Determining static elastic modulus of weak sandstone in Andalusian historical constructions from non-destructive tests: San Cristóbal's stone

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Abstract

This work presents a relationship between static and dynamic elastic moduli for San Cristóbal's stone, which was used to build some of the most representative historical constructions in Andalusia (Spain) during 15^{th}-18^{th} centuries, including religious, military and civil buildings. Numerical models are able to provide useful information in structural health assessment of historical constructions, but static elastic modulus is necessary to perform them. This is why it is particularly interesting to count on an equation to predict this parameter from others, such as dynamic elastic modulus, which can be obtained in situ and through tests based on wave propagation.

A new relationship is proposed after having shown that equations previously defined by other authors are not valid for San Cristóbal's stone. The prposed relationship in this work is based on a set of physical and mechanical experimental tests carried out in lab on 17 specimens directly extracted from support elements of Santiago's (Jerez de la Frontera, Cádiz-Spain). Linear, polynomial and nonlinear multiple regressions were considered, as well as the inclusion of other parameters, such as bulk density and porosity. However, an equation with a coefficient of determination of 0.95 was achieved with a simple regression where only dynamic elastic modulus was involved. This simple equation allows to predict static modulus of San Cristóbal's Stone with a high level of confidence and only from one parameter, that can be obtained in situ through non-destructive techniques and respectfully to built heritage.

Finally, a first approximation to the application on an ancient construction is provided. Six columns of the Monastery of San Jerónimo de Buenavista, in Seville (Spain) underwent tests based on the propagation of wave to determine *in situ* their dynamic elastic modulus. The *In situ* results for the dynamic elastic modulus are consistent with those obtained in lab.

Keywords:
historical constructions, dynamic tests, sandstone, non-destructive tests, San Cristóbal's stone, static elastic modulus, dynamic elastic modulus

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1. Introduction

When based on numerical models, most processes the structural assessment of historical constructions require the previous determination of structural materials mechanical properties such as Young's Modulus, also called elastic modulus. For this kind of assessments, the static value for the Young's Modulus (E_{st}) of rocks as constitutive material is needed to calculate both the stress and the strain states of historical buildings [1]. The most common process to obtain E_{st} has been to test drilled specimens in lab, trough the use of uniaxial compression tests. However, this process involves several limitations that are important to take into account in the case of historical constructions. The main one is its destructive character. As an action on the built heritage, the extraction of samples should not be extensive in this kind of constructions. This usually leads to have few specimens from which it is not possible to obtain representative values of E_{st}. On the other hand, the number of drilled specimens that would be necessary to obtain representative values is even higher when the construction is built using rocks especially sensitive to factors that can significantly affect E_{st}, such as moisture content or aging level [2]. They can considerably vary in the same building depending on the location of the rock in it, overall if the construction was built in stages.

Predictive methods make it possible to determine E_{st} from geomechanical parameters or rock indices by using analytic expressions that correlate E_{st} to them. Schmidt's hammer [3] rebound number, uniaxial compressive strength [4], P-wave velocity [5], dynamic Young's modulus (E_{dyn}) [6] and porosity or density [7] are the most common parameters or indices from which E_{st} can be predicted. The analytic expressions used to establish these correlations were traditionally obtained by applying regression methods that led to simple functions such as linear, logarithmic or exponential ones. Most of these functions were capable of providing fairly accurate predictions, with coefficients of determination greater than 0.90 in numerous cases.

The analytic expressions obtained by abovementioned methods must, in any event, be based on a database resulting from a different number of laboratory-tested rock specimens. These specimens are usually extracted from the same quarry as the stone that is the object of the study comes from. Therefore, they are usually unaltered specimens in good or optimum conditions and this situation does not coincide with the conditions of stone as part of a historical construction. In some cases, and in order to reproduce the real conditions of the historical stone, specimens are altered in laboratories before being tested. Thus, for example, some studies alter their moisture conditions [8, 9]. Likewise, other research reproduces different degrees of aging of the stone by means of heating the specimens [6].

San Cristóbal's stone was widely used to build some of the most representative constructions along the Guadalquivir Valley in Andalusia (Spain) during 15^{th}-18^{th} centuries. Nowadays, they constitute an important part of the protected Andalusian built heritage. Among these constructions, it includes religious, military and civil buildings are included. As some examples, the town hall, the Monastery of San Jerónimo de Buenavista [10], Hospital de las Cinco Llagas (Nowadays, Andalusian Parliament building), Charterhouse of Santa María de las Cuevas and much of the cathedral [11, 12], in Seville; the cathedral, the Atalaya Tower [13], San Dionisio's and Santiago's [14, 15], in Jerez de la Frontera (Cádiz); and the Mayor Priori Church, in El Puerto de Santa María (Cádiz) [12] (Fig.1).

Figure 1: Some examples of constructions built using San Cristóbal's stone: (a) Santiago Church (Jerez-Cádiz, 15^{th} - 17^{th} centuries); (b) Cathedral of Seville (Gothic period 15^{th} - 16^{th} centuries); (c) Monastery of San Jerónimo de Buenavista (Seville, 16^{th} - 17^{th} centuries).

Some of these constructions have recently required in-depth evaluations of their structural capacity skills after suffering different structural damages. Thus, for example, the cathedral of Seville was subjected to a study that eventually led to the replacement of two of its columns (2002-2009) [11]. On the other hand, and after having several cracks, injections were made into on different stone columns in San Dionisio and Santiago Churches and Monastery of San Jerónimo with the aim of improving its resistance [15, 16].

