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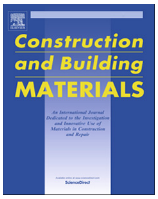

Control of structural intervention in the area of the Roman Theatre of Cadiz (Spain) by using non-destructive techniques

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HIGHLIGHTS

- Operational modal testing of a group of historical masonry buildings.
- Topographic control and ambient vibration tests are used for the control of a structural intervention.
- Updated FE model is used to control a structural intervention in different stages.
- Updated FE model is used to foresee the behaviour of a group of historical buildings after a structural intervention.

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ABSTRACT

This paper presents the dynamic characterization of a group of historical buildings located over the Roman Theatre of Cadiz (Spain), a piece of heritage that has been buried for centuries under the historical town. In 2010, an intervention began in order to excavate it, while respecting the buildings over it. The control of this intervention is being done by means of topographic control points and ambient vibration tests. On the basis of the results obtained from the dynamic tests, a finite element model was updated and subsequently used to check the current works and foresee the final behaviour of the complex. A brief description of the Roman theatre and the works to recover it, the methodology followed to control such works and the results obtained are the main goals of this paper.

1. Introduction

Nowadays, the preservation of the architectural heritage is a fundamental aspect in the cultural development of modern cities. This heritage has to be preserved and different technical works are often necessary to ensure its proper preservation. In this process, non-destructive techniques are an indispensable tool to provide information about the structural behaviour of the building at different stages of the works [1].

In the last decades, Operational Modal Analysis (OMA) method has consolidated as a non-destructive technique that allows the experimental estimation of the modal parameters of a structure from measurements of its dynamic response to ambient vibration only [1–3]. Due to the nature of historical buildings, the use of artificial elements such as impact hammers or shakers is usually not allowed and the tests are thus performed by using the ambient vibration survey technique. The main advantages of OMA are its low economic cost and the fact that excitation equipment is unnecessary. Because of these factors, OMA is currently recognised as quite as convenient technique for the control of structural interventions in this kind of buildings. This is possible due to the fact that alterations of geometrical dimensions, boundary conditions, mass and mechanical properties of materials, or the simultaneous occurrence of all these phenomena during the works, affect the dynamic behaviour of the structures. In this way, if the dynamic response of the structure is evaluated before, during and after the works by using ambient vibration tests, changes in its performance can be detected. The assumption that damage can be linked to a decrease of stiffness seems reasonable for this type of structures [1,3].

Furthermore, the modal properties provided by the application of operational modal analysis are convenient to check and update, if necessary, numerical models. It is assumed that the greatest difficulty for the analytical analysis of a historical building is the high

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level of uncertainty associated with many factors [1,4]. Due to this fact, slight modifications of the mechanical properties of the structural materials, the soil–structure interaction or even the building construction process, are usually the cause of great differences between the results obtained from an analytical analysis and others estimated experimentally by using ambient vibration tests. Thus, the FE model updating from the dynamic structural parameters identified experimentally allows the adjustment of these models in order to obtain a more accurate estimation of the actual behaviour of the structure [4,5]. In this way, updated FE models can be used to assess structural works and even to foresee the final performance of the historical building.

In recent years, many cases of application of ambient vibration tests can be found in historical buildings. Despite the fact that output-only modal tests are especially suitable for flexible systems, such as bell towers and minarets [5–8], several applications of OMA have already been performed in massive buildings [9,10]. However, the applications of OMA to evaluate and control structural interventions in this kind of buildings are more limited [11–13].

This paper investigates the dynamic characteristics of a group of historical buildings located over the Roman Theatre of Cadiz (Spain) (Fig. 1). The theatre is a piece of heritage that has been buried for centuries under the historical town (“Populo” district). Recently, an intervention began in order to dig it up while respecting the monumental landmarks over it. The intervention consisted in the construction of some vaults under the buildings using the grout umbrella technique. Due to the fact that the excavations were too close to several historical inhabited buildings, a dynamic control was planned in addition to a topographical control. This control is being carried out through the estimation of the dynamic parameters of the structure by using ambient vibration tests. In this way, the main goals of the paper focus on: (i) describing the Roman Theatre of Cadiz and the works that are in progress to recover it and (ii) describing the non-destructive techniques implemented to control the impact of the structural intervention on the surrounding buildings. For this latter purpose, special attention is paid to application of OMA, with a twofold aim. First, to estimate the modal parameters prior to the excavation works and at the completion of the initial stages of the works, since any significant alteration of such parameters would be indicative of undesirable changes in the structural behaviour of the complex. Second, the obtained experimental results permit to develop an updated finite element model that replicates the actual structural behaviour of the complex. This model can be subsequently used to foresee the final behaviour of the complex.

The paper is organised as follows: Section 2 summarizes the main characteristics of the Roman Theatre of Cadiz together with a historical overview and a brief description of the current intervention aimed at recovering the buried Roman Theatre. Section 3 is devoted to discuss the control of the intervention, mainly based on topographic control and ambient vibration tests. It further describes the initial finite element model developed to estimate the modal parameters of the system. Section 4 presents the updating of the finite element model based on the experimentally obtained modal parameters. The resulting finite element model is then applied to check the current intervention works and foresee its structural incidence in the surrounding buildings when completed. Finally, the conclusions to this work are drawn in Section 5.

2. The roman theatre of Cadiz

2.1. Historical aspects

The Theatre of Cadiz is a sample of Roman architecture still remaining in Cadiz, a coastal city placed in the south of Spain. It is one of the largest and oldest theatres in this country. It was constructed in the last part of the 1st century BC by the politician Lucius Cornelius Balbus the Younger and its diameter is approximately 120 m, which is similar to the diameter of those theatres constructed in Rome. At that time, Cadiz was the centre of a huge economic and military activity. Thus, the theatre was conceived as part of an urban renovation of the city intended to provide it with new public buildings and to enlarge it [14]. The materials used to construct the theatre were opus caementicium and masonry. These resistant and hardly recyclable materials prevented the fact that the theatre were later used as a quarry, as happened with other Roman buildings in the city. At the same time, they made the theatre a platform suitable to be used as the foundations of other buildings constructed later [15]. The history of Cadiz is marked by the fact that the city is placed in an isthmus. This fact restricts its enlargement due to the lack of available land. As a consequence, the city has grown up in superimposed layers. The theatre was abandoned in the 3rd century due to the declining economy caused by the Third Century crisis of the Roman Empire. In the Islamic years of the city, a fortress was partially built over the remains of the theatre [14,16].

The reconquest of Cadiz by the king Alfonso X in the 13th century meant the revival of the city of Cadiz. The reconstruction of the old and destroyed Islamic city gave as a result a citadel composed by several houses, the previous fortress that was rebuilt and a new church in the place of the old Mosque (currently known as the Old Cathedral or Santa Cruz Church) (Fig. 2). These buildings

Fig. 1. Area of the Roman Theatre of Cádiz (2010).

