{n-5}] = \frac{1}{\sqrt[6]{b_{n-5}}} [X_1, Y_{n-5}] = \sqrt[6]{b_{n-5}^5} X_4 = X_4^* \\ [X_1^*, Y_j^*] & = & [\frac{1}{\sqrt[6]{b_{n-5}}} X_1, b_{n-5} Y_j - b_j Y_{n-5}] = (\frac{b_{n-5}}{\sqrt[6]{b_{n-5}}} b_j - \frac{b_j}{\sqrt[6]{b_{n-5}}} b_{n-5}).X_4 = \\ & = & 0.X_4 = 0 \qquad 1 \leq j \leq n-6 \\ [Y_i^*, Y_j^*] & = & [Y_i, Y_j] = c_{ij} X_4 = c_{ij}.\frac{1}{\sqrt[6]{b_{n-5}^5}}.X_4^* = c_{ij}^*.X_4^* \qquad 1 \leq i < j \leq n-6. \end{array}$$

Y, en consecuencia, se obtiene la siguiente ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad & 1 \le i \le 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \le i < j \le n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 1 \le i < j \le n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{12,1} : \begin{cases} [X_0, X_i] &= X_{i+1} \quad & 1 \le i \le 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \end{cases}$$

* Si existe algún $c_{ij} \ne 0 \quad 1 \le i < j \le n-6$, se puede suponer $c_{12} \ne 0$. Basta con considerar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t \quad & 0 \le t \le 4 \\ Y_1^* &= Y_i & \\ Y_2^* &= Y_j & \\ Y_i^* &= Y_1 & \\ Y_j^* &= Y_2 & \\ Y_k^* &= Y_k & 1 \le k \le n-5 \quad k \notin \{1, 2, i, j\}. \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \qquad 3 \le k \le n-6$, sin más que aplicar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t \quad & 0 \le t \le 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 & \\ Y_2^* &= Y_2 & \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \le k \le n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad & 1 \le i \le 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \le i < j \le n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{12,2} : \begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [X_1, Y_{n-5}] &= X_4 \\ [Y_1, Y_2] &= X_4 \end{cases}$$

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-6$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-6$, y para demostrarlo es suficiente considerar cambios de base análogos a algunos anteriores. Se obtiene un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [Y_1, Y_2] &= X_4 \\ [Y_3, Y_4] &= X_4 \\ [Y_i, Y_j] &= c_{ij} X_4 \quad 5 \leq i < j \leq n-6. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{12,k}$, $\quad 1 \leq k \leq r-1$ ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [X_1, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 \quad 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij} X_4 \quad 2r-1 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{12,r} : \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-6$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con hacer el siguiente cambio de base:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_{2r}^* &= Y_j & \\ Y_i^* &= Y_{2r-1} & \\ Y_j^* &= Y_{2r} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r-1, 2r, i, j\}. \end{cases}$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \leq k \leq n-6$. Estas igualdades se consiguen al aplicar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* &= Y_{2r} & \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \leq i < j \leq n-6, \end{cases}$$

tratándose de una situación análoga a las ya consideradas.

Es evidente que el proceso finaliza cuando $r = E(\frac{n-6}{2}) + 1 = E(\frac{n-4}{2})$, por lo que en el caso: $b_{n-5} \neq 0$, surge la familia:

$$\begin{array}{lllll} \mathfrak{g}_n^{12,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & & 1 \leq r \leq E(\frac{n-4}{2}) \\ & [X_1, X_2] = Y_{n-5} & & & \\ & [X_1, Y_{n-5}] = X_4 & & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1. & & \end{array}$$

□

Las álgebras de Lie p-filiformes como extensiones por derivaciones

Las álgebras (n-4)-filiformes

En todo lo que sigue se supondrá que $\mathfrak{g}$ es un álgebra de Lie nilpotente, de dimensión n y sucesión característica $(4, 1, 1, \ldots, 1)$.

Existen solamente dos álgebras de sucesión característica $(4, 1)$: el álgebra modelo L_4 y el álgebra μ_4, cuyas leyes, en una base $\{X_0, X_1, X_2, X_3, X_4\}$, vienen dadas mediante

$$\begin{array}{rrcll} L_4: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ \mu_4: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, X_2] & = & X_4 & \end{array}$$

Cualquiera de ellas puede considerarse álgebra soporte de $\mathfrak{g}$ y, para contemplar ambas situaciones, su ley se denota por

$$\begin{array}{rrcll} \mathfrak{g}^*: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0, 1\}. \end{array}$$

Al ser $\mathfrak{g}$ un álgebra de sucesión característica $(4, 1, 1, \ldots, 1)$, se puede obtener una base de $\mathfrak{g}$ como una extensión de la base de $\mathfrak{g}^*$ añadiendo $n - 5$ vectores:

$Y_1, Y_2, \ldots, Y_{n-5}$ asociados a derivaciones adecuadas de $\mathfrak{g}^*$: la nula, las homogéneas (δ_1, δ_2) o alguna combinación lineal de ellas, cumpliéndose que

$$\begin{array}{ll} \delta_1(X_1) = X_3 & \delta_1(X_2) = X_4 \\ \delta_2(X_1) = X_4 & \end{array}$$

Teorema 3.4 *En dimensión $n, n \geq 8$, hay exactamente 6n-29 álgebras de Lie nilpotentes complejas, dos a dos no isomorfas y que se pueden obtener como extensiones por derivaciones de alguna de las dos álgebras de Lie filiformes de dimensión 5. Sus leyes se pueden expresar, respecto a una cierta base adaptada $\{X_0, X_1, X_2, X_3, X_4, Y_1, Y_2, \ldots, Y_{n-5}\}$, mediante*

$$\begin{array}{llcll} \mathfrak{g}_n^{1,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{2,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [X_1, X_2] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{3,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [X_1, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{4,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [X_1, X_2] & = & X_4 & \\ & [X_1, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{5,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{6,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-5}{2}) \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \leq k \leq r-1 & \\ & [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

$$
\mathfrak{g}_n^{7,r}: \quad \begin{aligned}
[X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 & \qquad 1 \leq r \leq E(\tfrac{n-5}{2}) \\
[X_1, X_2] &= X_4 & & \\
[X_1, Y_{n-5}] &= X_3 & & \\
[X_2, Y_{n-5}] &= X_4 & & \\
[Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 & \\
[Y_{2r-1}, Y_{n-5}] &= X_4 & &
\end{aligned}
$$

$$
\mathfrak{g}_n^{8,r}: \quad \begin{aligned}
[X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 & \qquad 1 \leq r \leq E(\tfrac{n-5}{2}) \\
[X_1, Y_{n-6}] &= X_4 & & \\
[X_1, Y_{n-5}] &= X_3 & & \\
[X_2, Y_{n-5}] &= X_4 & & \\
[Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 &
\end{aligned}
$$

$$
\mathfrak{g}_n^{9,r}: \quad \begin{aligned}
[X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 & \qquad 1 \leq r \leq E(\tfrac{n-6}{2}) \\
[X_1, Y_{n-6}] &= X_4 & & \\
[X_1, Y_{n-5}] &= X_3 & & \\
[X_2, Y_{n-5}] &= X_4 & & \\
[Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 & \\
[Y_{2r-1}, Y_{n-5}] &= X_4 & &
\end{aligned}
$$

$$
\mathfrak{g}_n^{10,r}: \quad \begin{aligned}
[X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 & \qquad 1 \leq r \leq E(\tfrac{n-4}{2}) \\
[X_1, X_2] &= Y_{n-5} & & \\
[Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 &
\end{aligned}
$$

$$
\mathfrak{g}_n^{11,r}: \quad \begin{aligned}
[X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 & \qquad 1 \leq r \leq E(\tfrac{n-5}{2}) \\
[X_1, X_2] &= Y_{n-5} & & \\
[X_1, Y_{n-6}] &= X_4 & & \\
[Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 &
\end{aligned}
$$

$$
\mathfrak{g}_n^{12,r}: \quad \begin{aligned}
[X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 & \qquad 1 \leq r \leq E(\tfrac{n-4}{2}) \\
[X_1, X_2] &= Y_{n-5} & & \\
[X_1, Y_{n-5}] &= X_4 & & \\
[Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1. &
\end{aligned}
$$

Solamente cuando n es impar:

$$\mathfrak{g}_n^{3,E(\frac{n-3}{2})} \simeq \mathfrak{g}_n^{1,E(\frac{n-3}{2})}$$

$$\mathfrak{g}_n^{4,E(\frac{n-3}{2})} \simeq \mathfrak{g}_n^{2,E(\frac{n-3}{2})}$$

$$\mathfrak{g}_n^{6,E(\frac{n-5}{2})} \simeq \mathfrak{g}_n^{5,E(\frac{n-3}{2})}.$$

Demostración:

Se ha demostrado en la memoria que el álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= \lambda_1 X_4 & \\ [X_1, Y_{n-5}] &= \lambda_2 X_3 + \lambda_3 X_4 + \lambda_2 \sum_{k=1}^{n-5} b_k Y_k & \\ [X_2, Y_{n-5}] &= \lambda_2 X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5, \end{cases}$$

cumpliéndose

$$\begin{cases} \lambda_1 \lambda_2 b_k &= 0 & 1 \leq k \leq n-5 \\ \sum_{k=2}^{n-5} b_k c_{1k} &= 0 & \\ \sum_{k=i+1}^{n-5} b_k c_{ik} &= \sum_{r=1}^{i-1} b_r c_{ri} & 2 \leq i \leq n-6 \\ \sum_{k=1}^{n-6} b_k c_{k,n-5} &= -\lambda_2^2 b_{n-5} & \end{cases}$$

y donde la terna $(\lambda_1, \lambda_2, \lambda_3)$ puede tomar los valores

$$\begin{array}{ll} (0,0,0) & \text{(caso 1)} \\ (0,0,1) & \text{(caso 2.1)} \\ (0,1,\alpha) & \text{(caso 2.2)} \\ (1,1,0). & \text{(caso 3).} \end{array}$$

Caso 1: $(\lambda_1, \lambda_2, \lambda_3) = (0, 0, 0)$

El álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \epsilon \in \{0, 1\} \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5, \end{cases}$$

cumpliéndose

$$\begin{cases} \sum_{k=2}^{n-5} b_k c_{1k} &= 0 \\ \sum_{k=i+1}^{n-5} b_k c_{ik} &= \sum_{r=1}^{i-1} b_r c_{ri} & 2 \leq i \leq n-6 \\ \sum_{k=1}^{n-6} b_k c_{k,n-5} &= 0. \end{cases}$$

Se encuentran las familias

$$\mathfrak{g}_n^{1,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-3}{2}) \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1, \end{array}$$

$$\mathfrak{g}_n^{2,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-3}{2}) \\ [X_1, X_2] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1 \qquad \text{y} \end{array}$$

$$\mathfrak{g}_n^{10,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-4}{2}) \\ [X_1, X_2] &=& Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1. \end{array}$$

Demostración

Hay que distinguir dos casos, según sean nulos todos los b_j $1 \leq j \leq n-5$ o bien exista alguno no nulo.

Caso 1.1: $b_j = 0 \qquad 1 \leq j \leq n-5$

El álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 1 \leq i < j \leq n-5,$ se obtienen las álgebras siguientes, dependiendo del valor de ϵ:

$$\mathfrak{g}_n^{1,1} : \left\{ [X_0, X_i] = X_{i+1} \qquad 1 \leq i \leq 3 \right.$$

$$\mathfrak{g}_n^{2,1} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= X_4 \end{cases}$$

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-5,$ se puede suponer siempre que $c_{12} \neq 0$. Basta con efectuar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= Y_i \\ Y_2^* &= Y_j \\ Y_i^* &= Y_1 \\ Y_j^* &= Y_2 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{1, 2, i, j\}. \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \qquad 3 \leq k \leq n-5,$ sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 \\ Y_2^* &= Y_2 \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-5, \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \leq i < j \leq n-5. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-5$, se obtienen las álgebras:

$$\mathfrak{g}_n^{1,2} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [Y_1, Y_2] &= X_4 & \end{cases}$$

$$\mathfrak{g}_n^{2,2} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \end{cases}$$

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-5$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-5$, y para demostrarlo es suficiente considerar cambios de base análogos a algunos anteriores. Se obtiene un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [Y_1, Y_2] &= X_4 & \\ [Y_3, Y_4] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 5 \leq i < j \leq n-5. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{1,k}$, $\mathfrak{g}_n^{2,k} \quad 1 \leq k \leq r-1$ ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r-1 \leq i < j \leq n-5. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-5$, se obtienen las álgebras siguientes, dependiendo del valor de ϵ:

$$\mathfrak{g}_n^{1,r} : \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \end{array}$$

$$\mathfrak{g}_n^{2,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-5$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con aplicar el siguiente cambio de base:

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* & = & Y_i & \\ Y_{2r}^* & = & Y_j & \\ Y_i^* & = & Y_{2r-1} & \\ Y_j^* & = & Y_{2r} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r-1, 2r, i, j\}. \end{array} \right.$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \leq k \leq n-5$, sin más que considerar el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* & = & \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* & = & Y_{2r} & \\ Y_k^* & = & Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-5, \end{array} \right.$$

y se obtiene el álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0, 1\} \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] & = & c_{ij} X_4 & 2r+1 \leq i < j \leq n-5 \end{array} \right.$$

llegándose, de nuevo, a una situación parecida a las ya consideradas.

Es evidente que el proceso finaliza cuando $r = E(\frac{n-5}{2}) + 1 = E(\frac{n-3}{2})$, por lo que en el caso $b_j = 0 \quad 1 \leq j \leq n-5$ surgen las familias

$$\mathfrak{g}_n^{1,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-3}{2}) \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \end{array}$$

$$\mathfrak{g}_n^{2,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-3}{2}) \\ [X_1, X_2] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1. \end{array}$$

Caso 1.2: $\exists j \in \{1, 2, \ldots, n-5\} : b_j \neq 0$

El álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \epsilon \in \{0, 1\} \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5. \end{cases}$$

Los cambios de base definidos por las relaciones

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{n-5}^* &= Y_j & \\ Y_j^* &= Y_{n-5} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{j, n-5\} \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \\ Y_{n-5}^* &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \end{cases}$$

permiten, el primero, suponer que $b_{n-5} \neq 0$ y, el segundo, obtener los siguientes productos corchete:

$$[X_0^*, X_i^*] = [X_0, X_i] = X_{i+1} = X_{i+1}^* \quad 1 \leq i \leq 3$$

$$[X_1^*, X_2^*] = [X_1, X_2] = \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k = Y_{n-5}^*$$

$$[Y_i^*, Y_j^*] = [Y_i, Y_j] = c_{ij} X_4 = c_{ij} X_4^* \quad 1 \leq i < j \leq n-6$$

$$[Y_1^*, Y_{n-5}^*] = [Y_1, \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k] = \sum_{k=2}^{n-5} b_k [Y_1, Y_k] = (\sum_{k=2}^{n-5} b_k c_{1k}) X_4 = 0.X_4^* = 0$$

$$\begin{aligned} [Y_i^*, Y_{n-5}^*] &= [Y_i, \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k] = (-\sum_{r=1}^{i-1} b_r c_{ri} + \sum_{k=i+1}^{n-5} b_k c_{ik}) X_4 = \\ &= 0.X_4^* = 0 \quad 2 \leq i \leq n-6 \end{aligned}$$

y, en consecuencia $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{10,1} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \end{cases}$$

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-6$, se puede suponer $c_{12} \neq 0$. Basta con efectuar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= Y_i & \\ Y_2^* &= Y_j & \\ Y_i^* &= Y_1 & \\ Y_j^* &= Y_2 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{1, 2, i, j\}. \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \qquad 3 \leq k \leq n-6$, y para demostrarlo es suficiente considerar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 & \\ Y_2^* &= Y_2 & \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{10,2} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [Y_1, Y_2] &= X_4 \end{cases}$$

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-6$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-6$, sin más que hacer cambios de base análogos a algunos anteriores. Se obtiene un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [Y_1, Y_2] &= X_4 \\ [Y_3, Y_4] &= X_4 \\ [Y_i, Y_j] &= c_{ij}X_4 & 5 \leq i < j \leq n-6. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{10,k}, \quad 1 \leq k \leq r-1$ ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij}X_4 & 2r-1 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{10,r} : \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &=& Y_{n-5} \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-6$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con aplicar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i \\ Y_{2r}^* &= Y_j \\ Y_i^* &= Y_{2r-1} \\ Y_j^* &= Y_{2r} \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r-1, 2r, i, j\}. \end{cases}$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \leq k \leq n-6$. Esto se consigue con el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* &= Y_{2r} & \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \leq i < j \leq n-6 \end{cases}$$

y se llega a una situación análoga a las ya consideradas.

Es evidente que el proceso finaliza cuando $r = E(\frac{n-6}{2}) + 1 = E(\frac{n-4}{2})$, por lo que cuando $\exists j \in \{1, 2, \ldots, n-5\} : b_j \neq 0$, surge la familia

$$\begin{array}{llcll} \mathfrak{g}_n^{10,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 3 & \quad 1 \leq r \leq E(\frac{n-4}{2}) \\ & [X_1, X_2] & = & Y_{n-5} & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \leq k \leq r-1. & \end{array}$$

□

Caso 2.1: $(\lambda_1, \lambda_2, \lambda_3) = (0, 0, 1)$

El álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \le i < j \le n-5, \end{cases}$$

cumpliéndose

$$\begin{cases} \sum_{k=2}^{n-5} b_k c_{1k} &= 0 & \\ \sum_{k=i+1}^{n-5} b_k c_{ik} &= \sum_{r=1}^{i-1} b_r c_{ri} & 2 \le i \le n-6 \\ \sum_{k=1}^{n-6} b_k c_{k,n-5} &= 0. & \end{cases}$$

Se encuentran las familias

$$\begin{array}{llcll} \mathfrak{g}_n^{3,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \le i \le 3 & 1 \le r \le E(\frac{n-3}{2}) \\ & [X_1, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \le k \le r-1, & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{4,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \le i \le 3 & 1 \le r \le E(\frac{n-3}{2}) \\ & [X_1, X_2] & = & X_4 & \\ & [X_1, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \le k \le r-1, & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{11,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \le i \le 3 & 1 \le r \le E(\frac{n-5}{2}) \\ & [X_1, X_2] & = & Y_{n-5} & \\ & [X_1, Y_{n-6}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \le k \le r-1 & \text{y} \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{12,r}: & [X_0, X_i] & = & X_{i+1} \quad 1 \le i \le 3 & 1 \le r \le E(\frac{n-4}{2}) \\ & [X_1, X_2] & = & Y_{n-5} & \\ & [X_1, Y_{n-5}] & = & X_4 & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 \quad 1 \le k \le r-1. & \end{array}$$

Demostración

Hay que distinguir dos casos, según sean nulos todos los b_j $1 \leq j \leq n-5$ o bien exista alguno no nulo.

Caso 2.1.1: $b_j = 0 \qquad 1 \leq j \leq n-5$

El álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5. \end{cases}$$

Los casos que se van a considerar ahora son, esquemáticamente, los siguientes:

a) $c_{ij} = 0 \quad 1 \leq i < j \leq n-6$ y se distingue según sea $c_{i,n-5} = 0$ $1 \leq i \leq n-6$ o exista algún $c_{i,n-5} \neq 0$.

b) existe $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-6$. Se puede suponer $c_{12} = 1$, $c_{1k} = 0 = c_{2k}$, $3 \leq k \leq n-5$.

b.1) $c_{ij} = 0 \quad 3 \leq i < j \leq n-6$ y se distingue según sean todos los $c_{i,n-5}$ nulos o no.

b.2) existe $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-6 \Rightarrow c_{34} = 1$, $c_{3k} = 0 = c_{4k}$, $5 \leq k \leq n-5$ (además de $c_{12} = 1$, $c_{1k} = 0 = c_{2k}$, $3 \leq k \leq n-5$, naturalmente) y así sucesivamente. En realidad, lo que se hace es un proceso de inducción finita.

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-5$, se obtienen las álgebras siguientes, dependiendo del valor de ϵ:

$$\mathfrak{g}_n^{3,1} : \begin{array}{lcl} [X_0, X_i] &=& X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &=& X_4 \end{array}$$

$$\mathfrak{g}_n^{4,1} : \begin{array}{lcl} [X_0, X_i] &=& X_{i+1} \quad 1 \leq i \leq 3 \\ [X_1, X_2] &=& X_4 \\ [X_1, Y_{n-5}] &=& X_4 \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $1 \leq i \leq n-6$, se puede suponer $c_{1,n-5} \neq 0$, sin más que aplicar el cambio de base definido por

$$\left\{\begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_1^* & = & Y_i & \\ Y_i^* & = & Y_1 & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-6 \quad k \notin \{1,i\} \\ Y_{n-5}^* & = & Y_{n-5} & \end{array}\right.$$

Subcaso: $\epsilon = 0$

La ley viene determinada por

$$\left\{\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_i, Y_{n-5}] & = & c_{i,n-5}X_4 & 1 \leq i \leq n-6. \end{array}\right.$$

con $c_{1,n-5} \neq 0$.

Aplicando el cambio de base dado por

$$\left\{\begin{array}{lcll} X_0^* & = & X_0 & \\ X_1^* & = & c_{1,n-5}X_1 - Y_1 & \\ X_t^* & = & c_{1,n-5}X_t & 2 \leq t \leq 4 \\ Y_1^* & = & Y_1 & \\ Y_i^* & = & c_{1,n-5}Y_i - c_{i,n-5}Y_1 & 2 \leq i \leq n-6 \\ Y_{n-5}^* & = & Y_{n-5} & \end{array}\right.$$

se obtiene que

$$\begin{array}{lcl} [X_0^*, X_1^*] & = & [X_0, c_{1,n-5}X_1 - Y_1] = c_{1,n-5}X_2 = X_2^* \\ [X_0^*, X_t^*] & = & [X_0, c_{1,n-5}X_t] = c_{1,n-5}X_{t+1} = X_{t+1}^* \quad 2 \leq t \leq 3 \\ [X_1^*, Y_{n-5}^*] & = & [c_{1,n-5}X_1 - Y_1, Y_{n-5}] = c_{1,n-5}[X_1, Y_{n-5}] - [Y_1, Y_{n-5}] = \\ & = & (c_{1,n-5} - c_{1,n-5}).X_4 = 0.X_4 = 0 \\ [Y_1^*, Y_{n-5}^*] & = & [Y_1, Y_{n-5}] = c_{1,n-5}X_4 = X_4^* \end{array}$$

$$\begin{aligned}[Y_i^*, Y_{n-5}^*] &= [c_{1,n-5}Y_i - c_{i,n-5}Y_1, Y_{n-5}] = c_{1,n-5}[Y_i, Y_{n-5}] - c_{i,n-5}[Y_1, Y_{n-5}] = \\ &= (c_{1,n-5}c_{i,n-5} - c_{i,n-5}c_{1,n-5}).X_4 = 0.X_4 = 0 \quad 2 \le i \le n-6\end{aligned}$$

y se consigue el álgebra de ley:

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \le i \le 3 \\ [Y_1, Y_{n-5}] = X_4 & \end{cases}$$

El cambio de base expresado por

$$\begin{cases} X_t^* = X_t & 0 \le t \le 4 \\ Y_1^* = Y_1 & \\ Y_2^* = Y_{n-5} & \\ Y_k^* = Y_k & 3 \le k \le n-6 \\ Y_{n-5}^* = Y_2 & \end{cases}$$

demuestra que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{1,2} : \begin{cases} [X_0, X_i] = X_{i+1} & 1 \le i \le 3 \\ [Y_1, Y_2] = X_4 & \end{cases}$$

ya obtenida anteriormente.

Subcaso: $\epsilon = 1$

La ley viene determinada por

$$\begin{cases} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_i, Y_{n-5}] & = & c_{i,n-5}X_4 & 1 \leq i \leq n-6. \end{cases}$$

con $c_{1,n-5} \neq 0$.

Con el cambio de base definido por

$$\begin{cases} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_1^* & = & \frac{1}{c_{1,n-5}}Y_1 & \\ Y_i^* & = & c_{1,n-5}Y_i - c_{i,n-5}Y_1 & 2 \leq i \leq n-6 \\ Y_{n-5}^* & = & Y_{n-5} & \end{cases}$$

se obtiene que

$$[X_0^*, X_i^*] = X_{i+1}^* \quad 1 \leq i \leq 3$$

$$[X_1^*, X_2^*] = X_4^*$$

$$[X_1^*, Y_{n-5}^*] = [X_1, Y_{n-5}] = X_4 = X_4^*$$

$$[Y_1^*, Y_{n-5}^*] = [\frac{1}{c_{1,n-5}}Y_1, Y_{n-5}] = \frac{1}{c_{1,n-5}}.c_{1,n-5}.X_4 = X_4 = X_4^*$$

$$\begin{aligned} [Y_i^*, Y_{n-5}^*] &= [c_{1,n-5}Y_i - c_{i,n-5}Y_1, Y_{n-5}] = c_{1,n-5}[Y_i, Y_{n-5}] - c_{i,n-5}[Y_1, Y_{n-5}] = \\ &= (c_{1,n-5}c_{i,n-5} - c_{i,n-5}c_{1,n-5}).X_4 = 0.X_4 = 0 \quad 2 \leq i \leq n-6 \end{aligned}$$

y queda demostrado que $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$\begin{cases} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_{n-5}] & = & X_4 & \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* & = & X_t & 0 \leq t \leq 4 \quad t \neq 1 \\ X_1^* & = & X_1 - Y_1 & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= Y_1 & \\ Y_2^* &= Y_{n-5} & \\ Y_k^* &= Y_k & 3 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_2 & \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{2,2} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \end{cases}$$

ya obtenida anteriormente.

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-6$, se puede suponer siempre que $c_{12} \neq 0$. Basta con efectuar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= Y_i & \\ Y_2^* &= Y_j & \\ Y_i^* &= Y_1 & \\ Y_j^* &= Y_2 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \quad k \notin \{1,2,i,j\} \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \qquad 3 \leq k \leq n-5$, sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 & \\ Y_2^* &= Y_2 & \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-5, \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \leq i < j \leq n-5. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \le i < j \le n-5$, se obtienen las álgebras

$$\mathfrak{g}_n^{3,2}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \end{array}$$

$$\mathfrak{g}_n^{4,2}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \le i < j \le n-6$ y existe algún $c_{i,n-5} \ne 0$ $3 \le i \le n-6$, se puede suponer $c_{3,n-5} \ne 0$, y considerando cambios de base análogos a algunos anteriores se demuestra que $\mathfrak{g}$ es isomorfa a alguna de las álgebras de leyes:

$$\mathfrak{g}_n^{1,3}: \left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [Y_1, Y_2] & = & X_4 & \\ [Y_3, Y_4] & = & X_4 & \end{array} \right.$$

$$\mathfrak{g}_n^{2,3}: \left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \\ [Y_3, Y_4] & = & X_4 & \end{array} \right.$$

ambas aparecidas anteriormente.

* Si existe algún $c_{ij} \ne 0 \quad 3 \le i < j \le n-6$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \le k \le n-5$, y se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \\ [Y_3, Y_4] & = & X_4 & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 5 \le i < j \le n-5. \end{array} \right.$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{4,k}$, $\mathfrak{g}_n^{5,k}$, $\mathfrak{g}_n^{1,k^*}$, $\mathfrak{g}_n^{2,k^*}$ $1 \leq k \leq r-1$ $1 \leq k^* \leq r$, ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r-1 \leq i < j \leq n-5. \end{cases}$$

* Si $c_{ij} = 0$ $\forall i,j$ $2r-1 \leq i < j \leq n-5$, se obtienen las álgebras siguientes:

$$\mathfrak{g}_n^{3,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1 \end{array}$$

$$\mathfrak{g}_n^{4,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &=& X_4 & \\ [X_1, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si $c_{ij} = 0$ $\forall i,j$ $2r-1 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $2r-1 \leq i \leq n-6$, se puede suponer $c_{2r-1,n-5} \neq 0$. Basta con efectuar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_i^* &= Y_{2r-1} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \quad k \notin \{2r-1, i\} \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Subcaso: $\epsilon = 0$

La ley viene determinada por

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_{n-5}] &= c_{i,n-5} X_4 & 2r-1 \leq i \leq n-6. \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_0^* & = X_0 & \\ X_1^* & = c_{2r-1,n-5}X_1 - Y_{2r-1} & \\ X_t^* & = c_{2r-1,n-5}X_t & 2 \le t \le 4 \\ Y_{2k-1}^* & = c_{2r-1,n-5}Y_{2k-1} & 1 \le k \le r-1 \\ Y_{2k}^* & = Y_{2k} & 1 \le k \le r-1 \\ Y_{2r-1}^* & = Y_{2r-1} & \\ Y_j^* & = c_{2r-1,n-5}Y_j - c_{j,n-5}Y_{2r-1} & 2r \le j \le n-6 \\ Y_{n-5}^* & = Y_{n-5} & \end{cases}$$

y

$$\begin{cases} X_t^* & = X_t & 0 \le t \le 4 \\ Y_{2r}^* & = Y_{n-5} & \\ Y_{n-5}^* & = Y_{2r} & \\ Y_k^* & = Y_k & 1 \le k \le n-6 \quad k \ne 2r \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{1,r+1}: \begin{array}{lll} [X_0, X_i] & = X_{i+1} & 1 \le i \le 3 \\ [Y_{2k-1}, Y_{2k}] & = X_4 & 1 \le k \le r \end{array}$$

ya obtenida anteriormente.