The properties of San Cristóbal's stone had not been deeply analysed before the interventions performed in these historical constructions. These interventions implied the opportunity for a first approximation of the main physical and mechanical propierties of this stone. Therefore, the following values were obtained from the extraction of stone specimens from some of the abovementioned buildings, such as Santiago Church and Monastery of San Jerónimo de Buenavista. Among these properties, the following ones stand out:

- Low bulk density (kg/m^3): 1710-1860 [14, 17, 18]
- High porosity (%): 22.4-27 [14, 17, 18]
- Compressive strength (MPa): 1.5-2.6 [14, 18]
- Elastic modulus (MPa): 4000-10000 [14]
- Moisture content affection: In saturated state, compressive strength showed decreases in between 25% and 40%. Likewise, a preliminary analysis performed by the authors reported that dynamic Young modulus reaches values between 20-35% lower with moisture levels of about 40% [19].

In the present work, the determination of E_{st} from the dynamic Young's modulus E_{dyn} is proposed for San Cristóbal's stone. In historical constructions, starting from E_{dyn} as reference parameter implies advantages over others from which E_{st} can be predicted. E_{dyn} can be determined *in situ* and by means of tests based on waves propagation, that is, tests with a non-destructive value character and respectful of the heritage protection of these historical buildings. From a static value of the Young's modulus of San Cristóbal's stone, structural assessments based on numerical models of a large number of representative historical buildings representative of Guadalquivir Valley in Andalucía can be performed. These kind of assessments would allow to obtain useful results to develop diagnostics and to make suitable decisions during intervention processes.

Different authors have stablished relationships between E_{st} and E_{dyn}. Table 1 shows the result of some of these investigations, carried out for different types of rocks. However, most of these studies refer to calculations with properties that differ from those characterized San Cristóbal's stone, such as the low density and the high porosity. A few specimens among those analysed by Eissa and Kazi reach density values around 1700 kg/m^3, that is, of the same range of San Cristóbal's stone density [20]. In their work, they determined two relationships between E_{st} and E_{dyn}. The first one (Table 1, (2)), was obtained from 174 test results, and gave a determination coefficient (R^2) equal to 0.71. On other hand, this coefficient reached values of 0.92 for the second relation (Table 1, (3)). Although this last relationship was defined from smaller number of specimens (76 observations), is interesting for the present study, for one of the parameters that includes is the bulk density, one of the most representative features of San Cristóbal's stone. Figure 2 plots equations (1)-(7) (Table 1) and indicates the range of the elastic modulus of San Cristóbal's stone.

Authors	**Year**	**Equation** (Mechanical range)	R^2	**Lithology**	**Physical range**
King [21]	1983	E_{st}=-29.5+1.26E_{dyn}(1) (40 GPa≤E_{dyn}≤120 GPa)	0.82	Igneous and metamorphic	Porosity: Most>1% Max=1.8%
Elissa and Kazi [20]	1988	E_{st}=-0.82+0.74E_{dyn}(2) $\log_{10}E_{st}=0.02+0.77\log_{10}(\rho_{bulk}E_{dyn})$(3) (4 GPa≤$E_{dyn}$≤130 GPa)	0.71 0.92	Several types	Density (kg/m^3): 1618–3320
Lacy [22]	1997	$E_{st}=0.442E_{dyn}+0.018E_{dyn}^2$(4) (0.7 GPa≤$E_{dyn}$≤6.2 GPa)	0.55	Sedimentary	-
Moradian et al.[5]	2009	$E_{st}=0.25E_{dyn}^{1.29}$(5) (4.98 GPa≤$E_{dyn}$≤83.89 GPa)	0.92	Limestone Sandstone Maristone	Density (kg/m^3): 2040-2920
Brotons et al.[6]	2014	E_{st}=-2.085+0.867E_{dyn}(6) (3 GPa≤E_{dyn}≤31 GPa)	0.96	Biocalcarenite	Bulk density (kg/m^3): 2100±700
Najibi et al.[23]	2015	$E_{st}=0.014E_{dyn}^{1.96}$(7) (13.7 GPa≤$E_{dyn}$≤77.4 GPa)	0.87	Biocalcarenite	Density (kg/m^3): 2100-2700

Table 1: Empirical relationships between static (E_{st}) and dynamic (E_{dyn}) Young's moduli