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Fig. 2. Plot plan of the Roman Theatre area in 1980 (in red demolished buildings). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

were the first pieces of the "Populo" district, which is considered as a cultural and historical area in the city of Cadiz [17]. After the death of the king Alfonso X in 1284, the development of the city slowed down [18].

At the end of the 16th century, the Old Cathedral and the castle were rebuilt due to the destruction caused by the sacking of the city by the Earl of Essex [19]. In the 18th century and only for some years, the castle became the Midshipman Academy and the Navy Observatory [20]. This is the last news of the castle until it was demolished in the 19th century.

At the beginning of the 20th century, this empty area was used to build a warehouse. In 1980, the remains of the Roman Theatre were accidentally discovered under the "Populo" district (Fig. 2). At that time, a great interest emerged about the building, with the aim of recovering its remains and restore them. From 1980 to 2008, a large part of the summa cavea of the theatre was unearthed and some buildings without historical value were demolished for this purpose (Fig. 3).

2.2. Current intervention

In 2010, the major part of the theatre was still lying under the blocks of the "Populo" district. This district is part of Cadiz heritage from the historical and artistic points of view: the Old Cathedral (also known as Santa Cruz Church), the remains of the city walls and gates, and some listed urban buildings as the house known as "Posada del Meson" (XVII century) are currently standing over the remains of the Roman Theatre (Fig. 4). Subsequently, a significant intervention was planned and launched to uncover the remains of the theatre without damaging the "Populo" district.

To this end, the proposal of the project was based on the construction of some vaults under the "Populo" blocks using grout injection [20]. These vaults are intended to support the buildings and allow the excavation beneath them (Fig. 5). The technique applied in this project is umbrella grouting, which is a well-known technique in tunnelling [21,22]. The vaults are created by attaching horizontal grout elements. The exact position of the grout injections is achieved by steel arches used as the pattern for the introduction of the sleeve port pipes (Fig. 6-left). The grout arches overlap among them by their cone shape forming a vault (Fig. 6-right). The vaults rest on pilasters built also through grout injection. In all injections cement-bentonite grouts are used. The first stage of the works consisted in performing 5 metre-long vaults, in order to subsequently remove the soil located under them during the second stage (Fig. 5). Currently, these two phases have already been completed. In developing future steps, these vaults will reach greater length and other ones will be carried out perpendicular to them. The final result will be the creation of a new vaulted space under the "Populo" blocks created by grouting umbrellas (see Section 4.3 and Fig. 18 therein). This last stage is still pending and hopefully will be executed in the next months.

3. Control of the intervention

The control of the intervention that is running on the Roman Theatre area is being carried out mainly by means of two non-destructive techniques. These are the topographic surveys and ambient vibration tests, as next described:

3.1. Topographic control

The topographic control technique was used during and after the injection process of the concrete vaults. A high-performance motorised total station was implemented to monitor the movements of these historical buildings during the works, with a precision of 0.05 mm (Fig. 7-left). In addition, twenty-six high precision

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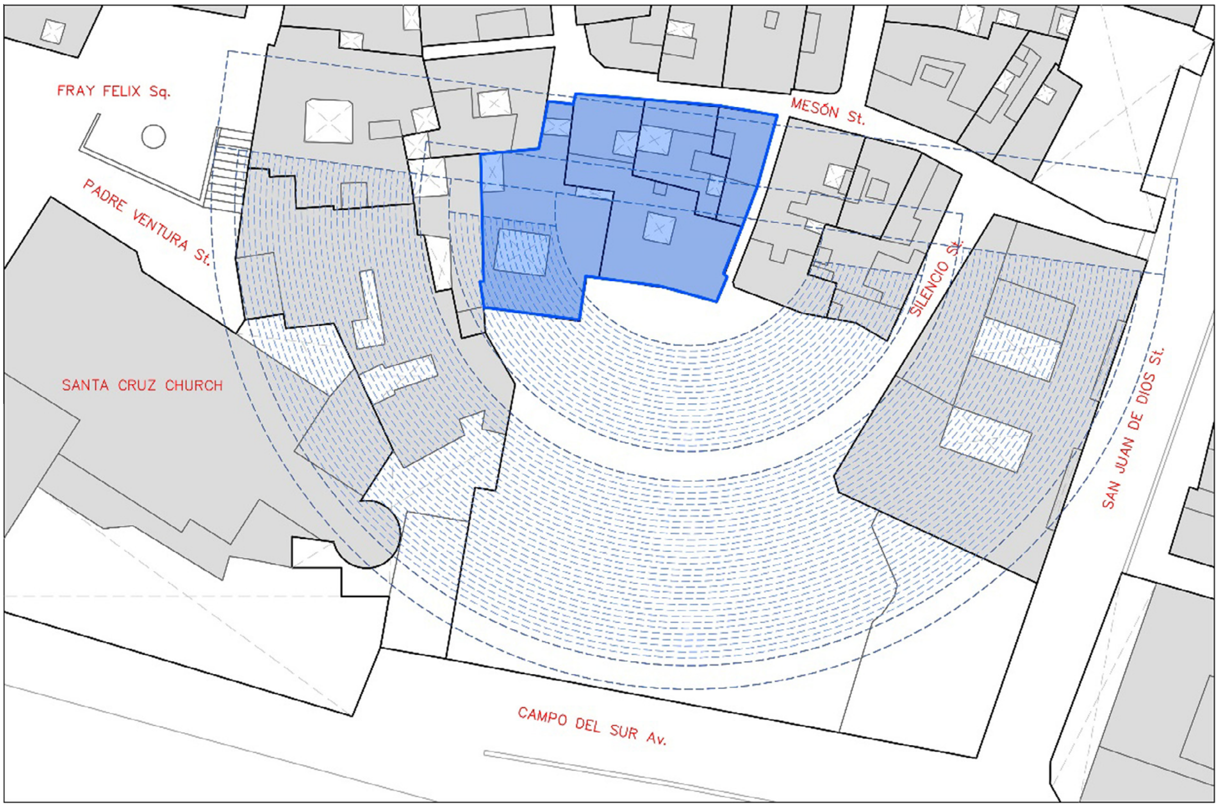

Fig. 3. Plot plan of the Roman Theatre area in 2010 (in blue analysed buildings). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 4. The Roman Theatre under the blocks of the "Populo" historical district.

targets (Fig. 7-right), whose disposition is reflected in Fig. 8, were used as measuring points. This equipment provides a real-time readout of all the movements registered in all three spatial directions. Furthermore, it is also equipped with an automatic alarm system, which will send a warning message to the mobile phones of the technical staff in case the displacements exceed the pre-established limits deformations (one millimetre).

The results collected during the execution of the injection umbrellas indicate insignificant movements that do not affect the structural stability of the buildings. The movements recorded between April 22, 2011 and March 27, 2012, execution period of one of these injection umbrellas, present in 99% of cases values less than one millimetre in both the vertical and the horizontal direction (Fig. 9).

