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FROM DESCRIPTIVE GEOMETRY TO ARCHITECTURAL GEOMETRY CONTRIBUTIONS BY CLASSIC AUTHORS TO THE NEW PARADIGM

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Abstract
The use of computation for architectural design, together with the application of digital fabrication techniques for materialisation, has brought a new paradigm where contributions by classic authors linked to Descriptive Geometry might seem inappropriate. This paper shows a case study of a full-scale prototype which proves how some of the classic geometric theorems and procedures can still play a role in the design computation era.

- Objectives: identify and analyse the usefulness of classic Descriptive Geometry contents within the computational design paradigm through the case study of a particular project.
- Methodology: Analysis of the conception, development and materialization processes, to identify the application and potential benefits of classic geometric contents.
- Results: at the different stages analysed, some classic geometric theorems and procedures were found to be useful and efficient.
- Practical implications: The implications of these findings involve both architectural practice and the academic sphere.
- Originality / value: Although initially bound to this case study, the methodology could be extrapolated to more complex projects to obtain more simple and efficient design algorithms.

Keywords: Geometry, Descriptive Geometry, Architectural Geometry, Theorems, Quadratic Surfaces, Algorithmic Design, Computational Design, Parametric Design, Digital Fabrication.

Resumen
De la Geometría Descriptiva a la Geometría Arquitectónica; Aportaciones de Autores Clásicos al Nuevo Paradigma
El uso de la computación en el diseño arquitectónico, junto con la aplicación de técnicas de fabricación digital para la materialización, ha traído un nuevo paradigma donde las aportaciones de los autores clásicos vinculados a la Geometría Descriptiva podrían parecer inadecuadas. Esta comunicación muestra un estudio de caso sobre un prototipo a escala natural que demuestra cómo algunos de los teoremas y trazados clásicos aún pueden tener un papel en la era de la computación.

- Objetivos: analizar la utilidad de contenidos de la Geometría Descriptiva clásica dentro del paradigma del diseño computacional a través del estudio de caso de un proyecto concreto.
- Metodología: Análisis de los procesos de concepción, desarrollo y materialización, para identificar la aplicación y beneficios potenciales de contenidos geométricos clásicos.
- Resultados: en las diferentes etapas analizadas, algunos procedimientos y teoremas geométricos clásicos resultaron ser útiles y eficientes.
- Repercusiones prácticas: Las repercusiones de los resultados obtenidos atañen tanto a la profesión arquitectónica como al ámbito académico.
- Originalidad / Valor: La metodología usada en este estudio de caso, podría extrapolarse a proyectos más complejos para obtener algoritmos de diseño más simples y eficientes.

Palabras clave: Geometría, Geometría Descriptiva, Geometría Arquitectónica, Teoremas, Superficies Cuádricas, Diseño Algorítmico, Diseño Computacional, Diseño Paramétrico, Fabricación Digital.

1. Introduction

The increasing use of digital technologies in architectural design a materialisation in the last decades has definitively opened a new architectural scenario which still holds many aspects to be explored. The feasibility of this new paradigm lies mainly on the existence of new digital tools. On the one hand, CAD tools, together with scripting possibilities to generate algorithms, allows the use of computation in design, in contrast to computerization [1]. The focus of this new design strategy has moved from the object to the process itself. Algorithms acquire the role of the new means of representation as the language that translates human thinking into the power of combination of computer-based processes. On the other hand, the development of digital fabrication tools and techniques have made way for that new paradigm to be materialised, facilitating a totally digital architectural process, from conception to execution [2].

This situation has implications both in the architectural practice and the academic sphere, meaning a significant change on the designer's mindset, very different from the conventional thinking in architectural design [3], which would need to be supported from the beginning of the undergraduate training period. This is the time when students, as future practitioners, mould their way of thinking, conceiving and expressing their architectural work, having the tools available a strong influence on the design and production of buildings [4] [5]. This is the reason why, in our department of Graphic Engineering in the University of Seville, we have been researching on the development of a new approach on how to teach and learn geometry for undergraduate first-year students. It challenges those taking their first steps on Architectural Geometry [6], under the integration of geometry itself, the use of digital tools for architectural design, and the application of digital fabrication techniques to construct a real architectural installation [7].