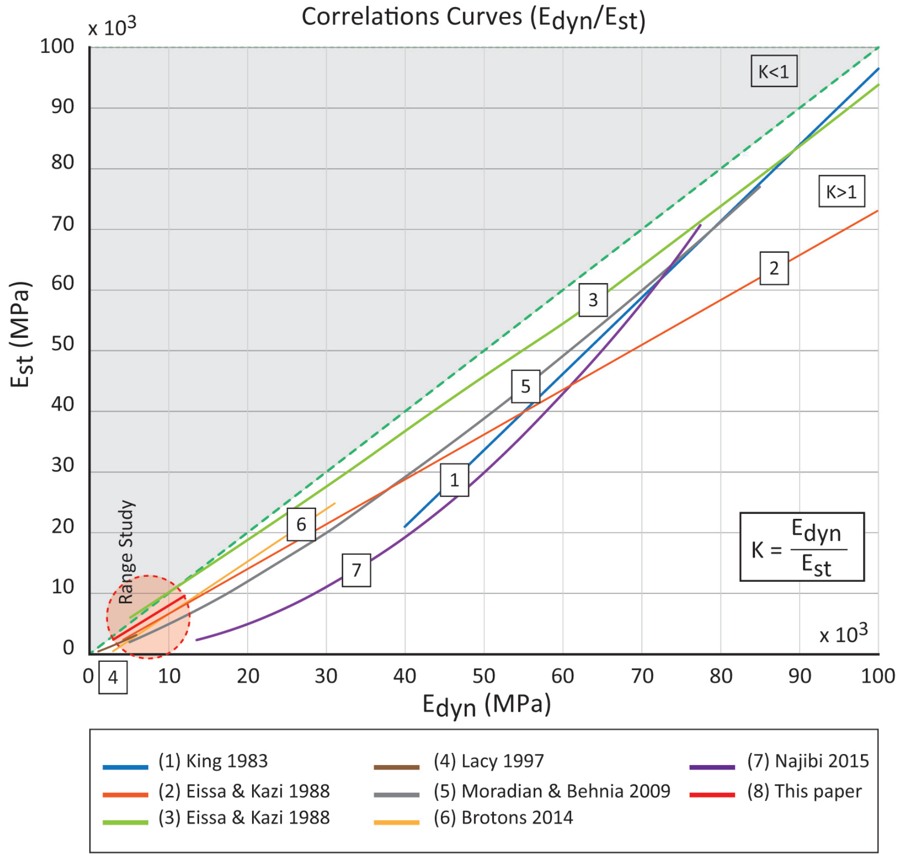

Figure 2: Relationship between static and dynamic elastic modulus. Plot of the equations (1) to (7) (Table 1).

The main aim of this study is to propose a particular correlation for obtaining the static Young's modulus of San Cristóbal's stone from its dynamic Young's modulus, throughout the testing in lab of original stone specimens from Santiago Church (Jerez de la Frontera, 16^{th}- 17^{th} centuries). It is included an earlier analysis of the validity for San Cristóbal's stone of some of the most relevant preceding relationships, such as those presented in Table 1. As a secondary aim, it is proposed the establishment of an order of magnitude between values of E_{dyn}, obtained in lab for San Cristóbal's stone, and those obtained *in situ* from certain support elements of Monastery of San Jerónimo (Seville, 16^{th}- 17^{th} centuries), built with the same stone (Fig.1).

To achieve the described aims, 17 specimens extracted from Santiago Church were mainly underwent to sonic wave tests in order to determine E_{dyn}. On the other hand, uniaxial compression tests were conducted to determine E_{st}. The obtained results were related to mathematical expressions proposed by other authors (Table 1) and finally an expression specifically proposed for San Cristóbal's stone was developed. In order to establish a reference about E_{dyn} values obtained *in situ* for San Cristóbal's stone in historical constructions, results extracted from sonic wave tests directly carried out on five columns of Monastery of San Jerónimo, in Seville, are also related to those obtained in the laboratory.

For the present study, it is a unique opportunity to have original stone specimens directly extracted from Santiago Church in Jerez de la Frontera (Cádiz). The specimens were extracted from support elements of the church (columns and walls) during the structural consolidation works that were carried out in it (2007). The obtained correlation between E_{st} and E_{dyn} is a valuable contribution, as it is useful to determine E_{st} for the structural analysis of several Andalusian buildings with a significant heritage value, by means of non-destructive tests that can be performed *in situ*.

2. Brief historical introduction and architectural configuration: Santiago Church and Monastery of San Jerónimo de Buenavista

Santiago Church (Jerez de la Frontera-Cádiz, 1496-1603). The church was built on an Islamic military construction, which was transformed into a Christian shrine after the Spanish Reconquest, in the 13^{th} century.

As a basic architectural configuration, the church is rectangular in plan, with three naves covered by ribbed vaults. The main nave is the central one, which is the highest. From the main rectangle in plan, the apse and several chapels protrude [24] (Fig.3). As a support structure, the perimeter wall and six isolated columns squared-in-plan stand out. Likewise, another couple of columns is disposed attached to walls that extend in parallel until the apse.

From the end of the 17^{th} century, Santiago Church has been presenting serious structural problems which led into important interventions. The following are some of the most important ones: reconstruction of columns 3 and 5 and the six vaults over them after they collapsed (1695); reconstruction of column 1, using calcarenite stone of greater compactness and resistance (1905); substitution of lower ashlars of column 6 (1928); reconstruction of column 4, including a softly reinforced concrete core, after its collapse (1962); consolidation of columns 2-8 and some walls by means of grout injection (2007-2014) [14, 15].