Subcaso: $\epsilon = 1$

La ley viene determinada por

$$\begin{cases} [X_0, X_i] & = X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = X_4 & \\ [X_1, Y_{n-5}] & = X_4 & \\ [Y_{2k-1}, Y_{2k}] & = X_4 & 1 \le k \le r-1 \\ [Y_i, Y_{n-5}] & = c_{i,n-5}X_4 & 2r-1 \le i \le n-6. \end{cases}$$

Con el cambio de base dado por

$$\begin{cases} X_t^* & = X_t & 0 \le t \le 4 \\ Y_k^* & = Y_k & 1 \le k \le 2r-2 \\ Y_{2r-1}^* & = \frac{1}{c_{2r-1,n-5}} Y_{2r-1} & \\ Y_i^* & = c_{2r-1,n-5}Y_i - c_{i,n-5}Y_{2r-1} & 2r \le i \le n-6 \\ Y_{n-5}^* & = Y_{n-5} & \end{cases}$$

se obtienen los siguientes productos corchete no nulos, salvo antisimetría:

$$\begin{cases} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{cases}$$

y haciendo sucesivamente los cambios

$$\begin{cases} X_t^* & = & X_t & 0 \leq t \leq 4 \quad t \neq 1 \\ X_1^* & = & X_1 - Y_{2r-1} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \end{cases}$$

y

$$\begin{cases} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{2r}^* & = & Y_{n-5} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-6 \quad k \neq 2r \\ Y_{n-5}^* & = & Y_{2r} & \end{cases}$$

se demuestra que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{2,r+1} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r. \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-6$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con aplicar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_{2r}^* &= Y_j & \\ Y_i^* &= Y_{2r-1} & \\ Y_j^* &= Y_{2r} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \qquad k \notin \{2r-1, 2r, i, j\} \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \leq k \leq n-5$. Esto se consigue con el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* &= Y_{2r} & \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-5, \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \leq i < j \leq n-5 \end{cases}$$

y se llega a una situación parecida a las ya consideradas.

El último paso del proceso se realiza cuando $r = E(\frac{n-5}{2})$, y se observa que $r+1 = E(\frac{n-3}{2})$. En consecuencia, cuando aparecen $\mathfrak{g}_n^{1,r+1}$ y $\mathfrak{g}_n^{2,r+1}$, se trata de álgebras ya obtenidas anteriormente. Por lo que en este caso, surgen las familias

$$\mathfrak{g}_n^{3,r}: \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-3}{2}) \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \end{array}$$

$$\begin{array}{llll} \mathfrak{g}_n^{4,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [X_1, X_2] = X_4 & & \\ & [X_1, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1. & \end{array}$$

Caso particular : n impar

Se cumple que $E(\frac{n-3}{2}) = \frac{n-3}{2}$.
El cambio de base

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ X_1^* = X_1 - Y_{n-6} & \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \end{cases}$$

aplicado al álgebra

$$\begin{array}{lll} \mathfrak{g}_n^{3,E(\frac{n-3}{2})}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, Y_{n-5}] = X_4 & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq \frac{n-5}{2} \end{array}$$

demuestra que $\mathfrak{g}_n^{3,E(\frac{n-3}{2})} \simeq \mathfrak{g}_n^{1,E(\frac{n-3}{2})}$, y
aplicado al álgebra

$$\begin{array}{lll} \mathfrak{g}_n^{4,E(\frac{n-3}{2})}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, X_2] = X_4 & \\ & [X_1, Y_{n-5}] = X_4 & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq \frac{n-5}{2} \end{array}$$

demuestra que $\mathfrak{g}_n^{4,E(\frac{n-3}{2})} \simeq \mathfrak{g}_n^{2,E(\frac{n-3}{2})}$.

Caso 2.1.2: $\exists j \in \{1, 2, \ldots, n-5\} : b_j \neq 0$

El álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5, \end{cases}$$

cumpliéndose:

$$\begin{cases} \sum_{k=2}^{n-5} b_k c_{1k} &= 0 & \\ \sum_{k=i+1}^{n-5} b_k c_{ik} &= \sum_{r=1}^{i-1} b_r c_{ri} & 2 \leq i \leq n-6 \\ \sum_{k=1}^{n-6} b_k c_{kn-5} &= 0. & \end{cases}$$

Subcaso: $b_{n-5} \neq 0$

Al efectuar el siguiente cambio de base:

$$\begin{cases} X_0^* &= \sqrt[3]{b_{n-5}} X_0 & \\ X_1^* &= \frac{1}{\sqrt[6]{b_{n-5}}} X_1 & \\ X_2^* &= \sqrt[6]{b_{n-5}} X_2 & \\ X_3^* &= \sqrt{b_{n-5}} X_3 & \\ X_4^* &= \sqrt[6]{b_{n-5}^5} X_4 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \\ Y_{n-5}^* &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \end{cases}$$

se obtiene que

$$[X_0^*, X_1^*] = [\sqrt[3]{b_{n-5}} X_0, \frac{1}{\sqrt[6]{b_{n-5}}} X_1] = \sqrt[6]{b_{n-5}} X_2 = X_2^*$$

$$[X_0^*, X_2^*] = [\sqrt[3]{b_{n-5}} X_0, \sqrt[6]{b_{n-5}} X_2] = \sqrt{b_{n-5}} X_3 = X_3^*$$

$$
\begin{aligned}
[X_0^*, X_3^*] &= [\sqrt[3]{b_{n-5}}X_0, \sqrt{b_{n-5}}X_3] = \sqrt[6]{b_{n-5}^5}X_4 = X_4^* \\
[X_1^*, X_2^*] &= [\tfrac{1}{\sqrt[6]{b_{n-5}}}X_1, \sqrt[6]{b_{n-5}}X_2] = [X_1, X_2] = \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k = Y_{n-5}^* \\
[X_1^*, Y_{n-5}^*] &= [\tfrac{1}{\sqrt[6]{b_{n-5}}}X_1, \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k] = \frac{b_{n-5}}{\sqrt[6]{b_{n-5}}}[X_1, Y_{n-5}] = \sqrt[6]{b_{n-5}^5}X_4 = X_4^* \\
[Y_1^*, Y_{n-5}^*] &= [Y_1, \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k] = (\sum_{k=2}^{n-5} b_k c_{1k})X_4 = 0.X_4 = 0 \\
[Y_i^*, Y_{n-5}^*] &= [Y_i, \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k] = (-\sum_{r=1}^{i-1} b_r c_{ri} + \sum_{k=i+1}^{n-5} b_k c_{ik}).X_4 = 0.X_4 = 0 \quad 2 \leq i \leq \\
[Y_i^*, Y_j^*] &= [Y_i, Y_j] = c_{ij}X_4 = c_{ij}.\tfrac{1}{\sqrt[6]{b_{n-5}^5}}.X_4^* = c_{ij}^*.X_4^* \quad 1 \leq i < j \leq n-6.
\end{aligned}
$$

Y, en consecuencia, se obtiene la siguiente ley:

$$
\begin{cases}
[X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] = Y_{n-5} & \\
[X_1, Y_{n-5}] = X_4 & \\
[Y_i, Y_j] = c_{ij}X_4 & 1 \leq i < j \leq n-6.
\end{cases}
$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 1 \leq i < j \leq n-6$, se obtiene el álgebra

$$
\mathfrak{g}_n^{12,1} : \begin{cases}
[X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] = Y_{n-5} & \\
[X_1, Y_{n-5}] = X_4 &
\end{cases}
$$

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-6$, se puede suponer $c_{12} \neq 0$. Basta con considerar el cambio de base expresado por

$$
\begin{cases}
X_t^* = X_t & 0 \leq t \leq 4 \\
Y_1^* = Y_i & \\
Y_2^* = Y_j & \\
Y_i^* = Y_1 & \\
Y_j^* = Y_2 & \\
Y_k^* = Y_k & 1 \leq k \leq n-5 \quad k \notin \{1, 2, i, j\}.
\end{cases}
$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k}$ $\quad 3 \leq k \leq n-6$, sin más que aplicar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 & \\ Y_2^* &= Y_2 & \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 3 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{12,2} : \begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \end{cases}$$

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-6$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-6$, y para demostrarlo es suficiente considerar cambios de base análogos a algunos anteriores. Se obtiene un álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_1, Y_2] &= X_4 & \\ [Y_3, Y_4] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 5 \leq i < j \leq n-6. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{12,k}$, $1 \leq k \leq r-1$ ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r-1 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2r-1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{12,r}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &=& Y_{n-5} & \\ [X_1, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-6$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con hacer el siguiente cambio de base:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_{2r}^* &= Y_j & \\ Y_i^* &= Y_{2r-1} & \\ Y_j^* &= Y_{2r} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r-1, 2r, i, j\}. \end{cases}$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \leq k \leq n-6$. Estas igualdades se consiguen al aplicar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* &= Y_{2r} & \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] &= c_{ij}X_4 & 2r+1 \leq i < j \leq n-6, \end{cases}$$

tratándose de una situación análoga a las ya consideradas.

Es evidente que el proceso finaliza cuando $r = E(\frac{n-6}{2}) + 1 = E(\frac{n-4}{2})$, por lo que en el caso: $b_{n-5} \neq 0$, surge la familia

$$\begin{array}{lllll} \mathfrak{g}_n^{12,r}: & [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-4}{2}) \\ & [X_1, X_2] &= Y_{n-5} & & \\ & [X_1, Y_{n-5}] &= X_4 & & \\ & [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1. & \end{array}$$

Subcaso: $b_{n-5} = 0$

Se cumple que $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$\left\{\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 + \sum_{k=1}^{n-6} b_k Y_k & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-5, \end{array}\right.$$

cumpliéndose:

$$\left\{\begin{array}{lcll} \sum_{k=2}^{n-6} b_k c_{1k} & = & 0 & \\ \sum_{k=i+1}^{n-6} b_k c_{ik} & = & \sum_{r=1}^{i-1} b_r c_{ri} & 2 \leq i \leq n-6 \\ \sum_{k=1}^{n-6} b_k c_{kn-5} & = & 0 & \end{array}\right.$$

y $\exists k \in \{1, 2, \ldots, n-6\} : b_k \neq 0$.

Los cambios de base sucesivos:

$$\left\{\begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{n-6}^* & = & Y_k & \\ Y_k^* & = & Y_{n-6} & \\ Y_j^* & = & Y_j & 1 \leq j \leq n-6 \quad j \notin \{k, n-6\} \\ Y_{n-5}^* & = & Y_{n-5} & \end{array}\right.$$

y

$$\left\{\begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_i^* & = & Y_i & 1 \leq i \leq n-7 \\ Y_{n-6}^* & = & \epsilon X_4 + \sum_{k=1}^{n-6} b_k Y_k & \\ Y_{n-5}^* & = & Y_{n-5} & \end{array}\right.$$

permiten, el primero, suponer $b_{n-6} \neq 0$, y, el segundo, obtener que

$$\begin{aligned}
[X_0^*, X_i^*] &= X_{i+1}^* \quad 1 \leq i \leq 3 \\
[X_1^*, X_2^*] &= Y_{n-6}^* \\
[X_1^*, Y_{n-5}^*] &= X_4^* \\
[Y_i^*, Y_{n-6}^*] &= [Y_i, \epsilon X_4 + \sum_{k=1}^{n-6} b_k Y_k] = (-\sum_{r=1}^{i-1} b_r c_{ri} + \sum_{k=i+1}^{n-6} b_k c_{ik}).X_4 = \\
&= 0.X_4 = 0 \quad 1 \leq i \leq n-7 \\
[Y_{n-6}^*, Y_{n-5}^*] &= [\epsilon X_4 + \sum_{k=1}^{n-6} b_k Y_k, Y_{n-5}] = (\sum_{k=1}^{n-6} b_k c_{kn-5}) X_4 = 0.X_4 = 0 \\
[Y_i^*, Y_j^*] &= [Y_i, Y_j] = c_{ij} X_4^* \quad 1 \leq i < j \leq n-7 \\
[Y_i^*, Y_{n-5}^*] &= [Y_i, Y_{n-5}] = c_{in-5} X_4^* \quad 1 \leq i \leq n-7.
\end{aligned}$$

Se deduce que $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$\left\{ \begin{array}{lcll}
[X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] & = & Y_{n-6} & \\
[X_1, Y_{n-5}] & = & X_4 & \\
[Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-7 \\
[Y_i, Y_{n-5}] & = & c_{in-5} X_4 & 1 \leq i \leq n-7
\end{array} \right.$$

y aplicando el cambio de base definido por

$$\left\{ \begin{array}{lcll}
X_t^* & = & X_t & 0 \leq t \leq 4 \\
Y_i^* & = & Y_i & 1 \leq i \leq n-7 \\
Y_{n-6}^* & = & Y_{n-5} & \\
Y_{n-5}^* & = & Y_{n-6} &
\end{array} \right.$$

se convierte en

$$\left\{ \begin{array}{lcll}
[X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] & = & Y_{n-5} & \\
[X_1, Y_{n-6}] & = & X_4 & \\
[Y_i, Y_j] & = & c_{ij} X_4 & 1 \leq i < j \leq n-6.
\end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,1}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-7$ y existe algún $c_{i,n-6} \neq 0$ $1 \leq i \leq n-7$, se puede suponer $c_{1,n-6} \neq 0$. Basta con aplicar el cambio de base siguiente:

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_1^* & = & Y_i & \\ Y_i^* & = & Y_1 & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \qquad k \notin \{1, i\}. \end{array} \right.$$

Al efectuar el cambio de base expresado por

$$\left\{ \begin{array}{lcll} X_0^* & = & X_0 & \\ X_1^* & = & c_{1,n-6}X_1 - Y_1 & \\ X_t^* & = & c_{1,n-6}X_t & 2 \leq t \leq 4 \\ Y_1^* & = & Y_1 & \\ Y_k^* & = & c_{1,n-6}Y_k - c_{k,n-6}Y_1 & 2 \leq k \leq n-7 \\ Y_{n-6}^* & = & Y_{n-6} & \\ Y_{n-5}^* & = & c_{1,n-6}^2 Y_{n-5} & \end{array} \right.$$

se obtiene que

$$\begin{array}{lcll} [X_0^*, X_1^*] & = & [X_0, c_{1,n-6}X_1 - Y_1] = c_{1,n-6}X_2 = X_2^* & \\ [X_0^*, X_t^*] & = & [X_0, c_{1,n-6}X_t] = c_{1,n-6}X_{t+1} = X_{t+1}^* & 2 \leq t \leq 3 \\ [X_1^*, X_2^*] & = & [c_{1,n-6}X_1 - Y_1, c_{1,n-6}X_2] = c_{1,n-6}^2 Y_{n-5} = Y_{n-5}^* & \\ [X_1, Y_{n-6}^*] & = & [c_{1,n-6}X_1 - Y_1, Y_{n-6}] = c_{1,n-6}X_4 - c_{1,n-6}X_4 = 0 & \\ [Y_k^*, Y_{n-6}] & = & [c_{1,n-6}Y_k - c_{k,n-6}Y_1, Y_{n-6}] = & \\ & = & (c_{1,n-6}c_{k,n-6} - c_{k,n-6}c_{1,n-6}).X_4 = 0.X_4 = 0 & 2 \leq k \leq n-7 \\ [Y_1^*, Y_{n-6}] & = & [Y_1, Y_{n-6}] = c_{1,n-6}X_4 = X_4^* & \end{array}$$

En consecuencia, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_1, Y_{n-6}] & = & X_4 & \end{array} \right.$$

y al hacer el cambio siguiente:

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ Y_2^* = Y_{n-6} & \\ Y_{n-6}^* = Y_2 & \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \quad k \notin \{2, n-6\}, \end{cases}$$

se transforma en

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] = Y_{n-5} & \\ [Y_1, Y_2] = X_4 & \end{cases}$$

que corresponde a $\mathfrak{g}_n^{10,2}$, ya obtenida anteriormente.

* Si existe algún $c_{ij} \neq 0 \quad 1 \leq i < j \leq n-7$, se puede suponer $c_{12} \neq 0$. Basta con considerar el cambio de base definido por

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ Y_1^* = Y_i & \\ Y_2^* = Y_j & \\ Y_i^* = Y_1 & \\ Y_j^* = Y_2 & \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \quad k \notin \{1, 2, i, j\}. \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \quad 3 \leq k \leq n-6$, sin más que aplicar el cambio de base dado por

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ Y_1^* = \frac{1}{c_{12}} Y_1 & \\ Y_2^* = Y_2 & \\ Y_k^* = Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \leq k \leq n-6 \\ Y_{n-5}^* = Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] = Y_{n-5} & \\ [X_1, Y_{n-6}] = X_4 & \\ [Y_1, Y_2] = X_4 & \\ [Y_i, Y_j] = c_{ij} X_4 & 3 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,2} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-7$, y existe algún $c_{i,n-6} \neq 0$ $3 \leq i \leq n-7$,se obtiene el álgebra

$$\mathfrak{g}_n^{10,3} : \left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_1, Y_2] & = & X_4 & \\ [Y_3, Y_4] & = & X_4 & \end{array} \right.$$

ya obtenida.

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-7$, se obtiene un álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \\ [Y_3, Y_4] & = & X_4 & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 5 \leq i < j \leq n-6. \end{array} \right.$$

Tanto en el caso anterior como en éste, se consiguen las leyes sin más que considerar cambios de base análogos a algunos anteriores.

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{11,k} \quad \mathfrak{g}_n^{10,k+1} \quad 1 \leq k \leq r-1$, o a un álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 2r-1 \leq i < j \leq n-6. \end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-7$ y existe algún $c_{i,n-6} \neq 0$ $2r-1 \leq i \leq n-7$, se puede suponer $c_{2r-1,n-6} \neq 0$. Basta con aplicar el cambio de base siguiente:

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* & = & Y_i & \\ Y_i^* & = & Y_{2r-1} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \qquad k \notin \{2r-1, i\}. \end{array} \right.$$

Al efectuar el cambio:

$$\left\{ \begin{array}{lcll} X_0^* & = & X_0 & \\ X_1^* & = & c_{2r-1,n-6}X_1 - Y_{2r-1} & \\ X_t^* & = & c_{2r-1,n-6}X_t & 2 \leq t \leq 4 \\ Y_k^* & = & c_{2r-1,n-6}Y_k & 1 \leq k \leq 2r-2 \quad k = \dot{2}+1 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 \quad k = \dot{2} \\ Y_{2r-1}^* & = & Y_{2r-1} & \\ Y_k^* & = & c_{2r-1,n-6}Y_k - c_{k,n-6}Y_{2r-1} & 2r \leq k \leq n-7 \\ Y_{n-6}^* & = & Y_{n-6} & \\ Y_{n-5}^* & = & c_{2r-1,n-6}^2 Y_{n-5} & \end{array} \right.$$

se obtienen los siguientes productos corchete no nulos, salvo antisimetría:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] & = & X_4 & \end{array} \right.$$

y al hacer:

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{2r}^* & = & Y_{n-6} & \\ Y_{n-6}^* & = & Y_{2r} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \qquad k \notin \{2r, n-6\}, \end{array} \right.$$

la ley anterior se transforma en

$$\begin{cases} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r \end{cases}$$

que corresponde a $\mathfrak{g}_n^{10,r+1}$, ya obtenida anteriormente.

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-7$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con aplicar el siguiente cambio de base:

$$\begin{cases} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* & = & Y_i & \\ Y_{2r}^* & = & Y_j & \\ Y_i^* & = & Y_{2r-1} & \\ Y_j^* & = & Y_{2r} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r-1, 2r, i, j\}. \end{cases}$$

Se puede, además, suponer $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \leq k \leq n-5$. Se consigue con el cambio de base dado por

$$\begin{cases} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* & = & \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* & = & Y_{2r} & \\ Y_k^* & = & Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-6 \\ Y_{n-5}^* & = & Y_{n-5} & \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r \\ [Y_i, Y_j] & = & c_{ij} X_4 & 2r+1 \leq i < j \leq n-6. \end{cases}$$

Se llega a una situación parecida a las ya analizadas.

En consecuencia, van apareciendo las álgebras nuevas:

$$\mathfrak{g}_n^{11,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \qquad 1 \leq r \leq E(\frac{n-7}{2}) - 1 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1, \end{array}$$

y también las ya obtenidas anteriormente:

$$\mathfrak{g}_n^{10,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \qquad 2 \leq r \leq E(\frac{n-7}{2}) \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1, \end{array}$$

y justo antes del último paso del proceso, se obtiene la siguiente ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1 \\ [Y_i, Y_j] & = & c_{ij} X_4 & 2E(\frac{n-7}{2}) - 1 \leq i < j \leq n-6. \end{array} \right.$$

A continuación, hay que diferenciar dos casos, dependiendo de la paridad de la dimensión de $\mathfrak{g}$.

Caso n par ($E(\frac{n-7}{2}) = \frac{n-8}{2}$)

* Si $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-7}{2}) - 1 = n - 9 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,E(\frac{n-7}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2E(\frac{n-7}{2}) - 1 = n - 9 \leq i < j \leq n-7$, y existe algún $c_{i,n-6} \neq 0 \quad n-9 \leq i \leq n-7$, se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-8}{2} = E(\frac{n-6}{2}) - 1, \end{array} \right.$$

que corresponde a $\mathfrak{g}_n^{10,E(\frac{n-6}{2})}$, ya conocida.

* Si existe algún $c_{ij} \neq 0 \quad 2E(\frac{n-7}{2}) - 1 = n-9 \leq i < j \leq n-7$, se puede suponer $c_{n-9,n-8} \neq 0$, y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-6}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq \frac{n-8}{2} = E(\frac{n-5}{2}) - 1 \\ [Y_{n-7}, Y_{n-6}] &= c_{n-7,n-6}X_4 & \end{cases}$$

- Si $c_{n-7,n-6} = 0$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,E(\frac{n-5}{2})} : \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &=& Y_{n-5} & \\ [X_1, Y_{n-6}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1. \end{array}$$

- Si $c_{n-7,n-6} \neq 0$, al aplicar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* &= X_1 - \frac{1}{c_{n-7,n-6}} Y_{n-7} & & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \neq n-7 \\ Y_{n-7}^* &= \frac{1}{c_{n-7,n-6}} Y_{n-7} & & \end{cases}$$

se obtiene la ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq \frac{n-6}{2} = E(\frac{n-4}{2}) - 1, \end{cases}$$

que corresponde a $\mathfrak{g}_n^{10,E(\frac{n-4}{2})}$, ya conocida.

Caso n impar $(E(\frac{n-7}{2}) = \frac{n-7}{2})$

El álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= Y_{n-5} & \\ [X_1, Y_{n-6}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1 \\ [Y_i, Y_j] &= c_{ij}X_4 & 2E(\frac{n-7}{2}) - 1 \leq i < j \leq n-6. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-7}{2}) - 1 = n-8 \leq i < j \leq n-6$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,E(\frac{n-7}{2})} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-7}{2}) - 1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-7}{2}) - 1 = n-8 \leq i < j \leq n-7 \Leftrightarrow c_{n-8,n-7} = 0$, y existe algún $c_{i,n-6} \neq 0 \quad n-8 \leq i \leq n-7$, se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-7}{2} = E(\frac{n-4}{2}) - 1, \end{array} \right.$$

que corresponde a $\mathfrak{g}_n^{10,E(\frac{n-4}{2})}$, ya conocida.

* Si existe algún $c_{ij} \neq 0 \quad 2E(\frac{n-7}{2}) - 1 = n-8 \leq i < j \leq n-7 \Leftrightarrow c_{n-8,n-7} \neq 0$, se obtiene el álgebra

$$\mathfrak{g}_n^{11,E(\frac{n-5}{2})} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & Y_{n-5} & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1. \end{array}$$

Como conclusión del caso: $b_{n-5} = 0$ y $\exists j \in \{1, 2, \ldots, n-6\} : b_j \neq 0$, se observa que surge la familia de álgebras siguiente:

$$\mathfrak{g}_n^{11,r} : \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \qquad\qquad 1 \leq r \leq E(\frac{n-5}{2}) \\ [X_1, X_2] & = & Y_{n-6} & \\ [X_1, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r - 1. \end{array}$$

□

Caso 2.2: $(\lambda_1, \lambda_2, \lambda_3) = (0, 1, \alpha)$

El álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 + \sum_{k=1}^{n-5} b_k Y_k & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 + \alpha X_4 + \sum_{k=1}^{n-5} b_k Y_k & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5, \end{cases}$$

cumpliéndose

$$\begin{cases} \sum_{k=2}^{n-5} b_k c_{1k} &= 0 & \\ \sum_{k=i+1}^{n-5} b_k c_{ik} &= \sum_{r=1}^{i-1} b_r c_{ri} & 2 \leq i \leq n-6 \\ \sum_{k=1}^{n-6} b_k c_{k,n-5} &= -b_{n-5} & \end{cases}$$

Se encuentran las familias

$$\begin{array}{llll} \mathfrak{g}_n^{5,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-3}{2}) \\ & [X_1, Y_{n-5}] = X_3 & & \\ & [X_2, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1, & \end{array}$$

$$\begin{array}{llll} \mathfrak{g}_n^{6,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-5}{2}) \\ & [X_1, Y_{n-5}] = X_3 & & \\ & [X_2, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & \\ & [Y_{2r-1}, Y_{n-5}] = X_4 & & \text{y} \end{array}$$

$$\begin{array}{llll} \mathfrak{g}_n^{7,r}: & [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-5}{2}) \\ & [X_1, X_2] = X_4 & & \\ & [X_1, Y_{n-5}] = X_3 & & \\ & [X_2, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 & \\ & [Y_{2r-1}, Y_{n-5}] = X_4 & & \end{array}$$

Demostración

$\forall A \in \mathbb{K}$ ($\mathbb{K} = \mathbb{R}$ ó $\mathbb{C}$), se cumple que $AX_0 + X_1 \notin [\mathfrak{g}, \mathfrak{g}]$ y puede ser vector característico. Su matriz adjunta es:

$$ad(AX_0 + X_1) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ -1 & A & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & A & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \epsilon & A & 0 & 0 & 0 & \dots & 0 & \alpha \\ 0 & 0 & b_1 & 0 & 0 & 0 & 0 & \dots & 0 & b_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & b_k & 0 & 0 & 0 & 0 & \dots & 0 & b_k \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & b_{n-5} & 0 & 0 & 0 & 0 & \dots & 0 & b_{n-5} \end{pmatrix}$$

Se elige A tal que $A \neq 0$ y $A \neq 1$, lo que es siempre posible, y al ser la sucesión característica $(4, 1, 1, \dots, 1)$ se tiene que

$$\begin{vmatrix} A & 0 & 0 & 0 \\ 0 & A & 0 & 1 \\ 0 & \epsilon & A & \alpha \\ 0 & b_k & 0 & b_k \end{vmatrix} = A^2(A-1)b_k = 0 \Rightarrow b_k = 0 \quad 1 \leq k \leq n-5.$$

En consecuencia, la ley de $\mathfrak{g}$ viene determinada por

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 + \alpha X_4 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \leq i < j \leq n-5. \end{cases}$$