It is the construction of that real architectural installation, in this case an ephemeral wooden gallery named The Caterpillar [8], what is used in this paper as a case to be studied. The project was intended to make students gain an insight into the use of the basic contents of Architectural Geometry through the various stages of a Project: from conception to physical execution. The integration of geometry learning in a digital fabrication environment holds centre stage. Geometry shows itself to be the link between digital design tools and physical architectural space through the use of digital fabrication devices and the execution of assembly and setting-up processes.

2. Objectives

For almost two centuries, since the systematization of Descriptive Geometry by Gaspard Monge in 1795 [9], this discipline has been taught, explicitly or implicitly, in architecture and building engineering schools worldwide. Its main role was to deal with three-dimensional geometries through the use of the two-dimensional means available at the time. With the two-dimensional limitation of the means, many planar constructions had to be developed under this paradigm to carry out operations on the relevant three-dimensional objects. That fostered the appearance of many authors who made contributions to the development of this paradigm with new procedures, drawing strategies and theorems establishing relationships between spatial operations and their planar representations and properties [10]. For the sake of this paper, these authors contributing to the development of the Descriptive Geometry paradigm before the widespread use of CAD systems are considered as classic authors.

Within the paradigm of architecture conceived by computational means [1], and with the use of 3D CAD systems and programming as tools for geometric control and development, certain classic Descriptive Geometry texts appear to be outside the scope of this application. Nevertheless, a wide range of geometrical contents inherited from these treatises can be reformulated and enhanced with the use of contemporary technologies to produce new and attractive results. Much work remains to be done on this transcription or reformulation of which the project presented in this paper is a sample [11].

The main aim of this paper is therefore to analyse the case of the project presented, to draw conclusions about the role of contents stemming from classic authors within a project carried out with the means of the new computational paradigm. In other words, to test the validity and usefulness of these contents which were thought to manipulate three-dimensional geometries from the limitations of two-dimensional means, and therefore the geometric constructions used. Could they be useful within a paradigm where the two-dimensional limitation does not exist anymore? This paper tries to answer this question with the case study of the mentioned project.

3. Methodology

In order to achieve the aim of this paper, the first suggested step is the description of the project itself which is used for the case study. It is a ten-meter-long and three-point-five-meter-high gallery, and its origin lies on the combination of geometric research and teaching innovation as it will be shown later. From a structural point of view, it is a set of fractions of conical surfaces which are not self-supporting individually. They are used as components that, once assembled, bring about the emergence of the desired structural behaviour without any kind of auxiliary structure. The deformations obtained with the finite elements analysis, and later on the real prototype, were all acceptable without the need of specific rigidifying elements. The final result was a self-supporting five-millimetre-thick plywood shell which gave form to the designed space (Fig. 1).

The second step is the analysis of how the geometry of this project was conceived and developed. Due to the academic environment in which it took place, and in order to test one of the classic theorems within a computational design environment, a classic theorem was taken as the starting point of the project to be implemented in an algorithm which generates the three-dimensional global form of the installation.

The third step is the analysis of the development of the algorithm, generated with the software Rhinoceros 3D and its plugin Grasshopper, trying to take advantage of another classic theorem which may simplify the algorithm by applying some of the two-dimensional properties of certain three-dimensional geometries (Fig. 1).

The two previous steps were not isolated but bearing in mind the most important and last step to be analysed, which is the materialization of the project. In this case, after the conception and development of the form with the use of computational design, the materialization takes place with the use of a digital fabrication tool, a three-axis milling machine.

