Proclus's Reception in the 16th Century: *Commentary on the First Book of Euclid's Elements*

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1. Introduction

I intend to analyse here some aspects of Proclus's reception in the 16th century, in particular of his *Commentary on the First Book of Euclid's Elements* (*In Euc.*)[1]. The impact of this text in Europe during the second half of the 16th century has been known for years, as it is one of the factors contributing to the birth of so-called 'modern science'[2]. I understand by 'modern science' the assumption of mathematics as a universal language and as an adequate method for physical science, as well as the rejection of the theory of Aristotelian science.

[1] Proclus, *In Euc.* (Friedlein 1873; trans. Morrow 1970).

[2] Crapulli 1969; Giacobbe 1972a, 1972b, 1977; De Pace 1993; Kessler 1995; Romano 1999; Marion 2000; Sasaki 2003; Rabouin 2009.

More specifically, I wish to specify to what extent the use of Proclus's *In Euc.* is faithful (or not) to Proclus's philosophy of mathematics, and in what aspects its reading is relevant to the evolution of the modern scientific method.

Unlike other Proclus texts (e.g. *Elements of Theology)*, the *In Euc.* arouses little interest in Byzantium[3], and is also not well known among Arabs or Latins. Only during the 14th century do we know of the growing interest in Byzantium for Proclus's scientific writings[4].

The *In Euc.* was well known in Italy in the 15th century[5]. For instance, Giorgio Valla included, without citing it, the prologue in his work *De rerum expetendarum et fugiendarum* (1501). His pupil Bartolomeo Zamberti prepared a Latin translation of *In Euc.,* without getting to edit it[6]. In any case, no specific attention was paid to it until Grynaeus (1533) published the *editio princeps* of Euclid's *Elements,* together with Proclus's commentary[7]. The interest in Proclus's commentary was also reasonable since it was the only Greek commentary available on the *Elements*.

Grynaeus not only edited the *In Euc.* Moreover, in a *Praefatio* to his edition[8], following Proclus, he presents mathematics as the universal science, and not simply as a particular subject. Along with dialectics, geometry is the paradigm of the scientific method, as it is the core of discursive rationality: it provides a true knowledge of the physical world (*machina mundi),* based on an *a priori* knowledge of the spirit.

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[3] Thion 2009.

[4] Cacouros 2000, 2007a, 2007b.

[5] It is in the library of Pico della Mirandola (Kibre 1936: *ms. grecus pap.* 219).

[6] Kessler 1995, 289-290.

[7] Grynaeus received the *In Euc.* from 'Ioann. Claymundus' (Oxford): Εὐκλείδου Στοιχεῖων, *Praefatio* (Apud Ioan. Heruagium 1533, a6).

[8] Kessler 1995, 290-294.

A second stage of the reception of this text takes place in 1560, when Barozzi publishes the Latin translation. Moreover, around this year, Barozzi himself dictated and published various lessons on the text.

However, the interest in the *In Euc.* increased when Alessandro Piccolomini published in 1547 a *Commentarium de certitudine mathematicarum disciplinarum*. As the title announces, Piccolomini asks if —as Averroes asserts in his commentary on Aristotle's Book II of *Metaphysica,* and the Aristotelian tradition with it— mathematics offers the first degree of certainty among all knowledge.

Piccolomini examined the reason why mathematics exhibits such maximum certainty in the face of all other knowledge. Among the various arguments that Piccolomini presented, he cited numerous arguments taken from Proclus's text. In other words, he used Proclus to argue in favour of the certainty of mathematics, always in the context of Aristotelian science.

Piccolomini's text generated a wide debate[9]. In this controversy, no one doubted that the first degree of certainty corresponds to mathematics, but the discussion was about the reasons why mathematics is most true.

In particular, Francesco Barozzi rejected Piccolomini's argument in his writing *Opusculum, in quo una oratio et duae quaestiones: altera de certitudine, et altera de medietate mathematicarum continentur* (1560). What is surprising is that, in contrast to Piccolomini, Barozzi also used Proclus in his arguments, showing more rigorously the precise meaning of Proclus's texts.

[9] The issue generated a much wider debate in which many other authors intervened. For instance, Pietro Catena, a mathematics professor from Padua, followed Piccolomini in 1556: Giacobbe 1973.

For half a century, other philosophers and mathematicians will take part in these discussions (Pereira, Clavius, Dasypodius, Van Roomen, Alsted), who, in different ways, will take sides in favour of Piccolomini's theses or, on the contrary, in favour of Barozzi.

In summary, Proclus's text is widely quoted in a debate —internal to Aristotelianism— about the certainty of mathematics. The opposing positions are:

1) Piccolomini refuses that mathematics has strictly scientific demonstrations (*demonstrationes potissimae*), because it moves in the field of accidents (quantity) and imagination. Therefore, it does not meet the requirements demanded by Aristotle in the *Analytica* for a strictly scientific demonstration.

2) Barozzi accepts the validity of mathematical demonstrations, arguing that mathematics is completely in line with the requirements that Aristotle demands of the sciences.

What is surprising about this debate is that there are so many references to Proclus's text in both cases.