Al aplicar el cambio de base dado por

$$\begin{cases} X_0^* &= X_0 - \frac{\alpha}{2} Y_{n-5} & \\ X_1^* &= X_1 & \\ X_2^* &= X_2 + \frac{\alpha}{2} X_3 + \frac{\alpha^2}{2} X_4 & \\ X_3^* &= X_3 + \alpha X_4 & \\ X_4^* &= X_4 & \\ Y_j^* &= Y_j - \frac{\alpha}{2} c_{j,n-5} X_3 & 1 \leq j \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

se obtiene que

$$
\begin{aligned}
[X_0^*, X_1^*] &= [X_0 - \tfrac{\alpha}{2}Y_{n-5}, X_1] = X_2 + \tfrac{\alpha}{2}X_3 + \tfrac{\alpha^2}{2}X_4 = X_2^* \\
[X_0^*, X_2^*] &= [X_0 - \tfrac{\alpha}{2}Y_{n-5}, X_2 + \tfrac{\alpha}{2}X_3 + \tfrac{\alpha^2}{2}X_4] = X_3 + \alpha X_4 = X_3^* \\
[X_0^*, X_3^*] &= [X_0 - \tfrac{\alpha}{2}Y_{n-5}, X_3 + \alpha X_4] = X_4 = X_4^* \\
[X_0^*, Y_j^*] &= [X_0 - \tfrac{\alpha}{2}Y_{n-5}, Y_j - \tfrac{\alpha}{2}c_{j,n-5}X_3] = \\
&= (-\tfrac{\alpha}{2}c_{j,n-5} + \tfrac{\alpha}{2}c_{j,n-5})X_4 = 0 \qquad 1 \leq j \leq n-6 \\
[X_1^*, X_2^*] &= [X_1, X_2 + \tfrac{\alpha}{2}X_3 + \tfrac{\alpha^2}{2}X_4] = \epsilon X_4 = \epsilon X_4^* \\
[X_1^*, Y_{n-5}^*] &= [X_1, Y_{n-5}] = X_3 + \alpha X_4 = X_3^* \\
[X_2^*, Y_{n-5}^*] &= [X_2 + \tfrac{\alpha}{2}X_3 + \tfrac{\alpha^2}{2}X_4, Y_{n-5}] = X_4 = X_4^* \\
[Y_i^*, Y_j^*] &= [Y_i - \tfrac{\alpha}{2}c_{i,n-5}X_3, Y_j - \tfrac{\alpha}{2}c_{j,n-5}X_3] = [Y_i, Y_j] = c_{ij}X_4 = \\
&= c_{ij}X_4^* \qquad 1 \leq i < j \leq n-6 \\
[Y_i^*, Y_{n-5}^*] &= [Y_i - \tfrac{\alpha}{2}c_{i,n-5}X_3, Y_{n-5}] = [Y_i, Y_{n-5}] = c_{i,n-5}X_4 = \\
&= c_{i,n-5}X_4^* \qquad 1 \leq i \leq n-6
\end{aligned}
$$

y, entonces, $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$
\begin{cases}
[X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] = \epsilon X_4 & \epsilon \in \{0, 1\} \\
[X_1, Y_{n-5}] = X_3 & \\
[X_2, Y_{n-5}] = X_4 & \\
[Y_i, Y_j] = c_{ij}X_4 & 1 \leq i < j \leq n-5.
\end{cases}
$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{array}{llll} \mathfrak{g}_n^{5,1}: & [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, Y_{n-5}] &= X_3 & \\ & [X_2, Y_{n-5}] &= X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-6$, y existe algún $c_{i,n-5} \neq 0$ $1 \leq i \leq n-6$, se puede suponer $c_{1,n-5} \neq 0$. Basta con aplicar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & \\ Y_1^* &= Y_i & & \\ Y_i^* &= Y_1 & & \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 & k \notin \{1, i\} \\ Y_{n-5}^* &= Y_{n-5} & & \end{cases}$$

Se puede, además, suponer que $c_{1,n-5} = 1$ y $c_{k,n-5} = 0 \quad 2 \leq k \leq n-6$, sin más que efectuar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{1,n-5}} Y_1 & \\ Y_k^* &= c_{1,n-5} Y_k - c_{k,n-5} Y_1 & 2 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtienen las álgebras siguientes, dependiendo del valor de ϵ:

$$\begin{array}{llll} \mathfrak{g}_n^{6,1}: & [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, Y_{n-5}] &= X_3 & \\ & [X_2, Y_{n-5}] &= X_4 & \\ & [Y_1, Y_{n-5}] &= X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,1}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_{n-5}] & = & X_4 & \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 1 \le i < j \le n-6$, se puede suponer $c_{12} \neq 0$. Basta con efectuar el cambio de base definido por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \le t \le 4 \\ Y_1^* & = & Y_i & \\ Y_2^* & = & Y_j & \\ Y_i^* & = & Y_1 & \\ Y_j^* & = & Y_2 & \\ Y_k^* & = & Y_k & 1 \le k \le n-6 \qquad k \notin \{1, 2, i, j\} \\ Y_{n-5}^* & = & Y_{n-5}. & \end{array} \right.$$

Se puede, además, suponer $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \quad 3 \le k \le n-5$, sin más que aplicar el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \le t \le 4 \\ Y_1^* & = & \frac{1}{c_{12}} Y_1 & \\ Y_2^* & = & Y_2 & \\ Y_k^* & = & Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \le k \le n-5, \end{array} \right.$$

y se obtiene el álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_1, Y_2] & = & X_4 & \\ [Y_i, Y_j] & = & c_{ij} X_4 & 3 \le i < j \le n-5. \end{array} \right.$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \le i < j \le n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{array}{llcll} \mathfrak{g}_n^{5,2}: & [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_1, Y_2] & = & X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i,j \quad 3 \le i < j \le n-6$, y existe algún $c_{i,n-5} \neq 0 \quad 3 \le i \le n-6$, se puede suponer $c_{3,n-5} \neq 0$.

Sin más que considerar cambios de base análogos a algunos anteriores se puede suponer que $c_{3,n-5} = 1$ y $c_{k,n-5} = 0 \quad 4 \le k \le n-6$ y se obtienen las álgebras

$$\begin{array}{llcll} \mathfrak{g}_n^{6,2}: & [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_1, Y_2] & = & X_4 & \\ & [Y_3, Y_{n-5}] & = & X_4 & \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{7,2}: & [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ & [X_1, X_2] & = & X_4 & \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_1, Y_2] & = & X_4 & \\ & [Y_3, Y_{n-5}] & = & X_4 & \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-6$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-5$, y la ley de $\mathfrak{g}$ viene expresada por

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_1, Y_2] &= X_4 \\ [Y_3, Y_4] &= X_4 \\ [Y_i, Y_j] &= c_{ij} X_4 & 5 \leq i < j \leq n-5. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{5,k}$, $\mathfrak{g}_n^{6,k}$, $\mathfrak{g}_n^{7,k} \quad 1 \leq k \leq r-1$ ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r-1 \leq i < j \leq n-5. \end{cases}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2r-1 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= X_1 + \epsilon Y_{n-5} \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\mathfrak{g}_n^{5,r} : \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &=& X_3 \\ [X_2, Y_{n-5}] &=& X_4 \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1. \end{array}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2r-1 \leq i < j \leq n-6$, y existe algún $c_{i,n-5} \neq 0$ $2r-1 \leq i \leq n-6$, se puede suponer $c_{2r-1,n-5} \neq 0$. Basta con hacer el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_i^* &= Y_{2r-1} & \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \quad k \notin \{2r-1, i\} \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Se puede, además, suponer que $c_{2r-1,n-5} = 1$ y $c_{k,n-5} = 0 \quad 2r \leq k \leq n-6$, sin más que considerar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,n-5}} Y_{2r-1} & \\ Y_j^* &= c_{2r-1,n-5} Y_j - c_{j,n-5} Y_{2r-1} & 2r \leq j \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

y se obtienen las álgebras siguientes:

$$\mathfrak{g}_n^{6,r} : \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] &=& X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,r} : \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &=& X_4 & \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_{2k-1}, Y_{2k}] &=& X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] &=& X_4 & \end{array}$$

* Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-6$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con aplicar el siguiente cambio de base:

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_{2r-1}^* &= Y_i & \\ Y_{2r}^* &= Y_j & \\ Y_i^* &= Y_{2r-1} & \\ Y_j^* &= Y_{2r} & \\ Y_k^* &= Y_k & 1 \le k \le n-6 \qquad k \notin \{2r-1, 2r, i, j\} \\ Y_{n-5}^* &= Y_{n-5}. & \end{cases}$$

Se puede, además, suponer

$$c_{2r-1,2r} = 1 \text{ y } c_{2r-1,k} = 0 = c_{2r,k} \quad 2r+1 \le k \le n-5,$$

sin más que efectuar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le 2r-2 \\ Y_{2r-1}^* &= \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* &= Y_{2r} & \\ Y_k^* &= Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \le k \le n-5, \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \le k \le r \\ [Y_i, Y_j] &= c_{ij} X_4 & 2r+1 \le i < j \le n-5, \end{cases}$$

y se llega a una situación parecida a las ya analizadas.

En consecuencia, aparecen las álgebras

$$\begin{array}{llll} \mathfrak{g}_n^{5,r}: & [X_0, X_i] = X_{i+1} & 1 \le i \le 3 & 1 \le r \le E(\frac{n-7}{2}) \\ & [X_1, Y_{n-5}] = X_3 & & \\ & [X_2, Y_{n-5}] = X_4 & & \\ & [Y_{2k-1}, Y_{2k}] = X_4 & 1 \le k \le r-1 & \end{array}$$

$$\mathfrak{g}_n^{6,r}: \begin{array}{lcll} [X_0,X_i] & = & X_{i+1} & 1\le i\le 3 \qquad 1\le r\le E(\frac{n-7}{2}) \\ [X_1,Y_{n-5}] & = & X_3 & \\ [X_2,Y_{n-5}] & = & X_4 & \\ [Y_{2k-1},Y_{2k}] & = & X_4 & 1\le k\le r-1 \\ [Y_{2r-1},Y_{n-5}] & = & X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,r}: \begin{array}{lcll} [X_0,X_i] & = & X_{i+1} & 1\le i\le 3 \qquad 1\le r\le E(\frac{n-7}{2}) \\ [X_1,X_2] & = & X_4 & \\ [X_1,Y_{n-5}] & = & X_3 & \\ [X_2,Y_{n-5}] & = & X_4 & \\ [Y_{2k-1},Y_{2k}] & = & X_4 & 1\le k\le r-1 \\ [Y_{2r-1},Y_{n-5}] & = & X_4 & \end{array}$$

y justo antes del último paso del proceso, se obtiene que $\mathfrak{g}$ puede ser isomorfa a un álgebra de ley:

$$\left\{ \begin{array}{lcll} [X_0,X_i] & = & X_{i+1} & 1\le i\le 3 \\ [X_1,X_2] & = & \epsilon X_4 & \epsilon\in\{0,1\} \\ [X_1,Y_{n-5}] & = & X_3 & \\ [X_2,Y_{n-5}] & = & X_4 & \\ [Y_{2k-1},Y_{2k}] & = & X_4 & 1\le k\le E(\frac{n-5}{2})-1 \\ [Y_i,Y_j] & = & c_{ij}X_4 & 2E(\frac{n-5}{2})-1\le i<j\le n-5. \end{array} \right.$$

* Si $c_{ij}=0 \quad \forall i,j \quad 2E(\frac{n-5}{2})-1\le i<j\le n-5$, se obtiene el álgebra

$$\mathfrak{g}_n^{5,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0,X_i] & = & X_{i+1} & 1\le i\le 3 \\ [X_1,Y_{n-5}] & = & X_3 & \\ [X_2,Y_{n-5}] & = & X_4 & \\ [Y_{2k-1},Y_{2k}] & = & X_4 & 1\le k\le E(\frac{n-5}{2})-1. \end{array}$$

* Si $2E(\frac{n-5}{2})-1\le n-7$ y $c_{ij}=0 \quad \forall i,j \quad 2E(\frac{n-5}{2})-1\le i<j\le n-6$, y

existe algún $c_{i,n-5} \neq 0 \quad 2E(\frac{n-5}{2}) - 1 \leq i \leq n-6$, se obtienen las álgebras

$$\mathfrak{g}_n^{6,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1 \\ [Y_{2E(\frac{n-5}{2})-1}, Y_{n-5}] & = & X_4 & \end{array}$$

y

$$\mathfrak{g}_n^{7,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1 \\ [Y_{2E(\frac{n-5}{2})-1}, Y_{n-5}] & = & X_4 & \end{array}$$

Y a continuación, hay que diferenciar dos posibilidades dependiendo de la paridad de n.

Caso: n par ($E(\frac{n-5}{2}) = \frac{n-6}{2}$)

* Si existe algún $c_{ij} \neq 0 \quad 2E(\frac{n-5}{2}) - 1 = n-7 \leq i < j \leq n-6 \Leftrightarrow c_{n-7,n-6} \neq 0$, se obtiene la ley:

$$\begin{cases} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq \frac{n-6}{2}. \end{cases}$$

Al aplicar el cambio de base definido por

$$\begin{cases} X_0^* & = & X_0 & \\ X_1^* & = & X_1 + \epsilon Y_{n-5} & \\ X_t^* & = & X_t & 2 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\mathfrak{g}_n^{5,E(\frac{n-3}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le E(\frac{n-3}{2}) - 1. \end{array}$$

Caso: n impar ($E(\frac{n-5}{2}) = \frac{n-5}{2}$)

* Si existe algún $c_{ij} \neq 0$ $2E(\frac{n-5}{2}) - 1 = n - 6 \le i < j \le n - 5 \Leftrightarrow c_{n-6,n-5} \neq 0$, y aplicando el cambio de base:

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \le t \le 4 \\ Y_k^* & = & Y_k & 1 \le k \le n-5 \quad k \neq n-6 \\ Y_{n-6}^* & = & \frac{1}{c_{n-6,n-5}} Y_{n-6} & \end{array} \right.$$

se obtiene la ley:

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le \frac{n-5}{2}. \end{array} \right.$$

* Si $\epsilon = 0$, dicha ley corresponde a

$$\mathfrak{g}_n^{5,E(\frac{n-3}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le E(\frac{n-3}{2}) - 1, \end{array}$$

álgebra que también aparece cuando n es par.

Se observa que en este caso (n impar), dicho álgebra $\mathfrak{g}_n^{5,E(\frac{n-3}{2})}$ coincide con $\mathfrak{g}_n^{6,E(\frac{n-5}{2})}$.

* Si $\epsilon = 1$, la ley corresponde a $\mathfrak{g}_n^{7,E(\frac{n-5}{2})}$, álgebra ya obtenida.

Entonces, se concluye que surgen las familias

$$\mathfrak{g}_n^{5,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-3}{2}) \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \end{array}$$

$$\mathfrak{g}_n^{6,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-5}{2}) \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

$$\mathfrak{g}_n^{7,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \le i \le 3 \qquad 1 \le r \le E(\frac{n-5}{2}) \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4. & \end{array}$$

y cuando n es impar, $\mathfrak{g}_n^{5,E(\frac{n-3}{2})} \simeq \mathfrak{g}_n^{6,E(\frac{n-5}{2})}$.

□

Caso 3: $(\lambda_1, \lambda_2, \lambda_3) = (1, 1, 0)$

Al cumplirse $\lambda_1\lambda_2 b_k = 0 \quad 1 \le k \le n-5$, se deduce que $b_k = 0 \quad 1 \le k \le n-5$ y, entonces, las demás restricciones se verifican trivialmente y el álgebra dada $\mathfrak{g}$ es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 1 \le i < j \le n-5. \end{cases}$$

Se encuentran las familias

$$\begin{array}{llcll} \mathfrak{g}_n^{8,r}: & [X_0, X_i] &=& X_{i+1} \quad 1 \le i \le 3 & 1 \le r \le E(\frac{n-5}{2}) \\ & [X_1, Y_{n-6}] &=& X_4 & \\ & [X_1, Y_{n-5}] &=& X_3 & \\ & [X_2, Y_{n-5}] &=& X_4 & \\ & [Y_{2k-1}, Y_{2k}] &=& X_4 \quad 1 \le k \le r-1 & \text{y} \end{array}$$

$$\begin{array}{llcll} \mathfrak{g}_n^{9,r}: & [X_0, X_i] &=& X_{i+1} \quad 1 \le i \le 3 & 1 \le r \le E(\frac{n-6}{2}) \\ & [X_1, Y_{n-6}] &=& X_4 & \\ & [X_1, Y_{n-5}] &=& X_3 & \\ & [X_2, Y_{n-5}] &=& X_4 & \\ & [Y_{2k-1}, Y_{2k}] &=& X_4 \quad 1 \le k \le r-1 & \\ & [Y_{2r-1}, Y_{n-5}] &=& X_4 & \end{array}$$

Demostración:

*** Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= X_1 + \epsilon Y_{n-5} \\ X_t^* &= X_t \qquad\qquad 2 \leq t \leq 4 \\ Y_k^* &= Y_k \qquad\qquad 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{aligned} \mathfrak{g}_n^{8,1}: \quad [X_0, X_i] &= X_{i+1} \qquad 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \end{aligned}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-7$, y existe algún $c_{i,n-6} \neq 0$ $1 \leq i \leq n-7$, se puede suponer $c_{1,n-6} \neq 0$. Basta con efectuar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t \qquad 0 \leq t \leq 4 \\ Y_1^* &= Y_i \\ Y_i^* &= Y_1 \\ Y_k^* &= Y_k \qquad 1 \leq k \leq n-7 \qquad k \notin \{1, i\} \\ Y_{n-6}^* &= Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

Se puede, además, suponer que $c_{1,n-6} = 1$ y $c_{k,n-6} = 0 \quad 2 \leq k \leq n-7$, sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t \qquad\qquad 0 \leq t \leq 4 \\ Y_1^* &= \frac{1}{c_{1,n-6}} Y_1 \\ Y_k^* &= c_{1,n-6} Y_k - c_{k,n-6} Y_1 \qquad 2 \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

y se obtiene la ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_{n-6}] &= X_4 & \\ [Y_i, Y_{n-5}] &= c_{i,n-5}X_4 & 1 \leq i \leq n-6. \end{cases}$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 1 \leq i \leq n-6$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_{n-6}] &= X_4 & \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* &= X_1 - Y_1 & & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2, n-6\} \\ Y_2^* &= Y_{n-6} & & \\ Y_{n-6}^* &= Y_2 & & \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a

$$\mathfrak{g}_n^{5,2}: \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_1, Y_2] &=& X_4 & \end{array}$$

ya obtenida anteriormente.

* Si existe algún $c_{i,n-5} \neq 0 \quad 2 \leq i \leq n-7$, se puede suponer $c_{2,n-5} \neq 0$. Basta con considerar el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_1^* &= Y_1 \\ Y_2^* &= Y_i \\ Y_i^* &= Y_2 \\ Y_k^* &= Y_k & 2 \leq k \leq n-7 \qquad k \notin \{2, i\} \\ Y_{n-6}^* &= Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

Se puede, además, suponer que

$$c_{1,n-5} = 0, \; c_{2,n-5} = 1, \; c_{k,n-5} = 0 \qquad 3 \leq k \leq n-7,$$

sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_0^* &= \sqrt{c_{2,n-5}} X_0 \\ X_1^* &= c_{2,n-5} X_1 \\ X_2^* &= \sqrt{c_{2,n-5}^3} X_2 \\ X_3^* &= c_{2,n-5}^2 X_3 \\ X_4^* &= \sqrt{c_{2,n-5}^5} X_4 \\ Y_k^* &= c_{2,n-5} Y_k - c_{k,n-5} Y_2 & 1 \leq k \leq n-7 \qquad k \neq 2 \\ Y_2^* &= \sqrt{c_{2,n-5}} Y_2 \\ Y_{n-6}^* &= \sqrt{c_{2,n-5}^3} Y_{n-6} \\ Y_{n-5}^* &= c_{2,n-5} Y_{n-5} \end{cases}$$

En efecto, se obtiene que

$$\begin{aligned} [X_0^*, X_1^*] &= [\sqrt{c_{2,n-5}} X_0, c_{2,n-5} X_1] = \sqrt{c_{2,n-5}^3} X_2 = X_2^* \\ [X_0^*, X_2^*] &= [\sqrt{c_{2,n-5}} X_0, \sqrt{c_{2,n-5}^3} X_2] = c_{2,n-5}^2 X_3 = X_3^* \\ [X_0^*, X_3^*] &= [\sqrt{c_{2,n-5}} X_0, c_{2,n-5}^2 X_3] = \sqrt{c_{2,n-5}^5} X_4 = X_4^* \\ [X_1^*, X_2^*] &= [c_{2,n-5} X_1, \sqrt{c_{2,n-5}^3} X_2] = \epsilon \sqrt{c_{2,n-5}^5} X_4 = \epsilon X_4^* \end{aligned}$$

$$
\begin{aligned}
[X_1^*, Y_{n-6}^*] &= [c_{2,n-5}X_1, \sqrt{c_{2,n-5}^3}Y_{n-6}] = \sqrt{c_{2,n-5}^5}X_4 = X_4^* \\
[X_1^*, Y_{n-5}^*] &= [c_{2,n-5}X_1, c_{2,n-5}Y_{n-5}] = c_{2,n-5}^2X_3 = X_3^* \\
[X_2^*, Y_{n-5}^*] &= [\sqrt{c_{2,n-5}^3}X_2, c_{2,n-5}Y_{n-5}] = \sqrt{c_{2,n-5}^5}X_4 = X_4^* \\
[Y_1^*, Y_{n-6}^*] &= [c_{2,n-5}Y_1 - c_{1,n-5}Y_2, \sqrt{c_{2,n-5}^3}Y_{n-6}] = \sqrt{c_{2n-5}^5}X_4 = X_4^* \\
[Y_2^*, Y_{n-5}^*] &= [\sqrt{c_{2,n-5}}Y_2, c_{2,n-5}Y_{n-5}] = \sqrt{c_{2,n-5}^5}X_4 = X_4^* \\
[Y_k^*, Y_{n-5}^*] &= [c_{2,n-5}Y_k - c_{k,n-5}Y_2, c_{2,n-5}Y_{n-5}] = (c_{2,n-5}^2c_{k,n-5} - c_{k,n-5}c_{2,n-5}^2)X_4 = \\
&= 0.X_4 = 0 \qquad 1 \leq k \leq n-7 \qquad k \neq 2 \\
[Y_{n-6}^*, Y_{n-5}^*] &= [\sqrt{c_{2,n-5}^3}Y_{n-6}, c_{2,n-5}Y_{n-5}] = \sqrt{c_{2,n-5}^5}.c_{n-6,n-5}.X_4 = \beta X_4,
\end{aligned}
$$

y la ley de $\mathfrak{g}$ es:

$$
\left\{
\begin{array}{lcll}
[X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0, 1\} \\
[X_1, Y_{n-6}] & = & X_4 & \\
[X_1, Y_{n-5}] & = & X_3 & \\
[X_2, Y_{n-5}] & = & X_4 & \\
[Y_1, Y_{n-6}] & = & X_4 & \\
[Y_2, Y_{n-5}] & = & X_4 & \\
[Y_{n-6}, Y_{n-5}] & = & \beta X_4 &
\end{array}
\right.
$$

Al aplicar el cambio de base siguiente:

$$
\left\{
\begin{array}{lcll}
X_t^* & = & X_t & 0 \leq t \leq 4 \\
Y_k^* & = & Y_k & 1 \leq k \leq n-7 \\
Y_{n-6}^* & = & Y_{n-6} - \beta Y_2 & \\
Y_{n-5}^* & = & Y_{n-5} &
\end{array}
\right.
$$

se transforma en

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_1, Y_{n-6}] &= X_4 \\ [Y_2, Y_{n-5}] &= X_4 \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* &= X_1 - Y_1 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5, \end{cases}$$

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{2, n-6\} \\ Y_2^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_2 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 & k \notin \{3, n-6\} \\ Y_3^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_3 \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_1, Y_2] &= X_4 \\ [Y_3, Y_{n-5}] &= X_4 \end{cases}$$

que corresponde a $\mathfrak{g}_n^{6,2}$ si $\epsilon = 0$ y a $\mathfrak{g}_n^{7,2}$ si $\epsilon = 1$, ambas ya obtenidas.

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2 \leq i \leq n-7$, pero $c_{1,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se puede suponer $c_{n-6,n-5} = 0$, sin más que aplicar el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} + c_{n-6,n-5} Y_1 \end{cases}$$

y se obtiene la ley determinada por

$$\begin{cases} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = & X_4 \\ [X_1, Y_{n-5}] & = & X_3 \\ [X_2, Y_{n-5}] & = & X_4 \\ [Y_1, Y_{n-6}] & = & X_4 \\ [Y_1, Y_{n-5}] & = & c_{1,n-5} X_4 \end{cases}$$

Al hacer el cambio de base dado por

$$\begin{cases} X_0^* & = & X_0 + \frac{c_{1,n-5}}{2} Y_{n-5} \\ X_1^* & = & X_1 \\ X_2^* & = & X_2 - \frac{c_{1,n-5}}{2} X_3 \\ X_3^* & = & X_3 - c_{1,n-5} X_4 \\ X_4^* & = & X_4 \\ Y_1^* & = & Y_1 + \frac{c_{1,n-5}^2}{2} X_3 \\ Y_k^* & = & Y_k & 2 \leq k \leq n-7 \\ Y_{n-6}^* & = & Y_{n-6} \\ Y_{n-5}^* & = & -c_{1,n-5} Y_{n-6} + Y_{n-5} \end{cases}$$

se obtiene que

$$\begin{aligned} [X_0^*, X_1^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, X_1] = X_2 - \tfrac{c_{1,n-5}}{2} X_3 = X_2^* \\ [X_0^*, X_2^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, X_2 - \tfrac{c_{1,n-5}}{2} X_3] = X_3 - c_{1,n-5} X_4 = X_3^* \\ [X_0^*, X_3^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, X_3 - c_{1,n-5} X_4] = X_4 = X_4^* \\ [X_0^*, Y_1^*] &= [X_0 + \tfrac{c_{1,n-5}}{2} Y_{n-5}, Y_1 + \tfrac{c_{1,n-5}^2}{2} X_3] = 0 \\ [X_1^*, Y_{n-6}^*] &= [X_1, Y_{n-6}] = X_4 = X_4^* \\ [X_1^*, Y_{n-5}^*] &= [X_1, -c_{1,n-5} Y_{n-6} + Y_{n-5}] = X_3 - c_{1,n-5} X_4 = X_3^* \end{aligned}$$

$$[X_2^*, Y_{n-5}^*] = [X_2 - \frac{c_{1,n-5}}{2} X_3, -c_{1,n-5}Y_{n-6} + Y_{n-5}] = X_4 = X_4^*$$

$$[Y_1^*, Y_{n-6}^*] = [Y_1 + \frac{c_{1,n-5}^2}{2} X_3, Y_{n-6}] = X_4 = X_4^*$$

$$[Y_1^*, Y_{n-5}^*] = [Y_1 + \frac{c_{1,n-5}^2}{2} X_3, -c_{1,n-5}Y_{n-6} + Y_{n-5}] = 0.$$

Y la ley de $\mathfrak{g}$ se convierte en

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] = \epsilon X_4 & \epsilon \in \{0, 1\} \\ [X_1, Y_{n-6}] = X_4 & \\ [X_1, Y_{n-5}] = X_3 & \\ [X_2, Y_{n-5}] = X_4 & \\ [Y_1, Y_{n-6}] = X_4 & \end{cases}$$

Esta situación ya ha sido analizada y resulta $\mathfrak{g} \simeq \mathfrak{g}_n^{5,2}$.

