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Digital-analog quantum simulations in quantum photonics

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Acknowledgments

To my parents for trusting and always wanting the best for me. Also to my friends, specially to my dear Delia and Alberto for joining me up even in the distance from Germany.

Abstract

Over the last few decades, quantum simulations turns out to be a quite suitable solution to the obstacles found when trying to simulate many-body quantum systems. Two different approaches have been developed for this purpose. Analog quantum simulation (AQS) allows to reach higher capacity to scale for the increasingly complex applications, while digital quantum simulation (DQS) provides universality in terms of unitary operations broadening that way different possible schemes to simulate. Furthermore, an hybrid approach called digital-analog quantum simulations (DAQS), has stepped in trying to harness the best opportunities of each approach. On the other hand, the prolific development of integrated quantum photonics (IQP) has become a state-of-art technology able to make increasingly tiny structures endowing them with stability and high scalability. We present here a review about the evolution of quantum simulations discussing different proposals and physical platforms, in addition to a description of the development of IQP and bulk optics and its possible implementation in quantum information processing. In the last part, we make reference first to different proposals for quantum simulations (DAQS and QS architectures based on photons). Eventually, we end up proposing different photonic DAQS schemes to achieve several types of entanglement. Between them, we highlight the acquisition of a genuine tripartite entanglement (GHZ state) with our second scheme.

Contents

1 Introduction

The increasing development of industry and technology has led scientific researches to certain tasks involving the performance of tremendously tedious computations. Fluids and molecular dynamics or galactic evolution are some applications where these calculations are used the most. One of the fields where we find them more are the simulations of quantum systems which are a fruitful activity commonly employed in condensed-matter physics, material science and molecular chemistry. This challenge has always been tried to be addressed using supercomputers [1]. Also known as high efficiency computers, they are specific devices with a capacity of computing extremely high specially designed for determined purposes. However, when it comes to describing the behaviour of a quantum system, the things turn out to be a little bit complicated. For instance, taking into account the classical architecture of these types of computers, if we wanted to simulate the evolution of a pure quantum state of N particles with half-integer spin, the computational power of the device would scale exponentially. This is because it works with bits, that is to say the unit of storage capacity (minimum unit of information) in classical computation that only takes two possible values: 0 and 1. Then, if the object were composed by a number of particles (not necessarily very high) like 50, the 2^{50} resulting coefficients would make completely impossible to deal with that huge amount of digits [2].

Nevertheless, in 1980 the first steps started to be taken so that this matter could be faced subsequently in a new and innovative way. This was the year that Paul Benioff presented the first microscopic quantum mechanical Hamiltonian model for computers, something like a traditional computer that in a certain way worked with some laws of the quantum mechanics [3]. Benioff put then the first stone to start building what we call today quantum computing, one of the most promising and interdisciplinary fields involving quantum physics which goal is to raise the manner in which we compute to another level through the construc-

tion of quantum computers.

This discipline makes use of certain quantum phenomena like superposition and quantum entanglement to develop a novel way of computing that uses another unit of information different to the classical bit called the qubit. A **qubit** is basically a 2-level quantum system that can properly be manipulated using new logical operations (according to quantum mechanics) so that it is possible to store and send information as we have already done with bits in classical computation. The key difference is that a qubit not necessarily has to take two discrete values (0 or 1). Instead, qubits take values in a continuous way so that they can be the two values at the same time (or not) due to the quantum wave function properties. In addition, through quantum entanglement different qubits can be linked (described by the same quantum state) and due to quantum paralelism as you increase the number of qubits, the computation power grows up exponentially (in contrast to the classical one which raises polynomially) [5].

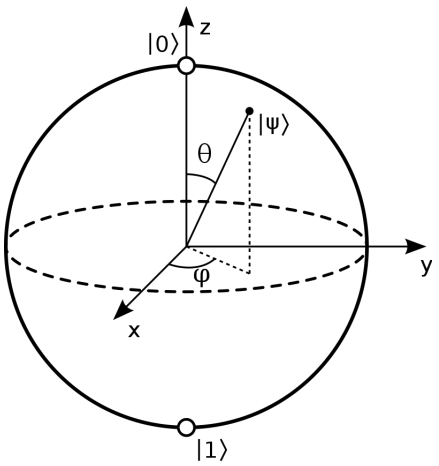

Figure 1: Graphic representation of a qubit as a Bloch sphere [4]

Notwithstanding, everything that has been commented is subject to a few requirements which actual (and future) science and engineering must acomplish. This is for example, that while bits physically are indeed optical and electrical pulses, qubits are in contrast subatomic particles, photons, electrons... whatever microscopic system (as explained above) which behaves according to quantum mechanics and that we are able to manipulate efficiently. Here is when the problem of decoherence gets into. This term refers to the fact that under specific circunstances, normally when the system is interacting with its environment, it loses

its quantum character and starts showing a completely classical behaviour. Taking into account that what we use to develop this new form of computation are these special quantum characteristics, it is a crucial question to avoid. The reality is that regardless the physical paradigm we use to tailor the qubits (either ion traps, nuclear spins, quantum dots, etc), temperatures near absolute zero are often needed to avoid thermal excitation. In spite of this, it is important to mention that in certain situations decoherence and thermal exitation can be used as an extra element of the experimental setup to reproduce the same effect but in the system of interest [6]. But overall, even taking under control a few of them using **quantum error correction**[1], as you increase the number of qubits this task at the present time is impossible to overcome satisfactorily for scaling levels as the ones got in classical computation along these past decades. It is true that not so long ago, a research group of quantum computing of Google was able to make effective an example of quantum supremacy using a high-fidelity processor called «Sycamore», to sample the output of a pseudo-random quantum circuit. That is to say, it has been able to overcome a computation task in 200 seconds which would be only done by a classical supercomputer in no less than 10000 years [8]. Nevertheless, the quantum processor had 53 qubits (not so much comparing as we commented above) and was developed exclusively to do this task. Furthermore, researchers of IBM published shortly after a web post arguing that «an ideal simulation of the same task can be performed on a classical system in 2.5 days and with far greater fidelity» [9]. Consecuently, it seems to be far from being a definitive quantum processor.

So if quantum computer is now a long-term aim and until we are able to achieve a completely developed universal quantum information processor, have we got any other alternative not so complex structurally but that allows us to

[1] **Quantum error-correction** is a theoretical line that uses the ideas of redundancy of information and the generalization of Shor's error-correction codes to protect quantum information from decoherence and noise taking the advantages of classical error-correction theory and the rules of the algebra of quantum mechanics [5, 7]

perform some tasks which the classical devices cannot? [2] The answer to this question turns out to be the quantum simulators; at least my purpose is to show you in the following pages why they are a quite feasible option and to delve into some of their types.

2 Quantum simulations

2.1 Classical simulations and the exponential explosion.

What a seasoned reader of this field is often used to seeing is that most of reviews start referring to certain talks given by the great Richard Feynman in the 60s and 80s [10, 11]. Altought this is the right cronological way to start talking about this topic, I think it is more ilustrative to begin with an analogy the way is done in [12].

It was in the ancient Greece when the first seeds were sown for what we now identify as simulations. In many situations it is more convenient to reduce the effects of a system to a less complex and more manipulable system instead of studying the former one as a whole. This idea focused on the comprehension of what they have over their heads all along the cloudless mediterranean nights led them to develop the first **orreries** [2]. Current ≪orreries≫ or modern simulators do not have to be necessarily such artifices. Instead, they can be physical systems with degrees of freedom we control and whose main magnitudes we can measure or on the other side they can be logical-mathematical models sustained by a certain computation power. The application of this approach has a lot of advantages like simplicity, reduction of resources and time spent and it is commonly employed in plenty of fields.

However, as we made reference in the introduction, simulating quantum sys-

[2] **Orrery**: Mechanical devices popularized in England in the eighteenth century whose purpose was to reproduce the dynamical behaviour of the main constituents of the Solar system. The simplest ones date from the ancient Greece, e.g. Antikythera mechanism.

tems directly through a classical approach results a quite difficult challenge. It is all about the excessive memory needed to contain the whole information of quantum states (mainly those concerned with large systems). The reason is nothing but the number of parameters employed to described these systems explicitly; quantity that increases exponentially with the size of the system, which usually corresponds to the degrees of freedom or the number of particles. And there is more, since proceeding with our case of the half-integer spin particle, if we consider taking into account the time evolution of these systems then the algebra of quantum mechanics leads us to exponentiated matrices 2^N by 2^N. Feynman called this phenomenon as «**exponential explosion**». Indeed, there are some well-known tools known as classic stochastic methods (e.g. Monte Carlo, coupled cluster methods, etc) which have been developed to overcome this barrier. And even though they are good aproximations, they have many limitations depending on the situation involved due to the so-called sign problem [3][13, 14].

Apparently, the solution to this question seems to be clarified in the previously mentioned talk of Richard Feynman in 1982 [11]. In his own words: «Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws». Such device should overcome the exponential explosion related to the external numerical computation of probability amplitudes through the characteristic natural inner evolution of a quantum system as in fact it is. «It would have the capacity to contain an exponentially large amount of information without using an exponentially large amount of physical resources, thus making it a natural tool to perform quantum simulation.» [13]. In spite of the vague description coined by Feynman, it took a decade until Seth Lloyd took a major step forward establishing that the conjecture was in fact right and therefore exists the possibility to program certain qubits performing operation through universal

[3]**Sign problem**: Expression which refers to the exponential growth of the statistical error resultant when applying phase space integral evaluation methods over nonpositive-semidefinite weight functions. This tends to happen specially with fermionic and frustrated systems using classical stochastic methods, like Monte Carlo, which works well as long as the function does not change the sign [13].

quantum gates acting as universal quantum simulators [13, 6] .