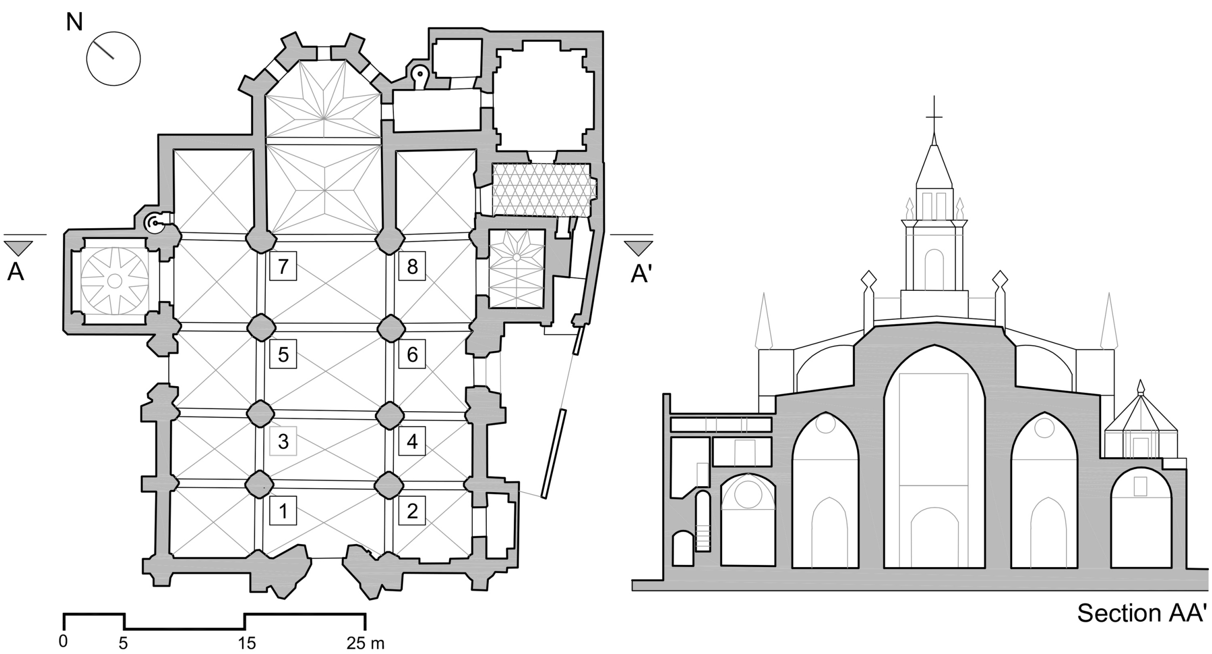

Figure 3: Santiago Church: Plant and cross section (See also Figure 1-a).

Monastery of San Jerónimo de Buenavista (Seville, 1414- 1597). In 1414, the construction of the Monastery of San Jerónimo began. The first part in being built was the gothic church (Fig. 4), followed by the eastern cloister (16^{th} century) and the main cloister. This last one was built using San Cristóbal's stone and under a renaissance style, and was completed in the 1580's. The top of the tower and the printing press were the last under construction. The construction of the complex was completed in 1597.

At the beginning of the 19th century, the monastery entered a period of decline. Nowadays, almost the entire main cloister remains standing, as well as the tower, two chapels and the staircase of the church (Fig. 4). After being declared a National Historic and Artistic Monument in 1964, the whole was subjected to consolidation works during 1960s to 1980's. Some of the most relevant interventions were developed in the northern wing of the main cloister: the column which had collapsed in 1969 located in front the tower was rebuilt using calcarenite stone; some columns were subjected to mortar injections; the filler over the upper gallery was replaced by concrete beams were added to prevent the collapse of the vaults and a concrete slab was added over the filling of the first floor [10].

Recently, two new wings were built attached to the remains of the main cloister to the east and south (Fig. 4). The aim was the enhancement of the historical construction and its rehabilitation as civil center. In 2013, and after the completion of the new construction, new damages were detected on the ancient structure. Nowadays, the complex is under analysis in order to assess the convenience of a new structural intervention.

Figure 4: Monastery of San Jerónimo de Buenavista: original architectural configuration, 1650 (left) and current architectural configuration, including new wings to the south and east of the historical remains (right). (See also Figure 01-c).

3. Methodology

To achieve the aims of this study, different tests were carried out both on 17 stone specimens in lab and on support elements (columns) *in situ*. Below is the description of the following items: samples analysed, tests carried out on them, studied columns and tests to which the latter were subjected.

Specimens description. The stone used to build Santiago Church in Jerez de la Frontera was obtained from San Cristóbal's quarry [24], located SW of Jerez, next to El Puerto de Santa María and in the Guadalete River Basin. It is a calcitic sandstone. The main components are: $CaCO_3$ (67%) and SiO_2 (30%), as an average composition [17].

Extraction	Specimen name	Depth of extraction, from the top (m)	Height (m)	Length (mm)	Diameter (mm)	Length/ diameter
Column 2	S01	11.20	12.80	164	63	2.6
Column 2	S02	19.27	4.73	165	69	2.4
Column 5	S03	5.15	18.85	215	83	2.6
Column 6	S04	2.18	21.82	183	82	2.2
Column 6	S05	4.60	19.40	168	83	2.0
Column 6	S06	6.80	17.20	194	83	2.3
Column 6	S07	9.27	14.73	180	82	2.2
Column 6	S08	11.55	12.45	204	83	2.5
Column 6	S09	12.85	11.15	202	83	2.4
Column 6	S10	21.21	2.79	144	71	2.0
Column 6	S11	22.03	1.97	159	71	2.2
Buttress A	S12	9.75	14.25	131	58	2.3
Buttress B	S13	11.26	12.74	149	61	2.4
Buttress C	S14	21.59	2.41	156	61	2.6
Buttress D	S15	1.60	22.40	155	62	2.5
Buttress D	S16	22.90	1.10	146	61	2.4
Buttress E	S17	2.67	21.33	149	62	2.4

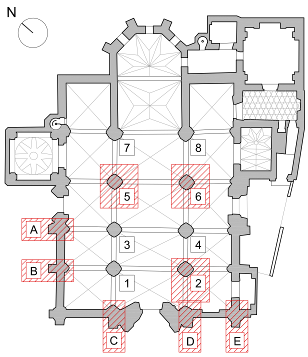

Table 2: Location and dimmensions of analyzed specimens from Santiago Church.

Most of the specimens that have been analyzed in this study are dated of late 15^{th} and early 16^{th} centuries, as the elements from which they were extracted. They were obtained from columns and walls of Santiago Church by controlled drillings from the upper section of the elements to their foundations, so the main direction of the cylindrical specimens follows and respects the longitudinal direction of the corresponding structural elements. Table 2 shows the corresponding location of each one of the analyzed specimens and their dimensions.

Tests on stone specimens. Prior of performing sonic tests to obtain E_{dyn} and uniaxial compression tests to obtain E_{st}, stone samples were dried in oven following the criteria that UNE- EN 1936/2007 standard contains [25].