*** Si $c_{ij} = 0 \quad \forall i,j \quad 1 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $1 \leq i \leq n-7$, se puede suponer $c_{1,n-5} \neq 0$. Basta con efectuar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t \qquad 0 \leq t \leq 4 \\ Y_1^* &= Y_i \\ Y_i^* &= Y_1 \\ Y_k^* &= Y_k \qquad 1 \leq k \leq n-7 \qquad k \notin \{1,i\} \\ Y_{n-6}^* &= Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

Al considerar

$$\begin{cases} X_0^* &= \sqrt[3]{c_{1,n-5}}X_0 \\ X_1^* &= \sqrt[3]{c_{1,n-5}^2}X_1 \\ X_2^* &= c_{1,n-5}X_2 \\ X_3^* &= \sqrt[3]{c_{1,n-5}^4}X_3 \\ X_4^* &= \sqrt[3]{c_{1,n-5}^5}X_4 \\ Y_1^* &= \sqrt[3]{c_{1,n-5}^2}Y_1 \\ Y_k^* &= c_{1,n-5}Y_k - c_{k,n-5}Y_1 \qquad 2 \leq k \leq n-6 \\ Y_{n-5}^* &= \sqrt[3]{c_{1,n-5}^2}Y_{n-5} \end{cases}$$

se obtiene que

$$[X_0^*, X_1^*] = [\sqrt[3]{c_{1,n-5}}X_0, \sqrt[3]{c_{1,n-5}^2}X_1] = c_{1,n-5}X_2 = X_2^*$$

$$[X_0^*, X_2^*] = [\sqrt[3]{c_{1,n-5}}X_0, c_{1,n-5}X_2] = \sqrt[3]{c_{1,n-5}^4}X_3 = X_3^*$$

$$[X_0^*, X_3^*] = [\sqrt[3]{c_{1,n-5}}X_0, \sqrt[3]{c_{1,n-5}^4}X_3] = \sqrt[3]{c_{1,n-5}^5}X_4 = X_4^*$$

$$[X_1^*, X_2^*] = [\sqrt[3]{c_{1,n-5}^2}X_1, c_{1,n-5}X_2] = \epsilon\sqrt[3]{c_{1,n-5}^5}X_4 = \epsilon X_4^*$$

$$[X_1^*, Y_{n-6}^*] = [\sqrt[3]{c_{1,n-5}^2}X_1, c_{1,n-5}Y_{n-6} - c_{n-6,n-5}Y_1] = \sqrt[3]{c_{1,n-5}^5}X_4 = X_4^*$$

$$[X_1^*, Y_{n-5}^*] = [\sqrt[3]{c_{1,n-5}^2}X_1, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \sqrt[3]{c_{1,n-5}^4}X_3 = X_3^*$$

$$[X_2^*, Y_{n-5}^*] = [c_{1,n-5}X_2, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \sqrt[3]{c_{1,n-5}^5}X_4 = X_4^*$$

$$[Y_1^*, Y_{n-5}^*] = [\sqrt[3]{c_{1,n-5}^2}Y_1, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \sqrt[3]{c_{1,n-5}^7}X_4 = \beta X_4^*$$

$$\begin{aligned}[Y_k^*, Y_{n-5}^*] &= [c_{1,n-5}Y_k - c_{k,n-5}Y_1, \sqrt[3]{c_{1,n-5}^2}Y_{n-5}] = \\ &= (\sqrt[3]{c_{1,n-5}^5}c_{k,n-5} - c_{k,n-5}\sqrt[3]{c_{1,n-5}^5})X_4 = 0.X_4 = 0 \quad 2 \leq k \leq n-6.\end{aligned}$$

La ley de $\mathfrak{g}$ es:

$$\left\{\begin{array}{lll} [X_0, X_i] = X_{i+1} & & 1 \leq i \leq 3 \\ [X_1, X_2] = \epsilon X_4 & & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] = X_4 & & \\ [X_1, Y_{n-5}] = X_3 & & \\ [X_2, Y_{n-5}] = X_4 & & \\ [Y_1, Y_{n-5}] = \beta X_4 & & \beta \neq 0. \end{array}\right.$$

Se puede suponer $\beta = 1$, sin más que hacer el cambio de base:

$$\left\{\begin{array}{lll} X_0^* &= \sqrt[5]{\beta}X_0 & \\ X_1^* &= \sqrt[5]{\beta^2}X_1 & \\ X_2^* &= \sqrt[5]{\beta^3}X_2 & \\ X_3^* &= \sqrt[5]{\beta^4}X_3 & \\ X_4^* &= \beta X_4 & \\ Y_1^* &= \frac{1}{\sqrt[5]{\beta^2}}Y_1 & \\ Y_k^* &= Y_k & 2 \leq k \leq n-7 \\ Y_{n-6}^* &= \sqrt[5]{\beta^3}Y_{n-6} & \\ Y_{n-5}^* &= \sqrt[5]{\beta^2}Y_{n-5} & \end{array}\right.$$

En efecto:

$$[X_0^*, X_1^*] = [\sqrt[5]{\beta}X_0, \sqrt[5]{\beta^2}X_1] = \sqrt[5]{\beta^3}X_2 = X_2^*$$

$$[X_0^*, X_2^*] = [\sqrt[5]{\beta}X_0, \sqrt[5]{\beta^3}X_2] = \sqrt[5]{\beta^4}X_3 = X_3^*$$

$$[X_0^*, X_3^*] = [\sqrt[5]{\beta}X_0, \sqrt[5]{\beta^4}X_3] = \beta X_4 = X_4^*$$

$$[X_1^*, X_2^*] = [\sqrt[5]{\beta^2}X_1, \sqrt[5]{\beta^3}X_2] = \epsilon\beta X_4 = \epsilon X_4^*$$

$$[X_1^*, Y_{n-6}^*] = [\sqrt[5]{\beta^2}X_1, \sqrt[5]{\beta^3}Y_{n-6}] = \beta X_4 = X_4^*$$

$$\begin{aligned}
[X_1^*, Y_{n-5}^*] &= [\sqrt[5]{\beta^2}X_1, \sqrt[5]{\beta^2}Y_{n-5}] = \sqrt[5]{\beta^4}X_3 = X_3^* \\
[X_2^*, Y_{n-5}^*] &= [\sqrt[5]{\beta^3}X_2, \sqrt[5]{\beta^2}Y_{n-5}] = \beta X_4 = X_4^* \\
[Y_1^*, Y_{n-5}^*] &= [\tfrac{1}{\sqrt[5]{\beta^2}}Y_1, \sqrt[5]{\beta^2}Y_{n-5}] = \beta X_4 = X_4^*
\end{aligned}$$

Por tanto, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases}
[X_0, X_i] = X_{i+1} & 1 \le i \le 3 \\
[X_1, X_2] = \epsilon X_4 & \epsilon \in \{0,1\} \\
[X_1, Y_{n-6}] = X_4 & \\
[X_1, Y_{n-5}] = X_3 & \\
[X_2, Y_{n-5}] = X_4 & \\
[Y_1, Y_{n-5}] = X_4 &
\end{cases}$$

Se puede suponer $\epsilon = 0$, sin más que aplicar el cambio de base:

$$\begin{cases}
X_t^* = X_t & 0 \le t \le 4 \quad t \neq 1 \\
X_1^* = X_1 + \epsilon Y_{n-5} & \\
Y_1^* = Y_1 + \epsilon Y_{n-6} & \\
Y_k^* = Y_k & 2 \le k \le n-5,
\end{cases}$$

y se obtiene el álgebra

$$\begin{aligned}
\mathfrak{g}_n^{9,1}: \quad [X_0, X_i] &= X_{i+1} \quad 1 \le i \le 3 \\
[X_1, Y_{n-6}] &= X_4 \\
[X_1, Y_{n-5}] &= X_3 \\
[X_2, Y_{n-5}] &= X_4 \\
[Y_1, Y_{n-5}] &= X_4
\end{aligned}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 1 \le i < j \le n-6$ y $c_{i,n-5} = 0 \quad \forall i \quad 1 \le i \le n-7$, pero $c_{n-6,n-5} \neq 0$ se distinguen dos casos, dependiendo del valor de ϵ.

Caso: $\epsilon = 0$

Se puede suponer $c_{n-6,n-5} = 1$, sin más que hacer el cambio de base:

$$\begin{cases} X_0^* = X_0 & \\ X_t^* = c_{n-6,n-5}X_t & 1 \leq t \leq 4 \\ Y_k^* = Y_k & 1 \leq k \leq n-5, \end{cases}$$

y la ley de $\mathfrak{g}$ viene determinada por

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] = X_4 & \\ [X_1, Y_{n-5}] = X_3 & \\ [X_2, Y_{n-5}] = X_4 & \\ [Y_{n-6}, Y_{n-5}] = X_4 & \end{cases}$$

Esta ley se consigue al aplicar a la ley

$$\begin{array}{lrcll} \mathfrak{g}_n^{7,1}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ & [X_1, X_2] & = & X_4 & \\ & [X_1, Y_{n-5}] & = & X_3 & \\ & [X_2, Y_{n-5}] & = & X_4 & \\ & [Y_1, Y_{n-5}] & = & X_4 & \end{array}$$

los siguientes cambios de base sucesivamente:

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ Y_1^* = Y_{n-6} & \\ Y_{n-6}^* = Y_1 & \\ Y_k^* = Y_k & 2 \leq k \leq n-5 \quad k \neq n-6 \end{cases} \quad \text{y}$$

$$\begin{cases} X_0^* = X_0 & & \\ X_1^* = -X_1 - Y_{n-5} & & \\ X_t^* = -X_t & 2 \leq t \leq 4 & \\ Y_k^* = -Y_k & 1 \leq k \leq n-7 & k = \dot{2}+1 \\ Y_k^* = Y_k & 1 \leq k \leq n-7 & k = \dot{2} \\ Y_{n-6}^* = -Y_{n-6} & & \\ Y_{n-5}^* = Y_{n-5} & & \end{cases}$$

En consecuencia, $\mathfrak{g} \simeq \mathfrak{g}_n^{7,1}$.

Caso: $\epsilon = 1 \quad c_{n-6,n-5} = 1$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* = X_t & 0 \le t \le 4 \quad t \ne 1 \\ X_1^* = X_1 + Y_{n-5} & \\ Y_k^* = Y_k & 1 \le k \le n-5 \end{cases}$$

y

$$\begin{cases} X_t^* = X_t & 0 \le t \le 4 \\ Y_1^* = Y_{n-6} & \\ Y_{n-6}^* = Y_1 & \\ Y_k^* = Y_k & 2 \le k \le n-5 \quad k \ne n-6 \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\begin{aligned} \mathfrak{g}_n^{6,1} : \; [X_0, X_i] &= X_{i+1} \qquad 1 \le i \le 3 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_1, Y_{n-5}] &= X_4 \end{aligned}$$

Caso: $\epsilon = 1 \quad c_{n-6,n-5} \neq 1$

Al aplicar el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5} \\ X_t^* &= -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_t & 2 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \neq n-6 \\ Y_{n-6}^* &= \frac{1}{c_{n-6,n-5}-1}Y_{n-6} \end{cases}$$

se obtiene que

$$[X_0^*, X_1^*] = [X_0, -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_2 = X_2^*$$

$$[X_0^*, X_t^*] = [X_0, -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_t] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_{t+1} = X_{t+1}^* \qquad 2 \leq t \leq 3$$

$$\begin{aligned} [X_1^*, X_2^*] &= [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}, -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_2] = \\ &= \frac{c_{n-6,n-5}^2}{4(1-c_{n-6,n-5})^2}X_4 + \frac{c_{n-6,n-5}^2-2c_{n-6,n-5}}{4(1-c_{n-6,n-5})^2}X_4 = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4 = X_4^* \end{aligned}$$

$$\begin{aligned} [X_1^*, Y_{n-6}^*] &= [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}, \frac{1}{c_{n-6,n-5}-1}Y_{n-6}] = \\ &= \frac{-c_{n-6,n-5}}{2(1-c_{n-6,n-5})(c_{n-6,n-5}-1)}X_4 - \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})(c_{n-6,n-5}-1)}X_4 = \\ &= -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4 = X_4^* \end{aligned}$$

$$[X_1^*, Y_{n-5}^*] = [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5}, Y_{n-5}] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_3 = X_3^*$$

$$[X_2^*, Y_{n-5}^*] = [-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_2, Y_{n-5}] = -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4 = X_4^*$$

$$[Y_{n-6}^*, Y_{n-5}^*] = [\frac{1}{c_{n-6,n-5}-1}Y_{n-6}, Y_{n-5}] = \frac{c_{n-6,n-5}}{c_{n-6,n-5}-1}X_4 = 2(-\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_4) = 2X_4^*$$

Por tanto, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= X_4 \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{n-6}, Y_{n-5}] &= 2X_4 \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_0^* &= X_0 \\ X_1^* &= 2X_1 + Y_{n-5} \\ X_t^* &= 2X_t & 2 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le n-5 \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_1^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_1 \\ Y_k^* &= Y_k & 2 \le k \le n-5 & k \ne n-6 \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\begin{aligned} \mathfrak{g}_n^{7,1}: \quad [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_1, Y_{n-5}] &= X_4 \end{aligned}$$

ya obtenida.

*** Si existe algún $c_{ij} \ne 0 \quad 1 \le i < j \le n-7$, se puede suponer $c_{12} \ne 0$. Basta con efectuar el cambio de base expresado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_1^* &= Y_i \\ Y_2^* &= Y_j \\ Y_i^* &= Y_1 \\ Y_j^* &= Y_2 \\ Y_k^* &= Y_k & 1 \le k \le n-7 & k \notin \{1,2,i,j\} \\ Y_{n-6}^* &= Y_{n-6} \\ Y_{n-5}^* &= Y_{n-5} \end{cases}$$

Se puede, además, suponer que $c_{12} = 1$ y $c_{1k} = 0 = c_{2k} \quad 3 \le k \le n-5$, sin más que hacer el cambio de base dado por

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_1^* &= \frac{1}{c_{12}} Y_1 & \\ Y_2^* &= Y_2 & \\ Y_k^* &= Y_k + \frac{c_{2k}}{c_{12}} Y_1 - \frac{c_{1k}}{c_{12}} Y_2 & 3 \le k \le n-5, \end{cases}$$

y se obtiene el álgebra de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_i, Y_j] &= c_{ij} X_4 & 3 \le i < j \le n-5. \end{cases}$$

*** Si $c_{ij} = 0 \quad \forall i, j \quad 3 \le i < j \le n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* &= X_0 & \\ X_1^* &= X_1 + \epsilon Y_{n-5} & \\ X_t^* &= X_t & 2 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{array}{llll} \mathfrak{g}_n^{8,2}: & [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ & [X_1, Y_{n-6}] &= X_4 & \\ & [X_1, Y_{n-5}] &= X_3 & \\ & [X_2, Y_{n-5}] &= X_4 & \\ & [Y_1, Y_2] &= X_4. & \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i, j \quad 3 \le i < j \le n-7$, y existe algún $c_{i,n-6} \neq 0$ $3 \le i \le n-7$, se puede suponer $c_{3,n-6} \neq 0$.

Sin más que considerar cambios de base análogos a algunos anteriores se puede suponer: $c_{3,n-6} = 1$ y $c_{k,n-6} = 0 \quad 4 \le k \le n-7$ y se obtiene la ley determinada por

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_1, Y_2] &= X_4 & \\ [Y_3, Y_{n-6}] &= X_4 & \\ [Y_i, Y_{n-5}] &= c_{i,n-5} X_4 & 3 \leq i \leq n-6. \end{cases}$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 3 \leq i \leq n-6$, se cumple que $\mathfrak{g}$ es isomorfa a $\mathfrak{g}_n^{5,3}$, ya obtenida.

* Si existe algún $c_{i,n-5} \neq 0 \quad 4 \leq i \leq n-7$, se cumple $\mathfrak{g} \simeq \mathfrak{g}_n^{6,3}$ si $\epsilon = 0$ y $\mathfrak{g} \simeq \mathfrak{g}_n^{7,3}$ si $\epsilon = 1$, ambas ya obtenidas.

* Si $c_{i,n-5} = 0 \quad \forall i \quad 4 \leq i \leq n-7$, pero $c_{3,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se cumple $\mathfrak{g} \simeq \mathfrak{g}_n^{5,3}$, ya obtenida.

*** Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $3 \leq i \leq n-7$, se obtiene el álgebra

$$\mathfrak{g}_n^{9,2} : \begin{array}{lcll} [X_0, X_i] &=& X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] &=& X_4 & \\ [X_1, Y_{n-5}] &=& X_3 & \\ [X_2, Y_{n-5}] &=& X_4 & \\ [Y_1, Y_2] &=& X_4 & \\ [Y_3, Y_{n-5}] &=& X_4. & \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 3 \leq i < j \leq n-6$ y $c_{i,n-5} = 0 \quad \forall i \quad 3 \leq i \leq n-7$, pero $c_{n-6,n-5} \neq 0$, se cumple que

$$\mathfrak{g} \simeq \mathfrak{g}_n^{7,2} \text{ si } \epsilon = 0$$

$$\mathfrak{g} \simeq \mathfrak{g}_n^{6,2} \text{ si } \epsilon = 1 \text{ y } c_{n-6,n-5} = 1$$

$$\mathfrak{g} \simeq \mathfrak{g}_n^{7,2} \text{ si } \epsilon = 1 \text{ y } c_{n-6,n-5} \neq 1.$$

*** Si existe algún $c_{ij} \neq 0 \quad 3 \leq i < j \leq n-7$, se puede suponer $c_{34} = 1$ y $c_{3k} = 0 = c_{4k} \quad 5 \leq k \leq n-5$. La ley de $\mathfrak{g}$ viene expresada por

$$\begin{cases} [X_0, X_i] & = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = X_4 & \\ [X_1, Y_{n-5}] & = X_3 & \\ [X_2, Y_{n-5}] & = X_4 & \\ [Y_1, Y_2] & = X_4 & \\ [Y_3, Y_4] & = X_4 & \\ [Y_i, Y_j] & = c_{ij} X_4 & 5 \leq i < j \leq n-5. \end{cases}$$

Si se continúa el proceso, resulta el álgebra $\mathfrak{g}$ dada isomorfa a alguna de las $\mathfrak{g}_n^{8,k}$, $\mathfrak{g}_n^{9,k}$, $\mathfrak{g}_n^{5,k^*}$, $\mathfrak{g}_n^{6,k^*}$, $\mathfrak{g}_n^{7,k^*}$ $\quad 1 \leq k \leq r-1 \quad 1 \leq k^* \leq r$, ó a una que tenga por ley:

$$\begin{cases} [X_0, X_i] & = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = X_4 & \\ [X_1, Y_{n-5}] & = X_3 & \\ [X_2, Y_{n-5}] & = X_4 & \\ [Y_{2k-1}, Y_{2k}] & = X_4 & 1 \leq k \leq r-1 \\ [Y_i, Y_j] & = c_{ij} X_4 & 2r-1 \leq i < j \leq n-5. \end{cases}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-5$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* & = X_0 & \\ X_1^* & = X_1 + \epsilon Y_{n-5} & \\ X_t^* & = X_t & 2 \leq t \leq 4 \\ Y_k^* & = Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{aligned} \mathfrak{g}_n^{8,r}: \quad [X_0, X_i] & = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = X_4 & \\ [X_1, Y_{n-5}] & = X_3 & \\ [X_2, Y_{n-5}] & = X_4 & \\ [Y_{2k-1}, Y_{2k}] & = X_4 & 1 \leq k \leq r-1. \end{aligned}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \leq i < j \leq n-7$ y existe algún $c_{i,n-6} \neq 0$ $2r-1 \leq i \leq n-7$, se puede suponer $c_{2r-1,n-6} \neq 0$. Basta con efectuar el cambio de base dado por

$$\left\{\begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* & = & Y_i & \\ Y_i^* & = & Y_{2r-1} & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-7 \quad k \notin \{2r-1, i\} \\ Y_{n-6}^* & = & Y_{n-6} & \\ Y_{n-5}^* & = & Y_{n-5} & \end{array}\right.$$

Se puede, además, suponer que $c_{2r-1,n-6} = 1$ y $c_{k,n-6} = 0 \quad 2r \leq k \leq n-7$, sin más que considerar el cambio de base dado por

$$\left\{\begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* & = & \frac{1}{c_{2r-1,n-6}} Y_{2r-1} & \\ Y_k^* & = & c_{2r-1,n-6} Y_k - c_{k,n-6} Y_{2r-1} & 2r \leq k \leq n-7 \\ Y_{n-6}^* & = & Y_{n-6} & \\ Y_{n-5}^* & = & Y_{n-5} & \end{array}\right.$$

y se obtiene la ley:

$$\left\{\begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] & = & X_4 & \\ [Y_i, Y_{n-5}] & = & c_{i,n-5} X_4 & 2r-1 \leq i \leq n-6. \end{array}\right.$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2r-1 \leq i \leq n-6$, y aplicando el cambio de base definido por

$$\begin{cases} X_0^* = X_0 & \\ X_1^* = X_1 + \epsilon Y_{n-5} & \\ X_t^* = X_t & 2 \leq t \leq 4 \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \end{cases}$$

se obtiene el álgebra

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] = X_4 & \\ [X_1, Y_{n-5}] = X_3 & \\ [X_2, Y_{n-5}] = X_4 & \\ [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] = X_4 & \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \quad t \neq 1 \\ X_1^* = X_1 - Y_{2r-1} & \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \end{cases}$$

y

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r, n-6\} \\ Y_{2r}^* = Y_{n-6} & \\ Y_{n-6}^* = Y_{2r} & \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a

$$\mathfrak{g}_n^{5,r+1} : \begin{array}{lll} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] = X_3 & \\ [X_2, Y_{n-5}] = X_4 & \\ [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r. \end{array}$$

* Si existe algún $c_{i,n-5} \neq 0 \quad 2r \leq i \leq n-7$, se puede suponer $c_{2r,n-5} \neq 0$. Basta con hacer el cambio de base definido por

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-1 \\ Y_{2r}^* &= Y_i & \\ Y_i^* &= Y_{2r} & \\ Y_k^* &= Y_k & 2r \leq k \leq n-7 \quad k \notin \{2r, i\} \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Se puede, además, suponer que

$$c_{2r-1,n-5} = 0, \ c_{2r,n-5} = 1 \text{ y } c_{k,n-5} = 0 \qquad 2r+1 \leq k \leq n-7,$$

sin más que aplicar el cambio de base expresado por

$$\begin{cases} X_0^* &= \sqrt{c_{2r,n-5}}X_0 & & \\ X_1^* &= c_{2r,n-5}X_1 & & \\ X_2^* &= \sqrt{c_{2r,n-5}^3}X_2 & & \\ X_3^* &= c_{2r,n-5}^2 X_3 & & \\ X_4^* &= \sqrt{c_{2r,n-5}^5}X_4 & & \\ Y_k^* &= \sqrt{c_{2r,n-5}^5}Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_k^* &= c_{2r,n-5}Y_k - c_{k,n-5}Y_{2r} & 2r-1 \leq k \leq n-7 & k \neq 2r \\ Y_{2r}^* &= \sqrt{c_{2r,n-5}}Y_{2r} & & \\ Y_{n-6}^* &= \sqrt{c_{2r,n-5}^3}Y_{n-6} & & \\ Y_{n-5}^* &= c_{2r,n-5}Y_{n-5} & & \end{cases}$$

Y, en consecuencia, la ley de $\mathfrak{g}$ es:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 & \\ [Y_{2r}, Y_{n-5}] &= X_4 & \\ [Y_{n-6}, Y_{n-5}] &= \beta X_4 & \end{cases}$$

Aplicando el cambio:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} - \beta Y_{2r} \\ Y_{n-5}^* &= Y_{n-5}, \end{cases}$$

se transforma en

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 \\ [Y_{2r}, Y_{n-5}] &= X_4 \end{cases}$$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \quad t \neq 1 \\ X_1^* &= X_1 - Y_{2r-1} \\ Y_k^* &= Y_k & 1 \leq k \leq n-5, \end{cases}$$

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r, n-6\} \\ Y_{2r}^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_{2r}, \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-5 \quad k \notin \{2r+1, n-6\} \\ Y_{2r+1}^* &= Y_{n-6} \\ Y_{n-6}^* &= Y_{2r+1} \end{cases}$$

demuestran que el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-5}] &= X_3 \\ [X_2, Y_{n-5}] &= X_4 \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r \\ [Y_{2r+1}, Y_{n-5}] &= X_4 \end{cases}$$

que corresponde a $\mathfrak{g}_n^{6,r+1}$ si $\epsilon = 0$ y a $\mathfrak{g}_n^{7,r+1}$ si $\epsilon = 1$, ambas ya obtenidas.

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2r \leq i \leq n-7$, pero $c_{2r-1,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se puede suponer $c_{n-6,n-5} = 0$, sin más que efectuar el cambio de base definido por

$$\left\{\begin{array}{lll} X_t^* &= X_t & 0 \leq t \leq 4 \\ Y_k^* &= Y_k & 1 \leq k \leq n-6 \\ Y_{n-5}^* &= Y_{n-5} + c_{n-6,n-5} Y_{2r-1} & \end{array}\right.$$

y se obtiene la ley determinada por

$$\left\{\begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-6}] &= X_4 & \\ [Y_{2r-1}, Y_{n-5}] &= c_{2r-1,n-5} X_4 & \end{array}\right.$$

Se puede suponer $c_{2r-1,n-5} = 0$, sin más que aplicar el cambio de base:

$$\left\{\begin{array}{lll} X_0^* &= X_0 + \frac{c_{2r-1,n-5}}{2} Y_{n-5} & \\ X_1^* &= X_1 & \\ X_2^* &= X_2 - \frac{c_{2r-1,n-5}}{2} X_3 & \\ X_3^* &= X_3 - c_{2r-1,n-5} X_4 & \\ X_4^* &= X_4 & \\ Y_k^* &= Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* &= Y_{2r-1} + \frac{c_{2r-1,n-5}^2}{2} X_3 & \\ Y_k^* &= Y_k & 2r \leq k \leq n-7 \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= -c_{2r-1,n-5} Y_{n-6} + Y_{n-5}. & \end{array}\right.$$

La situación que resulta ya ha sido analizada y resulta ser $\mathfrak{g}$ isomorfa a

$$\mathfrak{g}_n^{5,r+1}: \begin{array}{lll} [X_0, X_i] &= X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \leq k \leq r. \end{array}$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2r-1 \le i < j \le n-6$ y existe algún $c_{i,n-5} \neq 0$ $2r-1 \le i \le n-7$, se puede suponer $c_{2r-1,n-5} \neq 0$. Basta con considerar el cambio de base siguiente:

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 \\ Y_k^* &= Y_k & 1 \le k \le 2r-2 \\ Y_{2r-1}^* &= Y_i & \\ Y_i^* &= Y_{2r-1} & \\ Y_k^* &= Y_k & 2r-1 \le k \le n-7 \quad k \notin \{2r-1, i\} \\ Y_{n-6}^* &= Y_{n-6} & \\ Y_{n-5}^* &= Y_{n-5} & \end{cases}$$

Al considerar

$$\begin{cases} X_0^* &= \sqrt[3]{c_{2r-1,n-5}}X_0 & & \\ X_1^* &= \sqrt[3]{c_{2r-1,n-5}^2}X_1 & & \\ X_2^* &= c_{2r-1,n-5}X_2 & & \\ X_3^* &= \sqrt[3]{c_{2r-1,n-5}^4}X_3 & & \\ X_4^* &= \sqrt[3]{c_{2r-1,n-5}^5}X_4 & & \\ Y_k^* &= \sqrt[3]{c_{2r-1,n-5}^5}Y_k & 1 \le k \le 2r-2 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \le k \le 2r-2 & k = \dot{2} \\ Y_{2r-1}^* &= \sqrt[3]{c_{2r-1,n-5}^2}Y_{2r-1} & & \\ Y_k^* &= c_{2r-1,n-5}Y_k - c_{k,n-5}Y_{2r-1} & 2r \le k \le n-6 & \\ Y_{n-5}^* &= \sqrt[3]{c_{2r-1,n-5}^2}Y_{n-5} & & \end{cases}$$

se obtiene la ley:

$$\begin{cases} [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, X_2] &= \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] &= X_4 & \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] &= \beta X_4 & \beta \neq 0. \end{cases}$$

Se puede suponer $\beta = 1$, sin más que hacer el cambio de base:

$$\left\{\begin{array}{lllll} X_0^* & = & \sqrt[5]{\beta}X_0 & & \\ X_1^* & = & \sqrt[5]{\beta^2}X_1 & & \\ X_2^* & = & \sqrt[5]{\beta^3}X_2 & & \\ X_3^* & = & \sqrt[5]{\beta^4}X_3 & & \\ X_4^* & = & \beta X_4 & & \\ Y_k^* & = & \beta Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_{2r-1}^* & = & \frac{1}{\sqrt[5]{\beta^2}}Y_{2r-1} & & \\ Y_k^* & = & Y_k & 2r \leq k \leq n-7 & \\ Y_{n-6}^* & = & \sqrt[5]{\beta^3}Y_{n-6} & & \\ Y_{n-5}^* & = & \sqrt[5]{\beta^2}Y_{n-5}. & & \end{array}\right.$$

En consecuencia, el álgebra $\mathfrak{g}$ dada es isomorfa a una de ley:

$$\left\{\begin{array}{llll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}\right.$$

Se puede suponer $\epsilon = 0$, sin más que aplicar el cambio de base:

$$\left\{\begin{array}{lllll} X_t^* & = & X_t & 0 \leq t \leq 4 & t \neq 1 \\ X_1^* & = & X_1 + \epsilon Y_{n-5} & & \\ Y_{2r-1}^* & = & Y_{2r-1} + \epsilon Y_{n-6} & & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 & k \neq 2r-1, \end{array}\right.$$

y se obtiene el álgebra

$$\mathfrak{g}_n^{9,r}: \begin{array}{llll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

* Si $c_{ij} = 0 \quad \forall i, j \quad 2r-1 \leq i < j \leq n-6$ y $c_{i,n-5} = 0 \quad \forall i$ $2r-1 \leq i \leq n-7$, pero $c_{n-6,n-5} \neq 0$ se distinguen dos casos, dependiendo del valor de ϵ.