In my presentation, I will present in the following order. Firstly, I will summarise some aspects of the philosophy of mathematics in Neoplatonism, specifically in Iamblichus and Proclus. Secondly, I will examine the already mentioned controversy about the certainty of mathematics and the role played by Proclus's text in this debate; in particular, I will successively review the role of mathematics in 1) Aristotle, 2) Piccolomini, 3) Barozzi, 4) Clavius, 5) Pereira, and 6) finally I will mention succinctly the impact of *In Euc.* in an entirely mathematical context (Dasypodius, Van Roomen, Alsted). Finally, as a conclusion, I will present some considerations on the different readings of *In Euc*. in the 16th century.

2. Mathematics in Iamblichus and Proclus

The role of mathematics in Proclus belongs to the tradition of Greek mathematics[10], and more directly to the Platonic and Pythagorean tradition[11]. Proclus relies on the mathematical ideas of Porphyry[12], of Iamblichus (primarily in *De communi mathematica scientia*)[13] and of Syrianus[14] (in his commentary on Aristotle's *Metaphysica*, specifically the books M and N).

De communi mathematica scientia (Περὶ τῆς κοινῆς μαθηματικῆς ἐπιστήμης) is the third book of his work *On Pythagoreanism*. This work establishes that a universal mathematical science exists[15] with principles (limit and infinity)[16] that derive from dialectics[17]. The essence of the soul is constituted by mathematics[18]. In particular, Iamblichus shows the possible applications of mathematics to science, detailing various procedures of physical-mathematical science[19].

"The Pythagoreans found what is possible and impossible in the structure of the universe (τὰ δὲ δυνατὰ καὶ ἀδυνατα τῇ τοῦ κόσμου συστάσει) from what is possible and impossible in mathematics, [...] establishing the predictive science of nature (τὴν φυσιολογίαν

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[10] Sidoli 2018.

[11] O'Meara 1989.

[12] Mueller 1987a, 309-313.

[13] Mueller 1987b.

[14] O'Meara 1989, 119-141.

[15] Iamblichus, *Comm. Math*. 4, 5 (Romano 2006, 486).

[16] *Ibid*. 7, 29 (Romano 2006, 500).

[17] *Ibid*. 12, 26-27 (Romano 2006, 492).

[18] *Ibid*. 4, 10 (Romano 2006, 488).

[19] Sambursky 2009, 73.

τὴν προγνωστικὴν) from mathematics, turning mathematics into the principle of all that can be observed in the cosmos"[20].

Iamblichus's argument is that "ἐπεὶ γὰρ προτέρα ἐστὶ τῇ φύσει ἡ μαθηματική θεωρία […], διὰ τοῦτο καὶ τοὺς συλλογισμοὺς ποιεῖται ὡς ἐκ πρότερων αἰτίων ἀποδεικτικούς"[21]. The physical-mathematical science has various procedures, which Iamblichus briefly analyses[22], such as: abstraction, improvement, participation, division, comparison, and of course "the causal consideration (ἢ κατὰ τὴν αἰτίαν τὴν ἀπὸ τῶν προτέρων), when we put mathematical things as causes and examine how sensitive objects come from them"[23]. With these methods, concludes Iamblichus, "we can address mathematically everything in nature and in the world of the generation"[24].

These doctrines of Iamblichus are widely disseminated in the Neo-Platonic schools. They will immediately find an echo in Syrianus and Proclus. Later, in the 6th century, this interpretation of mathematics continued to be common, such as in Asclepius of Tralles, Eutocius of Ascalon[25] or Simplicius.

Centuries later in Byzantium, Psellos spread Iamblichus's ideas about mathematics, also transmitting some fragments of the books V-VII of *On Pythagoreanism* (which deal with arithmetic in physics, ethics and theology)[26].

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[20] Iamblichus, *Comm. Math*. 23, 73,22-74,1 (Romano 2006, 582).

[21] *Ibid*. 32, 93, 5-8 (Romano 2006, 610): "mathematics is prior to nature [...] and therefore it does its demonstrations from prior causes".

[22] *Ibid*. 32, 93,19-94,12 (Romano 2006, 610).

[23] *Ibid*. 32, 94, 7-10 (Romano 2006, 610).

[24] *Ibid*. 32, 94, 10-11 (Romano 2006, 610).

[25] Sasaki 2003, 296-301.

[26] O'Meara 1989, 53-91.

Shortly after, in the 12th century, Michael of Ephesus, considered nowadays the author of part of the commentary on Aristotle's *Metaphysica,* traditionally attributed exclusively to Alexander of Aphrodisias, insists on the unity and universality of mathematical science, which groups together the different branches of mathematics, such as arithmetic, geometry, music and astronomy. Specifically, he interprets ἡ καθόλου (E-1) as referring to universal mathematics (ἡ ἁπλῶς μαθηματική)[27], always subordinating this universality to that of the first philosophy. These comments by Michael of Ephesus will have a strong impact in the 16th century on account of their attribution to Alexander of Aphrodisias.

As far as Proclus is concerned, the core of his philosophy of mathematics is in the *In Euc*. And especially in his two prologues: one on general mathematics, and the other on geometry.

However, many of his other writings revolve around mathematics. For instance, the *Commentary on Plato's Timaeus,* the *Inquiry into Aristotle's Objections against the Timaeus*, the *Hypotyposis astronomicarum positionum* or the *Elements of Physics* include many considerations about the role of mathematics in physics.