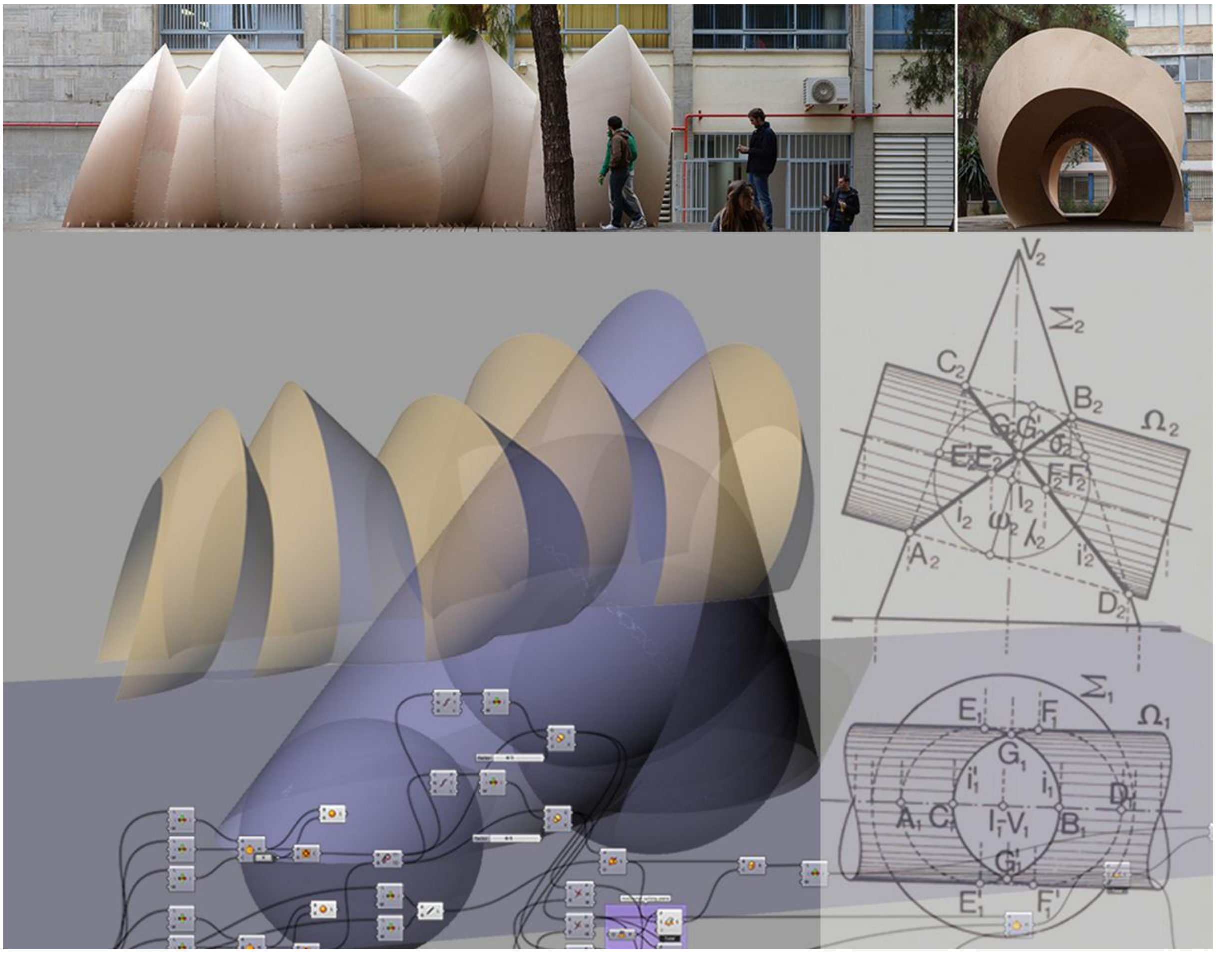

Fig. 1. Top: photographs of the installation. Bottom: images of the algorithmic design process in Grasshopper for Rhinoceros 3D and the representation, in one single multiview orthography projection, of two classic Descriptive Geometry theorems applied in the project.

4. Results

According to the last three steps exposed in the methodology, the results are given as follows in the same relevant three sections: the conception of the design, the development of the algorithm and the materialization of the installation.

4.1. Conception of the design

Among other aspects, a significant contribution of this project deals with the reinterpretation of the basic contents of traditional Descriptive Geometry via the use of digital means. Geometrical problems are addressed through the graphic thinking fostered by 3D CAD software and the algorithmic thinking needed for the use of parametric design tools [12]. Several theorems that stem from that tradition and contain basic contents of geometry were explored with digital tools to seek a source of architectural composition. One of the aims was that students acquired the necessary research and creative awareness to be led towards the creation of an architectural piece through the use of computation. In this case, a theorem about the intersection of quadratic surfaces by Gaspard Monge was used for the conceptual design of the project [13]. It essentially states that if two quadratic surfaces are circumscribed about a third common quadric, then the intersection curve decomposes into two planar curves (Fig.2).