Caso: $\epsilon = 0$

Se puede suponer $c_{n-6,n-5} = 1$, sin más que aplicar el cambio de base:

$$\begin{cases} X_0^* = X_0 & \\ X_t^* = c_{n-6,n-5} X_t & 1 \leq t \leq 4 \\ Y_k^* = c_{n-6,n-5} Y_k & 1 \leq k \leq 2r-2 \qquad k = \dot{2}+1 \\ Y_k^* = Y_k & 1 \leq k \leq 2r-2 \qquad k = \dot{2} \\ Y_k^* = Y_k & 2r-1 \leq k \leq n-5, \end{cases}$$

y la ley de $\mathfrak{g}$ viene determinada por

$$\begin{cases} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] = X_4 & \\ [X_1, Y_{n-5}] = X_3 & \\ [X_2, Y_{n-5}] = X_4 & \\ [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 \\ [Y_{n-6}, Y_{n-5}] = X_4 & \end{cases}$$

Esta ley se consigue al aplicar a

$$\mathfrak{g}_n^{7,r}: \begin{array}{lll} [X_0, X_i] = X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] = X_4 & \\ [X_1, Y_{n-5}] = X_3 & \\ [X_2, Y_{n-5}] = X_4 & \\ [Y_{2k-1}, Y_{2k}] = X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] = X_4 & \end{array}$$

los siguientes cambios de base sucesivamente:

$$\begin{cases} X_t^* = X_t & 0 \leq t \leq 4 \\ Y_{2r-1}^* = Y_{n-6} & \\ Y_{n-6}^* = Y_{2r-1} & \\ Y_k^* = Y_k & 1 \leq k \leq n-5 \qquad k \notin \{2r-1, n-6\} \end{cases}$$

y

$$\begin{cases} X_0^* &= X_0 & & \\ X_1^* &= -X_1 - Y_{n-5} & & \\ X_t^* &= -X_t & 2 \le t \le 4 & \\ Y_k^* &= -Y_k & 1 \le k \le n-7 & k = \dot{2}+1 \\ Y_k^* &= Y_k & 1 \le k \le n-7 & k = \dot{2} \\ Y_{n-6}^* &= -Y_{n-6} & & \\ Y_{n-5}^* &= Y_{n-5}. & & \end{cases}$$

En consecuencia, $\mathfrak{g} \simeq \mathfrak{g}_n^{7,r}$.

Caso: $\epsilon = 1 \quad c_{n-6,n-5} = 1$

Los cambios de base sucesivos:

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 & t \ne 1 \\ X_1^* &= X_1 + Y_{n-5} & & \\ Y_k^* &= Y_k & 1 \le k \le n-5 & \end{cases}$$

y

$$\begin{cases} X_t^* &= X_t & 0 \le t \le 4 & \\ Y_{2r-1}^* &= Y_{n-6} & & \\ Y_{n-6}^* &= Y_{2r-1} & & \\ Y_k^* &= Y_k & 1 \le k \le n-5 & k \notin \{2r-1, n-6\} \end{cases}$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\begin{aligned} \mathfrak{g}_n^{6,r} : \quad [X_0, X_i] &= X_{i+1} & 1 \le i \le 3 \\ [X_1, Y_{n-5}] &= X_3 & \\ [X_2, Y_{n-5}] &= X_4 & \\ [Y_{2k-1}, Y_{2k}] &= X_4 & 1 \le k \le r-1 \\ [Y_{2r-1}, Y_{n-5}] &= X_4 & \end{aligned}$$

Caso: $\epsilon = 1 \quad c_{n-6,n-5} \neq 1$

Al aplicar el cambio de base definido por

$$\left\{\begin{array}{lllll} X_0^* & = & X_0 & & \\ X_1^* & = & -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_1 + \frac{c_{n-6,n-5}-2}{2(1-c_{n-6,n-5})}Y_{n-5} & & \\ X_t^* & = & -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}X_t & 2 \leq t \leq 4 & \\ Y_k^* & = & -\frac{c_{n-6,n-5}}{2(1-c_{n-6,n-5})}Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_k^* & = & Y_k & 2r-1 \leq k \leq n-5 & k \neq n-6 \\ Y_{n-6}^* & = & \frac{1}{c_{n-6,n-5}-1}Y_{n-6} & & \end{array}\right.$$

Se obtiene que $\mathfrak{g}$ es isomorfa a un álgebra de ley:

$$\left\{\begin{array}{llll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{n-6}, Y_{n-5}] & = & 2X_4 & \end{array}\right.$$

Los cambios de base sucesivos:

$$\left\{\begin{array}{lllll} X_0^* & = & X_0 & & \\ X_1^* & = & 2X_1 + Y_{n-5} & & \\ X_t^* & = & 2X_t & 2 \leq t \leq 4 & \\ Y_k^* & = & 2Y_k & 1 \leq k \leq 2r-2 & k = \dot{2}+1 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 & k = \dot{2} \\ Y_k^* & = & Y_k & 2r-1 \leq k \leq n-5 & \end{array}\right.$$

y

$$\left\{\begin{array}{lllll} X_t^* & = & X_t & 0 \leq t \leq 4 & \\ Y_{2r-1}^* & = & Y_{n-6} & & \\ Y_{n-6}^* & = & Y_{2r-1} & & \\ Y_k^* & = & Y_k & 1 \leq k \leq n-5 & k \notin \{2r-1, n-6\} \end{array}\right.$$

demuestran que $\mathfrak{g}$ es isomorfa a

$$\mathfrak{g}_n^{7,r}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2r-1}, Y_{n-5}] & = & X_4 & \end{array}$$

*** Si existe algún $c_{ij} \neq 0 \quad 2r-1 \leq i < j \leq n-7$, se puede suponer $c_{2r-1,2r} \neq 0$. Basta con considerar el cambio de base expresado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* & = & Y_i & \\ Y_{2r}^* & = & Y_j & \\ Y_i^* & = & Y_{2r-1} & \\ Y_j^* & = & Y_{2r} & \\ Y_k^* & = & Y_k & 2r-1 \leq k \leq n-7 \quad k \notin \{2r-1, 2r, i, j\} \\ Y_{n-6}^* & = & Y_{n-6} & \\ Y_{n-5}^* & = & Y_{n-5} & \end{array} \right.$$

Se puede, además, suponer que $c_{2r-1,2r} = 1$ y $c_{2r-1,k} = 0 = c_{2r,k}$ $2r+1 \leq k \leq n-5$, sin más que hacer el cambio de base dado por

$$\left\{ \begin{array}{lcll} X_t^* & = & X_t & 0 \leq t \leq 4 \\ Y_k^* & = & Y_k & 1 \leq k \leq 2r-2 \\ Y_{2r-1}^* & = & \frac{1}{c_{2r-1,2r}} Y_{2r-1} & \\ Y_{2r}^* & = & Y_{2r} & \\ Y_k^* & = & Y_k + \frac{c_{2r,k}}{c_{2r-1,2r}} Y_{2r-1} - \frac{c_{2r-1,k}}{c_{2r-1,2r}} Y_{2r} & 2r+1 \leq k \leq n-5, \end{array} \right.$$

y se obtiene el álgebra de ley:

$$\left\{\begin{array}{lcll}
[X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\
[X_1, Y_{n-6}] & = & X_4 & \\
[X_1, Y_{n-5}] & = & X_3 & \\
[X_2, Y_{n-5}] & = & X_4 & \\
[Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r \\
[Y_i, Y_j] & = & c_{ij} X_4 & 2r+1 \leq i < j \leq n-5.
\end{array}\right.$$

Se llega a una situación análoga a las ya consideradas.

En consecuencia, aparecen las álgebras

$$\begin{array}{llclll}
\mathfrak{g}_n^{8,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-7}{2}) \\
& [X_1, Y_{n-6}] & = & X_4 & & \\
& [X_1, Y_{n-5}] & = & X_3 & & \\
& [X_2, Y_{n-5}] & = & X_4 & & \\
& [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 &
\end{array}$$

$$\begin{array}{llclll}
\mathfrak{g}_n^{9,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-7}{2}) \\
& [X_1, Y_{n-6}] & = & X_4 & & \\
& [X_1, Y_{n-5}] & = & X_3 & & \\
& [X_2, Y_{n-5}] & = & X_4 & & \\
& [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 & \\
& [Y_{2r-1}, Y_{n-5}] & = & X_4, & &
\end{array}$$

y justo antes del último paso del proceso, se obtiene que $\mathfrak{g}$ puede ser isomorfa a un álgebra de ley:

$$\left\{\begin{array}{lcll}
[X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\
[X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\
[X_1, Y_{n-6}] & = & X_4 & \\
[X_1, Y_{n-5}] & = & X_3 & \\
[X_2, Y_{n-5}] & = & X_4 & \\
[Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1 \\
[Y_i, Y_j] & = & c_{ij} X_4 & 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-5.
\end{array}\right.$$

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-5$, se obtiene el álgebra

$$\mathfrak{g}_n^{8,E(\frac{n-5}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-5}{2}) - 1. \end{array}$$

*** Si existe algún $c_{i,n-6} \neq 0 \quad 2E(\frac{n-5}{2}) - 1 \leq i \leq n-7$, se cumple que la ley de $\mathfrak{g}$ está determinada por

$$\left\{ \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, X_2] & = & \epsilon X_4 & \epsilon \in \{0,1\} \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 \\ [Y_{2E(\frac{n-5}{2})-1}, Y_{n-6}] & = & X_4 & \\ [Y_i, Y_{n-5}] & = & c_{in-5} X_4 & 2E(\frac{n-5}{2}) - 1 \leq i \leq n-6. \end{array} \right.$$

* Si $c_{i,n-5} = 0 \quad \forall i \quad 2E(\frac{n-5}{2}) - 1 \leq i \leq n-6$, se obtiene $\mathfrak{g} \simeq \mathfrak{g}_n^{5,E(\frac{n-3}{2})}$.

* Si $c_{2E(\frac{n-5}{2})-1,n-5} \neq 0$ ó $c_{n-6,n-5} \neq 0$, se obtiene $\mathfrak{g} \simeq \mathfrak{g}_n^{5,E(\frac{n-3}{2})}$.

*** Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-6$ y existe algún $c_{i,n-5} \neq 0$ $2E(\frac{n-5}{2}) - 1 \leq i \leq n-7$ (esta situación solamente ocurre cuando $E(\frac{n-5}{2}) = \frac{n-6}{2}$ $\Leftrightarrow$ *n par* $\Leftrightarrow \frac{n-6}{2} = E(\frac{n-6}{2})$), se obtiene el álgebra

$$\mathfrak{g}_n^{9,E(\frac{n-6}{2})}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 \\ [X_1, Y_{n-6}] & = & X_4 & \\ [X_1, Y_{n-5}] & = & X_3 & \\ [X_2, Y_{n-5}] & = & X_4 & \\ [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq E(\frac{n-6}{2}) - 1 \\ [Y_{2E(\frac{n-6}{2})-1}, Y_{n-5}] & = & X_4. & \end{array}$$

Cuando n es impar, la última álgebra de dicha familia es $\mathfrak{g}_n^{9,E(\frac{n-7}{2})} = \mathfrak{g}_n^{9,E(\frac{n-6}{2})}$.

* Si $c_{ij} = 0 \quad \forall i,j \quad 2E(\frac{n-5}{2}) - 1 \leq i < j \leq n-6$ y $c_{i,n-5} = 0 \quad \forall i$ $2E(\frac{n-5}{2}) - 1 \leq i \leq n-7$, pero $c_{n-6,n-5} \neq 0$ se distinguen dos casos, dependiendo del valor de ϵ.

$$\begin{array}{lll} \mathfrak{g} \simeq \mathfrak{g}_n^{7,E(\frac{n-5}{2})} & \text{si} \quad \epsilon = 0 & \\ \mathfrak{g} \simeq \mathfrak{g}_n^{6,E(\frac{n-5}{2})} & \text{si} \quad \epsilon = 0 & c_{n-6,n-5} = 1 \\ \mathfrak{g} \simeq \mathfrak{g}_n^{7,E(\frac{n-5}{2})} & \text{si} \quad \epsilon = 0 & c_{n-6,n-5} \neq 1\ . \end{array}$$

En consecuencia, se concluye que surgen las familias

$$\begin{array}{llclll} \mathfrak{g}_n^{8,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-5}{2}) \\ & [X_1, Y_{n-6}] & = & X_4 & & \\ & [X_1, Y_{n-5}] & = & X_3 & & \\ & [X_2, Y_{n-5}] & = & X_4 & & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 & \end{array}$$

$$\begin{array}{llclll} \mathfrak{g}_n^{9,r}: & [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 3 & 1 \leq r \leq E(\frac{n-6}{2}) \\ & [X_1, Y_{n-6}] & = & X_4 & & \\ & [X_1, Y_{n-5}] & = & X_3 & & \\ & [X_2, Y_{n-5}] & = & X_4 & & \\ & [Y_{2k-1}, Y_{2k}] & = & X_4 & 1 \leq k \leq r-1 & \\ & [Y_{2r-1}, Y_{n-5}] & = & X_4. & & \end{array}$$

□

Aplicaciones geométricas

Derivaciones de las álgebras de la familia $\mathfrak{g}_n^{2q-1}$, $1 \leq q \leq E(\frac{n-2}{2})$

Se designa por $\mathfrak{g}_n^{2q-1}$, $1 \leq q \leq E(\frac{n-2}{2})$, a la familia de álgebras de Lie $(n-3)$-filiformes, de dimensión n, de leyes

$$\mathfrak{g}_n^{2q-1}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 2 \\ [Y_{2k-1}, Y_{2k}] & = & X_3 & 1 \leq k \leq q-1 \end{array}$$

para cada $q \in \{1, 2, \ldots, E(\frac{n-2}{2})\}$.

Teorema 4.1. *Se verifica que*

$$dim(Der(\mathfrak{g}_n^{2q-1})) = \begin{cases} n^2 - 5n + 11 & \text{si} \quad q = 1 \\ \frac{4q^2+3q+5}{2} + (n-2q-2).(n-1) & \text{si} \quad q = \dot{2}+1 \quad q \geq 2 \\ \frac{4q^2+3q+6}{2} + (n-2q-2).(n-1) & \text{si} \quad q = \dot{2} \quad q \geq 2, \end{cases}$$

para $1 \leq q \leq E(\frac{n-2}{2})$.

Para cada q, la correspondiente álgebra se puede expresar como suma directa de dos álgebras:

$$\mathfrak{g}_n^{2q-1} = \mathfrak{h}_1^{2q-1} \oplus \mathfrak{h}_2^{2q-1}$$

donde

$$\mathfrak{h}_1^{2q-1} =< X_0, X_1, X_2, X_3, Y_1, Y_2, \ldots, Y_{2q-3}, Y_{2q-2} >$$
$$\mathfrak{h}_2^{2q-1} =< Y_{2q-1}, Y_{2q}, \ldots, Y_{n-4} > .$$

Se tiene, por tanto, que

$$Der(\mathfrak{g}_n^{2q-1}) = Der(\mathfrak{h}_1^{2q-1}) \oplus Der(\mathfrak{h}_2^{2q-1}) \oplus D(\mathfrak{h}_1^{2q-1}, \mathfrak{h}_2^{2q-1}) \oplus D(\mathfrak{h}_2^{2q-1}, \mathfrak{h}_1^{2q-1}).$$

Cálculo de $Der(\mathfrak{h}_1^{2q-1})$

Se considera la siguiente graduación de $\mathfrak{h}_1^{2q-1}$:

$$\mathfrak{h}_1^{2q-1} =< Y_1 > \oplus < Y_3 > \oplus < Y_5 > \oplus \ldots \oplus < Y_{2q-5} > \oplus < Y_{2q-3} > \oplus$$
$$\oplus < X_0 > \oplus < X_1 > \oplus < X_2 > \oplus < X_3, Y_{2q-2} > \oplus < Y_{2q-4} > \oplus$$
$$\oplus < Y_{2q-6} > \oplus \ldots \oplus < Y_6 > \oplus < Y_4 > \oplus < Y_2 >, \text{ donde}$$

$$\begin{array}{lcll} \mathfrak{g}_{-k} & = & < Y_{-2k+2q-3} > & 0 \leq k \leq q-2 \\ \mathfrak{g}_k & = & < X_{k-1} > & 1 \leq k \leq 3 \\ \mathfrak{g}_4 & = & < X_3, Y_{2q-2} > & \\ \mathfrak{g}_k & = & < Y_{-2k+2q+6} > & 5 \leq k \leq q+2. \end{array}$$

Sea $\bar{d}_1 \in Der(\mathfrak{h}_1^{2q-1})$. Entonces

$$\bar{d}_1 = \sum_{i \in Z} d_i$$

donde $d_i \in Der(\mathfrak{h}_1^{2q-1})$ y $d_i(\mathfrak{g}_j) \subset \mathfrak{g}_{i+j}$, siendo $\mathfrak{g}_k = \{0\}$ para $k < -q+2$ y $k > q+2$.

Como $d_{2q}(\mathfrak{g}_{-q+2}) \subset \mathfrak{g}_{q+2}$ y $d_{-2q}(\mathfrak{g}_{q+2}) \subset \mathfrak{g}_{-q+2}$, se deduce que

$$d_i = 0 \quad i > 2q \ , \ i < -2q \Rightarrow \bar{d}_1 = \sum_{i=-2q}^{2q} d_i$$

Habrá que expresar cada d_i , $-2q \leq i \leq 2q$, como una combinación lineal de un cierto conjunto, B_i , $-2q \leq i \leq 2q$, de derivaciones linealmente independientes de $\mathfrak{h}_1^{2q-1}$ cumpliéndose que

$$\bigcup_{i=-2q}^{2q} B_i$$

es una base de $Der(\mathfrak{h}_1^{2q-1})$ y, evidentemente,

$$dim(Der(\mathfrak{h}_1^{2q-1})) = \sum_{i=-2q}^{2q} dim(K < B_i >).$$

A continuación, se detalla cómo se obtienen las condiciones que resultan al exigir que cada d_i , $-2q \leq i \leq 2q$, sea una derivación.

Cálculo de $d_{-j} \quad q+3 \leq j \leq 2q$

Como $d_{-j}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-j} \quad \forall t$, se cumple que

$$d_{-j}(Y_{2k}) = \beta_{2k}^{j}.Y_{4q-2j-2k+3} \quad 1 \leq k \leq 2q-j+1.$$

Al exigir que d_{-j} sea derivación, se obtiene que

$$\begin{array}{l} d_{-j}([Y_{2k}, Y_{4q-2j-2k+4}]) = [d_{-j}(Y_{2k}), Y_{4q-2j-2k+4}] + [Y_{2k}, d_{-j}(Y_{4q-2j-2k+4})] \\ \quad 1 \leq k \leq 2q-j+1 \Rightarrow \\ \Rightarrow 0 = [\beta_{2k}^{j}.Y_{4q-2j-2k+3}, Y_{4q-2j-2k+4}] + [Y_{2k}, \beta_{4q-2j-2k+4}^{j}.Y_{2k-1}] = \\ = (\beta_{2k}^{j} - \beta_{4q-2j-2k+4}^{j}).X_3 \Rightarrow \beta_{2k}^{j} = \beta_{4q-2j-2k+4}^{j} \quad 1 \leq k \leq 2q-j+1. \end{array}$$

Cálculo de d_{-j} $\quad 4 \leq j \leq q+2$

Como $d_{-j}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-j} \quad \forall t$, se cumple que

$$\left\{\begin{array}{llll} d_{-j}(Y_{2k+2j-1}) & = & \beta^j_{2k+2j-1}.Y_{2k-1} & 1 \leq k \leq q-j-2 \\ d_{-j}(Y_{2q-3}) & = & \beta^j_{2q-3}.Y_{2q-2j-3} & \\ d_{-j}(X_0) & = & \bar{\alpha}^j_0.Y_{2q-2j-1} & \\ d_{-j}(X_1) & = & \bar{\alpha}^j_1.Y_{2q-2j+1} & \\ d_{-j}(Y_{2q-2}) & = & \beta^j_{2q-2}.Y_{2q-2j+5} & \\ d_{-j}(Y_{2k}) & = & \beta^j_{2k}.Y_{4q-2j-2k+3} & q-j+3 \leq k \leq q-2 \\ d_{-j}(Y_{2q-2j+4}) & = & \bar{\beta}^j_{2q-2j+4}.X_0 & \\ d_{-j}(Y_{2q-2j+2}) & = & \bar{\beta}^j_{2q-2j+2}.X_1 & \\ d_{-j}(Y_{2q-2j}) & = & \bar{\beta}^j_{2q-2j}.X_2 & \\ d_{-j}(Y_{2q-2j-2}) & = & \bar{\beta}^j_{2q-2j-2}.X_3 + \beta^j_{2q-2j-2}.Y_{2q-2} & \\ d_{-j}(Y_{2k}) & = & \beta^j_{2k}.Y_{2k+2j} & 1 \leq k \leq q-j-2. \end{array}\right.$$

Al exigir que d_{-j} sea derivación, se obtiene que

$* \ d_{-j}([X_0, Y_{2q-2j+2}]) = [d_{-j}(X_0), Y_{2q-2j+2}] + [X_0, d_{-j}(Y_{2q-2j+2})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2q-2j-1}, Y_{2q-2j+2}] + [X_0, \bar{\beta}^j_{2q-2j+2}.X_1] = \bar{\beta}^j_{2q-2j+2}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2q-2j+2} = 0 \Rightarrow d_{-j}(Y_{2q-2j+2}) = 0.$

$* \ d_{-j}([X_1, Y_{2q-2j+2}]) = [d_{-j}(X_1), Y_{2q-2j+2}] + [X_1, d_{-j}(Y_{2q-2j+2})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2q-2j+1}, Y_{2q-2j+2}] + [X_1, 0] \Rightarrow 0 = \bar{\alpha}^j_1.X_3 \Rightarrow \bar{\alpha}^j_1 = 0 \Rightarrow d_{-j}(X_1) = 0.$

$* \ d_{-j}([X_1, Y_{2q-2j+4}]) = [d_{-j}(X_1), Y_{2q-2j+4}] + [X_1, d_{-j}(Y_{2q-2j+4})] \Rightarrow$
$\Rightarrow 0 = [0, Y_{2q-2j+4}] + [X_1, \bar{\beta}^j_{2q-2j+4}.X_0] \Rightarrow 0 = -\bar{\beta}^j_{2q-2j+4}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2q-2j+4} = 0 \Rightarrow d_{-j}(Y_{2q-2j+4}) = 0.$

$* \ d_{-j}([X_0, Y_{2q-2j}]) = [d_{-j}(X_0), Y_{2q-2j}] + [X_0, d_{-j}(Y_{2q-2j})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2q-2j-1}, Y_{2q-2j}] + [X_0, \bar{\beta}^j_{2q-2j}.X_2] \Rightarrow 0 = (\bar{\alpha}^j_0 + \bar{\beta}^j_{2q-2j}).X_3 \Rightarrow \bar{\beta}^j_{2q-2j} = -\bar{\alpha}^j_0.$

$* \ d_{-j}([Y_{2k}, Y_{2k+2j-1}]) = [d_{-j}(Y_{2k}), Y_{2k+2j-1}] + [Y_{2k}, d_{-j}(Y_{2k+2j-1})] \quad 1 \leq k \leq q-j-2 \Rightarrow$
$\Rightarrow 0 = [\beta^j_{2k}.Y_{2k+2j}, Y_{2k+2j-1}] + [Y_{2k}, \beta^j_{2k+2j-1}.Y_{2k-1}] \Rightarrow 0 = (-\beta^j_{2k} - \beta^j_{2k+2j-1}).X_3 \Rightarrow$
$\Rightarrow \beta^j_{2k+2j-1} = -\beta^j_{2k} \quad 1 \leq k \leq q-j-2.$

$$* \; d_{-j}([Y_{2q-2j-2}, Y_{2q-3}]) = [d_{-j}(Y_{2q-2j-2}), Y_{2q-3}] + [Y_{2q-2j-2}, d_{-j}(Y_{2q-3})] \Rightarrow$$
$$\Rightarrow 0 = [\bar{\beta}^j_{2q-2j-2}.X_3 + \beta^j_{2q-2j-2}.Y_{2q-2}, Y_{2q-3}] + [Y_{2q-2j-2}, \beta^j_{2q-3}.Y_{2q-2j-3}] \Rightarrow$$
$$\Rightarrow 0 = (-\beta^j_{2q-2j-2} - \beta^j_{2q-3}).X_3 \Rightarrow \beta^j_{2q-3} = -\beta^j_{2q-2j-2}.$$

$$* \; d_{-j}([Y_{2q-2j+2k+2}, Y_{2q-2k+2}]) = [d_{-j}(Y_{2q-2j+2k+2}), Y_{2q-2k+2}] + [Y_{2q-2j+2k+2}, d_{-j}(Y_{2q-2k+2})$$
$$2 \leq k \leq E(\tfrac{j}{2}) \Rightarrow$$
$$\Rightarrow 0 = [\beta^j_{2q-2j+2k+2}.Y_{2q-2k+1}, Y_{2q-2k+2}] + [Y_{2q-2j+2k+2}, \beta^j_{2q-2k+2}.Y_{2q-2j+2k+1}] \Rightarrow$$
$$\Rightarrow 0 = (\beta^j_{2q-2j+2k+2} - \beta^j_{2q-2k+2}).X_3 \Rightarrow \beta^j_{2q-2j+2k+2} = \beta^j_{2q-2k+2} \quad 2 \leq k \leq E(\tfrac{j}{2}).$$

En consecuencia, se verifica que

$$\left\{ \begin{array}{lcll} d_{-j}(X_0) & = & \bar{\alpha}^j_0.Y_{2q-2j-1} & \\ d_{-j}(Y_{2k+2j-1}) & = & -\beta^j_{2k}.Y_{2k-1} & 1 \leq k \leq q-j-2 \\ d_{-j}(Y_{2k}) & = & \beta^j_{2k}.Y_{2k+2j} & 1 \leq k \leq q-j-2 \\ d_{-j}(Y_{2q-3}) & = & \beta^j_{2q-3}.Y_{2q-2j-3} & \\ d_{-j}(Y_{2q-2k+2}) & = & \beta^j_{2q-2k+2}.Y_{2q-2j+2k+1} & 2 \leq k \leq E(\frac{j}{2}) \\ d_{-j}(Y_{2q-2j+2k+2}) & = & \beta^j_{2q-2k+2}.Y_{2q-2k+1} & 2 \leq k \leq E(\frac{j}{2}) \\ d_{-j}(Y_{2q-2j}) & = & -\bar{\alpha}^j_0.X_2 & \\ d_{-j}(Y_{2q-2j-2}) & = & \bar{\beta}^j_{2q-2j-2}.X_3 - \beta^j_{2q-3}.Y_{2q-2} & \end{array} \right.$$

Cálculo de d_{-j} $\quad 1 \leq j \leq 3$

Como $d_{-j}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-j}$ $\quad \forall t$, se cumple que

$$\begin{cases} d_{-j}(Y_{2k+2j-1}) & = & \beta^j_{2k+2j-1}.Y_{2k-1} & 1 \leq k \leq q-j-2 \\ d_{-j}(Y_{2q-3}) & = & \beta^j_{2q-3}.Y_{2q-2j-3} & \\ d_{-j}(X_0) & = & \bar{\alpha}^j_0.Y_{2q-2j-1} & \\ d_{-j}(X_1) & = & \alpha^j_1\delta_{j1}.X_0 + \bar{\alpha}^j_1(1-\delta_{j1}).Y_{2q-2j+1} & \\ d_{-j}(Y_{2q-2j+4}) & = & \bar{\beta}^j_{2q-2j+4}.X_0 & \\ d_{-j}(Y_{2q-2j+2}) & = & \bar{\beta}^j_{2q-2j+2}.X_1 & \\ d_{-j}(Y_{2q-2j}) & = & \bar{\beta}^j_{2q-2j}.X_2 & \\ d_{-j}(Y_{2q-2j-2}) & = & \bar{\beta}^j_{2q-2j-2}.X_3 + \beta^j_{2q-2j-2}.Y_{2q-2} & \\ d_{-j}(Y_{2k}) & = & \beta^j_{2k}.Y_{2k+2j} & 1 \leq k \leq q-j-2. \end{cases}$$