I will schematically summarize some of Proclus's most characteristic theses, which are largely contained in the two prologues of *In Euc.*[28]:

1) Mathematical science, although subordinated to dialectics, which is the science of demonstration, contains *a priori* truths in the soul, truths that derive from divine numbers and, ultimately, from the first principles of limit and infinity.

2) Mathematical objects are projections of intelligible numbers on the soul, and specifically on the imagination. Thus, in contrast to Aristotle, mathematical science is not the result of abstraction but of the projection of the intelligible in the soul.

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[27] Alexander Aphrodisiensis, *In Metaph*. (Movia 2007, 447, 17).

[28] O'Meara 1989, 142-209; Cleary 2013; Garay 2018; Mueller 1987; Breton 1969.

3) The various mathematical sciences constitute a single common mathematical science, with the same principles.

4) The physical world is modelled by the demiurge according to ideal numbers, and therefore the cosmos is structured according to “physical numbers”, which can be abstracted. That is to say, mathematical models that depend on empirical observation are mere opinions and lack the exactitude and accuracy of the numbers projected on the soul. For instance, in *Hypotyposis* Proclus insists on the approximate character of astronomical models.

These and other statements by Proclus are obviously integrated into the Neoplatonic tradition and are largely alien to Aristotelianism. However, in the 16th century they were integrated into the Aristotelian tradition and generated a wide debate within Aristotelianism itself.

3. Proclus’s mathematics in the 16th century

Mathematics in Aristotle

There are some texts by Aristotle that focus the debates in the 16th century on mathematics. First of all, *Metaphysica* E-1, where he affirms that there are three theoretical sciences: physics, mathematics and theology. Mathematics deals with immobile beings, while physics deals with movable beings. Moreover, in E-1, 1026a 27 (and in K-7, 1064b 8-9) he seems to allude to a universal mathematics, common to all mathematical disciplines[29].

In *Metaphysica* M and N, Aristotle is very critical of Pythagoreans and Platonists who put numbers as principles of reality. He argues that numbers are neither an efficient cause, nor

[29] Rabouin 2009, 36-84; Sasaki 2003, 289-331.

material, nor formal, nor final of things[30]. He also reproaches them for making the relative (last and least of the categories) the first category, before the substance[31].

Furthermore, in the *Analytica* Aristotle establishes the requirements for a knowledge to be science in the strict sense. In *Analytica Posteriora* 1-27 he points out that the science that has less things as a starting point is more exact and earlier. He gives the example of arithmetic versus geometry, because the one is an entity without position, while the point is a unit with position. Therefore, a science that says both the "what" and the "why" (*demonstratio potissima: apódeixis haplôs, demostratio absoluta, simplicer*) is more accurate than the one that says separately the "what" and the "why". The issue in the 16th century will be whether the mathematical proof is a *demonstratio potissima*[32].

Moreover, the characteristic feature of science is that it knows the causes necessarily and not accidentally. Its demonstrations are based on the knowledge of why something is. Thus, a demonstration by signs will not be a scientific demonstration[33].

On the one hand, Aristotle supports the thesis of the irreducibility between arithmetic and geometry: "it is not possible to demonstrate passing from one genus to another. We cannot, for instance, prove geometrical truths by arithmetic"[34]; however, on the other hand, Aristotle also affirms that what concerns the proportions can be addressed with a single proof, independently of whether it is a question of numbers, lengths, times or volumes[35].

[30] 1092b 23-25.

[31] 1079a 14-17, 1088a 15-b5; Rabouin 2009, 61-67.

[32] Alexander Piccolomineus, *De certitudine* (Apud Traianum Curtium 1565, 81-96).

[33] *An. Post.* 1-2 and 6.

[34] *Ibid.* 1-7, 75a 38-39.

[35] *Ibid.* 1-5, 74a 17-25.

In summary, Aristotle is very critical of Platonism because of the primacy given to numbers and to the category of relative. He also proclaims the irreducibility of arithmetic and geometry, although there are some texts that seem to leave possibilities for general mathematics. And of course, he establishes that a science is only such if its demonstrations are based on the knowledge of the causes. Nevertheless, mathematics does not know the causes of physical beings.

Generally, the medieval Aristotelian tradition will reaffirm these doctrines over the centuries, regardless of the discrepancies between the various Aristotelianisms[36]. The problem in the 16th century, in view of the *In Euc.*, will be the extent to which Proclus's statements on mathematics are compatible with Aristotle.

Piccolomini

His writing on the certainty of mathematics[37] is the trigger for a broader debate about "universal mathematics" and its relation to metaphysics. He turns to Proclus to assert the existence of a mathematics common (*scientia communis*) to all branches of mathematics, with common principles, a common subject (imagination)[38] and common properties.

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[36] Sasaki 2003, 301-326. In any case, debates about the use of mathematics in physics, as well as the question of the certainty of mathematics, had been frequent in the history of Aristotelianism. Albert the Great, for example, claims, against the Platonists (perhaps Grosseteste and Bacon), that it is absolutely false that mathematical objects are the principles of physical beings. And Thomas Aquinas recognizes that mathematics has the highest degree of certainty compared to physics and theology.

[37] Alexander Piccolomineus, *De certitudine* (Apud Traianum Curtium 1565).