Though this series of events seem to lead inevitably to the quantum computer, we know currently that despite his early and novel development, it is not definitely necessary for performing quantum simulations. There is certainly room in Feynman's words for other kind of devices which can accomplish the same purpose in a more limited way. There is only one condition and this is that we have to desist from the concept of universality.

2.2 Controllable quantum systems as quantum simulators

The other way of tackling the issue is through the quantum simulators. They are proposed as controllable quantum devices which are able to mimic the behaviour (evolution) of other quantum systems. In contrast to the previous case, this type of equipment seeks to solve specific problems instead of being formulated as universal simulators. I will try to explain briefly his general operation by the means of an example using the Schrödinger equation as it is done in [13].

The simulation problem that we are going to address is finding out the state of a quantum system determining the value of certain physical magnitude in some time **t** through the description offered by its wave function $|\Psi\rangle$. For this purpose, we must solve the time-independent schrödinger equation:

$$i\hbar\frac{d}{dt}|\phi\rangle = H|\phi\rangle \tag{2.1}$$

whose solution is $|\phi(t)\rangle = e^{-\frac{iHt}{\hbar}}|\phi(0)\rangle$ where $|\phi(0)\rangle$ corresponds to the initial state of the system. For this reason the **main stages** of the process will be: preparation of an initial state, attainment of the final state after the time evolution and performing the measurement of some interesting quantity [14]. As regards the preparation of the initial states, there is currently some protocols to implement it efficiently as the ones cited below: Soklakov and Schack [15] developed an al-

gorithm to prepare arbitrary pure states requiring polynomial resources for the number of qubits which is based on Grover's quantum search algorithm. Furthermore, Wang, Ashhab and Nori [16] elaborated an efficient algorithm for pure states preparation of a molecular system «with polynomial scaling of the number of CNOT gates in terms of the number of the qubits». When we think in the process of measurement, the idea of applying **quantum state tomography**[4] crosses our minds. However, «this procedure requires resources that grow exponentially with the number of subsystems... As result, process tomography is an intractable procedure for characterizing the multi-qubit quantum systems» [17]. Instead, it will be more interesting to access to some physical properties like magnetization per lattice site, correlation functions or densities [2]. In this direction, the measurement issue has been surpassed in many ocassions [18, 19, 20, 21].

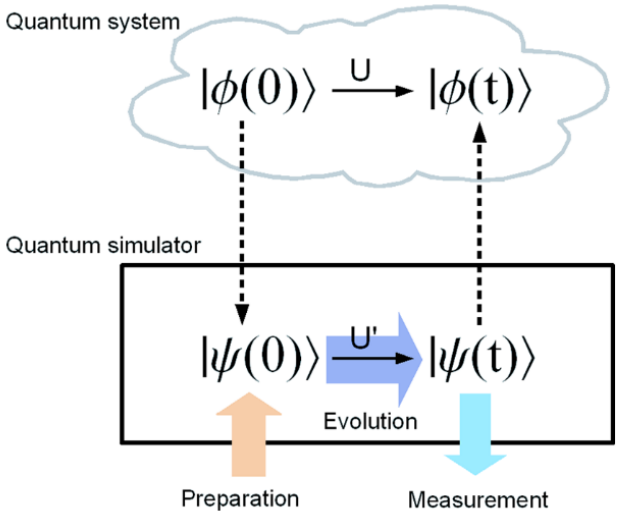

Figure 2: Sketch of the simulation process [13]

We make now the differentiation between our simulated system which evolves from $|\phi(0)\rangle$ to $|\phi(t)\rangle$ by means of the unitary transformation $U = e^{-\frac{iH_{syst}t}{\hbar}}$ and the controllable quantum simulator which in turn evolves from the initial state (pre-

[4]**Quantum state tomography**: Technique that consists of the fully characterizing quantum states through different processes of measurement applied to an ensemble of identical quantum states.

viously prepared) $|\psi(0)\rangle$ to $|\psi(t)\rangle$, which is its final state, also via the unitary transformation: $U' = e^{-\frac{iH_{sim}t}{\hbar}}$. While on the one hand our system is inaccesible or untestable experimentally, on the other hand the quantum simulator will be developed so as to ensure the existence of a practical experimental approach to address the three stages (2.2) commented above and get the sought information as if we were working with our system itself. The necessary condition for the simulation to happen is that there must be an accurate mapping between the system of interest and the simulator, that is, a biunivocal relation which takes from $|\phi(0)\rangle$, $|\phi(t)\rangle$ and U to $|\psi(0)\rangle$, $|\psi(t)\rangle$ and U' (represented in (2) by the dashed arrows).

This way we can envisage how this kind of devices work at large. However, to understand in a more complex way the process, advantages and handicaps and physical limitations, it is necessary to explore this topic through a fundamental classification depending on how the Hamiltonian of the system is mapped onto the physical structure is it built on.

2.3 The two approaches: DQS and AQS

There are two different approaches to fulfill the task we mention in the former section. On the one hand, we can map the Hamiltonian of the simulated system onto the one of our controlled system directly, so that our system mimics in a natural way the behavior of the system of interest. This is called **Analog quantum simulator (AQS)** and it exploits the similarities of both Hamiltonians and our access to the degrees of freedom of the simulator. On the other hand, it is possible to «use qubits to encode the state of the quantum system, "translate" its unitary evolution in terms of elementary quantum gates, and implement them in a circuit-based quantum computer» [14]. This kind of quantum simulator is given the name of **Digital quantum simulator (DQS)**. We are first starting to extend the latter.

Digital quantum simulators (DQS)

A good way to see what is a DQS is to continue with our scheme of (2). This kind of devices represents the unitary transformation U' through the implementation of sucessive single- and two-qubit gates over the previously encoded qubits in the specific physical platform. It is also known for being a universal approach, since it is possible to tailor whatever unitary operation in terms of universal quantum gates. The key question is that it may not be possible to fulfill this task efficiently, namely, employing polynomial resources. The reason is that though the precision (number of qubits) can be chosen arbitrarily high, it scales exponentially with the amount of quantum gates.This issue leads to the fact that there are some Hamiltonians which cannot be fitted this way. Actually, it is only feasible to simulate all the finite-dimensional local Hamiltonians; for example all local spin systems belong to this class, since they can be expresed as sum of many terms of local interactions. So it will be possible to simulate the Hamiltonians of the most theories of physics properly [13]. It must be noted too that in spite of what has been commented above, there are few non-local systems such as non local spin glasses that can also be studied this way and therefore efficiently simulated [6].

Continuing with its operation, it consists of the three same stages (2.2) commented as well in the previous section. Different protocols have been reached to initiate the states of the system under this approach: For example an algorithm for the realization of common chemical wave functions as starting points [22] or a more complex algorithm proposed by Wang, Ashab and Nori [23] capable of setting up initial states simulating the time evolution and the interaction with the ancilla qubits to prepare the energy eigenstates desired. However, the most important difference comes from the part related to the unitary evolution of the system as it is showed in [13, 5]. The usual many-body interaction Hamiltonian can be expresed as:

$$H = \sum_{j=1}^{N} H_j \tag{2.2}$$

We can find this type of Hamiltonian in the solid state physics, for instance the Hubbard's model or the Ising's one. The unitary evolution can be reduced to

$$U = \prod_{j=1}^{N} e^{-\frac{iH_j t}{\hbar}} \tag{2.3}$$

only if every term commutes with each other (what hardly ever happens). So to decompose the Hamiltonian properly it is necessary to split up the time evolution into a certain amount of short time steps $\Delta t = t/N$:

$$U = \prod_{j=1}^{N} e^{(-\frac{iH_j \Delta t}{\hbar})(t/\Delta t)} \tag{2.4}$$

and employing the first-order Trotter formula and $\Delta t \to 0$ we get that up to $O\left(N\left(\Delta t\right)^2\right) = O\left(t^2/N\right)$:

$$U = e^{-\frac{i\sum_{j=1} H_j \Delta t}{\hbar}} = \prod_{j=1}^{N} e^{-\frac{iH_j \Delta t}{\hbar}} + O(N(\Delta t)^2) \approx \prod_{j=1}^{N} e^{-\frac{iH_j \Delta t}{\hbar}} \tag{2.5}$$

The bad thing is that increasing the accuracy of the method implies to reduce the size of Δt and thus to increase the number of quantum gates, which leads to a inevitable scaling of the resources. Some researchers have tried to overcome this problem proposing higher order decompositions [24, 5] or trying to use randomness to increase the efficiency of the process [25], though the problem seems to persist.