Al exigir que d_{-j} sea derivación, se obtiene que

$* \ d_{-j}([X_1, X_2]) = [d_{-j}(X_1), X_2] + [X_1, d_{-j}(X_2)] \Rightarrow$
$\Rightarrow 0 = [\alpha^j_1\delta_{j1}.X_0 + \bar{\alpha}^j_1(1-\delta_{j1}).Y_{2q-2j+1}, X_2] + [X_1, 0] = \alpha^j_1\delta_{j1}.X_3 \Rightarrow$
$j = 1 \Rightarrow d_{-j}(X_1) = d_{-1}(X_1) = 0$
$j = 2, 3 \Rightarrow d_{-j}(X_1) = \bar{\alpha}^j_1.Y_{2q-2j+1}.$

$* \ d_{-j}([X_1, Y_{2q-2j+2}]) = [d_{-j}(X_1), Y_{2q-2j+2}] + [X_1, d_{-j}(Y_{2q-2j+2})] \quad 2 \leq j \leq 3 \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2q-2j+1}, Y_{2q-2j+2}] + [X_1, \bar{\beta}^j_{2q-2j+2}.X_1] \Rightarrow 0 = \bar{\alpha}^j_1.X_3 \Rightarrow$
$\Rightarrow \bar{\alpha}^j_1 = 0 \Rightarrow d_{-j}(X_1) = 0 \quad 2 \leq j \leq 3.$

$* \ d_{-j}([Y_{2k}, Y_{2k+2j-1}]) = [d_{-j}(Y_{2k}), Y_{2k+2j-1}] + [Y_{2k}, d_{-j}(Y_{2k+2j-1})] \quad 1 \leq k \leq q-j-2 \Rightarrow$
$\Rightarrow 0 = [\beta^j_{2k}.Y_{2k+2j}, Y_{2k+2j-1}] + [Y_{2k}, \beta^j_{2k+2j-1}.Y_{2k-1}] = (-\beta^j_{2k} - \beta^j_{2k+2j-1}).X_3 \Rightarrow$
$\Rightarrow \beta^j_{2k+2j-1} = -\beta^j_{2k} \quad 1 \leq k \leq q-j-2.$

$* \ d_{-j}([X_0, Y_{2q-2j+2}]) = [d_{-j}(X_0), Y_{2q-2j+2}] + [X_0, d_{-j}(Y_{2q-2j+2})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2q-2j-1}, Y_{2q-2j+2}] + [X_0, \bar{\beta}^j_{2q-2j+2}.X_1] \Rightarrow 0 = \bar{\beta}^j_{2q-2j+2}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2q-2j+2} = 0 \Rightarrow d_{-j}(Y_{2q-2j+2}) = 0.$

$* \ d_{-j}([X_0, Y_{2q-2j}]) = [d_{-j}(X_0), Y_{2q-2j}] + [X_0, d_{-j}(Y_{2q-2j})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2q-2j-1}, Y_{2q-2j}] + [X_0, \bar{\beta}^j_{2q-2j}.X_2] \Rightarrow 0 = (\bar{\alpha}^j_0 + \bar{\beta}^j_{2q-2j}).X_3 \Rightarrow \bar{\beta}^j_{2q-2j} = -\bar{\alpha}^j_0.$

$* \; d_{-j}([Y_{2q-2j-2}, Y_{2q-3}]) = [d_{-j}(Y_{2q-2j-2}), Y_{2q-3}] + [Y_{2q-2j-2}, d_{-j}(Y_{2q-3})] \Rightarrow$
$\Rightarrow 0 = [\bar{\beta}^j_{2q-2j-2}.X_3 + \beta^j_{2q-2j-2}.Y_{2q-2}, Y_{2q-3}] + [Y_{2q-2j-2}, \beta^j_{2q-3}.Y_{2q-2j-3}] =$
$= (-\beta^j_{2q-2j-2} - \beta^j_{2q-3}).X_3 \Rightarrow \beta^j_{2q-2j-2} = -\beta^j_{2q-3}.$

$* \; d_{-j}([X_1, Y_{2q-2j+4}]) = [d_{-j}(X_1), Y_{2q-2j+4}] + [X_1, d_{-j}(Y_{2q-2j+4})] \Rightarrow$
$\Rightarrow 0 = [0, Y_{2q-2j+4}] + [X_1, \bar{\beta}^j_{2q-2j+4}.X_0] \Rightarrow 0 = -\bar{\beta}^j_{2q-2j+4}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2q-2j+4} = 0 \Rightarrow d_{-j}(Y_{2q-2j+4}) = 0.$

En consecuencia, se verifica que

$$\begin{cases} d_{-j}(X_0) & = \bar{\alpha}^j_0.Y_{2q-2j-1} & \\ d_{-j}(Y_{2k+2j-1}) & = -\beta^j_{2k}.Y_{2k-1} & 1 \leq k \leq q-j-2 \\ d_{-j}(Y_{2k}) & = \beta^j_{2k}.Y_{2k+2j} & 1 \leq k \leq q-j-2 \\ d_{-j}(Y_{2q-2j}) & = -\bar{\alpha}^j_0.X_2 & \\ d_{-j}(Y_{2q-2j-2}) & = \bar{\beta}^j_{2q-2j-2}.X_3 - \beta^j_{2q-3}.Y_{2q-2} & \\ d_{-j}(Y_{2q-3}) & = \beta^j_{2q-3}.Y_{2q-2j-3} & \end{cases}$$

Cálculo de d_0

Como $d_0(\mathfrak{g}_t) \subset \mathfrak{g}_t \quad \forall t$, se cumple que

$$\left\{\begin{array}{lcll} d_0(Y_{2k-1}) & = & \beta^0_{2k-1}.Y_{2k-1} & 1 \leq k \leq q-1 \\ d_0(X_0) & = & \alpha^0_0.X_0 & \\ d_0(X_1) & = & \alpha^0_1.X_1 & \\ d_0(X_2) & = & \alpha^0_2.X_2 & \\ d_0(X_3) & = & \alpha^0_3.X_3 & \\ d_0(Y_{2k}) & = & \beta^0_{2k}.Y_{2k} & 1 \leq k \leq q-2 \\ d_0(Y_{2q-2}) & = & \bar{\beta}^0_{2q-2}.X_3 + \beta^0_{2q-2}.Y_{2q-2} & \end{array}\right.$$

Al exigir que d_0 sea derivación, se obtiene que

$* \ d_0([X_0, X_1]) = [d_0(X_0), X_1] + [X_0, d_0(X_1)] \Rightarrow$
$\Rightarrow \alpha^0_2.X_2 = d_0(X_2) = [\alpha^0_0.X_0, X_1] + [X_0, \alpha^0_1.X_1] = (\alpha^0_0 + \alpha^0_1).X_2 \Rightarrow$
$\Rightarrow \alpha^0_2 = \alpha^0_0 + \alpha^0_1.$

$* \ d_0([X_0, X_2]) = [d_0(X_0), X_2] + [X_0, d_0(X_2)] \Rightarrow$
$\Rightarrow \alpha^0_3.X_3 = d_0(X_3) = [\alpha^0_0.X_0, X_2] + [X_0, \alpha^0_2.X_2] = (\alpha^0_0 + \alpha^0_2).X_3 \Rightarrow$
$\Rightarrow \alpha^0_3 = \alpha^0_0 + \alpha^0_2 = \alpha^0_0 + (\alpha^0_0 + \alpha^0_1) \Rightarrow \alpha^0_3 = 2\alpha^0_0 + \alpha^0_1.$

$* \ d_0([Y_{2q-3}, Y_{2q-2}]) = [d_0(Y_{2q-3}), Y_{2q-2}] + [Y_{2q-3}, d_0(Y_{2q-2})] \Rightarrow$
$\Rightarrow \alpha^0_3.X_3 = d_0(X_3) = [\beta^0_{2q-3}.Y_{2q-3}, Y_{2q-2}] + [Y_{2q-3}, \bar{\beta}^0_{2q-2}.X_3 + \beta^0_{2q-2}.Y_{2q-2}] =$
$= (\beta^0_{2q-3} + \beta^0_{2q-2}).X_3 \Rightarrow \alpha^0_3 = \beta^0_{2q-3} + \beta^0_{2q-2} \Rightarrow$
$\Rightarrow 2\alpha^0_0 + \alpha^0_1 = \beta^0_{2q-3} + \beta^0_{2q-2} \Rightarrow \beta^0_{2q-3} = 2\alpha^0_0 + \alpha^0_1 - \beta^0_{2q-2}.$

$* \ d_0([Y_{2k-1}, Y_{2k}]) = [d_0(Y_{2k-1}), Y_{2k}] + [Y_{2k-1}, d_0(Y_{2k})] \quad 1 \leq k \leq q-2 \Rightarrow$
$\Rightarrow \alpha^0_3.X_3 = d_0(X_3) = [\beta^0_{2k-1}.Y_{2k-1}, Y_{2k}] + [Y_{2k-1}, \beta^0_{2k}.Y_{2k}] = (\beta^0_{2k-1} + \beta^0_{2k}).X_3 \Rightarrow$
$\Rightarrow \alpha^0_3 = \beta^0_{2k-1} + \beta^0_{2k} \Rightarrow 2\alpha^0_0 + \alpha^0_1 = \beta^0_{2k-1} + \beta^0_{2k} \Rightarrow \beta^0_{2k-1} = 2\alpha^0_0 + \alpha^0_1 - \beta^0_{2k} \quad 1 \leq k \leq q-2.$

En consecuencia, se verifica que

$$\left\{\begin{array}{lcll} d_0(X_0) & = & \alpha^0_0.X_0 & \\ d_0(X_1) & = & \alpha^0_1.X_1 & \\ d_0(X_2) & = & (\alpha^0_0 + \alpha^0_1).X_2 & \\ d_0(X_3) & = & (2\alpha^0_0 + \alpha^0_1).X_3 & \\ d_0(Y_{2k-1}) & = & (2\alpha^0_0 + \alpha^0_1 - \beta^0_{2k}).Y_{2k-1} & 1 \leq k \leq q-1 \\ d_0(Y_{2k}) & = & \beta^0_{2k}.Y_{2k} & 1 \leq k \leq q-2 \\ d_0(Y_{2q-2}) & = & \bar{\beta}^0_{2q-2}.X_3 + \beta^0_{2q-2}.Y_{2q-2} & \end{array}\right.$$

Cálculo de d_j $\quad 1 \leq j \leq 3$

Como $d_j(\mathfrak{g}_t) \subset \mathfrak{g}_{t+j} \quad \forall t$, se cumple que

$$\begin{cases} d_j(Y_{2k-1}) & = \beta^j_{2k-1}.Y_{2k+2j-1} \\ 1 \leq k \leq q-j-1 & \\ d_j(Y_{2q-2j-1}) & = \bar{\beta}^j_{2q-2j-1}.X_0 \\ d_j(Y_{2q-2j+1}) & = \bar{\beta}^j_{2q-2j+1}.X_1 \\ d_j(Y_{2q-2j+3}) & = \bar{\beta}^j_{2q-2j+3}.X_2 \\ d_j(X_0) & = \alpha^j_0.\delta_{j1}.X_1 + \alpha^j_0.\delta_{j2}.X_2 + \alpha^j_0.\delta_{j3}.X_3 + \bar{\alpha}^j_0.\delta_{j3}.Y_{2q-2} \\ d_j(X_1) & = \alpha^j_1.\delta_{j1}.X_2 + \alpha^j_1.\delta_{j2}.X_3 + \bar{\alpha}^j_1.\delta_{j2}.Y_{2q-2} + \bar{\alpha}^j_1.\delta_{j3}.Y_{2q-4} \\ d_j(X_2) & = \alpha^j_2.\delta_{j1}.X_3 \\ d_j(Y_{2q-2}) & = \beta^j_{2q-2}.\delta_{j1}.Y_{2q-4} + \beta^j_{2q-2}.\delta_{j2}.Y_{2q-6} + \beta^j_{2q-2}.\delta_{j3}.Y_{2q-8} \\ d_j(Y_{2k+2j}) & = \beta^j_{2k+2j}.Y_{2k} \\ 1 \leq k \leq q-j-1 & \end{cases}$$

Al exigir que d_j sea derivación, se obtiene que

$* \; d_j([X_0, X_1]) = [d_j(X_0), X_1] + [X_0, d_j(X_1)] \Rightarrow$
$\Rightarrow \alpha^j_2.\delta_{j1}.X_3 = d_j(X_2) = [\alpha^j_0.\delta_{j1}.X_1 + \alpha^j_0.\delta_{j2}.X_2 + \alpha^j_0.\delta_{j3}.X_3 + \bar{\alpha}^j_0.\delta_{j3}.Y_{2q-2}, X_1] +$
$+ [X_0, \alpha^j_1.\delta_{j1}.X_2 + \alpha^j_1.\delta_{j2}.X_3 + \bar{\alpha}^j_1.\delta_{j2}.Y_{2q-2} + \bar{\alpha}^j_1.\delta_{j3}.Y_{2q-4}] \Rightarrow$
$\Rightarrow \alpha^j_2.\delta_{j1}.X_3 = \alpha^j_1.\delta_{j1}.X_3 \Rightarrow \alpha^j_2.\delta_{j1} = \alpha^j_1.\delta_{j1} \Rightarrow d_j(X_2) = \alpha^j_1.\delta_{j1}.X_3.$

$* \; d_j([X_1, Y_{2q-2j-1}]) = [d_j(X_1), Y_{2q-2j-1}] + [X_1, d_j(Y_{2q-2j-1})] \Rightarrow$
$\Rightarrow 0 = [\alpha^j_1.\delta_{j1}.X_2 + \alpha^j_1.\delta_{j2}.X_3 + \bar{\alpha}^j_1.\delta_{j2}.Y_{2q-2} + \bar{\alpha}^j_1.\delta_{j3}.Y_{2q-4}, Y_{2q-2j-1}] +$
$+ [X_1, \bar{\beta}^j_{2q-2j-1}.X_0] \Rightarrow 0 = -\bar{\beta}^j_{2q-2j-1}.X_2 \Rightarrow \bar{\beta}^j_{2q-2j-1} = 0 \Rightarrow d_j(Y_{2q-2j-1}) = 0.$

$* \; d_j([X_0, Y_{2q-2j+1}]) = [d_j(X_0), Y_{2q-2j+1}] + [X_0, d_j(Y_{2q-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\alpha^j_0.\delta_{j1}.X_1 + \alpha^j_0.\delta_{j2}.X_2 + \alpha^j_0.\delta_{j3}.X_3 + \bar{\alpha}^j_0.\delta_{j3}.Y_{2q-2}, Y_{2q-2j+1}] +$
$+ [X_0, \bar{\beta}^j_{2q-2j+1}.X_1] \Rightarrow 0 = \bar{\beta}^j_{2q-2j+1}.X_2 \Rightarrow \bar{\beta}^j_{2q-2j+1} = 0 \Rightarrow d_j(Y_{2q-2j+1}) = 0.$

$* \; d_j([X_0, Y_{2q-2j+3}]) = [d_j(X_0), Y_{2q-2j+3}] + [X_0, d_j(Y_{2q-2j+3})] \Rightarrow$
$\Rightarrow 0 = [\alpha^j_0.\delta_{j1}.X_1 + \alpha^j_0.\delta_{j2}.X_2 + \alpha^j_0.\delta_{j3}.X_3 + \bar{\alpha}^j_0.\delta_{j3}.Y_{2q-2}, Y_{2q-2j+3}] +$
$+ [X_0, \bar{\beta}^j_{2q-2j+3}.X_2] \Rightarrow 0 = (-\bar{\alpha}^j_0.\delta_{j3} + \bar{\beta}^j_{2q-2j+3}).X_3 \Rightarrow \bar{\beta}^j_{2q-2j+3} = \bar{\alpha}^j_0.\delta_{j3}.$

$* \; d_j([X_1, Y_{2q-2j+1}]) = [d_j(X_1), Y_{2q-2j+1}] + [X_1, d_j(Y_{2q-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\alpha^j_1.\delta_{j1}.X_2 + \alpha^j_1.\delta_{j2}.X_3 + \bar{\alpha}^j_1.\delta_{j2}.Y_{2q-2} + \bar{\alpha}^j_1.\delta_{j3}.Y_{2q-4}, Y_{2q-2j+1}] + [X_1, 0] \Rightarrow$
$\Rightarrow 0 = (-\bar{\alpha}^j_1.\delta_{j2} - \bar{\alpha}^j_1.\delta_{j3}).X_3 \Rightarrow \bar{\alpha}^j_1.\delta_{j3} = \bar{\alpha}^j_1.\delta_{j2} \Rightarrow d_j(X_1) = \alpha^j_1.\delta_{j1}.X_2 + \alpha^j_1.\delta_{j2}.X_3.$

$* \ d_j([Y_{2k-1}, Y_{2k+2j}]) = [d_j(Y_{2k-1}), Y_{2k+2j}] + [Y_{2k-1}, d_j(Y_{2k+2j})] \quad 1 \leq k \leq q-j-1 \Rightarrow$
$\Rightarrow 0 = [\beta^j_{2k-1}.Y_{2k+2j-1}, Y_{2k+2j}] + [Y_{2k-1}, \beta^j_{2k+2j}.Y_{2k}] = (\beta^j_{2k-1} + \beta^j_{2k+2j}).X_3 \Rightarrow$
$\Rightarrow \beta^j_{2k+2j} = -\beta^j_{2k-1} \quad 1 \leq k \leq q-j-1.$

En consecuencia,se verifica que

$$\left\{\begin{array}{lcll} d_j(X_0) & = & \alpha^j_0.\delta_{j1}.X_1 + \alpha^j_0.\delta_{j2}.X_2 + \alpha^j_0.\delta_{j3}.X_3 + \bar{\alpha}^j_0.\delta_{j3}.Y_{2q-2} & \\ d_j(X_1) & = & \alpha^j_1.\delta_{j1}.X_2 + \alpha^j_1.\delta_{j2}.X_3 & \\ d_j(X_2) & = & \alpha^j_1.\delta_{j1}.X_3 & \\ d_j(Y_{2k-1}) & = & \beta^j_{2k-1}.Y_{2k+2j-1} & 1 \leq k \leq q-j-1 \\ d_j(Y_{2k+2j}) & = & -\beta^j_{2k-1}.Y_{2k} & 1 \leq k \leq q-j-1 \\ d_j(Y_{2q-2j+3}) & = & \bar{\alpha}^j_0.\delta_{j3}.X_2 & \end{array}\right.$$

Cálculo de d_j $\quad 4 \leq j \leq q+2$

Como $d_j(\mathfrak{g}_t) \subset \mathfrak{g}_{t+j} \quad \forall t$, se cumple que

$$\begin{cases} d_j(Y_{2k+2j}) & = & \beta^j_{2k+2j}.Y_{2k} & 1 \leq k \leq q-j-1 \\ d_j(X_1) & = & \bar{\alpha}^j_1.Y_{2q-2j+2} & \\ d_j(X_0) & = & \bar{\alpha}^j_0.Y_{2q-2j+4} & \\ d_j(Y_{2q-2j+2k+5}) & = & \beta^j_{2q-2j+2k+5}.Y_{2q-2k-2} & 1 \leq k \leq j-5 \\ d_j(Y_{2q-2j+5}) & = & \bar{\beta}^j_{2q-2j+5}.X_3 + \beta^j_{2q-2j+5}.Y_{2q-2} & \\ d_j(Y_{2q-2j+3}) & = & \bar{\beta}^j_{2q-2j+3}.X_2 & \\ d_j(Y_{2q-2j+1}) & = & \bar{\beta}^j_{2q-2j+1}.X_1 & \\ d_j(Y_{2q-2j-1}) & = & \bar{\beta}^j_{2q-2j-1}.X_0 & \\ d_j(Y_{2k-1}) & = & \beta^j_{2k-1}.Y_{2k+2j-1} & 1 \leq k \leq q-j-1 \\ (1-\delta_{j4}).d_j(Y_{2q-3}) & = & (1-\delta_{j4}).\beta^j_{2q-3}.Y_{2q-2j+6}. & \end{cases}$$

Al exigir que d_j sea derivación, se obtiene que

$*\ d_j([X_1, Y_{2q-2j+1}]) = [d_j(X_1), Y_{2q-2j+1}] + [X_1, d_j(Y_{2q-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2q-2j+2}, Y_{2q-2j+1}] + [X_1, \bar{\beta}^j_{2q-2j+1}.X_1] \Rightarrow 0 = -\bar{\alpha}^j_1.X_3 \Rightarrow$
$\Rightarrow \bar{\alpha}^j_1 = 0 \Rightarrow d_j(X_1) = 0.$

$*\ d_j([X_1, Y_{2q-2j-1}]) = [d_j(X_1), Y_{2q-2j-1}] + [X_1, d_j(Y_{2q-2j-1})] \Rightarrow$
$\Rightarrow 0 = [0, Y_{2q-2j-1}] + [X_1, \bar{\beta}^j_{2q-2j-1}.X_0] \Rightarrow 0 = -\bar{\beta}^j_{2q-2j-1}.X_2 \Rightarrow \bar{\beta}^j_{2q-2j-1} = 0 \Rightarrow$
$\Rightarrow d_j(Y_{2q-2j-1}) = 0.$

$*\ d_j([X_0, Y_{2q-2j+1}]) = [d_j(X_0), Y_{2q-2j+1}] + [X_0, d_j(Y_{2q-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2q-2j+4}, Y_{2q-2j+1}] + [X_0, \bar{\beta}^j_{2q-2j+1}.X_1] \Rightarrow 0 = \bar{\beta}^j_{2q-2j+1}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2q-2j+1} = 0 \Rightarrow d_j(Y_{2q-2j+1}) = 0.$

$*\ d_j([X_0, Y_{2q-2j+3}]) = [d_j(X_0), Y_{2q-2j+3}] + [X_0, d_j(Y_{2q-2j+3})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2q-2j+4}, Y_{2q-2j+3}] + [X_0, \bar{\beta}^j_{2q-2j+3}.X_2] \Rightarrow 0 = (-\bar{\alpha}^j_0 + \bar{\beta}^j_{2q-2j+3}).X_3 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2q-2j+3} = \bar{\alpha}^j_0.$

$*\ d_j([Y_{2q-2j+5}, Y_{2q-3}]) = [d_j(Y_{2q-2j+5}), Y_{2q-3}] + [Y_{2q-2j+5}, d_j(Y_{2q-3})] \quad j > 4 \Rightarrow$
$\Rightarrow 0 = [\bar{\beta}^j_{2q-2j+5}.X_3 + \beta^j_{2q-2j+5}.Y_{2q-2}, Y_{2q-3}] + [Y_{2q-2j+5}, \beta^j_{2q-3}.Y_{2q-2j+6}] \Rightarrow$
$\Rightarrow 0 = (-\beta^j_{2q-2j+5} + \beta^j_{2q-3}).X_3 \Rightarrow \beta^j_{2q-3} = \beta^j_{2q-2j+5} \quad (j \geq 4).$

$* \ d_j([Y_{2q-2j+2k+5}, Y_{2q-2k-3}]) = [d_j(Y_{2q-2j+2k+5}), Y_{2q-2k-3}] + [Y_{2q-2j+2k+5}, d_j(Y_{2q-2k-3})]$
$1 \leq k \leq E(\frac{j-4}{2}) \Rightarrow$
$\Rightarrow 0 = [\beta^j_{2q-2j+2k+5}.Y_{2q-2k-2}, Y_{2q-2k-3}] + [Y_{2q-2j+2k+5}, \beta^j_{2q-2k-3}.Y_{2q-2j+2k+6}] \Rightarrow$
$\Rightarrow 0 = (-\beta^j_{2q-2j+2k+5} + \beta^j_{2q-2k-3}).X_3 \Rightarrow \beta^j_{2q-2j+2k+5} = \beta^j_{2q-2k-3} \quad 1 \leq k \leq E(\frac{j-4}{2}).$

$* \ d_j([Y_{2k-1}, Y_{2k+2j}]) = [d_j(Y_{2k-1}), Y_{2k+2j}] + [Y_{2k-1}, d_j(Y_{2k+2j})] \quad 1 \leq k \leq q-j-1 \Rightarrow$
$\Rightarrow 0 = [\beta^j_{2k-1}.Y_{2k+2j-1}, Y_{2k+2j}] + [Y_{2k-1}, \beta^j_{2k+2j}.Y_{2k}] \Rightarrow 0 = (\beta^j_{2k-1} + \beta^j_{2k+2j}).X_3 \Rightarrow$
$\Rightarrow \beta^j_{2k+2j} = -\beta^j_{2k-1} \quad 1 \leq k \leq q-j-1.$

En consecuencia, se verifica que

$$\begin{cases} d_j(X_0) & = \bar{\alpha}^j_0.Y_{2q-2j+4} & \\ d_j(Y_{2k-1}) & = \beta^j_{2k-1}.Y_{2k+2j-1} & 1 \leq k \leq q-j-1 \\ d_j(Y_{2k+2j}) & = -\beta^j_{2k-1}.Y_{2k} & 1 \leq k \leq q-j-1 \\ d_j(Y_{2q-2k-3}) & = \beta^j_{2q-2k-3}.Y_{2q-2j+2k+6} & 1 \leq k \leq E(\frac{j-4}{2}) \\ d_j(Y_{2q-2j+2k+5}) & = \beta^j_{2q-2k-3}.Y_{2q-2k-2} & 1 \leq k \leq E(\frac{j-4}{2}) \\ d_j(Y_{2q-2j+3}) & = \bar{\alpha}^j_0.X_2 & \\ d_j(Y_{2q-2j+5}) & = \bar{\beta}^j_{2q-2j+5}.X_3 + \beta^j_{2q-3}.Y_{2q-2} & \\ (1-\delta_{j4}).d_j(Y_{2q-3}) & = (1-\delta_{j4}).\beta^j_{2q-3}.Y_{2q-2j+6} & \end{cases}$$

Cálculo de $d_j \quad q+3 \leq j \leq 2q$

Como $d_j(\mathfrak{g}_t) \subset \mathfrak{g}_{t+j} \quad \forall t$, se cumple que

$$d_j(Y_{2k-1}) = \beta^j_{2k-1}.Y_{4q-2j-2k+4} \quad 1 \leq k \leq 2q-j+1.$$

Al exigir que d_j sea derivación, se obtiene que

$d_j([Y_{2k-1}, Y_{4q-2j-2k+3}]) = [d_j(Y_{2k-1}), Y_{4q-2j-2k+3}] + [Y_{2k-1}, d_j(Y_{4q-2j-2k+3})] \quad 1 \leq k \leq 2q-j+1 \Rightarrow$
$\Rightarrow 0 = [\beta^j_{2k-1}.Y_{4q-2j-2k+4}, Y_{4q-2j-2k+3}] + [Y_{2k-1}, \beta^j_{4q-2j-2k+3}.Y_{2k}] =$
$= (-\beta^j_{2k-1} + \beta^j_{4q-2j-2k+3}).X_3 \Rightarrow \beta^j_{4q-2j-2k+3} = \beta^j_{2k-1} \quad 1 \leq k \leq 2q-j+1.$

□

Derivaciones de las álgebras de la familia $\mathfrak{g}_n^{2s}$, $1 \leq s \leq E(\frac{n-3}{2})$

Se designa por $\mathfrak{g}_n^{2s}$, $1 \leq s \leq E(\frac{n-3}{2})$, a la familia de álgebras de Lie $(n-3)$-filiformes, de dimensión n, de leyes

$$\mathfrak{g}_n^{2s}: \begin{array}{lcll} [X_0, X_i] & = & X_{i+1} & 1 \leq i \leq 2 \\ [X_1, Y_{n-4}] & = & X_3 & \\ [Y_{2k-1}, Y_{2k}] & = & X_3 & 1 \leq k \leq s-1 \end{array}$$

para cada $s \in \{1, 2, \ldots, E(\frac{n-3}{2})\}$.

Teorema 4.2. *Se verifica que*

$$dim(Der(\mathfrak{g}_n^{2s})) = \begin{cases} n^2 - 6n + 15 & \text{si} \quad s = 1 \\ \frac{4s^2+7s+7}{2} + (n-2s-3).(n-1) & \text{si} \quad s = \dot{2}+1 \quad s \geq 2 \\ \frac{4s^2+7s+8}{2} + (n-2s-3).(n-1) & \text{si} \quad s = \dot{2} \quad s \geq 2, \end{cases}$$

para $1 \leq s \leq E(\frac{n-3}{2})$.