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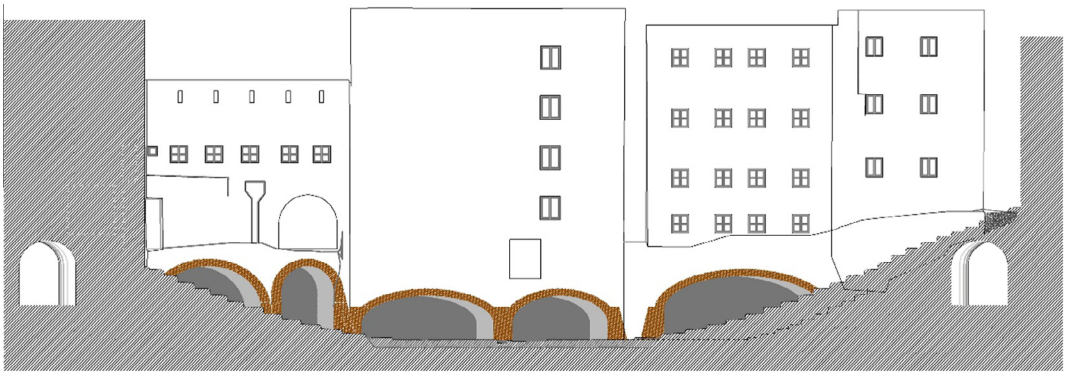

Fig. 5. Sketch of current stage of the intervention.

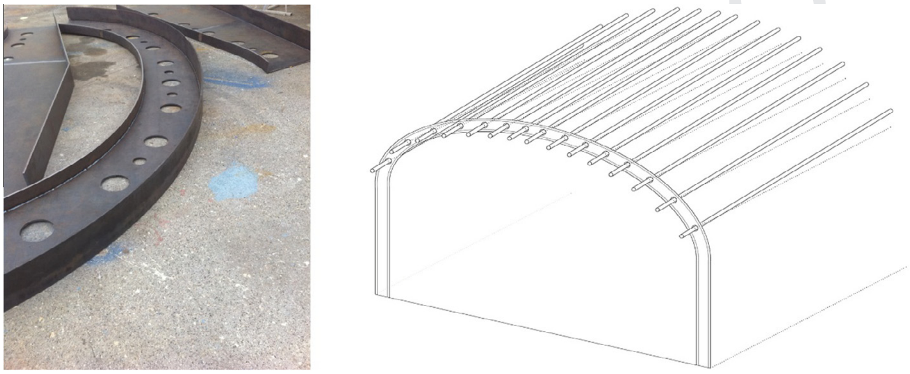

Fig. 6. Steel arches (left). Sketch of the construction of a vault using grout injections (right).

Fig. 7. Total station of topographical control (left). Target (right).

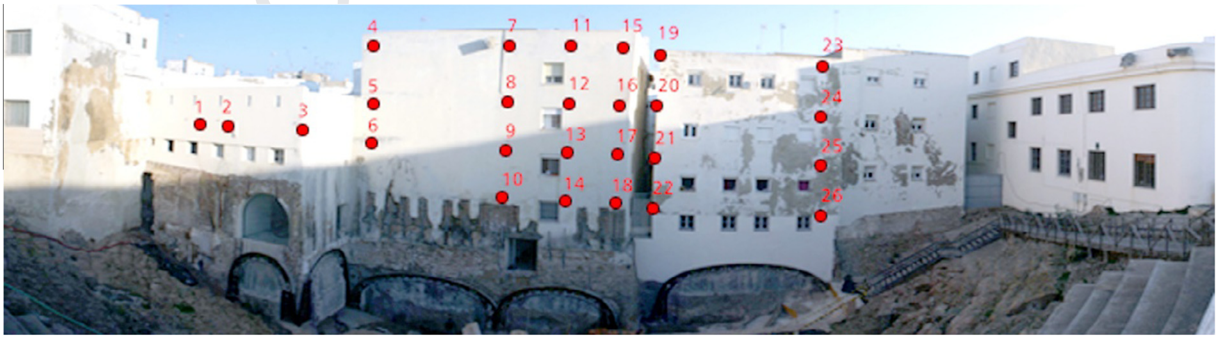

Fig. 8. Targets location.

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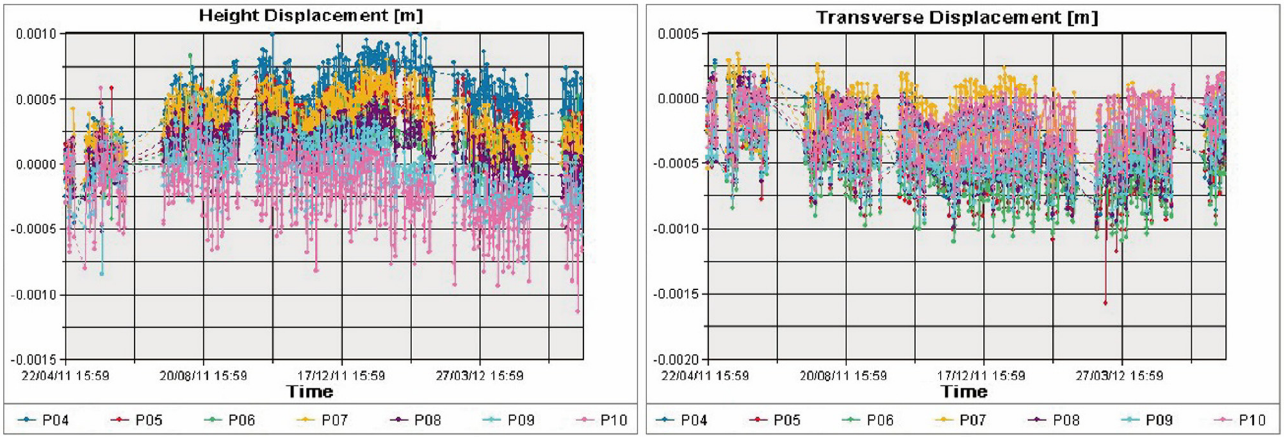

Fig. 9. Vertical (left) and lateral (right) movements of the points 4–10 during injection vault located just below.

3.2. Ambient vibration tests and modal identification

The intervention in the Roman Theatre area is also being controlled by ambient vibration tests. These tests were carried out between March 2011 and April 2013, with the aim of identifying the natural frequencies, mode shapes and damping ratios of the "Populo" blocks and their variation along the construction period. The control consisted in obtaining these modal parameters before and at different stages of the works, and comparing them. Significant changes in these parameters would indicate any alteration of the structural stiffness of the buildings located above the theatre. It is well known that structural stiffness is directly related with natural frequencies:

$$f_n = \sqrt{\frac{K_n}{m}} \tag{1}$$

where f_n is the natural frequency; m, the mass and K_n, the structural stiffness. Since the mass of the buildings remains constant, the relationship between the natural frequencies and the structural stiffness is straightforward.