The cylindrical specimens are different in size due to the followed process of drilling stone from columns and walls. However, the dimensions of every one of those that were tested guarantee that they can be used to apply the codes selected to perform the corresponding tests (Table 2). Thus, to obtain dynamic elastic modulus, it was used the standard UNE-EN 14146:2004 [26]. It requires the specimens to have a length to diameter ratio greater than 2. Likewise, static Young's modulus was obtained by applying the standard UNE-EN 14580:2006 [27]. For the application of this code, specimens must observe three conditions: their diameter must be greater than 50 mm, their slenderness must be between 2 and 4, and a ratio 10:1 is required for the ratio diameter to maximum crystal grain size. On the other hand, shorter slicing specimens were extracted from each of the main ones to determine their bulk density and porosity, ensuring that it results dimensions were within the requirements of the standard applied in any case.

Bulk density and porosity. The UNE- EN 1936/2007 standard [25] determines how to obtain these properties. Thus, the bulk density is obtained by means of the ratio of the weight of the dried specimen and its bulk volume. Likewise, porosity is the ratio between the volume of pores and the bulk volume of the specimen.

Sonic test. Dynamic Young's modulus E_{dyn} was obtained for each specimen from the analysis of the propagation of sonic waves through them and according to UNE- EN 14146:2004 standard [26]. It was used a V-E-400 Emodumeter device in order to obtain the longitudinal time-domain signal. An accelerometer located in the centre of one of the faces of each cylindrical specimen is used as a receiver. On the other hand, a hardened steel ball is used as an instant exciter, which generates the vibrations acting on the centre of the opposite face. To guarantee a good coupling between the accelerometer and the specimen, a visco - elastic couplant was used. Time-domain signals were uploaded from the storing Emodumeter device to a laptop. They were then analysed to identify the longitudinal resonance frequency of each measurement using the Fourier transform. According to UNE- EN 14146:2004 [26], each specimen was vibrated until three consecutive values of frequency are ± 60 Hz of the fundamental resonance frequency. E_{dyn} was determined from the fundamental resonance frequency and the bulk density [26].

Uniaxial compressive test. Static Young's modulus E_{st} was obtained using a MTS Criterion Electromechanical Test System of 100 KN load capacity (C45.105) and applying instructions of UNE-EN 14580:2006 [27]. On the other hand, a National Instruments device (cDAQ-9174, NI-9219), jointly with LVDTs Solartron AXR/2.5/S were used to measure strains during the mechanical test. Two softwares were used: MTS TestSuite TW, for controlling loading cycles, and Signal Express 2015, for measuring LVDTs deformation.

Two circumstances led to adapt instructions of UNE-EN 14580:2006 [27] related to loading. The first was the compressive resistance values obtained from specimens of San Cristobal's stone extracted from different ancient structures (see section 1) [14, 18] These values did not exceed 2.6 MPa. On the other hand, the aforementioned standard [30] assigns 0.5 $\pm$ 0.2 MPa/s as a speed to apply a load that reaches 1/3 times the compressive strength. These two circumstances supposed that a high speed in the loading application had to be considered (about 6s each charge-discharge cycle). To avoid effects of rapid loads, the instructions of UNE-EN 14580:2006 [27] were adapted. Thus, the load was applied so that a failure ocurred in a test time between 2 and 15 min, as indicated by the American Society for Testing and Materials (ASTM) [28]. During the test, three LVDTs were placed on each specimen. They were separated an angle of 120^o. This made it possible to take three measurements during each charge-discharge cycle. Then, an average value was considered as a result.

Location and description of support elements. Figure 5 shows the location of the six columns of the Monastery of San Jerónimo, in Seville, that were tested. They belong to the main cloister, which is the one that still remains. Archaeological studies have stated that the stone with which these columns were made proceed from San Cristóbal quarry, that is, the same one from which the stone samples analysed in this study were extracted [10]. These studies also established an approximation about the date of the construction of the different wings of the main cloister. Thus, columns at the northern wing were the first, dating from the first third of the 16^{th} century. In the last decade of the same century, the rest of the wings were built. Taking into account that columns from which the stone samples used in this study were extracted are dated from the second half of the 16^{th} century, it is presumed that the stone of both columns of the Monastery and columns of Santiago have a very similar age. Ground-penetrating radar carried out on them in 2014 showed that they present a quite homogeneous section without relevant discontinuities [16]. Having simple symmetry, their cross section measures about 1.20m per 1m (Fig. 5).

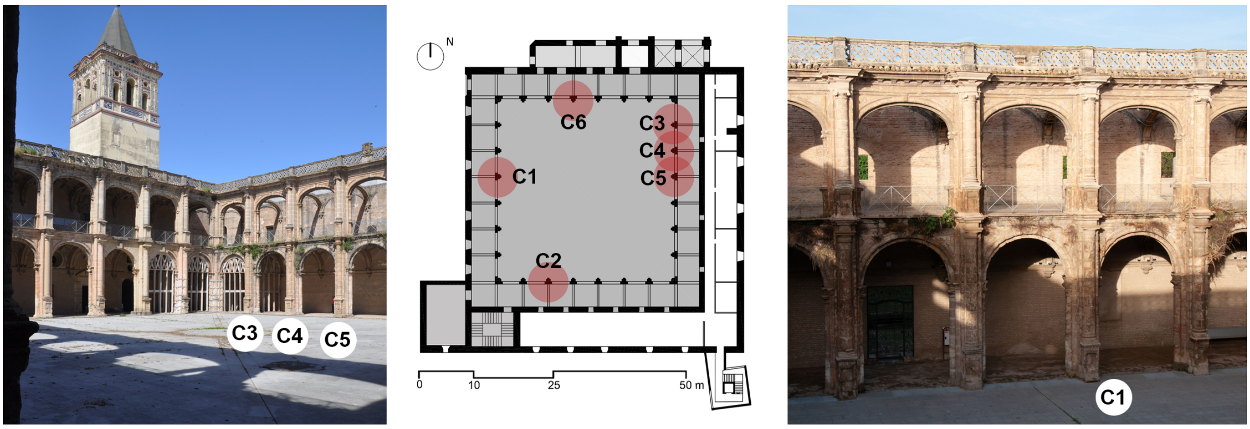

Figure 5: Monastery of San Jerónimo of Buenavista, Seville. Location of the five columns that were tested *in situ*.