For the generation law, an auxiliary sphere is considered as the common quadric, and two cones as the circumscribed surfaces (Fig. 2). The repetition of this trio along a desired path accomplishing certain intended conditions is the initial definition of the project. The spheres were placed at the vertices of a polygonal path with an elevation by way of zig-zag lines and a plan defining the route to follow. Moreover, the diameters of the spheres control and define the spaciousness of the project at every point of the path at two different levels: at the ground level and at the average human size. The result is a gallery composed of 10 fragments of conical surface defined by 6 auxiliary spheres and two boundaries for each shell, one at the intersection with the adjacent cones and the other one at the intersection with the ground (Fig. 3).

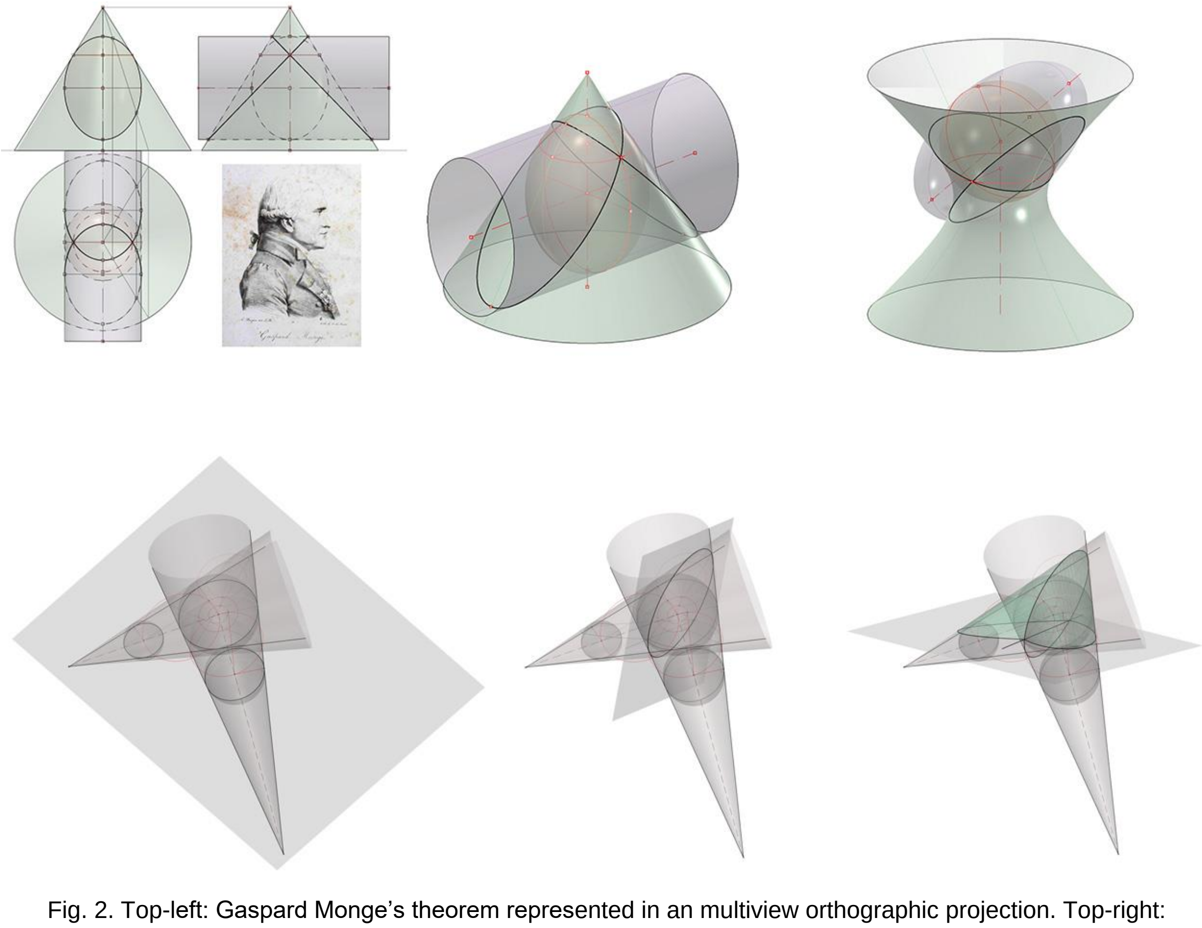

Fig. 2. Top-left: Gaspard Monge's theorem represented in an multiview orthographic projection. Top-right: perspective views of the application of the theorem to the intersection of different quadratic surfaces: cone-cylinder, hyperboloid-ellipsoid. Bottom: perspective views of three steps of the application of the theorem to the project, considering only two adjacent conical surfaces.

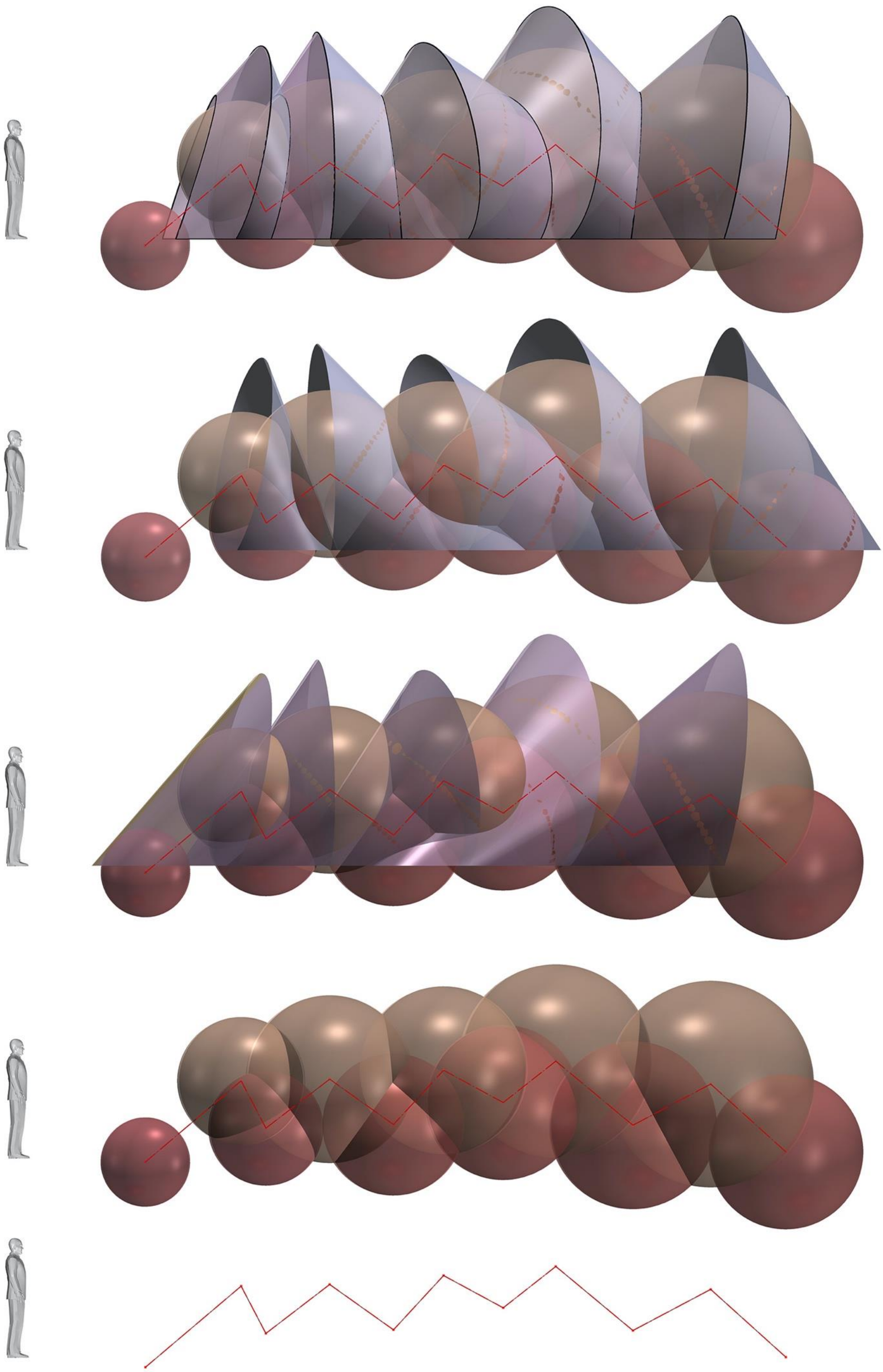

Fig. 3. From bottom to top, front views of the different steps taken for the generation of the project by implementing the theorem in the algorithm.