3.2.1. Initial finite element model

The application of the Operational Modal Analysis requires the creation of an initial model in order to both gain an estimation of the natural frequencies and check the position of the accelerometers [1]. This initial FE model was built with the Abaqus/CAE 6.13 Software [23] and corresponds to the first stage of the works (after the first execution phase of grout umbrellas). This is mainly composed by three components: buildings, vaults and soil. The buildings were modelled with shell elements, whilst solid elements were used to model the soil and the vaults (Fig. 10). The final model has 979,178 elements, 191,117 nodes and 666,177 degrees of freedom. The material properties were estimated from both the geotechnical study of the soil and related bibliography [24,25]. Thus, the following assumptions were made: the density of the masonry (load bearing and enclosing walls of the buildings), the concrete slabs of the buildings, the soil and the concrete vaults are 1800, 2350, 2000 and 2200 kg/m^3, respectively; the corresponding Young's modulus are 2500, 21,000, 1300 and 13,000 MPa, respectively; and a Poisson's ratio of 0.2 is adopted for all the components. A linear isotropic material model is considered for our purposes.

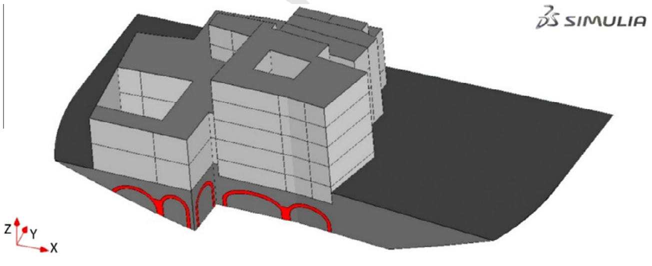

Fig. 10. FE model (after the first stage of the works).

As pointed out above, this initial finite element model has been used to locate the appropriate positions for the reference accelerometers and estimate the location with larger modal displacements. Fig. 11 illustrates the first vibration modes together with its corresponding natural frequencies.

3.2.2. Experimental set-up

Three dynamic modal identification tests were performed: before the works started (April 15, 2011), after the first execution phase of injection umbrellas was executed (April 20, 2012) and after removing the soil located under the vaults (April 6, 2013). The equipment used for these tests was composed by 8 uniaxial force balance accelerometers, with a bandwidth ranging from 0.01 to 200 Hz, a dynamic range of 140 dB, a sensitivity of 10V/g and 0.35 kg of weight (model ES-U2), connected by eight 40-metre coaxial cables to a twelve-channel data acquisition system with a 24-bit ADC, provided with anti-alias filters (model GRANITE). The equipment is manufactured by the company KINEMETRICS (Fig. 12).

Fig. 13 shows a schematic representation of the sensors layout. The total number of measuring points is 20. Four measuring points were set on each floor, two in the X direction and two in the Y direction, in order to capture the global vibration modes in both the longitudinal and lateral (transverse) direction of the buildings. Since only 8 accelerometers were available for the testing and 4 of them (placed at points 1–4) were held stationary for reference measurements, a series of four set-ups in each stage were necessary to cover all measuring points. In each of the setups, the accelerations were recorded with a sampling rate of 100 Hz and a sampling time of 12 min. These assumptions ensure that frequencies from 1 to 50 Hz would be properly recorded.

The same test planning was adopted in the three cases by using the same parameters and measuring points. Similar temperature and humidity conditions were also considered. It is to be noted that these aspects, such as important changes of humidity, could modify the frequencies up to 5% [11]. The excitation was always associated to environmental loads (Fig. 14).

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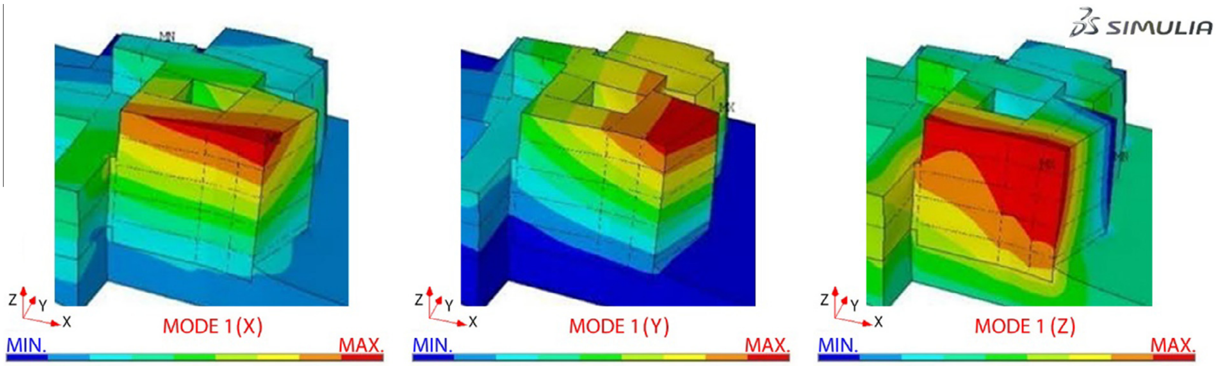

Fig. 11. Modal displacements of the first vibration mode (5.59 Hz) in *X*, *Y* and *Z* directions. Initial FE model.

Fig. 12. Measurement equipment.

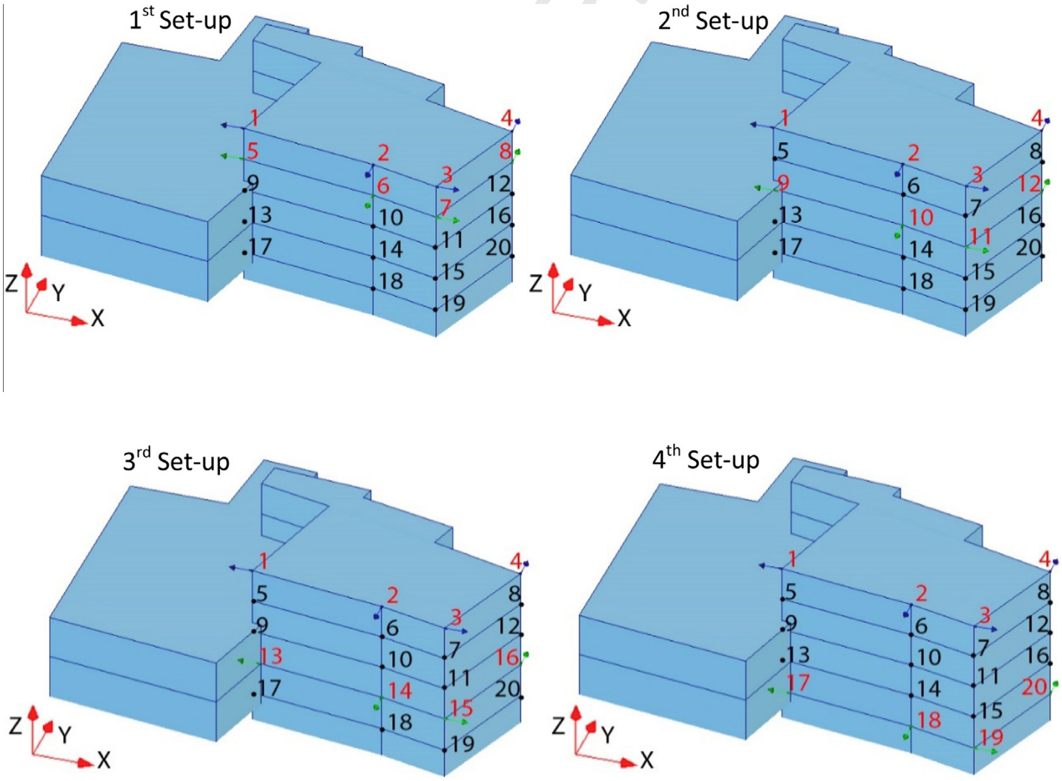

Fig. 13. The measuring set-ups and accelerometer locations and directions.