[38] *Ibid*. (Apud Traianum Curtium 1565, 96): "*Nam manifestissime Proclus in primo et 2. libro passim ostendit dari quandam scientiam communem ad illas duas, quae proprium subiectum,*

Although he accepts that mathematics provides the maximum degree of certainty, nevertheless, he rejects that mathematical demonstrations are perfect (*potissimae*)[39] because they do not say the cause: neither the efficient nor the final one[40], but rather the formal cause[41]. And to confirm his thesis, he refers to Proclus's criticism of Euclid for sometimes going to demonstrations by signs[42].

Therefore, for Piccolomini, mathematics cannot provide a scientific knowledge (for causes) of the physical world. This implies that the field of application of mathematics to physics is not as wide as that of (Aristotelian) logic or dialectics. Mathematical certainty cannot be demanded in all cases.

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et proprias passiones, propriaque principia sibi vendicat, et illas duas sibi subalternat". And then, he supports the authority of Proclus with that of Euclid (books 5-8) and with Aristotle's treatment of proportions (*An. Post.* 1-5).

[39] *Ibid*. (Apud Traianum Curtium 1565, 70): "*In quo, duo proponimus peragenda. Primum et rationibus et authoritatibus, demonstrare intendimus mathematicas demonstrationes non esse illas potissimas. […] Secundo vero loco […] qua mathematicae disciplinae in primo esse gradu certitudinis esse possunt*".

[40] *Ibid*. (Apud Traianum Curtium 1565, 101).

[41] *Ibid*. (Apud Traianum Curtium 1565, 102).

[42] *In Euc.* 206,12 ff.

The object of mathematics is the quantity abstracted from matter. Quantity is a very imperfect accident (*quantitas vero imperfectissima omnium accidentium*)[43] and it is entirely related to the first matter, so appears itself in an easy and certain way[44].

Piccolomini also turns to Proclus to affirm that mathematical objects are present in the imagination, that is, they have the space of the imagination as their foundation. And it is precisely quantity —abstract and present in the imagination: *quantum phantasiatum*— that gives mathematics a unitary basis, common to arithmetic and geometry (and generally to all mathematical disciplines).

> *Concludit ergo Proclus ex Platone, quod res ipsae mathematicae, de quibus fiunt demonstrationes, nec omnino in subiecto, sensibiles sunt, nec penitus in ipso liberatae, sed in ipsa phantasia reperiuntur figurae illae mathematicae, habita tamen occasione a quantitatibus in materia sensibili repertis. Intellectus autem, ex iis, quae in phantasia sunt in quantitatibus, rationes illas universales colligit. Materia ergo harum scientiarum, erit quantum ipsum, hoc modo, ut ita dicam, phantasiatum. Et id a plerisque, quamvis non satis proprie, materia intelligibilis nuncupatur*[45].

> Proclus concludes, according to Plato, that the same mathematical objects, from which the demonstrations are done, are neither entirely sensible in a subject, nor completely separate in the same subject, rather those mathematical figures are found in the

[43] Alexander Piccolomineus, *De certitudine* (Apud Traianum Curtium 1565, 104).

[44] *Ibid*. (Apud Traianum Curtium 1565, 107): "*Cum igitur abstractionis facilitas, […] sequitur quod res illae, quae ad nullam materiam in actu determinantur, sed cum denutata materia, coaeterna sunt, abstrahibiles erunt maxime, et iccirco fáciles cognitu, certae, ac manifeste*".

[45] *Ibid.* (Apud Traianum Curtium 1565, 97).

> imagination itself, making use of the quantities found in the sensible matter. The intellect forms the universal concepts from the quantities that are in the imagination. Therefore, the matter of these mathematical sciences is the same *quantum phantasiatum*. This matter of the imagination is called by many, although not very properly, '*materia intelligibilis'*.

If in the Aristotelian tradition intelligible matter is usually mentioned as the matter of mathematical objects, Piccolomini, following Proclus, sets the imagination as the proper place of mathematical objects. Strictly speaking, this is not a radical innovation, since Proclus —and now Piccolomini— had identified the Aristotelian *nous pathetikos* with imagination, so 'intelligible matter' would be equivalent to 'imagined matter'.

In summary, Piccolomini follows Proclus in some aspects, such as the affirmation of a universal mathematical science, common to arithmetic and geometry, as well as in situating imagination as the specifically mathematical faculty[46].

However, Piccolomini assumes —in agreement with Aristotle— that mathematical objects are known by abstraction from physical beings, and do not have an *a priori* character that is projected in the space of the imagination. Furthermore, he reconnects the certainty of mathematics to the link between quantity and first matter, so that this certainty comes from the easiness with which we perceive material objects, as opposed to the conviction of Proclus, who attributes the rigour and accuracy of mathematics to its *a priori* intelligibility.

Barozzi (Barocius), Francesco (1537-1604)

[46] Rabouin 2009, 201-209.

His knowledge of the *In Euc.* was more complete than Piccolomini's. Moreover, he was one of the greatest experts of Greek mathematics. He could therefore rectify some of Piccolomini's references to Proclus.

Barozzi can be considered as the restorer of Proclus's doctrines[47]:

1) He commented on the *In Euc.* in his opening discourse at the University of Padua in 1559: *Lectiones in Procli commentarios*. In this text, he opposes Piccolomini directly. (These *Lectiones* were published as an introduction in his later writing on the usefulness of mathematics in 1560: *Opusculum in quo una oratio, et duae questiones: altera de certitudine, et altera de medietate mathematicarum continentur*).