Tests on support elements. Five columns of the Monastery of San Jerónimo, in Seville, were subjected to ultraseismic tomography to obtain E_{dyn}. Two horizontal and quasi-orthogonal measures were performed in each one of the columns at a height of 2m (Fig. 6). E_{dyn} can be obtained from the velocity of both compressional waves (P-waves) and shear waves (S-waves). To obtain these velocities, a triaxial accelerometer registered waves, which were generated by high-frequency pulses (about 54 kHz) at opposite points. Knowing the starting moment of waves generation by means of a piezoelectric trigger, the distance travelled by waves was put into relation to the time that they spent on being registered, obtaining wave speeds. A density of 1800 kg/m^3 was taken to determine E_{dyn} [16].

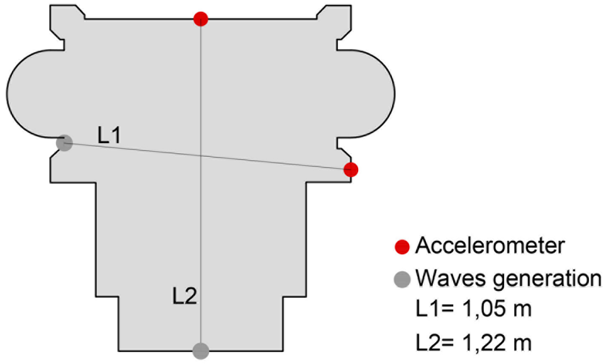

Figure 6: Cross section of columns. Ultraseismic tomography. Transits for the determination of E_{dyn}.

4. Results and discussion

4.1. Samples results

Table 3 shows the obtained results for 17 specimens extracted from Santiago Church with respect to bulk density, open porosity, static elastic modulus (E_{st}) and dynamic elastic modulus (E_{dyn}). Likewise, Figure 7 represents information about the distribution of the four studied parameters.

Specimen name	**Bulk density** (Kg/m^3)	**Porosity** (%)	$\mathbf{E_{st}}$ (MPa)	$\mathbf{E_{dyn}}$ (MPa)
S01	1760.75	34.49	5970	7509
S02	1843.26	29.58	10081	11543
S03	1759.05	34.73	4715	6357
S04	1742.76	35.08	4889	6401
S05	1831.65	31.65	9218	11948
S06	1693.98	36.97	4204	5212
S07	1777.66	33.88	5511	7459
S08	1655.74	38.66	3409	3755
S09	1867.62	30.75	6752	8370
S10	1766.65	34.08	3058	3990
S11	1791.88	33.03	7718	9579
S12	1761.35	34.30	5754	6155
S13	1752.66	34.5	5206	6687
S14	1798.32	33.11	6430	8787
S15	1753.07	34.83	4479	6007
S16	1737.72	35.25	3708	4891
S17	1861.19	30.68	4628	5113

Table 3: Results for specimens obtained from tests in lab (Santiago Church).

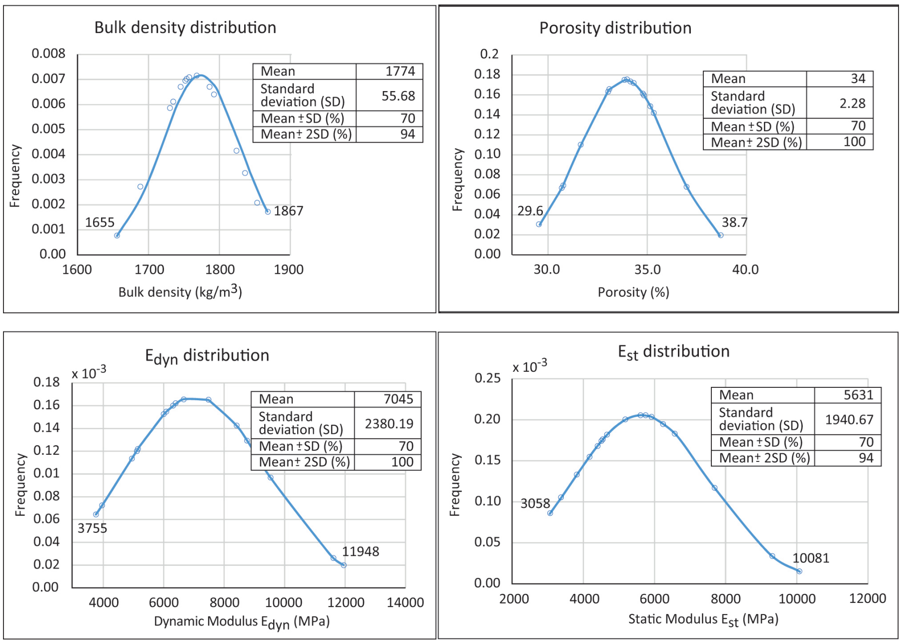

Figure 7: Tests results from Santiago Church stones. a) Bulk density, b) Porosity, c) Dynamic Young modulus, d) Static Young modulus.

Bulk density and porosity. Presenting an average of 1774 kg/m^3, obtained values for bulk density are within the range stablished by other authors for San Cristobal's stone [14, 17, 18], that is, between 1700-1860 kg/m^3 (see Section 1). However, the porosity values obtained by other authors are close to the lowest values extracted from this study, so the porosity is even higher than expected. A clear relationship has been found between bulk density and porosity, as shown in Figure 8. This relationship contributes to the validation of the results obtained for bulk density and porosity. Likewise, the normal distribution of the data is highlighted, with at least 68% of specimens in the Mean range ± SD and 96% of specimens in the Mean range ± 2·SD (note that an out of range constitutes the 16% of the sample) (Figure 7).