This methodology has been employed multiple times in several platforms: At the end of 1997 Abrams and Lloyd proposed the first complete quantum algorithm for the simulation of many-body fermi systems, which can be reproduced through photons in small cavities, electron spin or nuclear spin [26]. Salathé et al. [27] developed a procedure to simulate Ising and Heisenberg interacting spin models via two transmon-qubit circuit QED setup. Since Zoller and Cirac were able to design a two-qubit gate with trapped ions for the first time [28], it

has turned into a quite prolific platform to apply quantum computation «due to the long decoherence times and good control over quantum states» [29]. For instance, this group [30] proposed a digital quantum simulation of a minimal Ads/CFT model and reproduced «a simplified low-dimensional model of quantum gravity in advanced quantum platforms as trapped ions and superconducting circuits» as well. On the other hand, Lanyon et al. [31] realized the simulation of several spin systems employing sequences of 100 gates and 6 qubits via trapped ions. Another interesting platform quite suitable for DQS are nuclear spins in organic molecules applying nuclear magnetic resonance (NMR). Wen and Kong et al. [32] performed the restoration of the entanglements induced by PT- symmetric operation in a four qubit liquid nuclear magnetic resonance platform employing a PT- symmetric two-level operation in quantum computing previously demonstrated by them. Platforms aside, I would like to include an interesting application for DQS: In particular an image processing algorithm (3) where the pixels are modeled through a statistical operator which evolutes and whose information can be recovered turning it back to image [33, 29].

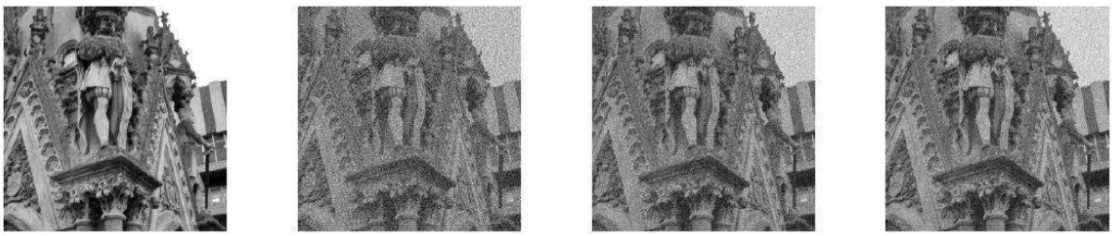

Figure 3: Digital quantum simulation for the reduction of noise in image processing [33, 29]

Summarizing, as Lloyd developed for the first time [6, 29], digital quantum simulation follows a universal scheme rich in flexibility that allows us to simulate multitude of systems finding out suitable decompositions in terms of universal quantum gates. However, the toughest problem that has to face is scalability, since as we have seen above, the proporcionality between accuracy and number

of gates implies an exponential scaling. In this regard, Dave Wecker calculated that, following a pure digital quantum approach, would be necessary no less than 50 logical qubits to be at the same level of classical simulation approaches and more than 100 in order to reach an important advantage [34, 29]. That is why it seems to be reasonable taking into account non-universal approaches and ≪in terms of practical implementation in the near future, AQS has the advantage, therefore most research groups studying quantum simulators are currently investigating them≫ [14].

Analog quantum simulators (AQS)

Leaving aside the feature of universality, the analogue simulators performs processes in a direct way so that the equations employed are approximately equal as the ones of the simulated systems. So its first advantage comes when we reject the universality, since this way they should be consigned to the description of specifically designed classes of problems [29]. As its operation is quite similar to the natural evolution of the system, it will not be necessary to take control of lots of individual elements along the time. This provides robustness and higher power of scalability to this approach than the digital one. The accuracy of the method is determined by the hardware the device is made of due to the fact that it works all the time as a closed box whereby we can extract information after the inner evolution through certain measurements. That is the reason it cannot be increased as well, so it will always depend on how the behavior of the simulator resembles the evolution of the system simulated. ≪This loosens the requirements to the analogue quantum hardware and allows to simplify the design, thus reducing the scale and complexity of the device and with it the sources of decoherence, compared to a digital quantum computer≫[29]. Another important advantage is that it is even possible to do AQ simulations in the presence of errors taking into account the extent of tolerance we are considering. For instance, we can get

a valuable answer leading our system to a phase transition [35] where certain conditions must theoretically be accomplished and so avoiding the loss of information due to the uncertainties in the control parameters [13]. Lastly, because of the similarities of both systems (simulator and simulated), it is reasonable to think that the state preparation must happen the same way, through a natural relaxation until reaching an equilibrium state. In addition, the measurements can be done directly and not via computational manipulations as in DQS. In spite of this, the literature seems to express the same opinion regarding these stages must be studied more in-depth [13].

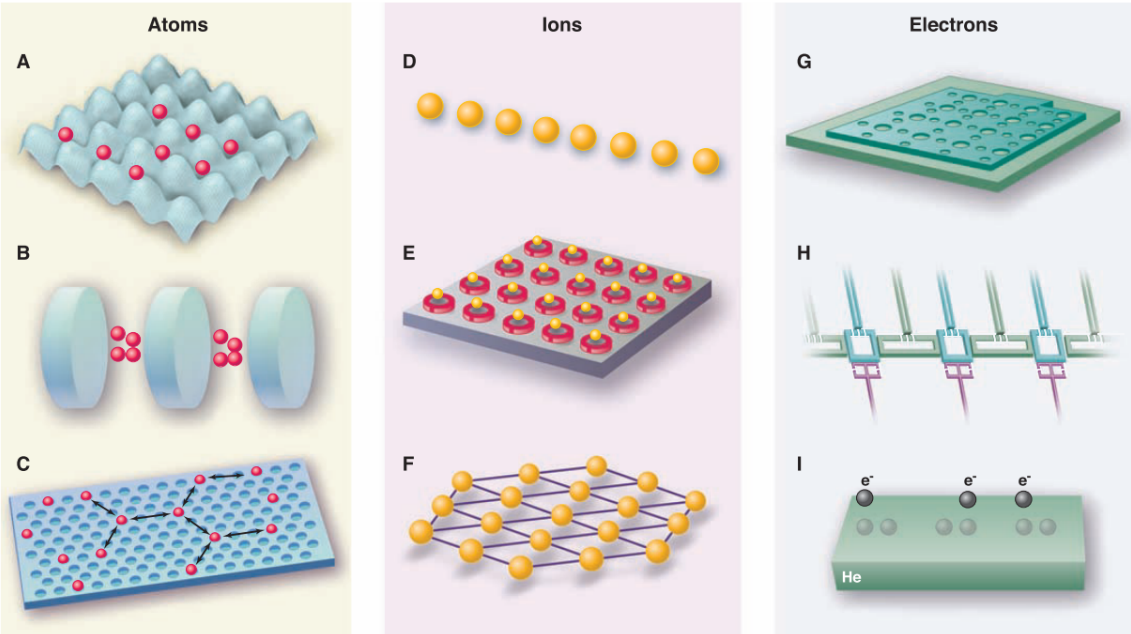

Figure 4: Different physical platforms employed for analogue quantum simulations: **A)** atoms in optical lattice [36, 37]; **B)** and **C)** arrays of cavities (1D and 2D) [38, 39]; **D)** linear ion chains [40, 41]; **E)** 2D arrays of planar ion traps [42]; **F)** 2D Coulomb crystals [43, 44]; **G)** electrons in quantum dots sustained by a 2D mesh [45, 46]; **H)** arrays of superconducting circuits [47, 48] and **I)** electrons trapped on the surface of liquid helium [49, 50]. Picture from [14].

It would be my desire to describe in more details the platforms presented in (4), but it goes beyond the scope of this essay. Notwithstanding, the interested reader can find several references in the caption of the picture for each physi-

cal platform. Now, I will comment below some systems simulated in AQS and their corresponding physical platform: To this day, atoms in optical lattice seems to be the most advanced platform for AQS employing more than one particle [13]. They are quite flexible systems and have enough controllable parameters. In addition, the optical potential employed can be adjusted so that changes in the geometry and dimesionality of the lattice may be obtained. For instance, the atoms interacting with each other can be reproduced via shifting the optical lattice in case of double lattices (in DQS) [51, 14]. There are here some advances in this branch [36, 52], an interesting article covering the progress of the discipline realizing a range of types of many-body Hamiltonians on the basis of the Hubbard's model [53] and finally a quite extensive review of the platform [54]. As in DQS, trapped ions turns out to be another valuable platform for AQS as well. When scalability appears, its implementation may find more difficulties at some extent. However, control of the decoherence can be accomplished with high fidelity and measuring individual atoms seems to carry certain ease, something quite particular comparing with other platforms [14]. For instance, Porras and Cirac [41] proposed a trapped ion system subjected to a laser which changing polarization and intensity is able to reproduce Ising and Heisenberg interactions between effective spins. Finally, electron distributions are another physical elements whose Hamiltonian is easy to obtain and whose algebraic expression can in turn be adapted to initiate AQS. On the one hand, there are quantum dots (QD) which can be made through overlaping of 2D electron gases (GaAs usually) and a 2D mesh or in a different way with the construction of layers via chemical growth over a semiconductor. A few advantages may be mentioned as the low relative temperatures to the Fermi temperature that can be reached or the intrinsic manifestation of long-range Coulomb interaction that happens in a natural way in this kind of technology [14]. An example of this can be found in [45] for the simulation of chemical reactions. On the other hand, it is possible to employ electrons as a carrier in superconducting circuits [47, 48] or suspended over a coating of

liquid helium [49, 50].