2) His first lessons in Padua were a commentary, line by line, on the first prologue of *In Euc*[48].

3) He made the first Latin translation of *In Euc*. (1560), employing more manuscripts than Grynaeus.

4) He was planning to write a commentary on the *In Euc*., but there is no record that he ever wrote it. We do keep the notes and glosses in the margins of his Latin version[49].

In any case, against Piccolomini, he places the certainty of mathematics in his demonstrations, which do state the material and formal causes. The authority of Aristotle, Proclus, and others, confirms that mathematics demonstrates through formal and material causes.

[47] In the non-philosophical, but mathematical field, we must add Dasypodius, to whom I will mention later.

[48] The text has been published by De Pace 1993, 336-430.

[49] Crapulli 1969, 57.

> *Quod scilicet mathematicae demonstrationes potissimae minime sint, quia per causam non fiunt, dicimus quod falsum est mathematicas demonstrationes a causa non fieri. Concedimus enim eas neque per efficientem neque per finalem fieri causam, non tamen concedimus neque etiam per materialem, formalem causam fieri*[50].

> We say it is false that mathematical proofs are minimally *potissimae* proofs because they are not made through the cause. We concede that mathematical proofs are not made through the efficient or the final cause, but we do not concede that they are not made through the material and the formal cause.

Likewise, he insists with Proclus on the usefulness of mathematics for all knowledge, especially for physics, and also as a propaedeutic for metaphysics and theology. He points out the superiority of mathematical intelligible objects over physical objects. And he establishes the place of mathematics among the sciences: it occupies an intermediate place between theology and physics[51]. *Quoad nos* is superior to theology, *quoad se* is superior to physics.

Mathematics is thus entirely suitable for the study of the physical world (against Piccolomini), according again to Proclus. Its certainty comes not only from the intelligibility of its objects but also from the scientific validity of its demonstrations. Mathematics organises in this way a logical order that complements traditional Aristotelian logic and provides the highest accuracy and certainty.

Clavius, Christophorus (1538-1612)

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[50] Barocius, *Oratio* (E. G. B. 1560, 21).

[51] Barocius, *Il Commentario* (De Pace 1993).

Clavius's importance in the reception of Proclus is due to his leadership in teaching mathematics within the Society of Jesus[52]. And, consequently, in the formation of Descartes in La Flèche[53].

In the debate about the certainty of mathematics, Clavius[54], in his *Prolegomena* to his edition of Euclid's *Elements*, is clearly placed next to Barozzi, with whom he also had a close friendship[55].

In the *Prolegomena* (section *Nobilitas Atque Praestantia Scientiarum Mathematicarum*), after pointing out the intermediate place of mathematics between metaphysics and physics, "*ut recte a Proclo probatur*", Clavius proclaims that, if the dignity of a science is measured by the certainty of its demonstrations, then mathematics is the science of greatest dignity[56]. He notes that in metaphysics there are many doubts, which imply many different interpretations of Aristotle, as opposed to the soundness of mathematical proofs.

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[52] Romano 1999.

[53] Sasaki 2003, 50-63; Kessler 1995; Marion 2000; Rabouin 2009. Clavius played a central role in the development of La Flèche's curriculum. He wrote a lot of mathematical manuals. At least, two were important to Descartes: a version of Euclid's *Elements,* prefaced and annotated by Clavius (1st ed. 1574, and 2nd greatly expanded in 1584); and his *Algebra* (1608).

[54] Sasaki 2003, 50-63.

[55] For instance, on January 29, 1586, Clavius wrote to Barozzi in relation to the recent publication of his *Cosmographia*: "I admired your Proclus in 1560 and now I admire your fluent and erudite *Cosmographia*" (cited by Sasaki 2003, 51).

[56] Clavius, *Prolegomena* (Apud Bartholomaeum Grassium 1589, 14): "*Si vero nobilitas, atque praestantia scientia ex certitudine demonstrationum, quibus utitur, sit iudicanda; haud dubie Mathematicae disciplinae caeteras omnes principem habebunt locum. Demonstrant enim omnia, de quibus suscipiunt disputationem, firmissimis rationes*".

Therefore, "*primus locus inter alias scientias omnes sit concedendus*". While dialectics only reaches the most probable position, mathematics determines with firm reasons the true conclusion[57].

Clavius's emphasis is also connected to his rejection of the increasing Pyrrhonian scepticism. It is visible, for instance, in his correspondence with Francisco Sánchez (author of *Quod nihil scitur*, 1581), who writes a letter to Clavius, arguing against the certainty of mathematics[58]. Clavius sides with realism in the physical-mathematical explanation of reality, as also will Kepler.

The chapter of the *Prolegomena* entitled "Diverse uses of mathematical disciplines" reminds us that to reach metaphysics we must go through mathematics ("*ut eleganter Proclus ostendit"*)[59], the same as Augustine or Gregory of Nazianzus. Overall, all sciences and arts need mathematics "*ut perspicue docet Proclus*"[60].

Proclus's influence on Clavius[61] and his Aristotelian environment of the Society of Jesus is visible years later in his disciple Blancanus (*Aristotelis loca mathematica,* 1615), who is dependent on Barozzi's doctrine and has many references to Proclus[62].

Pereira, Benedict (c. 1535-1610)

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[57] *Ibid.* (Apud Bartholomaeum Grassium 1589, 24).

[58] Sasaki 2003, 58.