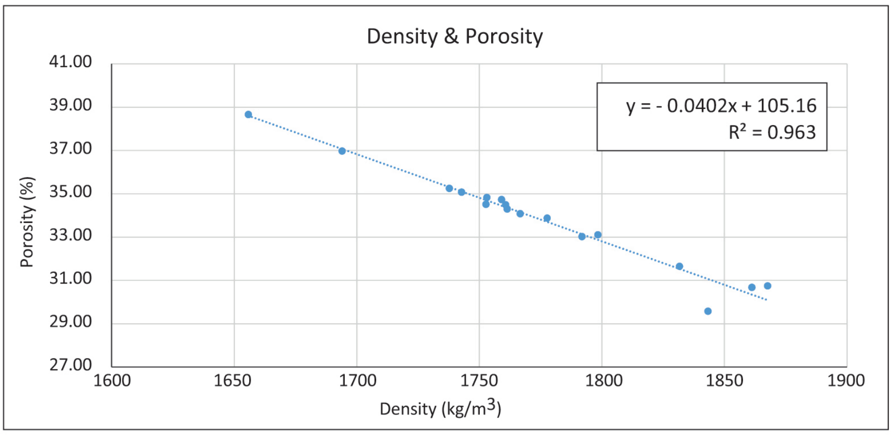

Figure 8: Relationship between bulk density and open porosity.

Static and dynamic elastic modulus. Figure 9 represents E_{dyn} and E_{st} of each specimen studied in this paper. Likewise, it shows the relationship between experimental results and the equations proposed by other authors. The values that were obtained for E_{dyn} are particularly low in the case of San Cristóbal's stone. It can be observed that only a few authors, such as Eissa and Kazi [20], Moradian and Benhia [5] and Brotons et al. [6], included in their studies specimens whose dynamic moduli are around the values obtained for San Cristóbal's stone. Figure 9 also shows that present data deviate from equations previously proposed.

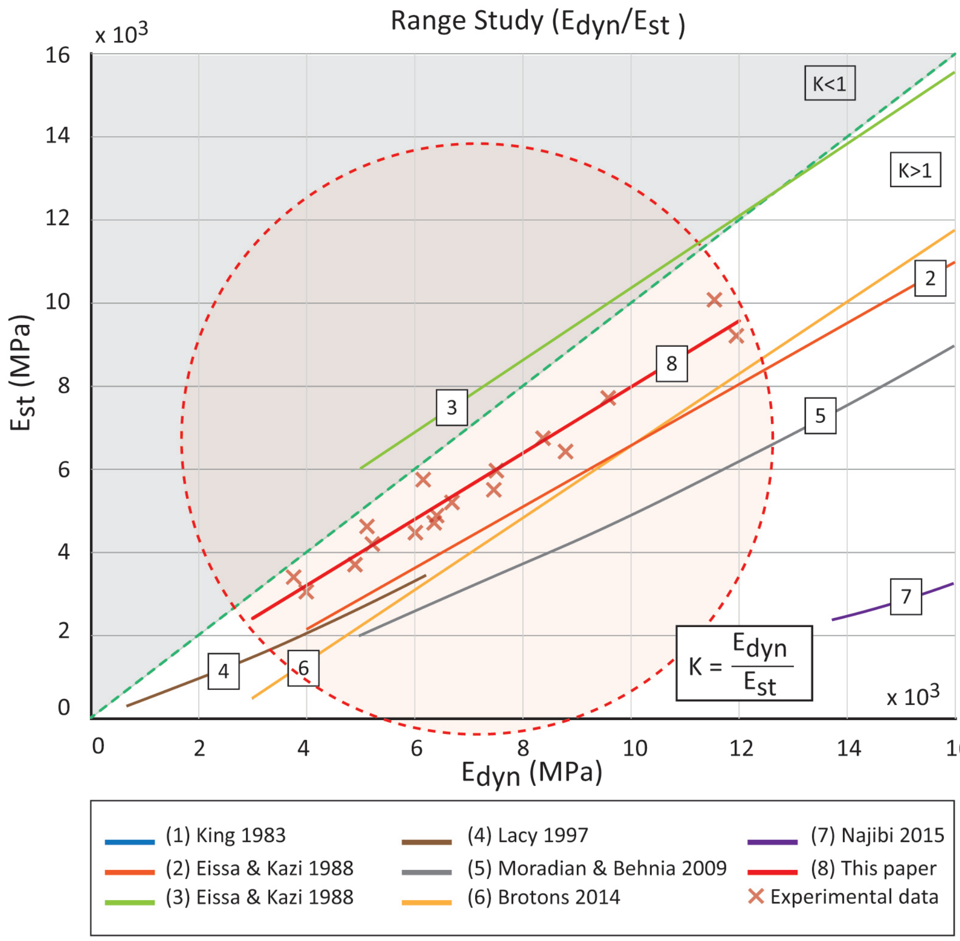

Figure 9: E_{dyn} and E_{st} of each studied specimen and their relationship to equations proposed by other authors.

4.2. Obtained results in columns

In general, there is no substantial difference between results obtained from orthogonal directions read in each column. As Table 4 shows, columns C4 and C5 present the greatest difference (a SD of 630 MPa and 550 MPa respectively). On the other hand, columns C2, C3 and C6 present a low difference between orthogonal readings (a SD lower than 200 MPa).