4.2. Development

Working with the students for the development of the project, and simultaneous to the graphic thinking used for the conception of the design, algorithmic thinking is also developed to address the parametric definition through a visual programming tool: Grasshopper for Rhinoceros 3D. This tool perfectly couples the two different approaches to geometry [14]. Therefore, once the geometrical principle of the project is defined, the next phase consists of translating the geometrical relationships into an algorithm for the parametric definition. After this task is concluded, every student can introduce variations on the input parameters to explore several design possibilities. Due to their lack of training on the use of such parametric tool, students are gradually introduced and guided by the lectures until the definition is satisfactory. Finally, a single result is selected for construction by the whole group.

Instead of facing the parametric definition from a three-dimensional point of view, another classic theorem is used to reduce the complexity of the global form to simple planar constructions every three of the initial auxiliary spheres. These planar constructions are drawn on the plane defined by the centres of every three consecutive spheres, which is a symmetry plane of the two surfaces, making the algorithm both simpler and more efficient than if the strategy had been searching for an algorithm to generate the form as a three-dimensional whole. The theorem applied in this case states that if the intersection between two quadratic surfaces is composed of two planar curves and they have a symmetry plane, the orthogonal projection of the curves onto the symmetry plane is a set of two straight-line segments [13]. Its application allows the designer to generate the algorithm with simple planar constructions on the plane defined by the centres of the spheres, where the spheres are represented by circles, the circumscribed cones by straight lines tangent to these circles and the intersections curves by two straight-line segments (Fig. 4).