Para cada s, la correspondiente álgebra se puede expresar como suma directa de dos álgebras:

$$\mathfrak{g}_n^{2s} = \mathfrak{h}_1^{2s} \oplus \mathfrak{h}_2^{2s}$$

donde

$$\begin{array}{l} \mathfrak{h}_1^{2s} =< X_0, X_1, X_2, X_3, Y_{n-4}, Y_1, \ldots, Y_{2s-3}, Y_{2s-2} > \\ \mathfrak{h}_2^{2s} =< Y_{2s-1}, Y_{2s}, \ldots, Y_{n-5} > . \end{array}$$

Se deduce que

$$Der(\mathfrak{g}_n^{2s}) = Der(\mathfrak{h}_1^{2s}) \oplus Der(\mathfrak{h}_2^{2s}) \oplus D(\mathfrak{h}_1^{2s}, \mathfrak{h}_2^{2s}) \oplus D(\mathfrak{h}_2^{2s}, \mathfrak{h}_1^{2s}).$$

Cálculo de $Der(\mathfrak{h}_1^{2s})$

Se considera la siguiente graduación de $\mathfrak{h}_1^{2s}$:

$\mathfrak{h}_1^{2s} =< Y_1 > \oplus < Y_3 > \oplus < Y_5 > \oplus \ldots \oplus < Y_{2s-5} > \oplus < Y_{2s-3} > \oplus$
$\oplus < X_0 > \oplus < X_1, Y_{n-4} > \oplus < X_2 > \oplus < X_3, Y_{2s-2} > \oplus < Y_{2s-4} > \oplus$
$\oplus < Y_{2s-6} > \oplus \ldots \oplus < Y_6 > \oplus < Y_4 > \oplus < Y_2 >$, donde

$$\begin{array}{lcll} \mathfrak{g}_{-k} & = & < Y_{-2k+2s-3} > & 0 \leq k \leq s-2 \\ \mathfrak{g}_k & = & < X_{k-1} > & k = 1, 3 \\ \mathfrak{g}_2 & = & < X_1, Y_{n-4} > & \\ \mathfrak{g}_4 & = & < X_3, Y_{2s-2} > & \\ \mathfrak{g}_k & = & < Y_{-2k+2s+6} > & 5 \leq k \leq s+2. \end{array}$$

Sea $\bar{d}_1 \in Der(\mathfrak{h}_1^{2s})$. Entonces

$$\bar{d}_1 = \sum_{i \in Z} d_i$$

donde $d_i \in Der(\mathfrak{h}_1^{2s})$ y $d_i(\mathfrak{g}_j) \subset \mathfrak{g}_{i+j}$, siendo $\mathfrak{g}_k = \{0\}$ para $k < -s+2$ y $k > s+2$.

Como $d_{2s}(\mathfrak{g}_{-s+2}) \subset \mathfrak{g}_{s+2}$ y $d_{-2s}(\mathfrak{g}_{s+2}) \subset \mathfrak{g}_{-s+2}$, se deduce que

$$d_i = 0 \quad i > 2s \ , \ i < -2s \Rightarrow \bar{d}_1 = \sum_{i=-2s}^{2s} d_i$$

Habrá que expresar cada d_i , $-2s \leq i \leq 2s$, como una combinación lineal de un cierto conjunto B_i , $-2s \leq i \leq 2s$, de derivaciones linealmente independientes de $\mathfrak{h}_1^{2s}$ cumpliéndose que

$$\bigcup_{i=-2s}^{2s} B_i$$

es una base de $Der(\mathfrak{h}_1^{2s})$ y, evidentemente,

$$dim(Der(\mathfrak{h}_1^{2s})) = \sum_{i=-2s}^{2s} dim(K < B_i >).$$

A continuación, se detallan las condiciones iniciales que deben satisfacer las d_i , $-2s \leq i \leq 2s$, y que se deducen de $d_i(\mathfrak{g}_j) \subset \mathfrak{g}_{i+j}$, como también las posteriores que resultan al exigir que cada d_i sea, efectivamente, una derivación.

Cálculo de d_{-j} $\quad s+3 \leq j \leq 2s$

Como $d_{-j}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-j}$ $\quad \forall t$, se cumple que

$$d_{-j}(Y_{2k}) = \beta^j_{2k}.Y_{4s-2j-2k+3} \quad 1 \leq k \leq 2s-j+1.$$

Al exigir que d_{-j} sea derivación, se obtiene que

$$d_{-j}([Y_{2k}, Y_{4s-2j-2k+4}]) = [d_{-j}(Y_{2k}), Y_{4s-2j-2k+4}] + [Y_{2k}, d_{-j}(Y_{4s-2j-2k+4})]$$
$$1 \leq k \leq 2s-j+1 \Rightarrow \beta^j_{2k} = \beta^j_{4s-2j-2k+4} \quad 1 \leq k \leq 2s-j+1.$$

Cálculo de d_{-j} $\quad 4 \leq j \leq s+2$

Como $d_{-j}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-j} \quad \forall t$, se cumple que

$$\begin{cases} d_{-j}(Y_{2k+2j-1}) &= \beta^j_{2k+2j-1}.Y_{2k-1} & 1 \leq k \leq s-j-2 \\ d_{-j}(Y_{2s-3}) &= \beta^j_{2s-3}.Y_{2s-2j-3} & \\ d_{-j}(X_0) &= \bar{\alpha}^j_0.Y_{2s-2j-1} & \\ d_{-j}(X_1) &= \bar{\alpha}^j_1.Y_{2s-2j+1} & \\ d_{-j}(Y_{n-4}) &= \beta^j_{n-4}.Y_{2s-2j+1} & \\ d_{-j}(Y_{2s-2}) &= \beta^j_{2s-2}.Y_{2s-2j+5} & \\ d_{-j}(Y_{2k}) &= \beta^j_{2k}.Y_{4s-2j-2k+3} & s-j+3 \leq k \leq s-2 \\ d_{-j}(Y_{2s-2j+4}) &= \bar{\beta}^j_{2s-2j+4}.X_0 & \\ d_{-j}(Y_{2s-2j+2}) &= \bar{\beta}^j_{2s-2j+2}.X_1 + \beta^j_{2s-2j+2}.Y_{n-4} & \\ d_{-j}(Y_{2s-2j}) &= \bar{\beta}^j_{2s-2j}.X_2 & \\ d_{-j}(Y_{2s-2j-2}) &= \bar{\beta}^j_{2s-2j-2}.X_3 + \beta^j_{2s-2j-2}.Y_{2s-2} & \\ d_{-j}(Y_{2k}) &= \beta^j_{2k}.Y_{2k+2j} & 1 \leq k \leq s-j-2. \end{cases}$$

Al exigir que d_{-j} sea derivación, se obtiene que

* $d_{-j}([X_0, Y_{2s-2j+2}]) = [d_{-j}(X_0), Y_{2s-2j+2}] + [X_0, d_{-j}(Y_{2s-2j+2})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2s-2j-1}, Y_{2s-2j+2}] + [X_0, \bar{\beta}^j_{2s-2j+2}.X_1 + \beta^j_{2s-2j+2}.Y_{n-4}] = \bar{\beta}^j_{2s-2j+2}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2s-2j+2} = 0 \Rightarrow d_{-j}(Y_{2s-2j+2}) = \beta^j_{2s-2j+2}.Y_{n-4}.$

* $d_{-j}([Y_{2s-2j+2}, Y_{n-4}]) = [d_{-j}(Y_{2s-2j+2}), Y_{n-4}] + [Y_{2s-2j+2}, d_{-j}(Y_{n-4})] \Rightarrow$
$\Rightarrow 0 = [\beta^j_{2s-2j+2}.Y_{n-4}, Y_{n-4}] + [Y_{2s-2j+2}, \beta^j_{n-4}.Y_{2s-2j+1}] \Rightarrow$
$\Rightarrow 0 = -\beta^j_{n-4}.X_3 \Rightarrow \beta^j_{n-4} = 0 \Rightarrow d_{-j}(Y_{n-4}) = 0.$

* $d_{-j}([X_1, Y_{2s-2j+2}]) = [d_{-j}(X_1), Y_{2s-2j+2}] + [X_1, d_{-j}(Y_{2s-2j+2})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2s-2j+1}, Y_{2s-2j+2}] + [X_1, \beta^j_{2s-2j+2}.Y_{n-4}] \Rightarrow 0 = (\bar{\alpha}^j_1 + \beta^j_{2s-2j+2}).X_3 \Rightarrow$
$\Rightarrow \beta^j_{2s-2j+2} = -\bar{\alpha}^j_1.$

* $d_{-j}([X_1, Y_{2s-2j+4}]) = [d_{-j}(X_1), Y_{2s-2j+4}] + [X_1, d_{-j}(Y_{2s-2j+4})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2s-2j+1}, Y_{2s-2j+4}] + [X_1, \bar{\beta}^j_{2s-2j+4}.X_0] \Rightarrow 0 = -\bar{\beta}^j_{2s-2j+4}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2s-2j+4} = 0 \Rightarrow d_{-j}(Y_{2s-2j+4}) = 0.$

* $d_{-j}([X_0, Y_{2s-2j}]) = [d_{-j}(X_0), Y_{2s-2j}] + [X_0, d_{-j}(Y_{2s-2j})] \Rightarrow \bar{\beta}^j_{2s-2j} = -\bar{\alpha}^j_0.$

$* \; d_{-j}([Y_{2k}, Y_{2k+2j-1}]) = [d_{-j}(Y_{2k}), Y_{2k+2j-1}] + [Y_{2k}, d_{-j}(Y_{2k+2j-1})] \quad 1 \leq k \leq s-j-2 \Rightarrow$
$\Rightarrow \beta^j_{2k+2j-1} = -\beta^j_{2k} \quad 1 \leq k \leq s-j-2.$

$* \; d_{-j}([Y_{2s-2j-2}, Y_{2s-3}]) = [d_{-j}(Y_{2s-2j-2}), Y_{2s-3}] + [Y_{2s-2j-2}, d_{-j}(Y_{2s-3})] \Rightarrow$
$\Rightarrow \beta^j_{2s-2j-2} = -\beta^j_{2s-3}.$

$* \; d_{-j}([Y_{2s-2j+2k+2}, Y_{2s-2k+2}]) = [d_{-j}(Y_{2s-2j+2k+2}), Y_{2s-2k+2}] + [Y_{2s-2j+2k+2}, d_{-j}(Y_{2s-2k+2}$
$2 \leq k \leq E(\frac{j}{2}) \Rightarrow \beta^j_{2s-2j+2k+2} = \beta^j_{2s-2k+2} \quad 2 \leq k \leq E(\frac{j}{2}).$

En consecuencia, se verifica que

$$\left\{ \begin{array}{lcll} d_{-j}(X_0) & = & \bar{\alpha}^j_0 . Y_{2s-2j-1} & \\ d_{-j}(X_1) & = & \bar{\alpha}^j_1 . Y_{2s-2j+1} & \\ d_{-j}(Y_{2k+2j-1}) & = & -\beta^j_{2k} . Y_{2k-1} & 1 \leq k \leq s-j-2 \\ d_{-j}(Y_{2k}) & = & \beta^j_{2k} . Y_{2k+2j} & 1 \leq k \leq s-j-2 \\ d_{-j}(Y_{2s-3}) & = & \beta^j_{2s-3} . Y_{2s-2j-3} & \\ d_{-j}(Y_{2s-2k+2}) & = & \beta^j_{2s-2k+2} . Y_{2s-2j+2k+1} & 2 \leq k \leq E(\frac{j}{2}) \\ d_{-j}(Y_{2s-2j+2k+2}) & = & \beta^j_{2s-2k+2} . Y_{2s-2k+1} & 2 \leq k \leq E(\frac{j}{2}) \\ d_{-j}(Y_{2s-2j+2}) & = & -\bar{\alpha}^j_1 . Y_{n-4} & \\ d_{-j}(Y_{2s-2j}) & = & -\bar{\alpha}^j_0 . X_2 & \\ d_{-j}(Y_{2s-2j-2}) & = & \bar{\beta}^j_{2s-2j-2} . X_3 - \beta^j_{2s-3} . Y_{2s-2}. & \end{array} \right.$$

Cálculo de $d_{-j}\quad 1 \le j \le 3$

Como $d_{-j}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-j}\quad \forall t$, se cumple que

$$\begin{cases} d_{-j}(Y_{2k+2j-1}) & = & \beta^j_{2k+2j-1}.Y_{2k-1} & 1 \le k \le s-j-2 \\ d_{-j}(Y_{2s-3}) & = & \beta^j_{2s-3}.Y_{2s-2j-3} & \\ d_{-j}(X_0) & = & \bar{\alpha}^j_0.Y_{2s-2j-1} & \\ d_{-j}(X_1) & = & \alpha^j_1\delta_{j1}.X_0 + \bar{\alpha}^j_1(1-\delta_{j1}).Y_{2s-2j+1} & \\ d_{-j}(Y_{n-4}) & = & \bar{\beta}^j_{n-4}\delta_{j1}.X_0 + \beta^j_{n-4}(1-\delta_{j1}).Y_{2s-2j+1} & \\ d_{-j}(Y_{2s-2j+4}) & = & \bar{\beta}^j_{2s-2j+4}.X_0 & \\ d_{-j}(Y_{2s-2j+2}) & = & \bar{\beta}^j_{2s-2j+2}.X_1 + \beta^j_{2s-2j+2}.Y_{n-4} & \\ d_{-j}(Y_{2s-2j}) & = & \bar{\beta}^j_{2s-2j}.X_2 & \\ d_{-j}(Y_{2s-2j-2}) & = & \bar{\beta}^j_{2s-2j-2}.X_3 + \beta^j_{2s-2j-2}.Y_{2s-2} & \\ d_{-j}(Y_{2k}) & = & \beta^j_{2k}.Y_{2k+2j} & 1 \le k \le s-j-2. \end{cases}$$

Al exigir que d_{-j} sea derivación, se obtiene que

$* \ d_{-j}([X_1, X_2]) = [d_{-j}(X_1), X_2] + [X_1, d_{-j}(X_2)] \Rightarrow$
$\Rightarrow 0 = [\alpha^j_1\delta_{j1}.X_0 + \bar{\alpha}^j_1(1-\delta_{j1}).Y_{2s-2j+1}, X_2] + [X_1, 0] = \alpha^j_1\delta_{j1}.X_3 \Rightarrow$
$\Rightarrow 0 = \alpha^j_1\delta_{j1} \Rightarrow$
$j = 1 \Rightarrow \delta_{j1} = 1 \Rightarrow \alpha^j_1 = 0 \Rightarrow d_{-j}(X_1) = d_{-1}(X_1) = 0$
$j = 2,3 \Rightarrow\Rightarrow \delta_{j1} = 0 \Rightarrow d_{-j}(X_1) = \bar{\alpha}^j_1.Y_{2s-2j+1} \Rightarrow$
$\Rightarrow d_{-j}(X_1) = \bar{\alpha}^j_1(1-\delta_{j1}).Y_{2s-2j+1}\quad 1 \le j \le 3.$

$* \ d_{-j}([X_1, Y_{2s-2j+2}]) = [d_{-j}(X_1), Y_{2s-2j+2}] + [X_1, d_{-j}(Y_{2s-2j+2})]\quad 2 \le j \le 3 \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2s-2j+1}, Y_{2s-2j+2}] + [X_1, \bar{\beta}^j_{2s-2j+2}.X_1 + \beta^j_{2s-2j+2}.Y_{n-4}] \Rightarrow$
$\Rightarrow 0 = (\bar{\alpha}^j_1 + \beta^j_{2s-2j+2}).X_3 \Rightarrow \beta^j_{2s-2j+2} = -\bar{\alpha}^j_1 \Rightarrow$
$\Rightarrow d_{-j}(Y_{2s-2j+2}) = \bar{\beta}^j_{2s-2j+2}.X_1 - \bar{\alpha}^j_1.Y_{n-4}\quad 2 \le j \le 3.$

$* \ d_{-j}([Y_{2k}, Y_{2k+2j-1}]) = [d_{-j}(Y_{2k}), Y_{2k+2j-1}] + [Y_{2k}, d_{-j}(Y_{2k+2j-1})]\quad 1 \le k \le s-j-2 \Rightarrow$
$\Rightarrow \beta^j_{2k+2j-1} = -\beta^j_{2k}\quad 1 \le k \le s-j-2.$

$* \ d_{-j}([X_0, Y_{2s-2j+2}]) = [d_{-j}(X_0), Y_{2s-2j+2}] + [X_0, d_{-j}(Y_{2s-2j+2})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2s-2j-1}, Y_{2s-2j+2}] + [X_0, \bar{\beta}^j_{2s-2j+2}.X_1 - \bar{\alpha}^j_1.Y_{n-4}] \Rightarrow 0 = \bar{\beta}^j_{2s-2j+2}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2s-2j+2} = 0 \Rightarrow d_{-j}(Y_{2s-2j+2}) = -\bar{\alpha}^j_1.Y_{n-4}.$

$* \ d_{-j}([X_0, Y_{2s-2j}]) = [d_{-j}(X_0), Y_{2s-2j}] + [X_0, d_{-j}(Y_{2s-2j})] \Rightarrow \bar{\beta}^j_{2s-2j} = -\bar{\alpha}^j_0.$

$* \ d_{-j}([Y_{2s-2j-2}, Y_{2s-3}]) = [d_{-j}(Y_{2s-2j-2}), Y_{2s-3}] + [Y_{2s-2j-2}, d_{-j}(Y_{2s-3})] \Rightarrow$
$\Rightarrow \beta^j_{2s-2j-2} = -\beta^j_{2s-3}.$

$* \ d_{-j}([X_1, Y_{2s-2j+4}]) = [d_{-j}(X_1), Y_{2s-2j+4}] + [X_1, d_{-j}(Y_{2s-2j+4})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1(1-\delta_{j1}).Y_{2s-2j+1}, Y_{2s-2j+4}] + [X_1, \bar{\beta}^j_{2s-2j+4}.X_0] \Rightarrow 0 = -\bar{\beta}^j_{2s-2j+4}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2s-2j+4} = 0 \Rightarrow d_{-j}(Y_{2s-2j+4}) = 0.$

$* \ d_{-j}([X_2, Y_{n-4}]) = [d_{-j}(X_2), Y_{n-4}] + [X_2, d_{-j}(Y_{n-4})] \Rightarrow$
$\Rightarrow 0 = [0, Y_{n-4}] + [X_2, \bar{\beta}^j_{n-4}\delta_{j1}.X_0 + \beta^j_{n-4}(1-\delta_{j1}).Y_{2s-2j+1}] \Rightarrow$
$\Rightarrow 0 = -\bar{\beta}^j_{n-4}\delta_{j1}.X_3 \Rightarrow \bar{\beta}^j_{n-4}\delta_{j1} = 0 \Rightarrow d_{-j}(Y_{n-4}) = \beta^j_{n-4}(1-\delta_{j1}).Y_{2s-2j+1}.$

$* \ d_{-j}([Y_{2s-2j+2}, Y_{n-4}]) = [d_{-j}(Y_{2s-2j+2}), Y_{n-4}] + [Y_{2s-2j+2}, d_{-j}(Y_{n-4})] \Rightarrow$
$\Rightarrow 0 = [-\bar{\alpha}^j_1.Y_{n-4}, Y_{n-4}] + [Y_{2s-2j+2}, \beta^j_{n-4}(1-\delta_{j1}).Y_{2s-2j+1}] \Rightarrow$
$\Rightarrow 0 = -\beta^j_{n-4}(1-\delta_{j1}).X_3 \Rightarrow \beta^j_{n-4}(1-\delta_{j1}) = 0 \Rightarrow d_{-j}(Y_{n-4}) = 0.$

En consecuencia,se verifica que

$$\left\{\begin{array}{lcll} d_{-j}(X_0) & = & \bar{\alpha}^j_0.Y_{2s-2j-1} & \\ d_{-j}(X_1) & = & \bar{\alpha}^j_1(1-\delta_{j1}).Y_{2s-2j+1} & \\ d_{-j}(Y_{2k+2j-1}) & = & -\beta^j_{2k}.Y_{2k-1} & 1 \le k \le s-j-2 \\ d_{-j}(Y_{2k}) & = & \beta^j_{2k}.Y_{2k+2j} & 1 \le k \le s-j-2 \\ d_{-j}(Y_{2s-2j+2}) & = & -\bar{\alpha}^j_1.Y_{n-4} & \\ d_{-j}(Y_{2s-2j}) & = & -\bar{\alpha}^j_0.X_2 & \\ d_{-j}(Y_{2s-2j-2}) & = & \bar{\beta}^j_{2s-2j-2}.X_3 - \beta^j_{2s-3}.Y_{2s-2} & \\ d_{-j}(Y_{2s-3}) & = & \beta^j_{2s-3}.Y_{2s-2j-3}. & \end{array}\right.$$

Cálculo de d_0

Como $d_0(\mathfrak{g}_t) \subset \mathfrak{g}_t \quad \forall t$, se cumple que

$$\begin{cases} d_0(Y_{2k-1}) &= \beta^0_{2k-1}.Y_{2k-1} & 1 \leq k \leq s-1 \\ d_0(X_0) &= \alpha^0_0.X_0 \\ d_0(X_1) &= \alpha^0_1.X_1 + \bar{\alpha}^0_1.Y_{n-4} \\ d_0(Y_{n-4}) &= \bar{\beta}^0_{n-4}.X_1 + \beta^0_{n-4}.Y_{n-4} \\ d_0(X_2) &= \alpha^0_2.X_2 \\ d_0(X_3) &= \alpha^0_3.X_3 \\ d_0(Y_{2k}) &= \beta^0_{2k}.Y_{2k} & 1 \leq k \leq s-2 \\ d_0(Y_{2s-2}) &= \bar{\beta}^0_{2s-2}.X_3 + \beta^0_{2s-2}.Y_{2s-2}. \end{cases}$$

Al exigir que d_0 sea derivación, se obtiene que

$* \ d_0([X_0, X_1]) = [d_0(X_0), X_1] + [X_0, d_0(X_1)] \Rightarrow$
$\Rightarrow \alpha^0_2.X_2 = d_0(X_2) = [\alpha^0_0.X_0, X_1] + [X_0, \alpha^0_1.X_1 + \bar{\alpha}^0_1.Y_{n-4}] = (\alpha^0_0 + \alpha^0_1).X_2 \Rightarrow$
$\Rightarrow \alpha^0_2 = \alpha^0_0 + \alpha^0_1.$

$* \ d_0([X_0, X_2]) = [d_0(X_0), X_2] + [X_0, d_0(X_2)] \Rightarrow \alpha^0_3 = 2\alpha^0_0 + \alpha^0_1.$

$* \ d_0([X_0, Y_{n-4}]) = [d_0(X_0), Y_{n-4}] + [X_0, d_0(Y_{n-4})] \Rightarrow$
$\Rightarrow 0 = [\alpha^0_0.X_0, Y_{n-4}] + [X_0, \bar{\beta}^0_{n-4}.X_1 + \beta^0_{n-4}.Y_{n-4}] \Rightarrow$
$\Rightarrow 0 = \bar{\beta}^0_{n-4}.X_2 \Rightarrow \bar{\beta}^0_{n-4} = 0 \Rightarrow d_0(Y_{n-4}) = \beta^0_{n-4}.Y_{n-4}.$

$* \ d_0([X_1, Y_{n-4}]) = [d_0(X_1), Y_{n-4}] + [X_1, d_0(Y_{n-4})] \Rightarrow$
$\Rightarrow \alpha^0_3.X_3 = d_0(X_3) = [\alpha^0_1.X_1 + \bar{\alpha}^0_1.Y_{n-4}, Y_{n-4}] + [X_1, \beta^0_{n-4}.Y_{n-4}] \Rightarrow$
$\Rightarrow \alpha^0_3.X_3 = (\alpha^0_1 + \beta^0_{n-4}).X_3 \Rightarrow \alpha^0_3 = \alpha^0_1 + \beta^0_{n-4} \Rightarrow$
$\Rightarrow 2\alpha^0_0 + \alpha^0_1 = \alpha^0_1 + \beta^0_{n-4} \Rightarrow \beta^0_{n-4} = 2\alpha^0_0.$

$* \ d_0([Y_{2s-3}, Y_{2s-2}]) = [d_0(Y_{2s-3}), Y_{2s-2}] + [Y_{2s-3}, d_0(Y_{2s-2})] \Rightarrow$
$\Rightarrow \beta^0_{2s-3} = 2\alpha^0_0 + \alpha^0_1 - \beta^0_{2s-2}.$

$* \ d_0([Y_{2k-1}, Y_{2k}]) = [d_0(Y_{2k-1}), Y_{2k}] + [Y_{2k-1}, d_0(Y_{2k})] \quad 1 \leq k \leq s-2 \Rightarrow$
$\Rightarrow \beta^0_{2k-1} = 2\alpha^0_0 + \alpha^0_1 - \beta^0_{2k} \quad 1 \leq k \leq s-2.$

En consecuencia, se verifica que

$$\begin{cases} d_0(X_0) &= \alpha_0^0.X_0 \\ d_0(X_1) &= \alpha_1^0.X_1 + \bar{\alpha}_1^0.Y_{n-4} \\ d_0(X_2) &= (\alpha_0^0 + \alpha_1^0).X_2 \\ d_0(X_3) &= (2\alpha_0^0 + \alpha_1^0).X_3 \\ d_0(Y_{2k-1}) &= (2\alpha_0^0 + \alpha_1^0 - \beta_{2k}^0).Y_{2k-1} \qquad 1 \leq k \leq s-1 \\ d_0(Y_{2k}) &= \beta_{2k}^0.Y_{2k} \qquad 1 \leq k \leq s-2 \\ d_0(Y_{2s-2}) &= \bar{\beta}_{2s-2}^0.X_3 + \beta_{2s-2}^0.Y_{2s-2} \\ d_0(Y_{n-4}) &= 2\alpha_0^0.Y_{n-4}. \end{cases}$$

Cálculo de d_j $\quad 1 \leq j \leq 3$

Como $d_j(\mathfrak{g}_t) \subset \mathfrak{g}_{t+j} \quad \forall t$, se cumple que

$$\begin{cases} d_j(Y_{2k-1}) &= \beta_{2k-1}^j.Y_{2k+2j-1} \qquad 1 \leq k \leq s-j-1 \\ d_j(Y_{2s-2j-1}) &= \bar{\beta}_{2s-2j-1}^j.X_0 \\ d_j(Y_{2s-2j+1}) &= \bar{\beta}_{2s-2j+1}^j.X_1 + \beta_{2s-2j+1}^j.Y_{n-4} \\ d_j(Y_{2s-2j+3}) &= \bar{\beta}_{2s-2j+3}^j.X_2 \\ d_j(X_0) &= \alpha_0^j.\delta_{j1}.X_1 + \bar{\alpha}_0^j.\delta_{j1}.Y_{n-4} + \alpha_0^j.\delta_{j2}.X_2 + \alpha_0^j.\delta_{j3}.X_3 + \bar{\alpha}_0^j.\delta_{j3}.Y_{2s-2} \\ d_j(X_1) &= \alpha_1^j.\delta_{j1}.X_2 + \alpha_1^j.\delta_{j2}.X_3 + \bar{\alpha}_1^j.\delta_{j2}.Y_{2s-2} + \bar{\alpha}_1^j.\delta_{j3}.Y_{2s-4} \\ d_j(Y_{n-4}) &= \bar{\beta}_{n-4}^j.\delta_{j1}.X_2 + \bar{\beta}_{n-4}^j.\delta_{j2}.X_3 + \beta_{n-4}^j.\delta_{j2}.Y_{2s-2} + \beta_{n-4}^j.\delta_{j3}.Y_{2s-4} \\ d_j(X_2) &= \alpha_2^j.\delta_{j1}.X_3 \\ d_j(Y_{2k+2j}) &= \beta_{2k+2j}^j.Y_{2k} \qquad 1 \leq k \leq s-j-1. \end{cases}$$

Al exigir que d_j sea derivación, se obtiene que

$* \ d_j([X_0, X_1]) = [d_j(X_0), X_1] + [X_0, d_j(X_1)] \Rightarrow$
$\Rightarrow d_j(X_2) = [\alpha_0^j.\delta_{j1}.X_1 + \bar{\alpha}_0^j.\delta_{j1}.Y_{n-4} + \alpha_0^j.\delta_{j2}.X_2 + \alpha_0^j.\delta_{j3}.X_3 + \bar{\alpha}_0^j.\delta_{j3}.Y_{2s-2}, X_1] +$
$+ [X_0, \alpha_1^j.\delta_{j1}.X_2 + \alpha_1^j.\delta_{j2}.X_3 + \bar{\alpha}_1^j.\delta_{j2}.Y_{2s-2} + \bar{\alpha}_1^j.\delta_{j3}.Y_{2s-4}] \Rightarrow$
$\Rightarrow \alpha_2^j.\delta_{j1}.X_3 = (-\bar{\alpha}_0^j.\delta_{j1} + \alpha_1^j.\delta_{j1}).X_3 \Rightarrow \alpha_2^j.\delta_{j1} = -\bar{\alpha}_0^j.\delta_{j1} + \alpha_1^j.\delta_{j1} \Rightarrow$
$\Rightarrow d_j(X_2) = (-\bar{\alpha}_0^j + \alpha_1^j).\delta_{j1}.X_3.$