3.2.3. Data processing and operational modal analysis

The data obtained in-situ was processed with the software ARTEMIS [26] using two different identification methods: the Enhanced Frequency Domain Decomposition (EFDD) technique (Fig. 15) [27] and the Stochastic Subspace Identification (SSI) method [28,29]. In order to perform a more accurate analysis and

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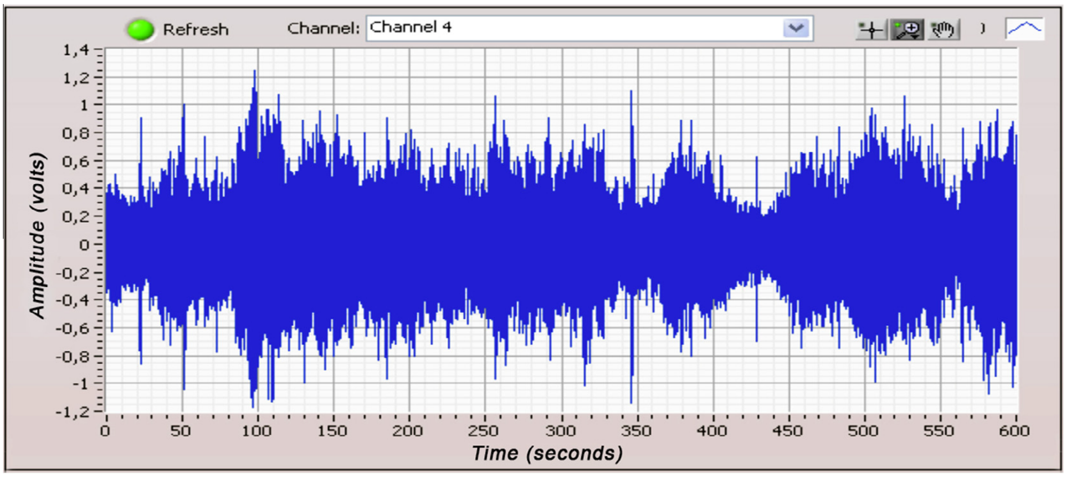

Fig. 14. Time history response (example of level of ambient excitation – accelerometer four, 2nd set-up).

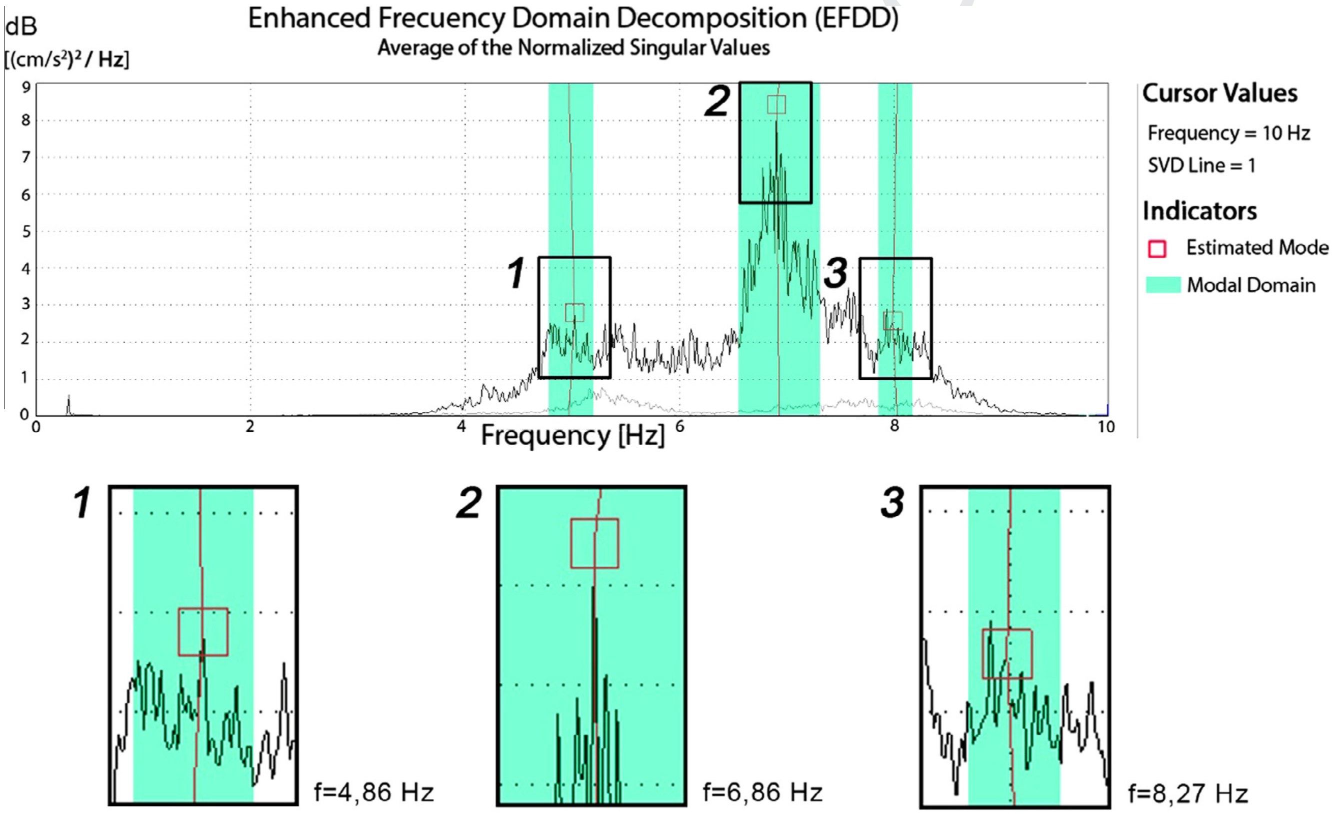

Fig. 15. Example of ambient test set up (EFDD).

to take into consideration that the expected natural frequencies are below 10 Hz, a decimation factor of 10 was previously applied.