2.4 Digital-analog quantum simulations (a brief introduction)

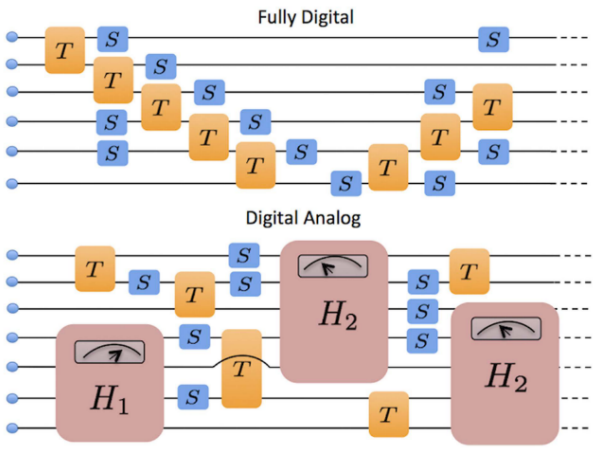

Figure 5: Scheme of a fully digital circuit vs a digital-analog one [55]

Apart from what we have described in the former section, there is also another novel form of quantum simulations that has risen in the last decade from merging both paradigms: the digital and the analog one. It is called digital-analog quantum simulation. We have seen that the digital approach is rich in flexibility and universal while the analog one has got a great robustness to scalability which is important given the necessity of carrying out simulations over increasingly greater size systems. ≪Analog blocks provide a scalable structure by reducing the number of gates and hence the experimental error, while digital steps enhances the variety of possible interactions≫ [56]. While the constituents of a pure AQ circuits are the Hamiltonian blocks (5) that warrant the direct mapping between the evolution of both systems (simulator and simulated), the digital ones are made of single and two-qubit operation which set up a way to fulfill local operations and so providing versatility [55]. This is going to be the approach that we will follow in the fourth section. But before then, it is crucial to develop the formalism and the different tools we are going to employ belonging to the field of quantum photonics.

3 Quantum photonics

Quantum photonics is a scientific area focused on the description and application of the quantum nature of light (photons). Along centuries, **Optics** aimed to clarify the complete nature of light through different approaches. Subsequently, it turned out that depending on the physical paradigm in which the light was tested, it could show a wave nature (light diffraction performed by Thomas Young) or a behavior closer to a particle (photoelectric effect by Albert Einstein). It was necessary to wait until the second half of the twentieth century when the term photonics was coined after the great progress in telecommunications because of the discoveries of the laser and the optical fiber, once the scientific community was able to understand and apply the different quantum effects of light in the incipient technologies.

On the other hand, **quantum information science (QIS)** is a transversal area of study involving physics and engineering. Its aim is exploiting the chance of getting computational progress and functionality taking advantages of analysing, storing, processing and manipulating information encoded intrinsecally in measurable magnitudes of quantum systems, i.e. degrees of freedom (DoF). Some of the rising technologies conceived by this discipline are: **Quantum key distributions (QKD)**[5], that are actually being commercially available, which allow a secure private communication through an encriptation sustained in quantum mechanics. A quite interesting feature of this design is that the presence of an eavesdropper can be detected because of their disturbance cannot avoid the inevitable collapse of the wave function of the system. Another useful applications are quantum metrology (focused on harnessing quantum phenomena to increase precision to limits not reached), quantum lithography and of course the devel-

[5] **Quantum key distributions:** As traditional key distributions, they are employed as a secret code exchanged between two talkers in order to mantain an encrypted conversation. The fundamental difference is that this kind of protocols allows to detect the presence of a third party via quantum mechanicals laws.

opement of a quantum computer [57].

Photonics development has shown that ≪quantum effects are particularly easy to observe in optical systems≫ [58], that is why it seems that it is going to occupy a main role in this area [57]. Photons appear to be great low-noise systems quite suitable to carry quantum information. These 'flying' qubits tend to show a weak coupling with environment which leads to avoid decoherence problems involving interaction processes with matter. From this fact rises the non-necessity of keeping the system in high vacuum at a temperature near to absolute zero [59].

3.1 Quantum circuits and logic gates

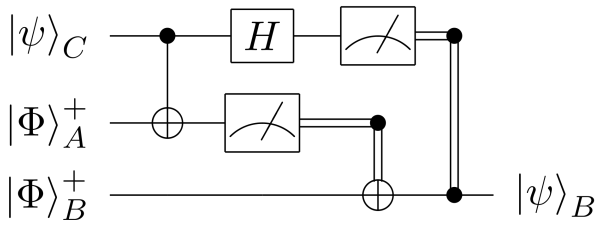

Figure 6: A quite representative quantum teleportation circuit [60].

It is appropiate to digress a bit from the conceptual line followed and to develop briefly the model employed in QIS to represent quantum algorithms and quantum information devices functioning. Quantum circuit paradigm [61] is similar to the classical one employed in traditional computer science. Sustained in a variant of Penrose graphical notation of tensors, It consists on a series of previously prepared quantum state inputs in the beginning of the horizontal lines (the qubits themselves) which are subjected to single or multiple-qubits operations (quantum gates) until they are measured (represented by the simbolic box next to the H box in (6)) letting the final states as outputs. Time runs across the circuit from left to right as the sequence of events take place. Quantum gates are reversible unitary transformations that can be applied over one or more qubits. As qubits can be seen as dual vectors in a Hilbert Space, quantum gates can be represented as well as square unitary matrices which can act

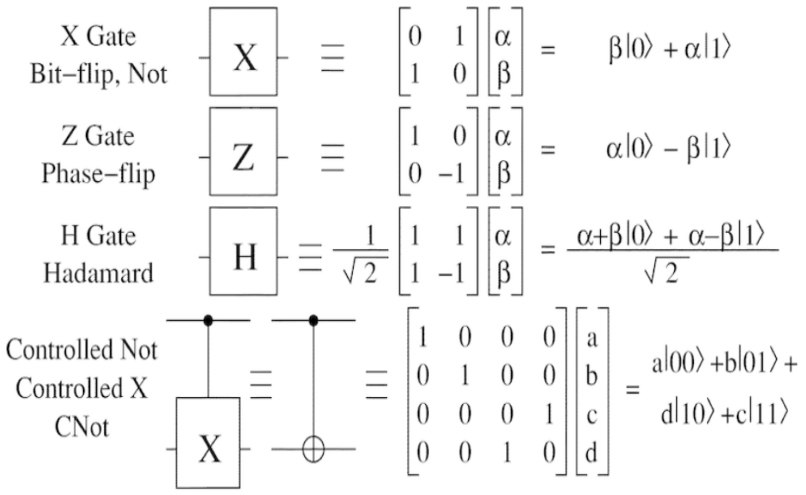

Figure 7: Common quantum gates expressed symbolically and in unitary matrix representation [62].

in many different ways (i.e. changing phases, exchanging states, or even coupling different qubits regarding a specific operation). There are a plethora of quantum gates, athough we will mention some of the most employed (7). X-gate exchanges the amplitud probabilities of the states while Z-gate produces a phase flip over the second state $|1\rangle$. On the other hand, the Hadamard gate (H) is a very special gate which is responsible for creating a superposition over single qubits. This is $|0\rangle \rightarrow |0\rangle + |1\rangle$ and $|1\rangle \rightarrow |0\rangle - |1\rangle$. Notwithstanding, given that they are reversible operations, the same gate applied in the opposite way gives $|0\rangle + |1\rangle \rightarrow |0\rangle$ and $|0\rangle - |1\rangle \rightarrow |1\rangle$. Finally, $CNot$ is a two-qubit gate subjected to the value of a control-qubit and applied over a target-qubit. It is also known as a universal quantum gate, because every circuit can be reduced to a combination of $CNot$ gates and certain rotacions over the qubits [5]. It lets the target-qubit unchanged if the control-qubit results $|0\rangle_C$, while it changes it by the opposite basis ($|0\rangle_T \rightarrow |1\rangle_T$ or $|1\rangle_T \rightarrow |0\rangle_T$) state when the control-qubit results $|1\rangle_C$. This gate is often used to entangle quantum states.

3.2 CNOT implementation case and LOQC

One of the challenges of optical quantum computing is to tailor two-qubit entangling logic gates [63]. The most representative example is the realization of a CNOT gate mentioned above. Following the way Jeremy O'Brien did in [57], this operation can be fulfilled by means of a Mach-Zehnder interferometer (MZI) (8). There are two optical channels to encode the target-qubit and they are mixed at the beginning and at the end by a 50% reflecting beamsplitter (acts the way as a Hadamard gate does). Quite similar as what we saw in the last lines of 3.1; if the control-qubit results in $|0\rangle_C$, the system let the target-photon unchanged, since the non-linear ((π) phase shift) operation is not realized and only the classical interferences in the beamsplitters take place: $|0\rangle_T^I \rightarrow |0\rangle_T^F$ or $|1\rangle_T^I \rightarrow |1\rangle_T^F$ (I and F means initial and final). However, if the control-qubit results in $|1\rangle_C$, then the non-linear operation is applied as Z-gate making $|0\rangle_T + |1\rangle_T \rightarrow |0\rangle_T - |1\rangle_T$ or $|0\rangle_T - |1\rangle_T \rightarrow |0\rangle_T + |1\rangle_T$. Later, in the second beamsplitter, the state isolated by the Hadamard gate would be the opposite to the one obtained if the phase shift would not have happened: $|0\rangle_T^I \rightarrow |1\rangle_T^F$ or $|1\rangle_T^I \rightarrow |0\rangle_T^F$. So this way, it would have been able to achieve a CNOT operation satisfactorily. The unavoidable drawback is to implement experimentally a phase shift like that. We need to say, however, that certain lines of research have made some progress: on the one hand, studying single atoms in finesse optical cavities [64, 65] and on the other hand, analysing Kerr-type non-linearities obtained via electromagneti-

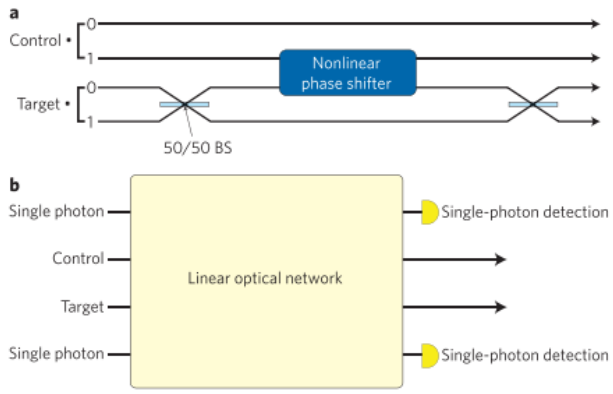

Figure 8: **a)** Possible scheme of a CNOT gate (MZI). **b)** Representation of a CNOT gate following the KLM scheme [57].

cally induced transparency [66].