[59] Clavius, *Prolegomena* (Apud Bartholomaeum Grassium 1589, 14).

[60] *Ibid.* (Apud Bartholomaeum Grassium 1589, 15).

[61] Sasaki 2003, 50: "The most remarkable characteristic of Clavius' *Prolegomena* seems to be that the author was deeply influenced by the Neoplatonist Proclus's philosophy of mathematics developed in the latter's *Commentary of the First Book of Euclid's Elements*".

[62] Giacobbe 1976.

He was a Jesuit and professor in Rome, like Clavius. He published *De communibus omnium rerum naturalium principiis et affectionibus* (1562)[63], where he developed some of Piccolomini's theses in the context of Aristotelian natural philosophy and where he habitually resorted to the authority of Proclus to confirm his claims.

Pereira moves around the question raised by Piccolomini in his *Commentarium de certitudine mathematicarum* (1547): on the one hand, he insists ,with Aristotle and Averroes, that mathematical demonstrations "*sunt in primo ordine certitudinis*"[64]; on the other hand, with Piccolomini, he denies that this maximum certainty is due to the perfection of its demonstrations[65]. "*Quantitas quae tractatur a Mathematico non est forma quidditativa rei*"[66]: mathematics does not deal with the essence of what is real. Its demonstrations do not show the formal cause[67].

First of all, he rejects the univocal use of the term 'science' both when referring to the theoretical and practical sciences, and when applied to physics, mathematics and metaphysics. Furthermore, he openly denies that mathematics is a science:

> *Mea opinio est, Mathematicas disciplinas non esse proprie scientias: in quam opinionem adducor tum aliis, tum hoc uno maximo argumento. Scire est rem per causam cognoscere propter quam res est; et scientia est demonstrationis effectus: demonstratio autem (loquor de perfectissimo demonstrationis genere) constare debet*

[63] Pererius, *De communibus* (Apud Micaëlem Sonnium 1579). It seems that the first edition dates back to 1562, but there are no traces of that edition: the first known dates back to 1576.

[64] *Ibid.* (Apud Micaëlem Sonnium 1579, 54).

[65] *Ibid.* (Apud Micaëlem Sonnium 1579, 118-122).

[66] *Ibid.* (Apud Micaëlem Sonnium 1579, 114-115).

[67] *Ibid.* (Apud Micaëlem Sonnium 1579, 114-118).

ex his quae sunt per se, et propria eius quod demonstratur; quae vero sunt per accidens, et communia, excluduntur a perfectis demonstrationibus, sed Mathematicus neque considerat essentiam quantitatis, neque affectiones eius tractat prout manent ex tali essentia, neque declarat eas per propias causas, propter quas insunt quantitati, neque conficit demonstrationes suas ex praedictis propriis, et per se; sed ex communibus, et per accidens, ergo doctrina mathematica non est proprie scientia[68].

My opinion is that the mathematical disciplines are not properly science. In favour of my opinion there are many arguments, but mainly this maximum argument: science is knowing one thing by means of the cause for which this thing is. Science is an effect of the demonstration. And demonstration (I am referring to the most perfect genus of demonstration) must consist of that which is *per se,* and of its demonstrated properties; however, that which is *per accident,* as well as common accidents, must be excluded from perfect demonstrations. But the mathematician does not consider the essence of quantity, nor does he examine them as coming from such essence, nor does he disclose them by means of their own causes for which they belong to quantity, nor does he make his demonstrations from the above-mentioned properties; his demonstrations arise from the common, and through the accidental. Therefore, the mathematical doctrine is not properly a science.

In support of this idea, Pereira turns to Plato, who does not call mathematics ‘science’ or ‘intelligence’ but only *cogitatio,* since his arguments come from certain hypotheses (*ex quibusdam subpositionibus*). And he adds: “*In quam sententiam multa scribit Proclus in I. lib.*

[68] *Ibid.* (Apud Micaëlem Sonnium 1579, 40).

suorum Commentarium in Euclidem"[69]. Proclus would thus confirm Pereira's thesis that mathematics is not science.

Of course, like Proclus, he asserts that the unity of mathematics is subordinated to the unity of first philosophy. Metaphysics is the first science. This is also affirmed by Plato, who puts dialectics as the first science, and confirmed by Proclus, who calls it "*omnium scientiarum capacissima*"[70]. However, neither the unity of metaphysics nor the unity of the various mathematical disciplines imply univocity: the diversity of the categories of being excludes such univocity.

According to Piccolomini, he affirms that quantity is the first accident of the natural substance "*non solum quia coaeva est materiae primae, per se nec generabilis nec corruptibilis, sed etiam quod proxime inhaeret materiae, quae nisi affecta sit quantitate, caeterorum accidentium nullum poteri accipere*"[71].

Furthermore, he states that mathematics studies quantity in itself (*per se)* and not in relation to substance: that is, it does not need to abstract quantity from substance: "*affectiones quae a mathematico demonstrantur de quantitate, non ei conveniunt in ordine ad*

[69] *Ibid.* (Apud Micaëlem Sonnium 1579, 40).

[70] *Ibid.* (Apud Micaëlem Sonnium 1579, 43).