In this way, the modal frequencies, the damping ratios and the mode shapes were obtained and later validated using the Modal Assurance Criterion (MAC) between the EFDD and SSI results. The MAC value is defined as follows [28]:

$$MAC_{j,k} = \frac{\left(\varphi_j^T \cdot \varphi_k\right)^2}{\left(\varphi_j^T \cdot \varphi_j\right) \cdot \left(\varphi_k^T \cdot \varphi_k\right)} \tag{2}$$

where φ_j and φ_k are the two modes to be compared and T denotes the transpose. A good correlation between two modes is achieved when the value of its M.A.C. ratio is greater than 0.85. Finally, the results of the data processing in each stage are presented in Tables 1–3.

As it may be seen from the tables above, the ambient vibration tests allowed identifying accurately the first three modes in a frequency range from 0 to 10 Hz. The frequencies were identified with relative errors lower than 1.5%, taking as reference the results of the SSI method. The results of the damping ratio have higher variability (up to 63%) and the average modal damping ratios of the buildings are 1.62% and 3% for EFDD and SSI techniques, respectively. With respect to the mode shapes, the MAC values were always higher than 0.94 for all collected data. In the three experimental campaigns, the first and second modes correspond

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Table 1
Results of the experimental modal analysis tests before the works (April 15, 2011): natural frequencies (f), damping ratios (ξ) and standard variances (Std.).

	SSI				EFDD				MAC
	f (Hz)	Std. f	ξ (%)	Std. ξ	f (Hz)	Std. f	ξ (%)	Std. ξ	
Mode 1	4.83	0.03	3.43	0.50	4.77 (1.2%)	0.08	2.2 (35%)	0.59	0.95
Mode 2	6.88	0.01	2.36	0.28	6.78 (1.4%)	0.10	1.1 (53%)	0.24	0.94
Mode 3	8.25	0.02	3.21	0.18	8.26 (0.1%)	0.13	1.2 (63%)	0.21	0.94

The percentage in parenthesis indicates the relative error taking as reference the results of the SSI method.

Table 2
Results after the first stage of the works (April 20, 2012): natural frequencies (f), damping ratios (ξ) and standard variances (Std.).

	SSI				EFDD				MAC
	f (Hz)	Std. f	ξ (%)	Std. ξ	f (Hz)	Std. f	ξ (%)	Std. ξ	
Mode 1	4.86	0.02	3.74	0.71	4.86 (0.1%)	0.05	2.2 (41%)	1.30	0.99
Mode 2	6.92	0.05	2.16	0.21	6.86 (0.8%)	0.03	0.8 (62%)	0.44	0.99
Mode 3	8.30	0.06	2.81	0.55	8.27 (0.4%)	0.08	1.8 (36%)	0.11	0.98

The percentage in parenthesis indicates the relative error taking as reference the results of the SSI method.

Table 3
Results after the second stage of the works (April 6, 2013): natural frequencies (f), damping ratios (ξ) and standard variances (Std.).

	SSI				EFDD				MAC
	f (Hz)	Std. f	ξ (%)	Std. ξ	f (Hz)	Std. f	ξ (%)	Std. ξ	
Mode 1	4.85	0.04	3.70	0.61	4.85 (0.1%)	0.06	2 (45%)	1.10	0.97
Mode 2	6.90	0.02	2.66	0.22	6.86 (0.5%)	0.05	1.7 (62%)	0.24	0.98
Mode 3	8.26	0.08	3.12	0.45	8.23 (0.4%)	0.10	1.6 (36%)	0.21	0.95

The percentage in parenthesis indicates the relative error taking as reference the results of the SSI method.

to translation of the buildings while the third one corresponds to a rotational mode (Fig. 16).

Finally, the variation among the three project stages of the natural frequencies associated with the different vibration modes of the blocks were compared as a control measure. As can be observed in Table 4, the differences between the natural frequencies are always lower than 1%. It can therefore be concluded that the structural stiffness of the buildings are not being affected by the current works.

4. Model updating

On the basis of the results obtained from the ambient vibration tests, the initial finite element model was updated and subsequently used to foresee the final behaviour of the complex. One of the first difficulties that can be detected in finite element models of historical buildings is the high level of uncertainty associated with many factors affecting the actual behaviour of the structure. Often the inaccuracies in this model will arise because of poorly known boundary conditions, unknown material properties or simplification in the modelling. These uncertainties in the modelling process cause the predicted dynamics of a structure to be different from the measured dynamics of the real structure. If accurate measured data is available, then this data could be used to improve the numerical model in general and some uncertain parameters of the model in particular.

Table 4
Comparison of frequencies (Hz) associated with each experimental campaign.

	$f_{(\text{before the works})}$ [a]	$f_{(\text{after 1st stage})}$ [b]	$f_{(\text{after 2nd stage})}$ [c]	Max.% difference
Mode 1	4.83	4.86	4.85	0.61
Mode 2	6.88	6.92	6.90	0.57
Mode 3	8.25	8.30	8.26	0.60

Results of natural frequencies obtained from the SSI method.
[a] April 15, 2011.
[b] April 20, 2012.
[c] April 6, 2013.

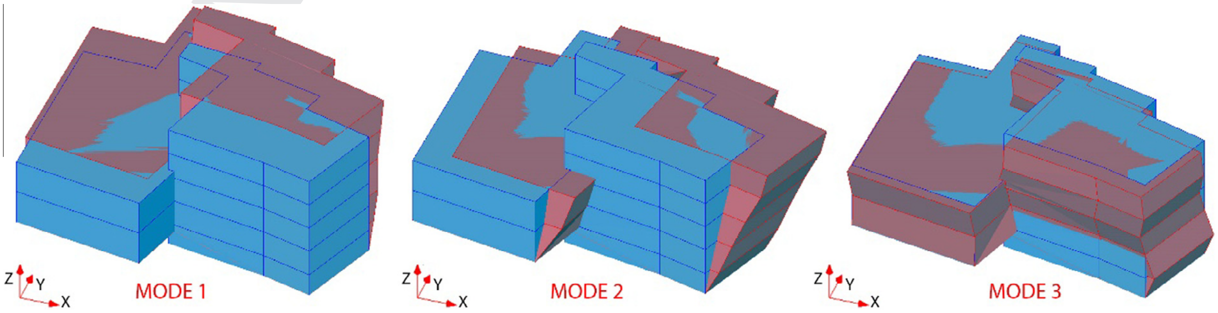

Fig. 16. Mode shapes associated with the experimental results (SSI).