In 2001, Knill, Laflamme and Milburn made a great progress showing that it was possible to perform scalable quantum computing just employing single-photon sources and detectors through linear optical networks, i.e. **Linear optical quantum computing (LOQC)** [58, 57]. Linear optics can be distinguished by counting with the superposition principle as one of their main properties and by being frequency preserving and completely opposite to non-linear optics (in which these properties may be violated). One of the most important breakthroughs achieved by linear optics has been the detection of gravitational waves in LIGO experiments based on a Michelson interferometer [67]. The novelty of the KLM scheme is the addition of ancilla photons, which were actually not part of the computation, to supply the now nonexistent non-linear interaction. As can be seen in (8) b), four photons (control, target and two ancillas) combine their paths inside a linear optical network (acts as interferometer) giving rise to quantum interferences. This way the target and control-qubit get out the system with the operation implemented and with the two ancilla photons expected to be measured. However, detection only takes place probabilistically, so the gate works in a non-deterministic way. Although this probability can be boosted resorting to quantum teleportation, the amount of resources needed to perform the operation employing non-deterministic interactions results too large for large-scale experiments [57]. There is another approach employing atom-cavity systems which allow photons interact deterministically [68] and still, «multiple high-fidelity deterministic single-photon sources remains a major challenge» [57].

3.3 Bulk optics vs Integrated quantum photonics

Within this field, we can find different types of equipments. When we talk about **bulk optics**, we mean to the traditional components that have been developed as optics progresses. Some examples of these devices can be found on the left

of (9) and ranging from a classical interferometer to more modern optical fibers. One example of quite useful technique which applies bulk optics is Optical coherence tomography (OTC), which is indeed employed in the medical field (e.g. Optometry). Bulk optics has also been employed in realizing fundamental demonstrations or more specific tests and it is still delivering important results [59, 69, 70]. Notwithstanding, these equipments tend to be unwieldy and employ huge benches which requires lots of stability in every individual component making very difficult the development of fully scalable quantum optics devices and impractical for any kind of application devised to take place out of laboratory [71].

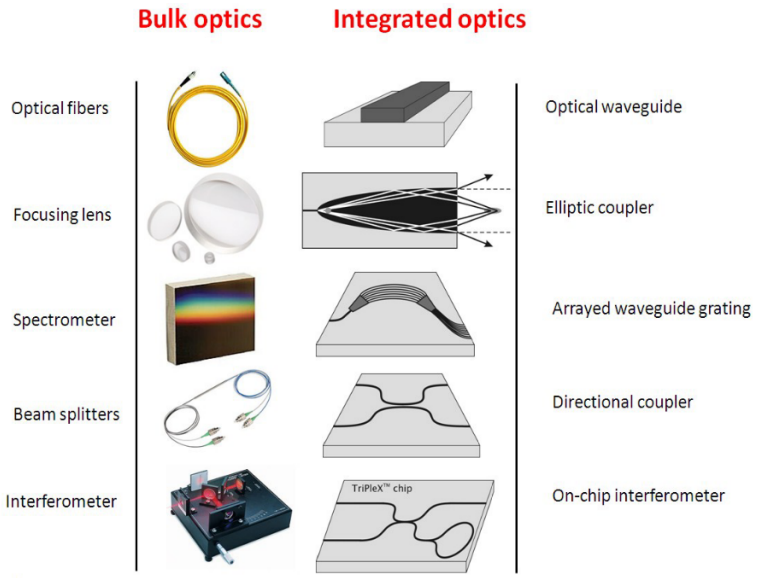

Figure 9: Equivalent components of each system [72]

In the last two decades, **integrated quantum photonics (IQP)** seems to have taken the lead being able to overcome certain drawbacks that result when using bulk optics equipments. Some even talk about a «paradigm shift from bulk optics to integrated photonics» [73]. The increasing refining of nanofabrication techniques allows a feasible miniaturization of IQP structures providing enough functionality and scaling ratio as the needed to support large-scale quantum comunication process with thousand of photons [59]. There is a clear equivalence

between components of bulk optics and IQP: the classical beamsplitter often employed to reproduce the effect of a Hadamard gate, has as analogous in the directional couplers (DC) present in IQP structures; the same way that all bulk components have their counterparts possibly implemented into a IQP chip. In fact, this is one of the main reasons why IQP is leading the way, since it provides more compact and smaller systems, reducing costs and more robust regarding stability and loss of photon problems, improving that way the fidelities reached [72]. While relative phase shifts are produced in bulk optics employing classical phase shifters, this can be realized in IQP through changes in the refractive index of one arm inside a MZI. In fact, «a combination of one MZI and two additional phases enables an arbitrary SU(2) unitary transformation» [59].

3.4 Optical qubits encoding in photons

When it comes to make qubits via quantum states of light there are a few approaches, which harness different physical magnitudes, that do this task with certain advantages and drawbacks. Even though there are another techniques based in multiphoton encoding (e.g. continuous variable encoding [74], **cluster states**[6] [75], etc), we will cover the ones that consist in single-photon encoding (most widely employed) the way

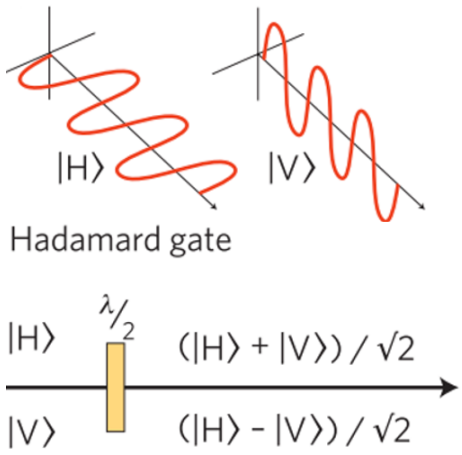

Figure 10: polarization encoding of a single photon and superposition of states (Hadamard gate) performed by a half-waveplate [57].

[6] **Cluster states:** Special approach that involves many qubits producing a massive entanglement arranged in the form of a lattice. This idea is quite useful for the one-way quantum computer method in which «single-qubit measurements, from left to right, ultimately leave the rightmost column of qubits in the answer state» [63].

it is done in [73].

Polarization encoding

Between the different properties of the electromagnetic waves, propagation modes provides information about the behavior of the electric and magnetic fields during the wave displacement. Assuming that we deal with the polarization of a photon in a single spatial mode, it is possible to construct a logical qubit as:

$$|\phi\rangle = \alpha\,|H\rangle + \beta\,|V\rangle \tag{3.1}$$

where H and V represents the horizontal and vertical polarization ways of a photon and α and β their corresponding probability amplitudes (10). This procedure has as advantages the fact that both logical states are equally affected by common mechanical errors as loss of photons or path-length mismatch. Furthermore, this approach also shines for easily manipulating qubits; in the case of single-qubits applying no more than quarter or half-waveplates and for large number of entangled states using polarising beamsplitters. The main condition for this to happen correctly is that all the components of the optical set up must be polarization-preserving; this way the logical state will not be modified unintentionally.

Dual rail encoding

In this case we assume a single photon encoding a logical qubit through two spatial modes. Being $|i,j\rangle$ a two-mode state and the coefficients i and j the number of photons found in each mode. The single-photon state would be:

$$|\phi\rangle = \alpha\,|0,1\rangle + \beta\,|1,0\rangle \tag{3.2}$$

In this case, arbitrary single-qubit operation can be fulfilled applying phase-shifters and beamsplitters. On the other hand this framework can be merely mapped to

the polarization one via polarising beamsplitters (or directional coupler in IQP) dividing the H and V components into two spatial modes. Due to the evolution of each basis state in a different spacial mode, path-length mismatch is an issue to take into account. By contrast, photon loss problem is more easily detected in this approach than in the polarization one since in the latter the loss can be misunderstood with one of the states.

Time-bin encoding

In this last approach, the flight distance of the photon is divided into discrete bins which forms an orthogonal basis. This way it is possible to store information in qubits through the flight time of photons moving in an individual space mode and with a fixed polarization:

$$|\phi\rangle = \alpha\,|0\rangle_t\,|1\rangle_{t+\tau} + \beta\,|1\rangle_t\,|0\rangle_{t+\tau} \tag{3.3}$$

$|0\rangle_t, |1\rangle_t$ represent states of a photon in the vacuum for a flight time t and τ is the discrete value in which we have splitted the arrival time. This magnitude must be large enough so that photons arrival do not overlap and then avoiding any perturbation in the orthogonality of the state space. This approach has certain advantages since it requires less resources and it is possible to introduce many time-bin qubits in a single spatial mode. On the other hand, single-qubit operation cannot be performed the way it is done in the former lines since the nature of this qubit underlies in the time in which the photon is flying until it is measured, forcing

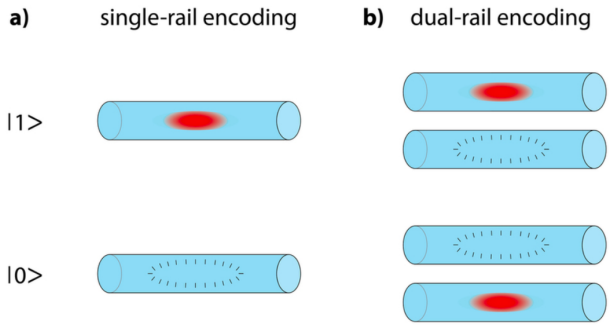

Figure 11: Single and dual-rail encoding schematic [76].

to calling upon to fast switching techniques (though it is possible to set up beam-splitters in regions before time reaches τ). It is suitable to add that it is possible to implement an encoding mode based on frequency as well.