[71] *Ibid.* (Apud Micaëlem Sonnium 1579, 546): "the quantity is not only coeternal with the first matter, which cannot be generated and is incorruptible *per se,* but it is the one that is most immediately in the matter, which, if it were not affected by the quantity, could not accept any other accident".

substantiam, sed per se; ut esse divisibilem, commensurabilem, proportionabilem, aequalem vel inaequalem”[72].

That is to say, the relations established by quantity are not established to determine the substance of something, but are alien to the essence of physical beings. Equality or inequality, commensurability, the proportions or analogies between some substances and others are external to the substances, they do not penetrate into the essence of each thing. Mathematics establishes exclusively the relations between substances, but these relations are alien to the essence of physical beings. Mathematics analyses relations, not substances. Aristotelian syllogistic differs from mathematical argumentation. Aristotle’s scientific demonstration deals with essences, while mathematical demonstrations deal only with relations.

Nevertheless, mathematics precedes physics *secundum nos,* because the mathematical principles are known by themselves, without the need of a long experience as in physics (and cites Proclus in this regard[73], since mathematical science receives more than any other discipline the denomination of μάθησις, knowledge). Mathematics, therefore, is learned with great ease “*a pueris et rudibus*”[74]. Mathematics is “*certissimae, evidentissimae et facillimae*” because quantity is perceived by all the senses[75]. In addition, mathematics precedes physics

[72] *Ibid.* (Apud Micaëlem Sonnium 1579, 587): “The features demonstrated by the mathematician do not refer to the quantity in its relation to the substance, but to the quantity in itself: as being divisible, commensurable, proportionable, equal or unequal”.

[73] *Ibid.* (Apud Micaëlem Sonnium 1579, 55).

[74] *Ibid*. (Apud Micaëlem Sonnium 1579, 55).

[75] *Ibid.* (Apud Micaëlem Sonnium 1579, 121).

secundum naturam, since mathematics studies the quantity in itself, whereas physics studies the mobile and sensitive being, which is determined by quantity[76].

Mathematical definitions do not express essential predicates but only relations: "*definitiones mathematicae non sunt definitiones essentiales, sed descriptiones quaedam accidentariae, et affectiones quae demonstrantur in huiusmodi scientiis, magna ex parte sunt respectus et relationes quaedam extrinsecus advenientes quantitati vel figurae […] Neque vero medium, quod ponitur in demonstrationibus mathematicis, dici potest esse causa formalis*"[77].

Pereira emphasises that the quantity, an attribute proper to mathematical objects, is known by abstraction. In contrast to Proclus, he categorically affirms Aristotelian abstractionism: the quantity is separated by abstraction, from the extension already existing in nature[78].

Pereira by no means concludes in the discredit of mathematics, quite the opposite, he emphasises its syntactic and methodological autonomy: it is not science in the sense demanded by Aristotle, but it is an accurate and certain language that shows the relations between things.

In summary, Pereira relies on Proclus to affirm the existence of a "universal mathematics", which, without being strictly a science, offers a universal language that can express the relations between all natural beings.

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[76] *Ibid.* (Apud Micaëlem Sonnium 1579, 55).

[77] *Ibid.* (Apud Micaëlem Sonnium 1579, 115): "Mathematical definitions are not essential definitions, but, in a certain way, accidental descriptions; and the features shown in sciences of this kind are largely extrinsic relations that supervene to the quantity or the figure [...]. But it cannot be said that the formal cause is the middle ground in mathematical demonstrations".

[78] *Ibid.* (Apud Micaëlem Sonnium 1579, 57-58).

Dasypodius, Van Roomen, Alsted

Finally, I would like to point out that the reception of *In Euc.* is not limited to the authors already mentioned in the Aristotelian controversy about the certainty of mathematics. Concretely, in the specifically mathematical field, and outside Italy, there are some mathematicians who widely disseminate the Proclus's text and many of his theses[79]. I will briefly focus on three of them.

First of all, Dasypodius (Konrad Rauchfuss, 1532-1600), together with Barozzi, is the other great restorer of Proclus's doctrines. He held the chair of mathematics at the Academy of Strasbourg (one of the academic institutions of Protestant humanism) for forty years.

In his writings on the *universalis mathematica* (between 1564 and 1593), he constantly uses Proclus's *In Euc.*[80], although sometimes without explicitly citing it. The first references are found in 1564 in some *scholia* annexed to his edition of the *Elements*. In 1570 he published a manual of mathematics, where he reproduced, without citing it, almost entirely the two prologues of *In Euc*. Moreover, in the *Protheoria mathematica* (1593) he frequently refers to Proclus, bringing the universality of mathematics closer to the universality of metaphysics[81].

[79] Sasaki 2003, 342-343: "*Proclus's Commentary on the First Book of Euclid's Elements* stimulated some philosophers and mathematicians of the Renaissance to restore and construct a novel philosophy of mathematics different from that of the Aristotelians. When this commentary on Euclid was brought into the Renaissance, the idea of 'common mathematics' advocated in it was naturally connected with mathematics in the ancient world. That mathematical concept, however, would find a new expression in the mathematical avant-garde of the sixteenth century".

[80] Crapulli 1969, 31, 90.