Column	Distance (m)	Time-P (s)	Time-S (s)	VP (m/s)	VS (m/s)	E_{dyn} (MPa)
C1	1.22	0.721	1.426	1694	856	3510
	1.05	0.646	1.303	1634	810	3160
C2	1.22	0.693	1.334	1763	915	3970
	1.05	0.563	1.124	1874	939	4230
C3	1.22	0.797	1.590	1532	768	2830
	1.05	0.691	1.404	1526	751	2720
C4	1.22	0.786	1.578	1554	774	2880
	1.05	0.792	1.650	1332	639	1990
C5	1.22	0.901	1.788	1337	673	2170
	1.05	0.673	1.344	1568	785	2950
C6	1.22	0.632	1.318	1930	926	4170
	1.05	0.567	1.100	1862	959	4370

Table 4: Results for columns obtained from tests in situ (Monastery of San Jerónimo de Buenavista).

From results showed in Table 4, two groups of columns can be identified. The first group is constituted by those columns whose dynamic elastic moduli are less than 3000 MPa (columns C3, C4 and C5). Secondly, there are columns which present values for E_{dyn} higher than 3000 MPa.

The columns in the first group are located contiguously along the eastern wing of the cloister and close to the area that presents the highest level of structural damage in the ancient building (north-east corner) (Fig. 5) [16].

From the tests on columns of the second group (C1, C2 and C6), values for E_{dyn} between 3160-4370 MPa were obtained. They match with the lowest dynamic elastic moduli obtained for the analysed specimens from Santiago Church (see Table 4 and Figure 7).

Considering previous studies which demonstrated the general decrease of E_{dyn} in stones with moisture content with respect to their dried state [9, 19], and the fact that the columns had a moisture content of around 40% when they were tested [16], it can be considered that values for E_{dyn} obtained in this study *in situ* are consistent with those obtained in the laboratory. This helps to validate results of the tests carried out.

4.3. About relationships between E_{dyn} and E_{st} for San Cristóbal's stone

Figure 9 shows that results obtained for San Cristóbal's stone do not match with equations proposed by other authors for specimens whose elastic moduli are within the same range as San Cristóbal's stone.

Once it has been confirmed that specific properties of San Cristóbal's stone would require a specific equation to correlate its static and dynamic moduli, different options were assessed in order to achieve it. Table 5 gives the main equations that were obtained and the corresponding coefficient of determination.

	Equation	Type	R^2
(8)	$E_{st} = 0.0283 + 0.7952*E_{dyn}$	Simple regression	0.95
(9)	$E_{st} = 0.3511 + 0.0004*E_{dyn}*\rho_{bulk}$	Simple regression	0.95
(10)	E_{st} = -0.6889 + $0.0268*E_{dyn}*P$	Simple regression	0.89
(11)	$E_{st} = 1.2510 + 0.4496*E_{dyn} + 0.0221*E_{dyn}^2$	Polynomial regression	0.96
(12)	$E_{st} = 1.30 + 0.0002*E_{dyn}*\rho_{bulk} + 5.34\cdot10^{-9}*(E_{dyn}*\rho_{bulk})^2$	Polynomial regression	0.96
(13)	$E_{st} = 1.7075 + 0.0006*E_{dyn}*P + 0.0001*(E_{dyn}*P)^2$	Polynomial regression	0.90
(14)	$E_{st} = -2.9196 + 0.7692*E_{dyn} + 0.0017*\rho_{bulk}$	Nonlinear regression	0.95

E_{dyn}: Dynamic modulus (MPa); E_{st}: Static modulus (MPa); ρ_{bulk}: bulk density (kg/m^3); P: porosity (%)

Table 5: Equations of correlation between E_{dyn} and E_{st} for San Cristóbal's stone.

The results show how simple regressions gives equations with a coefficient of determination (R^2) as high as those obtained by using polynomial or nonlinear ones, reaching coefficients of 0.95. Likewise, considering other parameters beyond E_{dyn} to predict E_{st} is not relevant, due to equations reach the highest values for R^2 by means of including the dynamic elastic modulus as unique parameter.

5. Conclusions

The interest of finding a relationship between dynamic and static moduli (E_{dyn} and E_{st}) of San Cristóbal's stone, used to build a wide representation of Andalusian architectural heritage, is related to the possibility of predicting a mechanical property necessary to perform structural analyses of historical constructions (E_{st}) from a parameter that can be obtained *in situ* by non-destructive tests (E_{dyn}).

Studied physical and mechanical properties of San Cristóbal's stone locate it out of the ranges associated to the majority of the stones previously analysed by other authors who determined different relationships between E_{dyn} and E_{st}.

This work has demonstrated that authors who studied stones with elastic moduli as low as San Cristóbal's stone found equations to correlate E_{dyn} and E_{st} that are not valid for this one.

From tests on specimens extracted from support elements of Santiago Church (16^{th} century), a relationship has been obtained between Edyn and Est for San Cristóbal's stone with a high determination coefficient (R^2= 0.95). Despite other parameters characteristic of this stone were considered to predict Est, such as bulk density and porosity, a simple equation was chosen using as few parameters as possible due to the high determination coefficient achieved using only E_{dyn} and the fact that including them does not imply substantial improvements in results.

Results from the experimental campaign carried out to obtain in situ E_{dyn} in Monastery of San Jerónimo de Buenavista (Seville, 16^{th} century) are consistent with those obtained in laboratory from specimens. However, having regard to the fact that the laboratory tests were carried out on dry stone and tests in situ on stone with its moisture content, the extrapolation of results obtained in situ to E_{st} values useful in numerical models could be improved by quantifying influence of the moisture content on E_{dyn} in San Cristóbal's stone.

In light of the results obtained in the present work, two lines have been opened as future research: (i) the quantification of the influence of the moisture content on mechanical properties of San Cristóbal's stone; (ii) the effect of time on mechanical properties of San Cristóbal's stone by studying new stone extracted from the quarry.

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