3.5 Sources and detectors of photons

Single-photon sources

We can find two predominant ways of producing single-photons for optical quantum computing: on the one hand, parametric photon-pair sources and on the other hand, quantum dot (QD) single-photon sources [59]. The former one divides in turn into spontaneous four-wave mixing (SFWM) and spontaneous parametric down conversion (SPDC). SFWM generally consists on a non-linear process where two photons are pumped (usually by a laser) creating a pair of photons (signal and idler) whose values of wavelength are determined by the fulfilled conditions of conservation of energy and momentum [77]. Similarly, SPDC is a process where a non-linear crystal is pumped by a laser source, so that with a low probability, the mentioned pump is finally absorbed by the crystal producing another two signal and idler photons with lower energy than the incident radiation [73]. This method is non-deterministic (probability aprox. 5-10%) and furthermore it can be improved employing multiplexing techniques [70, 59].

QD sources represent instead a quite versatile deterministic single-photon source. However, the most important problem they suffer is the lack of distinguishability which is fundamental for the quantum interferences, needed in QIP, to happen [73]. In spite of this, it is trying to overcome this applying resonant excitation through lasers as can be seen in this research about boson sampling [78]. There are also another solid state single-photon sources like single neutral atoms and ions, single molecules and color centers [79].

Photon detectors

There is an available large number of photodetector designs based on different technological paradigms (e.g. field-effect transistors based detector or superconducting tunnel junction based detector) [79]. We have chosen to highlight two types of photodetectors: photon-number-resolving (PNR) detectors and non-photon-number-resolving detectors. One is able to detect the cuantitative number of photons applying measurement projectors discerning that way the optical modes involved:

$$\prod_n = |n\rangle \langle n| , \forall\, n \epsilon Z^+$$

while the other one is only able to detect photon presence:

$$\prod_{off} = |0\rangle \langle 0| \; and \prod_{on} = I - \prod_{off}$$

where I means identity and $\prod_{off}$ and $\prod_{on}$ the detection operation of absence and presence of photon [73]. An usual example of non-PNR detector and possibly one of the most widely employed in quantum information protocols is avalanche photodiode detector (APD). Broadly speaking, this design is based on a semiconductor material (e.g. Silicon) which produces an electron avalanche when interacting with the incident radiation, deriving an electrical current that may be measured. It provides low dark counts rate, flexibility towards variable spectrum of frequencies, very suitable efficiency and a considerable costs reducing. However, it faces problems involved the photon loss generated by the deadtime between electron avalanche resettings. There are another non-PNR designs faster and more efficient than APD like the superconducting nanowire SPD, although requiring special low temperatures isolations turn them into more bulky and expensive equipments.

We can find in the same way some examples of PNR detectors with extraordinarily high efficiencies. For instance, the visible light photon counter (VLPC).

Quite similar to APD, it gets a proportionality between the electrical pulse and the number of detected photons because of the presence of arsenic dopping in its layer what results in photon absorption and an electrical pulse always of the same magnitude. The drawback is that, although it does not require an special ambience as SNSPD, it presents certain problems suffering higher dark count rates and undergoing low speeds [73].

4 Digital-analog quantum simulations with photons

4.1 Proposals for quantum simulations with quantum photonics

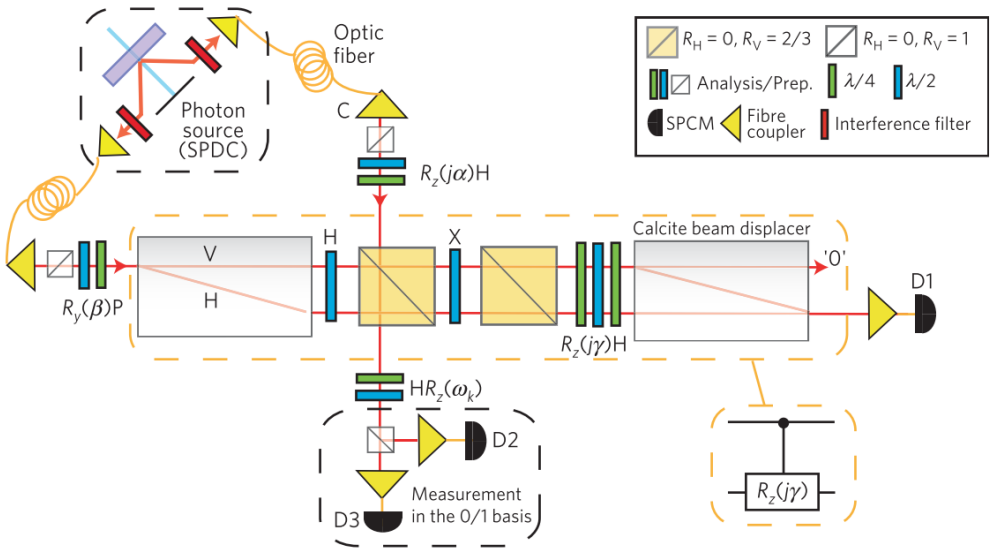

Figure 12: First experiment in quantum chemistry with a quantum information processor [12, 80]

In the previous section we showed that quantum photonics is a magnificent platform for quantum simulating. Photons turn out to be particles that do not interact easily with each other making them decoherence-free, well manipulated and addressed through free space and waveguides and potentially scalable [12]. To prove that fact, we will comment different proposals for quantum simulation

with photons published in the recent years.

4.1.1 Simulation of Hydrogen energy spectrum

The field of quantum chemistry seems to be one of the main sandboxes for quantum simulations in the last decades. This is explained by the necessity of this approach to overcome the difficulties turned up when calculating band structures or simulating wave function of different elements or molecules; i.e. exponential scaling with the size of the system (number of basis functions). In 2009, Lanyon et al. presented the first quantum simulation of the molecule of H_2 through a two-qubit quantum iterative phase estimation algorithm (IPEA) using entanglement gates [80]. Employing the algorithm developed by Aspuru et al. [81], in turn based on previous Lloyd's proposal [6, 26], they were able to do this task with polynomial resources. They designed the scheme with bulk optics components represented in (12) using polarization of photons as the chosen DoF to encode the qubits. Two entangled photons are generated via SPDC, connected to the equipment through optical fiber and then coupled spatially into the two different polarization (vertical and horizontal) serving one as control and the other one as target. The rest of the scheme consist of different wave plates ($\lambda/2$ and $\lambda/4$) which allows to implement single-qubit operation like rotations (R_y and R_z), X-gates and superpositions through Hadamard gates. The circuit works successfully when detection of both photons takes place in the counting modules D1 and D3 or D2 and D3. The basis is chosen in such a manner that the Hamiltonian is block diagonal and can be decomposed in 2x2 sub-matrices, for which it is only necessary one qubit to represent the wave function. Besides, implementing an IPEA algorithm reduces the number of qubits and elements of the circuit and allows to obtain the energy eigenvalues after the application of a time-evolution operator:

$$e^{-\frac{iHt}{\hbar}}|\psi\rangle = e^{-\frac{iEt}{\hbar}}|\psi\rangle = e^{-i2\pi\phi}|\psi\rangle \tag{4.1}$$

So estimating the phase for each eigenstate leads to a estimation of each energy or eigenvalue. In words of the authors: «the ground state energy obtained at the equilibrium bond length, 1.3886 a_0 (where a_0 is the Bohr radius), is −0.20399±0.00001 E_h, which agrees exactly with the result obtained on a classical computer» [80]. This shows the well-functioning of the simulation.

4.1.2 Discrete single-photon quantum walk

Random walk is a quite important tool of probability theory that consist of the determination of the position of an object which realizes certain random displacements of some distance with a certain probability. There is also a quantum analog approach called quantum walk (QW) which spreads this idea to the quantum world. While in the classical one, randomness appeared because of the stochastic transition between different states of the movement of a particle, in QW it is caused by the interrelation of phenomena as superposition of states, application of non-random unitary operations and the collapse of the wave function through state measurement. As its classical analogous, It results quite important for a plehora of fields, like chemistry, biology, and quantum computation. For instance, Grover search algorithm, one of the most important algorithm in quantum computation, can be viewed as a quantum walk algorithm [83].

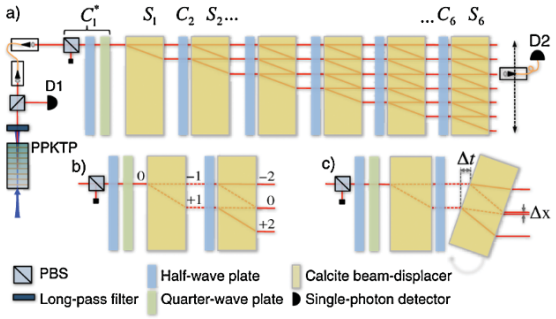

Figure 13: Scheme of the single-photon quantum walk [82].