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4.1. Basics of FE model updating

The FE model updating based on the experimental results of natural frequencies and vibration modes may be performed of the structure from two different perspectives, following either direct or indirect methods. In the early years of this technique, the adjustment of the FE model was performed directly through the introduction of changes in the mass and stiffness matrices of the structure, what has the advantage of allowing an adjustment between the numerical model and the experimental data through a direct algorithm without the need of iterating. However, this methodology has as main disadvantage that the updating process is performed without necessarily involving the physical knowledge of the problem. This drawback caused the later appearance of other family of methods, iterative methods [30,31], where the model updating arises from the changes applied on some well-defined structural physical parameters selected by the users. In this case, the modified parameters are not linearly related to the modal parameters, so that the adjustment process requires the use of optimization algorithms for non-linear problems, thus being necessary to undergo an iterative process. A straightforward manner to perform the FE model updating is to define as objective function the minimization of the relative differences between the experimental and numerical modal parameters. The equation resulting from this aim is usually formulated as a least square problem:

$$l(\theta) = \frac{1}{2}\cdot\sum_{j=1}^{m} w_j \cdot \left[z_{NUM,j}(\theta) - z_{EXP,j}\right]^2 = \frac{1}{2}\sum_{j=1}^{m} w_j \cdot r_j(\theta)^2 \qquad (3)$$

where $z_{NUM,j}(\theta)$ are the magnitudes obtained from the numerical model, which are related to the physical parameters of the model, θ (modulus of elasticity, soil stiffness ...), θ being the object of the adjustment. The variables $z_{EXP,j}$ represent the same magnitudes obtained from experimental data. The differences between the experimental and numerical parameters are denoted as residues, $r_j(\theta)$. It is advisable that the number of residues, $m = m_f + m_s$ (with m_f being the number of natural frequencies considered and m_s being the number of the coordinates of the vibration modes considered), is greater than the number of variables adjusted, θ. A weight variable w_j is established for each residue to take into account the different reliability of the identified modal parameters. This later approach will be the one implemented in our study. Both residues (from natural frequencies and vibration modes coordinates) are applied in the above Eq. (3) according to the following expressions:

$$r_{f,j}(\theta) = \frac{f_{NUM,j}(\theta) - f_{EXP,j}}{f_{EXP,j}}, \quad j = 1, 2, \ldots, m_f \qquad (4)$$

where $f_{NUM,j}(\theta)$ and $f_{EXP,j}$ are the numerical and experimental natural frequencies of the structure; and

$$r_{s,j}(\theta) = \frac{\varphi^{l}_{NUM,j}(\theta)}{\varphi^{r}_{NUM,j}(\theta)} - \frac{\varphi^{l}_{EXP,j}}{\varphi^{r}_{EXP,j}}, \quad j = 1, 2, \ldots, m_s \qquad (5)$$

where $\varphi^{l}_{NUM,j}(\theta)$ and $\varphi^{r}_{NUM,j}(\theta)$ are the considered and reference component of the numerical vibration mode j, and $\varphi^{l}_{EXP,j}$ and $\varphi^{r}_{EXP,j}$ are the same components of the experimental vibration mode j. The above objective function is to be minimized by the application of an optimization algorithm. Either local or global optimization algorithms could be considered. For our purposes, global algorithms are implemented due to their robustness and controlled dependence on the initial point selected to initiate the search process. In particular, genetic algorithms have been used for the present study.

At the end, the accuracy of the updating process is checked through the comparison of the experimental and numerical natural frequencies and modal shapes by computing: the relative difference between frequencies $\Delta f_j = r_{fs,j} \cdot 100$ (%), and the MAC value computed between the experimental and numerical vibration modes. For our purposes we will consider that a good correlation between two modes is achieved when the $MAC_{NUMj,EXPj}$ value is greater than 0.85 and $\Delta f_j \leq 5\%$.

4.2. Application of FE model updating

Typically, the finite element model updating process begins with a previous manual adjustment of the natural frequencies in order to facilitate the optimization process of the least square problem. In this case, given the relatively limited differences between the experimental and numerical (obtained from the preliminary FE model) natural frequencies, as well as the good initial correlation observed between the numerical and experimental modal shapes, this phase has been omitted during the present updating process. Next, the physical parameters of the structure that have a stronger influence on its dynamic behaviour should be selected as updating parameters. To this end, a sensitivity study has been performed among the possible material parameters, leading to the three following parameters to be updated: the Young's modulus of the masonry (E_m), the soil (E_s) and the concrete vaults (E_v). It should be mentioned here that modifying Poisson's ratio has little influence on the dynamic properties of the system, as compared with the effect of modifying Young's modulus. For instance, while a 50% increase in the masonry Young's modulus (to E_m = 3750 MPa) leads to about a 18% increment of the natural frequencies predicted by the initial FE model for the first three modes of vibration, a 50% increase in the masonry Poisson's ratio (to ν_m = 0.3) leads to just about a 1.3% increment of those natural frequencies. The selection of more parameters would unnecessarily complicate the physical understanding of the updating process. Given the good quality of the experimental data, the three identified vibration modes were chosen for the updating process. Both measured natural frequencies and modal coordinate values were taken into account. Therefore, in total 63 residual components were selected for model updating (three identified natural frequencies and twenty coordinates of each identified vibration mode). To take into account the lower reliability of the identified mode shapes in comparison with the measured natural frequencies, the weight variable for the natural frequencies was set to $w_f = 1.00$ whilst for the modal coordinates values it was set to $w_s = 0.10$ [30]. Finally, the finite element model updating has been performed through the implementation of the optimization

Table 5
Summary of the results of the FE model updating process: parameters.

Updating parameter	Initial value	Optimizing interval of values		Updated value
		Lower bound	Upper bound	
E_m (MPa)	2800	1500	3000	2100
E_s (MPa)	1300	500	1500	780
E_v (MPa)	13,000	7000	18,000	10,100

Table 6
Comparison of frequencies (Hz) obtained experimentally (f_{SSI}) and analytically (f_{FEM}).

Modes	$f_{SSI\ (after\ 1st\ stage)}$	$f_{(Initial\ FEM)}$	$f_{(FEM\ UPDATED)}$	MAC value (f_{SSI}-$f_{FEM\ UPDATED}$)
Mode 1	4.86	5.59 (15%)	4.88 (0.41%)	0.99
Mode 2	6.92	7.88 (13.9%)	6.88 (0.57%)	0.96
Mode 3	8.30	9.36 (12.8%)	8.38 (0.96%)	0.88

The percentage in parenthesis indicates the relative error taking as reference the results of the SSI method.

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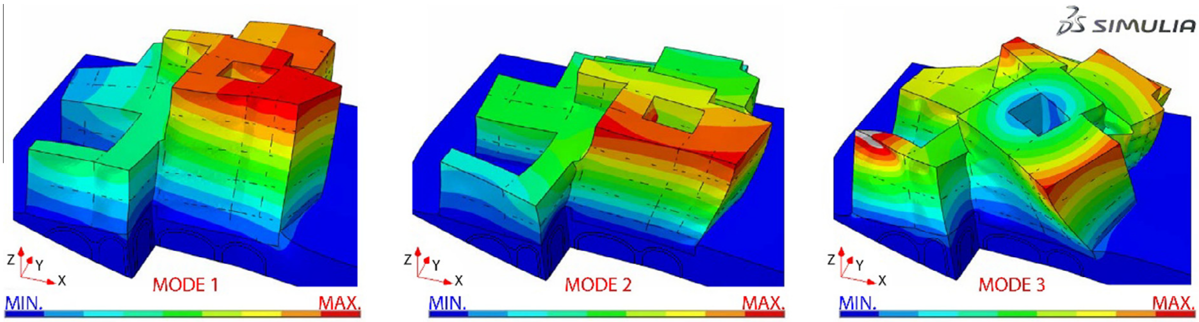

Fig. 17. Mode shapes associated with the numerical results (FEM UPDATED).