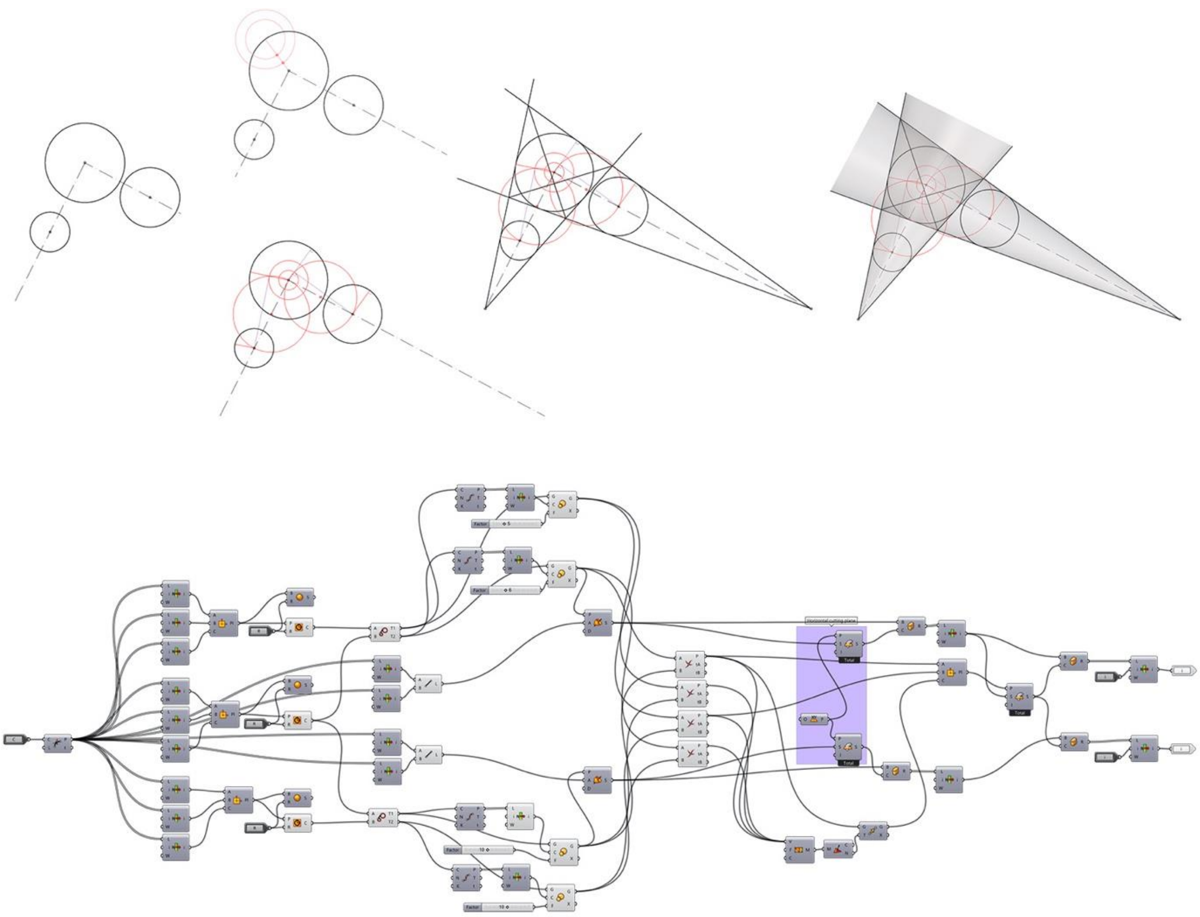

Fig. 4. Bottom: Grasshopper definition (bottom). Top: Orthogonal views of the symmetry plane at different steps of the definition. Due to the use of the theorem, the 3D surfaces can be defined by 2D constructions to finally use their solution to model the 3D surfaces with the minimal use of resources. Spheres are represented by circles, circumscribed cones are represented by lines tangent to these circles and the intersections curves are represented by straight-line segments, as stated by the theorem.

4.3. Materialization

In contrast with conventional materialization approaches, digital fabrication techniques, and subsequent assembly processes, rely on a detailed and accurate definition of construction elements and assembly strategies that must be present from the design stage. Craft quality depends less on the production phase and is more present in the design and development phases [15]. Hence this involves the need of conceptual geometric resources to be applied in the development of the design to perfectly define all the aspects, such as addressing constructive solutions, determining true sizes and shapes of elements, developing surfaces and optimising material. In this project, some of these resources also stem from classic Descriptive Geometry as shown in the following paragraphs.

Although not only associated to Descriptive Geometry, but also to antique techniques, the use of reduced scale physical models is still a common and necessary practice within the new paradigm. It even holds more importance to test assembly processes now than it was for the architectural practice of some decades ago. In this project, before the 1:1 scale prototype, there was a checking phase in which two models were constructed to detect possible mistakes or improvements: one on a 1:10 scale and another on a 1:4 scale (Fig. 5). This made students have to think and handle with fluency the concept of scale and its association with the different purposes and materials used for prototyping. Not only did students acquire geometrical competences but they were also aware of the importance of the need for geometric control before starting the digital fabrication stage, since the correct definition of the fabrication documentation has a direct repercussion on the desired result of the architectural installation [16].

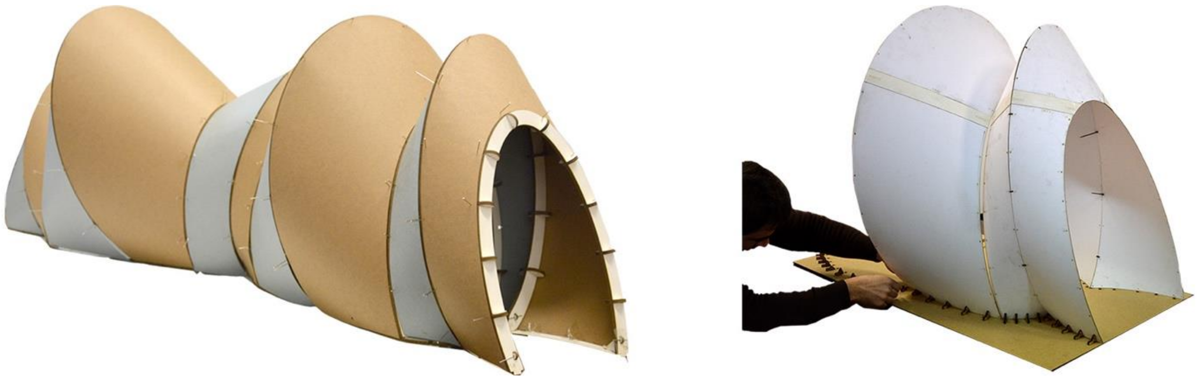

Fig. 5. Physical models before the fabrication of the full-scale prototype. Left: scale 1:10. Right: scale 1:4.