In 2010, Broome et al. [82] developed an innovative proposal for quantum

walk employing single-photons in space through bulk optics elements. The approach raised an equipment able to carry out quantum walk with single-photons with precise control and harnessing the robustness afforded by intereferometers. In addition, the schematic includes the possibility of studying the quantum-to-classical transition introducing a controlled amount of tunable decoherence. This effect can be added making certain intentional misalignments, for example causing «a temporal delay and a transversal mode mismatch between interfering wave packets» when adjusting with a non zero relative angle the neighboring calcite beam-displacers. If we see (13), it turns out to be a schematic similar to (12): SPDC single-photon source, bulk optics elements like wave plates or polarizing beamsplitters to implement unitary operations over the qubits (Hadamard operator or a corresponding shift operator) and SP detectors where measurements take place. In addition, the qubits encoding is realized in the polarization mode of the photons as well. Although increasing the number of steps beyond six seems to degrade the gathered data because of certain problems of unwanted phase shifting due to optical surface are not perfectly planar, the proposal allows to study the abovementioned transition of both approaches showing the expected probability distributions in each case, classical and quantum walk.

4.2 Proposals for digital-analog quantum simulations so far

As we have already introduced in section 2.4, the hybrid digital-analog approach for quantum simulations is a new perspective explored in the recent years which finds prolific development because of certain research groups mainly located in organisations like the UPV/EHU and the IKERBASQUE, Basque Foundation for Science. Although it is a relatively young methodology, it is possible to enumerate a couple of different proposals that can be highlighted in the direction of overcoming problems found when one of the simulation strategies is the only one applied.

One of the first DAQS proposals was presented in 2014 by Mezzacapo et al. [84] and consisted of an hybrid simulation of Rabi and Dicke models in the platform of superconducting circuits. Rabi model follows the theoretical line that describes the most fundamental quantum interaction between light and matter «consisting of the dipolar coupling of a two-level system with a single radiation mode» [84]. The Dicke model can be seen as a generalization of Rabi model to N two-level systems.

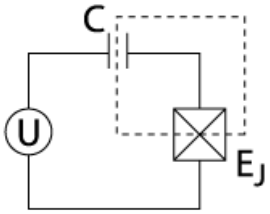

Figure 14: Charge qubit equivalent circuit [85].

The digital-analog approach obtains advantages since for example on the one hand, the scheme is flexible enough to reanalyze the digital decomposition of the larger parts of the dynamics reducing the number of elementary interactions and gaining that way efficiency. On the other hand, some interactions can be added as analog block, e.g. employing a harmonic oscillator to simulate a bosonic field, providing thus certain degree of scalability. According to the authors: «We find that all physical regimes, in particular those which are impossible to realize in typical cavity QED setups, can be simulated via unitary decomposition into digital steps». The simulation is carried out in a quantum electrodynamic circuit employing a transmon qubit. This device is a sofisticated design of a superconducting charge qubit which presents a substantial reduction of the sensitivity to charge noise. Sustained in the Josephson effect (via Josephson junction), the ratio between the energy associated to the junction (superconductor-insulator-superconductor) and the energy inversely proportional to the total capacitance of the circuit, increases making that the charge noise decreases due to energy levels turn into independent with respect to the electrical charge across the junction.

Another example to highlight is the line followed by Arrazola et al. [55] in 2016. They presented a DAQS to simulate spin models in the platform of trapped ions. This approach allows to represent a certain amount of spin models (e.g. Ising model, Heisenberg model, etc) but reducing significantly the number of gates than in a fully digital one. In this case, the analog blocks correspond to the spin-spin interaction easily accesible in trapped ions (represented by an effective spin-spin Hamiltonian H_{XX} and H_{YY} involving more degrees of freedom), while the digital steps occur through elementary local spin rotations realized by the carrier transitions. The authors showed through a numerical analysis that the fidelities for longer interaction ranges in the DA approach turns out to be substantially higher than in the fully digital (as can be seen in (15)) making the former one more advantageous to simulate certain spin models like the Heisenberg model.

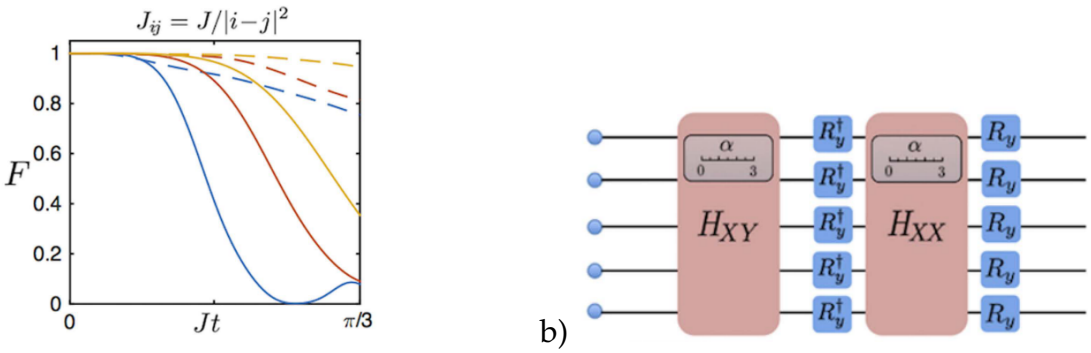

Figure 15: **a)** Fidelity loss determinated numerically for a fully digital scheme (solid lines) and a digital-analog approach (dashed lines).Blue, orange and yellow refer to the number of Trotter steps (one, two and three). **b)** Trotter decomposition of a DAQS protocol for the Heisenberg model [55].

They also claim that it is expected that a larger amount of ions can be employed compared to the purely digital approaches. This way, « the natural continuation of this research line is to explore how other models could benefit from the DAQS technique» [55].

Another propositions have been made as well as for example: a DAQS using the cross-resonance effect [86] and proposal for the simulation of other kind of

fermionic models (e.g. Fermi-Hubbard model) [87].

4.3 Digital-analog quantum simulations scheme proposals with photons

In the last section, we want to propose possible schemes melting the two fundamental topics reviewed in this thesis. As starting point, we resort to a paper published in 2012 by Shadbolt et al. [88]. These researchers provided the scheme of a reconfigurable photonic quantum circuit by means of integrated waveguide platforms (16). It consisted of a two qubits gate with a certain number Hadamard gates together with 8 rotations operations coupled through a CNOT gate. This is physically implemented encoding both qubits as photons in dual rails previously generated via type-I SPDC. While the rotations are implemented by voltage-controlled thermo-optic phase shifters, directional couplers are employed in first place, to reproduce the superposition operations of Hadamard gates ($\eta = 1/2$)[7] and in second place, to put a CNOT gate into effect in the middle band of the scheme ($\eta = 1/3$). The CNOT operation is implemented as a non-deterministic linear optical gate with probability 1/9 according to the methods described in section 3.2.

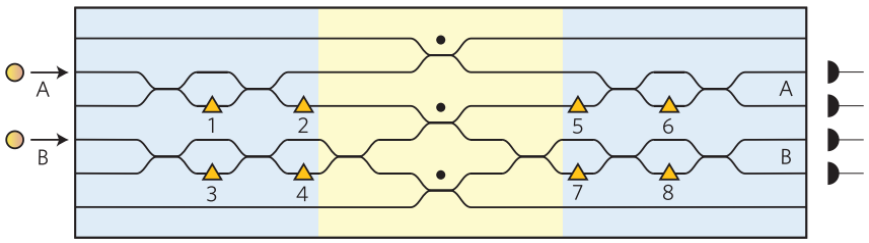

Figure 16: **Reconfigurable two entangled photon quantum circuit**. The first part is useful to prepare initial states, (the third part is its mirror image) while the second one is employed to apply a CNOT operation over the qubits which are finally measured by the end of the circuit [88].

[7] η represents the ratio of photons reflected and transmitted by the directional coupler

The feasible reconfiguration of the circuit fixing different values for the phase shifters allowed them to produce maximally two-qubit entangled states (all the Bell states) and mixed states, to apply quantum state tomography and to realize a Bell inequality manifold as well. According to the authors, they performed all these applications in a straightforward way with high fidelity because of the «interferometric stability of integrated optics which makes path-encoding of qubits a natural choice» [88] in contrast with an hypothetical bulk optics scheme where polarization encoding is more suitable and in turn increases significantly the required resources.

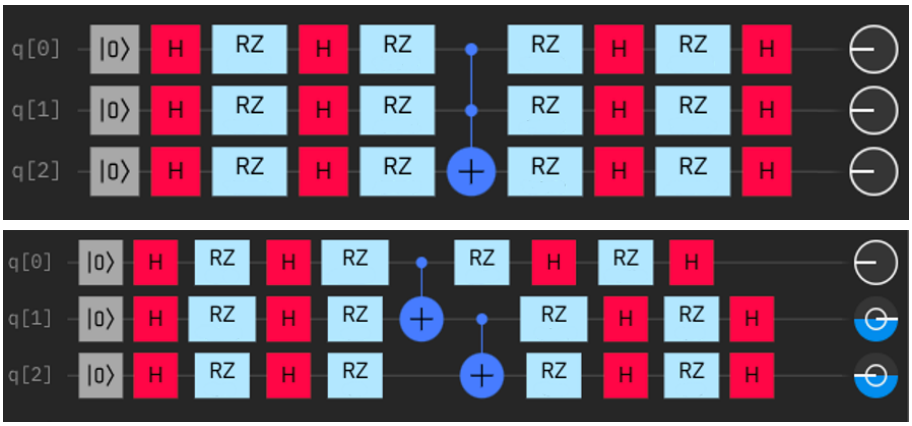

Figure 17: **Circuit scheme of the three-qubits reconfigurable quantum gates**. First circuit employing a Toffoli gate and the second one resorting conversely to two consecutive CNOT gates. Own image produced through the IBM quantum composer [89].