$* \ d_j([X_1, Y_{2s-2j-1}]) = [d_j(X_1), Y_{2s-2j-1}] + [X_1, d_j(Y_{2s-2j-1})] \Rightarrow$
$\Rightarrow \bar{\beta}_{2s-2j-1}^j = 0 \Rightarrow d_j(Y_{2s-2j-1}) = 0.$

$* \ d_j([X_0, Y_{2s-2j+1}]) = [d_j(X_0), Y_{2s-2j+1}] + [X_0, d_j(Y_{2s-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\alpha_0^j.\delta_{j1}.X_1 + \bar{\alpha}_0^j.\delta_{j1}.Y_{n-4} + \alpha_0^j.\delta_{j2}.X_2 + \alpha_0^j.\delta_{j3}.X_3 + \bar{\alpha}_0^j.\delta_{j3}.Y_{2s-2}, Y_{2s-2j+1}]+$
$+ [X_0, \bar{\beta}_{2s-2j+1}^j.X_1 + \beta_{2s-2j+1}^j.Y_{n-4}] \Rightarrow 0 = \bar{\beta}_{2s-2j+1}^j.X_2 \Rightarrow \bar{\beta}_{2s-2j+1}^j = 0 \Rightarrow$
$\Rightarrow d_j(Y_{2s-2j+1}) = \beta_{2s-2j+1}^j.Y_{n-4}.$

$* \ d_j([X_0, Y_{2s-2j+3}]) = [d_j(X_0), Y_{2s-2j+3}] + [X_0, d_j(Y_{2s-2j+3})] \Rightarrow$
$\Rightarrow 0 = [\alpha_0^j.\delta_{j1}.X_1 + \bar{\alpha}_0^j.\delta_{j1}.Y_{n-4} + \alpha_0^j.\delta_{j2}.X_2 + \alpha_0^j.\delta_{j3}.X_3 + \bar{\alpha}_0^j.\delta_{j3}.Y_{2s-2}, Y_{2s-2j+3}]+$
$+ [X_0, \bar{\beta}_{2s-2j+3}^j.X_2] \Rightarrow 0 = (-\bar{\alpha}_0^j.\delta_{j3} + \bar{\beta}_{2s-2j+3}^j).X_3 \Rightarrow \bar{\beta}_{2s-2j+3}^j = \bar{\alpha}_0^j.\delta_{j3}.$

$* \ d_j([X_1, Y_{2s-2j+1}]) = [d_j(X_1), Y_{2s-2j+1}] + [X_1, d_j(Y_{2s-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\alpha_1^j.\delta_{j1}.X_2 + \alpha_1^j.\delta_{j2}.X_3 + \bar{\alpha}_1^j.\delta_{j2}.Y_{2s-2} + \bar{\alpha}_1^j.\delta_{j3}.Y_{2s-4}, Y_{2s-2j+1}]+$
$+ [X_1, \beta_{2s-2j+1}^j.Y_{n-4}] \Rightarrow 0 = (-\bar{\alpha}_1^j.\delta_{j2} - \bar{\alpha}_1^j.\delta_{j3} + \beta_{2s-2j+1}^j).X_3 \Rightarrow$
$\Rightarrow \beta_{2s-2j+1}^j = \bar{\alpha}_1^j.\delta_{j2} + \bar{\alpha}_1^j.\delta_{j3} \Rightarrow d_j(Y_{2s-2j+1}) = (\bar{\alpha}_1^j.\delta_{j2} + \bar{\alpha}_1^j.\delta_{j3}).Y_{n-4}.$

$* \ d_j([Y_{2k-1}, Y_{2k+2j}]) = [d_j(Y_{2k-1}), Y_{2k+2j}] + [Y_{2k-1}, d_j(Y_{2k+2j})] \quad 1 \le k \le s-j-1 \Rightarrow$
$\Rightarrow \beta_{2k+2j}^j = -\beta_{2k-1}^j \quad 1 \le k \le s-j-1.$

$* \ d_j([X_0, Y_{n-4}]) = [d_j(X_0), Y_{n-4}] + [X_0, d_j(Y_{n-4})] \Rightarrow$
$\Rightarrow 0 = [\alpha_0^j.\delta_{j1}.X_1 + \bar{\alpha}_0^j.\delta_{j1}.Y_{n-4} + \alpha_0^j.\delta_{j2}.X_2 + \alpha_0^j.\delta_{j3}.X_3 + \bar{\alpha}_0^j.\delta_{j3}.Y_{2s-2}, Y_{n-4}]+$
$+ [X_0, \bar{\beta}_{n-4}^j.\delta_{j1}.X_2 + \bar{\beta}_{n-4}^j.\delta_{j2}.X_3 + \beta_{n-4}^j.\delta_{j2}.Y_{2s-2} + \beta_{n-4}^j.\delta_{j3}.Y_{2s-4}] \Rightarrow$
$\Rightarrow 0 = (\alpha_0^j.\delta_{j1} + \bar{\beta}_{n-4}^j.\delta_{j1}).X_3 \Rightarrow \bar{\beta}_{n-4}^j.\delta_{j1} = -\alpha_0^j.\delta_{j1}.$

$* \ d_j([Y_{2s-2j+1}, Y_{n-4}]) = [d_j(Y_{2s-2j+1}), Y_{n-4}] + [Y_{2s-2j+1}, d_j(Y_{n-4})] \Rightarrow$
$\Rightarrow 0 = [(\bar{\alpha}_1^j.\delta_{j2} + \bar{\alpha}_1^j.\delta_{j3}).Y_{n-4}, Y_{n-4}] + [Y_{2s-2j+1}, -\alpha_0^j.\delta_{j1}.X_2 + \bar{\beta}_{n-4}^j.\delta_{j2}.X_3+$
$+ \beta_{n-4}^j.\delta_{j2}.Y_{2s-2} + \beta_{n-4}^j.\delta_{j3}.Y_{2s-4}] \Rightarrow 0 = (\beta_{n-4}^j.\delta_{j2} + \beta_{n-4}^j.\delta_{j3}).X_3 \Rightarrow$
$\Rightarrow \beta_{n-4}^j.\delta_{j2} + \beta_{n-4}^j.\delta_{j3} = 0 \Rightarrow \beta_{n-4}^j.\delta_{j2} = 0 \quad , \quad \beta_{n-4}^j.\delta_{j3} = 0 \Rightarrow$
$\Rightarrow d_j(Y_{n-4}) = -\alpha_0^j.\delta_{j1}.X_2 + \bar{\beta}_{n-4}^j.\delta_{j2}.X_3.$

En consecuencia, se verifica que

$$\begin{cases} d_j(X_0) &= \alpha_0^j.\delta_{j1}.X_1 + \bar{\alpha}_0^j.\delta_{j1}.Y_{n-4} + \alpha_0^j.\delta_{j2}.X_2 + \alpha_0^j.\delta_{j3}.X_3 + \bar{\alpha}_0^j.\delta_{j3}.Y_{2s-2} \\ d_j(X_1) &= \alpha_1^j.\delta_{j1}.X_2 + \alpha_1^j.\delta_{j2}.X_3 + \bar{\alpha}_1^j.\delta_{j2}.Y_{2s-2} + \bar{\alpha}_1^j.\delta_{j3}.Y_{2s-4} \\ d_j(X_2) &= (-\bar{\alpha}_0^j + \alpha_1^j).\delta_{j1}.X_3 \\ d_j(Y_{2k-1}) &= \beta_{2k-1}^j.Y_{2k+2j-1} \qquad 1 \le k \le s-j-1 \\ d_j(Y_{2k+2j}) &= -\beta_{2k-1}^j.Y_{2k} \qquad 1 \le k \le s-j-1 \\ d_j(Y_{2s-2j+1}) &= (\bar{\alpha}_1^j.\delta_{j2} + \bar{\alpha}_1^j.\delta_{j3}).Y_{n-4} \\ d_j(Y_{2s-2j+3}) &= \bar{\alpha}_0^j.\delta_{j3}.X_2 \\ d_j(Y_{n-4}) &= -\alpha_0^j.\delta_{j1}.X_2 + \bar{\beta}_{n-4}^j.\delta_{j2}.X_3. \end{cases}$$

Cálculo de d_j $\quad 4 \leq j \leq s+2$

Como $d_j(\mathfrak{g}_t) \subset \mathfrak{g}_{t+j} \quad \forall t$, se cumple que

$$\begin{cases} d_j(Y_{2k+2j}) & = & \beta^j_{2k+2j}.Y_{2k} & 1 \leq k \leq s-j-1 \\ d_j(X_1) & = & \bar{\alpha}^j_1.Y_{2s-2j+2} & \\ d_j(Y_{n-4}) & = & \beta^j_{n-4}.Y_{2s-2j+2} & \\ d_j(X_0) & = & \bar{\alpha}^j_0.Y_{2s-2j+4} & \\ d_j(Y_{2s-2j+2k+5}) & = & \beta^j_{2s-2j+2k+5}.Y_{2s-2k-2} & 1 \leq k \leq j-5 \\ d_j(Y_{2s-2j+5}) & = & \bar{\beta}^j_{2s-2j+5}.X_3 + \beta^j_{2s-2j+5}.Y_{2s-2} & \\ d_j(Y_{2s-2j+3}) & = & \bar{\beta}^j_{2s-2j+3}.X_2 & \\ d_j(Y_{2s-2j+1}) & = & \bar{\beta}^j_{2s-2j+1}.X_1 + \beta^j_{2s-2j+1}.Y_{n-4} & \\ d_j(Y_{2s-2j-1}) & = & \bar{\beta}^j_{2s-2j-1}.X_0 & \\ d_j(Y_{2k-1}) & = & \beta^j_{2k-1}.Y_{2k+2j-1} & 1 \leq k \leq s-j-1 \\ (1-\delta_{j4}).d_j(Y_{2s-3}) & = & (1-\delta_{j4}).\beta^j_{2s-3}.Y_{2s-2j+6}. & \end{cases}$$

Al exigir que d_j sea derivación, se obtiene que

$*\ d_j([X_1, Y_{2s-2j+1}]) = [d_j(X_1), Y_{2s-2j+1}] + [X_1, d_j(Y_{2s-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2s-2j+2}, Y_{2s-2j+1}] + [X_1, \bar{\beta}^j_{2s-2j+1}.X_1 + \beta^j_{2s-2j+1}.Y_{n-4}] \Rightarrow$
$\Rightarrow 0 = (-\bar{\alpha}^j_1 + \beta^j_{2s-2j+1}).X_3 \Rightarrow \beta^j_{2s-2j+1} = \bar{\alpha}^j_1.$

$*\ d_j([X_1, Y_{2s-2j-1}]) = [d_j(X_1), Y_{2s-2j-1}] + [X_1, d_j(Y_{2s-2j-1})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{2s-2j+2}, Y_{2s-2j-1}] + [X_1, \bar{\beta}^j_{2s-2j-1}.X_0] \Rightarrow 0 = -\bar{\beta}^j_{2s-2j-1}.X_2 \Rightarrow \bar{\beta}^j_{2s-2j-1} = 0 \Rightarrow$
$\Rightarrow d_j(Y_{2s-2j-1}) = 0.$

$*\ d_j([X_0, Y_{2s-2j+1}]) = [d_j(X_0), Y_{2s-2j+1}] + [X_0, d_j(Y_{2s-2j+1})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_0.Y_{2s-2j+4}, Y_{2s-2j+1}] + [X_0, \bar{\beta}^j_{2s-2j+1}.X_1 + \bar{\alpha}^j_1.Y_{n-4}] \Rightarrow 0 = \bar{\beta}^j_{2s-2j+1}.X_2 \Rightarrow$
$\Rightarrow \bar{\beta}^j_{2s-2j+1} = 0 \Rightarrow d_j(Y_{2s-2j+1}) = \bar{\alpha}^j_1.Y_{n-4}.$

$*\ d_j([X_0, Y_{2s-2j+3}]) = [d_j(X_0), Y_{2s-2j+3}] + [X_0, d_j(Y_{2s-2j+3})] \Rightarrow \bar{\beta}^j_{2s-2j+3} = \bar{\alpha}^j_0.$

$*\ d_j([Y_{2s-2j+5}, Y_{2s-3}]) = [d_j(Y_{2s-2j+5}), Y_{2s-3}] + [Y_{2s-2j+5}, d_j(Y_{2s-3})] \quad j > 4 \Rightarrow$
$\Rightarrow \beta^j_{2s-2j+5} = \beta^j_{2s-3} \quad (j \geq 4).$

$*\ d_j([Y_{2s-2j+2k+5}, Y_{2s-2k-3}]) = [d_j(Y_{2s-2j+2k+5}), Y_{2s-2k-3}] + [Y_{2s-2j+2k+5}, d_j(Y_{2s-2k-3})]$
$1 \leq k \leq E(\frac{j-4}{2}) \Rightarrow \beta^j_{2s-2j+2k+5} = \beta^j_{2s-2k-3} \quad 1 \leq k \leq E(\frac{j-4}{2}).$

$* \ d_j([Y_{2k-1}, Y_{2k+2j}]) = [d_j(Y_{2k-1}), Y_{2k+2j}] + [Y_{2k-1}, d_j(Y_{2k+2j})] \quad 1 \leq k \leq s-j-1 \Rightarrow$
$\Rightarrow \beta^j_{2k+2j} = -\beta^j_{2k-1} \quad 1 \leq k \leq s-j-1.$

$* \ d_j([Y_{2s-2j+1}, Y_{n-4}]) = [d_j(Y_{2s-2j+1}), Y_{n-4}] + [Y_{2s-2j+1}, d_j(Y_{n-4})] \Rightarrow$
$\Rightarrow 0 = [\bar{\alpha}^j_1.Y_{n-4}, Y_{n-4}] + [Y_{2s-2j+1}, \beta^j_{n-4}.Y_{2s-2j+2}] \Rightarrow$
$\Rightarrow 0 = \beta^j_{n-4}.X_3 \Rightarrow \beta^j_{n-4} = 0 \Rightarrow d_j(Y_{n-4}) = 0.$

En consecuencia, se verifica que

$$\begin{cases} d_j(X_0) &= \bar{\alpha}^j_0.Y_{2s-2j+4} & \\ d_j(X_1) &= \bar{\alpha}^j_1.Y_{2s-2j+2} & \\ d_j(Y_{2k-1}) &= \beta^j_{2k-1}.Y_{2k+2j-1} & 1 \leq k \leq s-j-1 \\ d_j(Y_{2k+2j}) &= -\beta^j_{2k-1}.Y_{2k} & 1 \leq k \leq s-j-1 \\ d_j(Y_{2s-2k-3}) &= \beta^j_{2s-2k-3}.Y_{2s-2j+2k+6} & 1 \leq k \leq E(\frac{j-4}{2}) \\ d_j(Y_{2s-2j+2k+5}) &= \beta^j_{2s-2k-3}.Y_{2s-2k-2} & 1 \leq k \leq E(\frac{j-4}{2}) \\ d_j(Y_{2s-2j+1}) &= \bar{\alpha}^j_1.Y_{n-4} & \\ d_j(Y_{2s-2j+3}) &= \bar{\alpha}^j_0.X_2 & \\ d_j(Y_{2s-2j+5}) &= \bar{\beta}^j_{2s-2j+5}.X_3 + \beta^j_{2s-3}.Y_{2s-2} & \\ (1-\delta_{j4}).d_j(Y_{2s-3}) &= (1-\delta_{j4}).\beta^j_{2s-3}.Y_{2s-2j+6}. & \end{cases}$$

Cálculo de d_j $\quad s+3 \leq j \leq 2s$

Como $d_j(\mathfrak{g}_t) \subset \mathfrak{g}_{t+j} \quad \forall t$, se cumple que

$$d_j(Y_{2k-1}) = \beta^j_{2k-1}.Y_{4s-2j-2k+4} \quad 1 \leq k \leq 2s-j+1.$$

Al exigir que d_j sea derivación, se verifica que

$d_j([Y_{2k-1}, Y_{4s-2j-2k+3}]) = [d_j(Y_{2k-1}), Y_{4s-2j-2k+3}] + [Y_{2k-1}, d_j(Y_{4s-2j-2k+3})]$
$1 \leq k \leq 2s-j+1 \Rightarrow \beta^j_{4s-2j-2k+3} = \beta^j_{2k-1} \quad 1 \leq k \leq 2s-j+1.$

□

Derivaciones del álgebra $\mathfrak{g}_n^{n-2}$

Se designa por $\mathfrak{g}_n^{n-2}$ al álgebra de Lie $(n-3)$-filiforme de dimensión n y de ley

$$\mathfrak{g}_n^{n-2}: \begin{array}{rcl} [X_0, X_i] & = & X_{i+1} \quad 1 \leq i \leq 2 \\ [X_1, X_2] & = & Y_{n-4} \end{array}$$

Teorema 4.3. *Se verifica que*

$$dim(Der(\mathfrak{g}_n^{n-2})) = n^2 - 6n + 15.$$

Dicha álgebra se puede expresar como suma directa de dos álgebras:

$$\mathfrak{g}_n^{n-2} = \mathfrak{h}_1^{n-2} \oplus \mathfrak{h}_2^{n-2}$$

donde

$$\mathfrak{h}_1^{n-2} =< X_0, X_1, X_2, X_3, Y_{n-4} > \text{ y } \mathfrak{h}_2^{n-2} =< Y_1, Y_2, \ldots, Y_{n-5} > .$$

Se deduce que

$$Der(\mathfrak{g}_n^{n-2}) = Der(\mathfrak{h}_1^{n-2}) \oplus Der(\mathfrak{h}_2^{n-2}) \oplus D(\mathfrak{h}_1^{n-2}, \mathfrak{h}_2^{n-2}) \oplus D(\mathfrak{h}_2^{n-2}, \mathfrak{h}_1^{n-2}).$$

Cálculo de $Der(\mathfrak{h}_1^{n-2})$

Se considera la siguiente graduación de $\mathfrak{h}_1^{n-2}$:

$\mathfrak{h}_1^{n-2} =< X_0 > \oplus < X_1 > \oplus < X_2 > \oplus < X_3 > \oplus < Y_{n-4} >$, donde

$$\begin{array}{rcl} \mathfrak{g}_k & = & < X_{k-1} > \quad 1 \leq k \leq 4 \\ \mathfrak{g}_5 & = & < Y_{n-4} > . \end{array}$$

Sea $\bar{d}_1 \in Der(\mathfrak{h}_1^{n-2})$. Entonces:

$$\bar{d}_1 = \sum_{i \in Z} d_i$$

donde $d_i \in Der(\mathfrak{h}_1^{n-2})$ y $d_i(\mathfrak{g}_j) \subset \mathfrak{g}_{i+j}$, siendo $\mathfrak{g}_k = \{0\}$ para $k < 1$ y $k > 5$.

Como $d_4(\mathfrak{g}_1) \subset \mathfrak{g}_5$ y $d_{-4}(\mathfrak{g}_5) \subset \mathfrak{g}_1$, se deduce que:

$$d_i = 0 \quad i > 4 \ , \ i < -4 \Rightarrow \bar{d_1} = \sum_{i=-4}^{4} d_i$$

Habrá que expresar cada d_i , $-4 \leq i \leq 4$, como una combinación lineal de un cierto conjunto B_i , $-4 \leq i \leq 4$, de derivaciones linealmente independientes de $\mathfrak{h}_1^{n-2}$, cumpliéndose que

$$\bigcup_{i=-4}^{4} B_i$$

es una base de $Der(\mathfrak{h}_1^{n-2})$ y, evidentemente,

$$dim(Der(\mathfrak{h}_1^{n-2})) = \sum_{i=-4}^{4} dim(K < B_i >).$$

A continuación, se detallan las condiciones iniciales que deben satisfacer las d_i , $-4 \leq i \leq 4$, y que se deducen de $d_i(\mathfrak{g}_j) \subset \mathfrak{g}_{i+j}$, como también las posteriores que resultan al exigir que cada d_i sea, efectivamente, una derivación.

Cálculo de d_{-j} $\quad 2 \leq j \leq 4$

Como $d_{-j}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-j}$ $\quad \forall t$, se cumple que

$$\begin{cases} d_{-j}(X_i) & = & 0 \qquad 0 \leq i \leq 3 \\ d_{-j}(Y_{n-4}) & = & 0. \end{cases}$$

Se deduce que $d_{-j} = 0$.

Cálculo de d_{-1}

Como $d_{-1}(\mathfrak{g}_t) \subset \mathfrak{g}_{t-1}$ $\quad \forall t$, se cumple que

$$\begin{cases} d_{-1}(X_1) & = & \alpha_1 X_0 \\ d_{-1}(Y_{n-4}) & = & \bar{\beta}_{n-4} X_3. \end{cases}$$

Al exigir que d_{-1} sea derivación, se obtiene que

$* \ d_{-1}([X_1, X_2]) = [d_{-1}(X_1), X_2] + [X_1, d_{-1}(X_2)] \Rightarrow$
$\Rightarrow d_{-1}(Y_{n-4}) = [\alpha_1 X_0, X_2] + [X_1, 0] \Rightarrow \bar{\beta}_{n-4} X_3 = \alpha_1 X_3 \Rightarrow \bar{\beta}_{n-4} = \alpha_1.$

En consecuencia, se verifica que

$$\begin{cases} d_{-1}(X_1) & = & \alpha_1 X_0 \\ d_{-1}(Y_{n-4}) & = & \alpha_1 X_3. \end{cases}$$

Cálculo de d_0

Como $d_0(\mathfrak{g}_t) \subset \mathfrak{g}_t \quad \forall t$, se cumple que

$$\begin{cases} d_0(X_i) & = & \alpha_i X_i & 0 \leq i \leq 3 \\ d_0(Y_{n-4}) & = & \beta_{n-4} Y_{n-4}. \end{cases}$$

Al exigir que d_0 sea derivación, se obtiene que

$* \ d_0([X_0, X_2]) = [d_0(X_0), X_2] + [X_0, d_0(X_2)] \Rightarrow$
$\Rightarrow \alpha_3 X_3 = d_0(X_3) = [\alpha_0 X_0, X_2] + [X_0, \alpha_2 X_2] = (\alpha_0 + \alpha_2).X_3 \Rightarrow \alpha_3 = \alpha_0 + \alpha_2.$

$* \ d_0([X_0, X_1]) = [d_0(X_0), X_1] + [X_0, d_0(X_1)] \Rightarrow$
$\Rightarrow \alpha_2 X_2 = d_0(X_2) = [\alpha_0 X_0, X_1] + [X_0, \alpha_1 X_1] = (\alpha_0 + \alpha_1).X_2 \Rightarrow$
$\Rightarrow \alpha_2 = \alpha_0 + \alpha_1 \Rightarrow \alpha_3 = \alpha_0 + (\alpha_0 + \alpha_1) \Rightarrow \alpha_3 = 2\alpha_0 + \alpha_1.$

$* \ d_0([X_1, X_2]) = [d_0(X_1), X_2] + [X_1, d_0(X_2)] \Rightarrow$
$\Rightarrow \beta_{n-4}.Y_{n-4} = d_0(Y_{n-4}) = [\alpha_1 X_1, X_2] + [X_1, \alpha_2 X_2] = (\alpha_1 + \alpha_2).Y_{n-4} \Rightarrow$
$\Rightarrow \beta_{n-4} = \alpha_1 + \alpha_2 \Rightarrow \beta_{n-4} = \alpha_1 + (\alpha_0 + \alpha_1) \Rightarrow \beta_{n-4} = \alpha_0 + 2\alpha_1.$

En consecuencia, se verifica que

$$\begin{cases} d_0(X_i) & = & \alpha_i X_i & 0 \leq i \leq 1 \\ d_0(X_2) & = & (\alpha_0 + \alpha_1).X_2 \\ d_0(X_3) & = & (2\alpha_0 + \alpha_1).X_3 \\ d_0(Y_{n-4}) & = & (\alpha_0 + 2\alpha_1).Y_{n-4}. \end{cases}$$

Cálculo de d_1

Como $d_1(\mathfrak{g}_t) \subset \mathfrak{g}_{t+1} \quad \forall t$, se cumple que

$$\begin{cases} d_1(X_i) &= \alpha_i X_{i+1} \qquad 0 \leq i \leq 2 \\ d_1(X_3) &= \bar{\alpha}_3 Y_{n-4}. \end{cases}$$

Al exigir que d_1 sea derivación, se obtiene que

$* \ d_1([X_0, X_2]) = [d_1(X_0), X_2] + [X_0, d_1(X_2)] \Rightarrow$
$\Rightarrow \bar{\alpha}_3 Y_{n-4} = d_1(X_3) = [\alpha_0 X_1, X_2] + [X_0, \alpha_2 X_3] = \alpha_0 Y_{n-4} \Rightarrow \bar{\alpha}_3 = \alpha_0.$

$* \ d_1([X_0, X_1]) = [d_1(X_0), X_1] + [X_0, d_1(X_1)] \Rightarrow$
$\Rightarrow \alpha_2 X_3 = d_1(X_2) = [\alpha_0 X_1, X_1] + [X_0, \alpha_1 X_2] = \alpha_1 . X_3 \Rightarrow \alpha_2 = \alpha_1.$

En consecuencia, se verifica que

$$\begin{cases} d_1(X_i) &= \alpha_i X_{i+1} \qquad 0 \leq i \leq 1 \\ d_1(X_2) &= \alpha_1 X_3 \\ d_1(X_3) &= \alpha_0 Y_{n-4}. \end{cases}$$

Cálculo de d_2

Como $d_2(\mathfrak{g}_t) \subset \mathfrak{g}_{t+2} \quad \forall t$, se cumple que

$$\begin{cases} d_2(X_i) &= \alpha_i X_{i+2} \qquad 0 \leq i \leq 1 \\ d_2(X_2) &= \bar{\alpha}_2 Y_{n-4}. \end{cases}$$

Al exigir que d_2 sea derivación, se obtiene que

$* \ d_2([X_0, X_1]) = [d_2(X_0), X_1] + [X_0, d_2(X_1)] \Rightarrow$
$\Rightarrow \bar{\alpha}_2 Y_{n-4} = d_2(X_2) = [\alpha_0 X_2, X_1] + [X_0, \alpha_1 X_3] = -\alpha_0 Y_{n-4} \Rightarrow \bar{\alpha}_2 = -\alpha_0.$

En consecuencia, se verifica que

$$\begin{cases} d_2(X_i) &= \alpha_i X_{i+2} \qquad 0 \leq i \leq 1 \\ d_2(X_2) &= -\alpha_0 Y_{n-4}. \end{cases}$$

Cálculo de d_3

Como $d_3(\mathfrak{g}_t) \subset \mathfrak{g}_{t+3} \quad \forall t$, se cumple que

$$\begin{cases} d_3(X_0) &= \alpha_0 X_3 \\ d_3(X_1) &= \bar{\alpha}_1 Y_{n-4}. \end{cases}$$

Se verifica, evidentemente, que d_3 es una derivación.

Cálculo de d_4

Como $d_4(\mathfrak{g}_t) \subset \mathfrak{g}_{t+4} \quad \forall t$, se cumple:

$$d_4(X_0) = \bar{\alpha_0} Y_{n-4}.$$

Se verifica, evidentemente, que d_4 es una derivación.

□

UNIVERSIDAD DE SEVILLA

Reunido el Tribunal integrado por los abajo firmantes el día de la fecha, para juzgar la Tesis Doctoral de Jesús M. Cabezas Martínez de Aragón titulada Una generalización de las álgebras de Lie filiformes.

acordó otorgarle la calificación de Apto Cum Laude

Sevilla, 30 de enero 1997

El Vocal,

Yu. KHAKIMDJANOV

El Vocal,

Michel Goze

El Presidente

F.J. ARMENDI

El Secretario,

A. Jiménez

El Doctorado,

Jesús Mª Cabezas Mtz. de Aragón

JESÚS Mª CABEZAS MTZ. DE ARAGÓN

Firmado:
Francisco J. Echarte