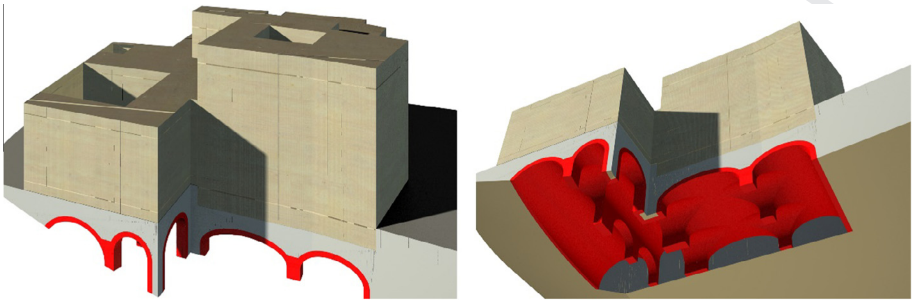

Fig. 18. Final stage of the works (FINAL FEM).

Table 7
Comparison of frequencies (Hz) obtained before (f_{SSI}) and after ($f_{FINAL\ FEM}$) the project.

Modes	f_{SSI} (before the works)	$f_{FINAL\ FEM}$ (final stage)	MAC value
Mode 1	4.83	4.87 (0.82%)	0.99
Mode 2	6.88	6.85 (0.43%)	0.95
Mode 3	8.25	8.37 (1.45%)	0.86

The percentage in parenthesis indicates the relative error taking as reference the results of the SSI method.

algorithm (genetic algorithm) in the Matlab software [32], considering lower and upper bounds for the updating variables, as next indicated. Table 5 presents the summary of the updating process and show its importance for obtaining reliable models that replicate the real response of the structures. As shown in Table 5, the final values of the updating parameters can differ in over 40% in comparison to the values considered for the initial model.

Table 6 confirms the high correspondence of the results of the calibrated model and the experimental dynamic tests. This clearly shows that the updated frequencies are close to the experimental frequencies (lower than 1%).

In Table 6, it can also be observed that the results of the mode shapes (Fig. 17) are also of high quality since in the whole cases the Modal Assurance Criterion (MAC values) are relatively close to the unit value (and always higher than 0.85).

4.3. Forecast of the final stability at the end of the works

Once the numerical model has been updated as described in previous section, it is further employed to include the geometrical modifications proposed for the final stage of the works, that is: build the remaining vaults and proceed to empty the soil beneath the new grout umbrellas (Fig. 18). Subsequently, the resulting FE model will permit to estimate the effect of the projected intervention on the historical buildings above the theatre.

To this end, a new FE modal analysis was performed and the obtained results were compared with the experimental results obtained before the works started (Table 1), in order to foresee the expected behaviour of the historical buildings complex. These results are summarized in Table 7 and confirm that the structural intervention in the theatre should not modify significantly the dynamic behaviour of the buildings, since only small variations

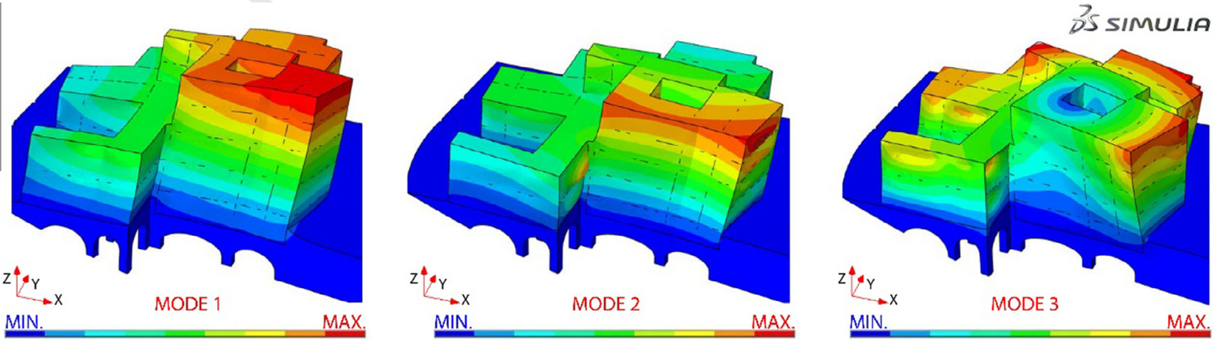

Fig. 19. Mode shapes associated with the numerical results (FINAL FEM).

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in the frequencies before and after the project are anticipated. The mode shapes are also similar to those obtained in the initial model (Fig. 19) with a good MAC correlation (Table 7). Thus, it is expected that the overall intervention project will not modify significantly the structural stiffness of the buildings. At any event, a final OMA testing campaign is planned at the end of the works in order to confirm the FE model predictions.

5. Conclusions

This paper focuses on the control of the works that are currently running in the area of the Roman Theatre of Cadiz (Spain) to unearth its remains. A vast part of the theatre actually lays beneath existing buildings of the historical ‘‘Populo” district, which must be kept. Thus, the project to recover the theatre proposed to build several vaults under those buildings by means of umbrella grouting, to excavate later beneath the vaults and up to the theatre level.

To ensure that the works do not affect the buildings on top of the excavation area, non-destructive control techniques are applied. Namely topographical control and Operational Modal Analysis. On one hand, the results collected by topographical surveys indicate insignificant movements that do not affect the structural stability of the buildings. On the other hand, by means of ambient vibration tests, the dynamic behaviour of the complex has been assessed before the works, after the execution of the initial grout injection umbrellas (first stage of the works) and after removing the soil located under those vaults (second stage). The recorded frequencies and mode shapes exhibit minimal modifications in the dynamic behaviour of the buildings after both stages of the works, so that the current works are not significantly affecting their structural stiffness.

Subsequently, the experimental dynamic results (frequencies and mode shapes) have been used to update a finite element model. Initially, the mechanical properties were set from the available data and the bibliography –when such data were not available- and later adjusted/updated by comparing the dynamic behaviour predicted by the numerical model with that observed from the data collected in-situ. Genetic algorithms, as implemented in Matlab software, were selected for the optimization process. The updated FE model exhibits natural frequencies for the first three modes within 1% of the experimental ones, with the modes showing a good correlation of MAC values as well (ranging from 0.99 for the first mode of vibration to 0.88 for the third mode). Once updated the FE model, the expected geometrical changes were included to simulate the final state of the works. This has permitted to foresee the dynamic behaviour of the historical buildings at the end of the works. A new modal analysis based on the latter FE model anticipates that the structural stiffness of the buildings should not be modified by the intervention project, with insignificant differences between the modal frequencies obtained experimentally before the works and numerically after such works are completed. Therefore, OMA and finite element updating have proven as valuable non-destructive tools to both control and anticipate the effect of structural interventions involving historical masonry structures.

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