Taking into account this framework, our proposition can be seen as **digital-analog schemes of a three-qubits reconfigurable quantum gate** that couples two Shadbolt reconfigurable gates through a **Toffoli gate**[8] and through two consecutive CNOT gates. Each Shadbolt gate can be seen as an analog blocks and in particular, the Toffoli gate just as the CNOT gates can act equally as analog blocks providing scalability while single-qubit operation performed by the phase shifters adds universality in terms of available operations over the qubits. As

[8]**Toffoli gate:** Also known as a CCNOT gate, it is a 3-qubits quantum gate which acts as a CNOT gate where 2 qubits works as control-qubits and the last one as a target-qubit

showed in (17), it is composed by twelve pairs of Hadamard gates together with rotations over the z-basis ϕ_{1-12} which are applied on the initial state: $|000\rangle$. Central element acts in both scheme on the one hand, as a connecting element between the two big Shadbolt blocks and on the other hand, as a multiple-qubits interaction which, in combination with the phase shifters, allows to implement generic unitary operations for this configuration.

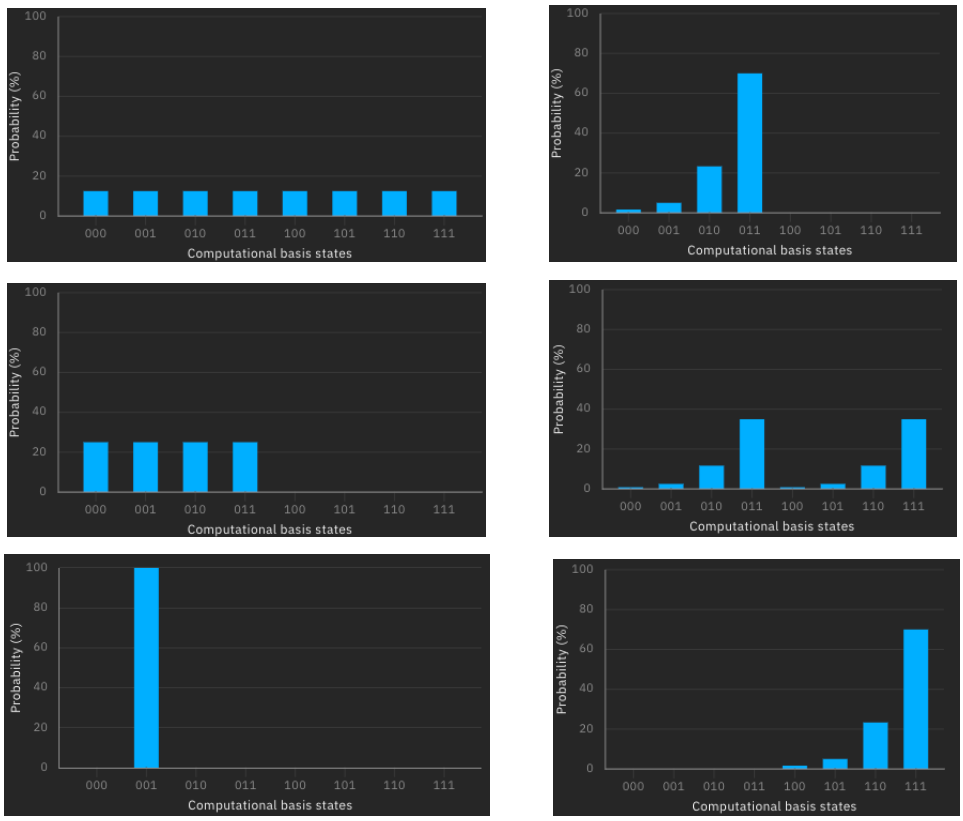

Figure 18: Probabilities of states for different values of the phase shifters in the first scheme. Simulated via IBM quantum composer [89]

As regards the possible physical implementation of these circuits, the previous proposal has been proved to be experimentally developed as described above. In addition, certain progress has been made in respect of reproducing a Toffoli gate experimentally. For instance, we have here two proposal for its implementation in bulk optics [90] and as far as our approach, in integrated photonics [91]. There-

fore, since each of its elements can be feasibly constructed in IP, this provides certain hope for an hypothetical experimental reproduction. Notwithstanding, certain problems tend to appear when coupling different elements related to decoherence and for example path-length mismatch (associated to dual rail encoding), that cannot be discarted.

Our intention has also been to test them trying to understand which kind of operations or possible applications could be performed and to define each limitations. Fixing different values of the phase shifters for the first scheme, we have obtained several interesting configurations. In (18) we can observe some examples: The three graphs on the right side show linear combinations with different weights and have been reached just modifying $\phi_{12} = 0, \frac{\pi}{2}, \pi$, which certainly exhibits a clear sample of its sensitivity. Furthermore, it is possible to obtain single qubits states and linear combination of two qubits the same way was possible for the scheme of [88].

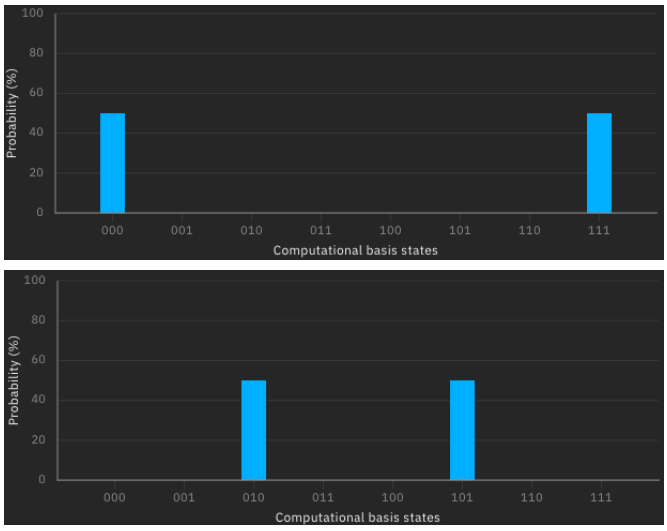

Figure 19: Probabilities of states for different values of the phase shifters in the second scheme. **GHZ state** in the first plot and another tripartite entangled state in the second one. Simulated via IBM quantum composer [89]

We had in addition as a goal, the attainment of genuine tripartite entanglemet. For example, reaching a Greenberger-Horne-Zeilinger (GHZ) state:

$$|GHZ\rangle = \left(\frac{|000\rangle + |111\rangle}{\sqrt{2}}\right) \tag{4.2}$$

This is a special genuine tripartite entanglement where none of the qubits is factorized in any of the other ones. This state has quite high non-classical properties and was first studied in 1989 and employed to test Bell inequalities [92]. However, no matter how much we changed the parameters in this scheme, we always found an obstacle. That is the reason why we decided to try with the second scheme letting the two CNOT gates configuration to provide a broader spectrum of outputs. This design not only allowed us to implement similar operations as the described in (18), but we were able to obtain genuine tripartite entanglement. As we can see in (19) in the first plot we obtained the probabilities associated to the GHZ state.

5 Conclusions

Throughout the different sections of this study, we have covered extensive fields which are currently making progress to this day. As we exposed in the first section, the long-term goal of a quantum computer does not confront directly with the possibility of resort to another approaches trying to reduce the complexity of certain computation tasks. After all the proposals presented, we can conclude that it is quite clear that quantum simulations is more than a suitable strategy to overcome the problematic exponential explosion of resources associated to the size of quantum systems.

When it comes to perform this theoretical designs in physical platforms, there are many currently-available physical systems to do tasks of this nature depending on what we are investigating and on the framework we are working in. Photons highlight for being one of the most employed platform in quantum information processing due to the versatility granted for qubits encoding by all its DoF

and their robustness against decoherence. Although many proposals have been (and are currently being) reported employing bulk optics equipments for implementing this technology, it seems that the fascinating progress made in IQP in the last decade, is leading to a context where this technological approach will be the only one that will provide enough scalability to perform circuits like these ones over larger and more complex systems.

Finally, we have showed some examples of how the DA approach harnesses the best part of both simulation strategies. Our circuit propositions are a simple example of this fact. It is possible to keep delving into this design trying to get new results like for example, getting a W state:

$$|W\rangle = \left(\frac{|001\rangle + |010\rangle + |100\rangle}{\sqrt{3}}\right) \tag{5.1}$$

or looking for a S state:

$$|S\rangle = \frac{1}{\sqrt{6}}\left(|001\rangle + |010\rangle + |100\rangle\right) + \frac{1}{\sqrt{2}}|111\rangle \tag{5.2}$$

presented by Anwer et al. [93]. Apparently, this entangled state seems to be quite useful to produce maximal non-locality through measurement required in experiments for proving violations of Bell inequalities. It turns out to be another alternative to the already mentioned state like GHZ or the W one. However, taking into account the maximum extension of this thesis, we believe that it is out of the scope of it. Even so, it would be a very suitable continuation of this study.

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