[81] Crapulli 1969, 89-90: "Nell'arco della produzione letteraria di Dasipodio, dall' *Elementum I* del 1564 alla *Protheoria mathematica* del 1593 il rilievo di una disciplina che faccia riscontro

Also relevant for the Proclus's reception is Adriaan Van Roomen *(Adrianus Romanus*: 1561-1615), a Belgian mathematician who worked at the University of Würzburg since 1593. Van Roomen was aware of the debates of Piccolomini and Barozzi about the certainty of mathematics. He published *Apologia pro Archimede* in 1597, second part of a work with a more general title: *In Archimedis Circuli Dimensionem Expositio et Analysis*[82]. He followed, for the most part, Clavius's teachings, also resorting to Proclus's authority[83].

Van Roomen defended the existence of a *prima mathematica* or *mathesis universalis*, which provides a basis for arithmetic and geometry, whose core is in the treatment of proportions[84]. As Alsted would later do, Van Roomen referred to Benedict Pereira for the

alla *mathesis universalis* rimane sempre nell'ambito dell'interpretazione di un unico testo classico, il *Commento al I libro degli Elementi* di Proclo, mutuando dalla polivalenza delle considerazioni una caratterizzazione ambigua ed oscillante nell'involucro mutevole di espressioni come *universalis mathematica cognitio et doctrina*, *universalis* μαθηματική, *universalis disciplina mathematica*, *universalis mathematica scientia*, *communis scientia universalis*".

[82] Crapulli 1969, 209-242.

[83] Sasaki 2003, 348.

[84] The most complete definition of *mathesis universalis* is that of Van Roomen: "*geometriae et arithmeticae communis est scientia quae quantitatem generaliter uti mensurabilem considerat […] ad quam spectant affectiones communes omnibus quantitatibus […] non abstractis tantum ut numeris et magnitudinibus, sed concretis etiam, uti temporibus, sonis, vocibus, locis, motibus, potentiis…proportiones eas quae spectant ad analogias […] ad scientiam aliquam universalem iure merito pertinere existimandum est*" (cited by Crapulli 1969, 146).

notion of *mathesis universalis*[85]. Van Roomen said that he was the only philosopher who had dealt with *mathesis universalis* before him.

Van Roomen did not cite Proclus when referring to the *prima mathesis*, he cited Eutocius of Ascalon[86], but, in doing so, he gathered arguments from Proclus. Van Roomen said that Eutocius, like other Greek mathematicians, constantly used arithmetic to solve geometrical problems[87].

In addition to Dasypodius and Van Roomen, Proclus's doctrines were spread in mathematical circles through the writings of Johann Heinrich Alsted (1588-1638), who was linked to the University of Leiden, a leading institution in the Protestant sphere. The most characteristic feature of Alsted (as he recognised) was the demand for method and system, applied to all sciences and knowledge. This encyclopaedic and systematic character is what made him so popular in Europe[88].

4. Conclusions

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[85] The text cited by Van Roomen and Alsted is: "*Quemadmodum non est dubium quin sit aliqua scientia mathematica communis, quae debeat speculari affectiones communes magnitudini et numero, quae tamen scientia, a mathematicis non numeratur distincta a Geometria et Aritmetica*", in: Pererius, *De communibus* (Apud Micaëlem Sonnium 1579, 57): "there is no doubt that there is some common mathematical science, which must study the common properties of magnitude and number, which however is not considered by mathematicians as different from geometry or arithmetic".

[86] Eutocius of Ascalon was a mathematician who lived around the 6th century. He wrote a commentary on Apollonius's *Conics* and was probably a disciple of Ammonius Hermiae.

[87] Crapulli 1969, 31.

[88] Crapulli 1969, 125-143.

The *In Euc.* generated in Aristotelian circles an intense debate about the status of mathematics in the second half of the sixteenth century. In particular, it served as a reference to dissolve the irreducibility between arithmetic and geometry, and to affirm the existence of a universal mathematics.

There was a wide controversy about the status of this universal mathematics: specifically, whether it was science according to the demands of Aristotle's *Analytica,* or whether it was not. And in that case, whether it was a universal language capable of expressing and articulating the relations of all physical beings. In both senses —mathematics as a science or only as a universal language—, the *In Euc.* served as a source of authority.

As a universal language, universal mathematics displaced Aristotelian logic as the methodology of science. Not because mathematical proofs showed the essential causes of the physical world, but because they showed the relations between all natural beings.

Moreover, mathematics relegated metaphysics to a secondary position as universal knowledge. Not because it could discover the essences of all things, but because it showed their relations. In this way, the ontology of substance was transmuted into the ontology of relations.

Without a doubt, Proclus's position was not this: firstly, because according to Proclus, ideal mathematical objects are maximally intelligible, cause and essence of physical objects; secondly, because mathematics acquires its validity not through abstraction but through an *a priori* projection; thirdly, because the weakness of mathematical physics comes from the instability of physical objects, not from the insufficiency of its mathematical proofs.

But Proclus had pushed mathematics into new territories: firstly, because of the high value given to the relation (cf. *Elements of Theology,* 103: everything is related to everything in its own way); secondly, because of the affirmation of a universal mathematics, beyond the Aristotelian division by genus, a universality that structures everything that is real and makes a mathematical ontology possible; and thirdly, because of the role given to the imagination as the proper place of mathematical science.

After Piccolomini, Barozzi and Pereira, the debate about the status of mathematics turned into a strictly mathematical discussion, in which mathematicians such as Dasypodius, Van Roomen or Alsted continued to refer, explicitly or implicitly, to Proclus's commentary on Euclid. In this sense, Descartes's *Regulae ad directionem ingenii* can be read from this perspective.

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