Another necessary resource stemming from traditional practice and especially developed by classic authors is the use of planar sections or projections to solve certain problems or to obtain relevant information from a design object. In this project is particularly notable the use of planar sections to determine the angle between two kinds of surfaces, the conical ones and the ground plane. The aim was to determine, not only the angle, but the exact shape of the planar section to design a series of wedges with two main functions; being one of them the structural constraints of the global shell with the ground, and the second one, a necessary auxiliary element during the assembly process so that the conical surfaces acquired the design shape after the manual bending. The planes where located perpendicularly to the intersection line of the shell with the ground and evenly spaced for each conical fraction. This common practice in classic Descriptive Geometry is also used here, although in this case it is taken to a new level by automating the task, due to the number of necessary wedges and the variable angle of the surfaces along the intersection line. This operation not only involves algorithmic design skills, but also the classic knowledge related to geometric properties of surfaces and their intersections curves, such as the Frenet-Serret frame used in this case to define the automation [17] [18] (Fig. 6).

Finally, the last task being worth mentioning in this project for the purpose of this paper is the creation of graphical documentation for the assembly and setting up of the structure that explains tasks in two different, although related, environments. One of them is the laboratory where the pieces are fabricated, labelled, organised and even partially assembled. The other one is the project site where the final assembly takes place. In both cases, graphic documentation, understood in classical terms, although generated by digital means, is necessary to specify the tasks that must be undertaken by the different work teams (Fig. 6). The division itself of the assembly and setting up activities within each work team shows the importance of the use of graphic expression with standard codes that every team member can understand.

Fig. 6. Top three rows: composition of the installation on the site. Middle row: automation to determine the shape of the wedges. Bottom row: Development of the surfaces and graphic documentation for digital fabrication.

5. Conclusions

Although many of the treatises on Descriptive Geometry currently appear to be obsolete, this paper shows how they might still have a place within the computational design and digital fabrication paradigm of architecture. On the one hand, much of their contents, translated from conventional representations to parametric tools, can provide the starting point for the redefinition and exploration of new forms with the power of computational methods. On the other hand, these treatises also contribute towards simple solutions for geometrical problems.

In the professional sphere, it has been pointed out the importance of classic geometrical knowledge and competence throughout the different stages of a real architectural project, even within the computational design and digital fabrication paradigm. The studied project shows how geometry is present from the initial creative task, based on the use of the aforementioned theorem of quadratic surfaces, to the last activities related to fabrication and assembly. Especially nowadays, when architecture is subject to strong demands, any resources designers may have at hand to improve the workflow and the efficiency of algorithms and solutions adopted is most welcome, and this article proves how some of the classic contents of Descriptive Geometry might hold some of those necessary resources. On another note, conventional graphic communication and documentation, although now generated with digital means, move their central role from design to the fabrication and assembly processes, where the graphic codes obtain special importance so that the various devices, agents and teams involved can better understand their own tasks to be performed within the production system.

The demands and needs of architecture practitioners must also be reflected in the academic sphere, what implies that changes on the learning systems should also be undertaken. These changes are especially important at the beginning of the undergraduate training period, which is the time when students mould their way of conceiving, developing, representing and materialising architecture. This project proves that this implementation is feasible in two different aspects: first, the way in which classic geometry is blend, and even rendered, within digital environments. Second, it hast been tested that the first year of the undergraduate training period is not too early, as some faculty colleagues think, to introduce students into the computational design and digital fabrication paradigm, when it is done in the way shown in this project. Moreover, digital fabrication processes leading to the construction of a full-scale prototype provide an effective and stimulating environment for students attain hands-on experience with the geometrical competences of the new components of Architectural Geometry.

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