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SESSION 2

DAYLIGHTING

SOME CONSIDERATIONS ON THE ENVIRONMENTAL PROPERTIES OF DOMES

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ABSTRACT. With this issue we intend to promote a different attitude towards architecture and especially bioclimatic architecture. It is time to react against the increasing trends for standardization of "right angles" in building construction. If bioclimatism has come of age for architectural practice, it should resume the search for richer and more complex new forms so constant, on the other hand, in nature and in the proposals of the past.

1. INTRODUCTION

1.1 Conceptual properties

Much has been written about the geometrical, aesthetic and even mystic features of the dome; in words of Giedion /1/ the dome is a typical phenomenon from the second space conception in architecture, that of the radiance of interior space. The Roman Pantheon can be a relevant example of this conception. Inside that building for instance daylighting reveals, according to Burckhardt /2/, visions of the unattainable. Then, the domes of Bernini, Borromini, Guarini and Vittone added important precisions to the original notion and as stated by Wittkower /3/ those buildings became the measure of themselves for they had no equal. Nowadays, the use of this typology has seriously declined mainly with the integration of "urban" concepts in building design and the ebb of spiritual centers.

1.2 Environmental properties

Historically, the dome belonged to the basic repertoire of most vernacular cultures. Its outstanding structural capacities constituted the basis for the membrane and shell theory. Its thermal properties also, have become rather well known in the last few years, the low surface to volume ratio which prevents thermal dissipation, the maximum exposure to night cooling in arid regions and the enhanced air-flow through top-vents are typical examples. Should all this meet with the former conceptual properties we would enter the third idea of space, the radiance between interior and exterior; the building itself, from its spaciousness, reaches out for the environment. However, in the aim to retrieve the new dome typology, a major difficulty arises when analyzing the daylighting potential, as the available tools are unsuitable for curvilinear geometries. That extreme we will try to solve from now on.

2. CONFIGURATION FACTORS

2.1 Rectangular surface source

Let us consider the luminance due to a Lambertian surface:

$$L= \frac{dI_\theta}{dS*\cos\theta}$$

and the illuminance formula:

$$dE= \frac{dI_\theta*\cos\theta}{l^2}$$

Substituting into this expression dI_θ we obtain:

$$dE= \frac{L*dS*\cos^2\theta}{l^2}$$

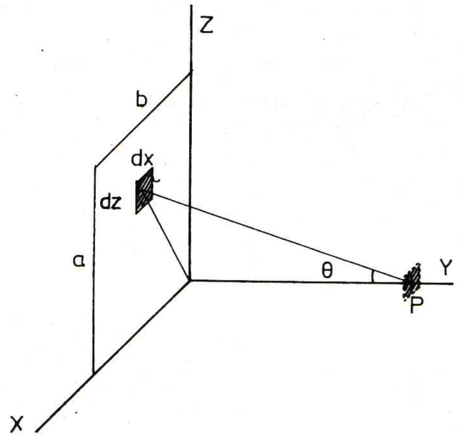

Fig1.rectangular surface. cartesian coordinates

Using Cartesian coordinates, as illustrated in figure 1, this can be written:

$$dE= L *\frac{y^2\, dx\, dz}{(x^2+y^2+z^2)^2}$$

Which, over the domain of the rectangular surface becomes,

$$E = L \int_0^a \int_0^b \frac{y^2\, dx\, dz}{(x^2+y^2+z^2)^2}$$

If $a=\sqrt{y^2+z^2}$ and x=x, this integral follows the rule:

$$\int \frac{dx}{X^2} = \frac{x}{2a^2X} + \frac{1}{2a^3} Y$$

Where $X=a^2+x^2$ and Y=arctg (x/a)

Consequently

$$\int_0^b \frac{y^2\, dx}{(x^2+y^2+z^2)^2}$$

becomes

$$\left[y^2 \left(\frac{x}{2\, z^2+y^2((\sqrt{z^2+y^2})^2+x^2)} + \frac{1}{2(\sqrt{z^2+y^2})^3} \operatorname{arctg} \frac{x}{\sqrt{z^2+y^2}} \right) \right]_0^b$$

Substituting we have

$$\frac{1}{2} y^2 \left[\frac{b}{b^2(z^2+y^2)+(z^2+y^2)^2} + \frac{1}{(\sqrt{z^2+y^2})^3} \operatorname{arctg} \frac{b}{\sqrt{z^2+y^2}} \right]$$

Integrating with respect to z we obtain

$$\frac{1}{2} y^2 \int_0^a \left(\frac{b}{b^2(z^2+y^2)+(z^2+y^2)^2} + \frac{1}{(\sqrt{z^2+y^2})^3} \operatorname{arctg} \frac{b}{\sqrt{z^2+y^2}} \right) dz$$

The first fraction can be reduced to

$$\frac{-1}{b} \left[\int_0^a \frac{dz}{b^2+z^2+y^2} - \int_0^a \frac{dz}{z^2+y^2} \right]$$

both belonging to the type $\int \frac{dx}{X} = \frac{1}{a} Y$

With $X=a^2+z^2$ and Y=arctg (z/a)

Therefore putting $a=\sqrt{y^2+b^2}$ in the first term and a=y in the second we receive

$$\frac{-1}{b} \left[\frac{1}{\sqrt{y^2+b^2}} \operatorname{arctg} \frac{z}{\sqrt{y^2+b^2}} - \frac{1}{y} \operatorname{arctg} \frac{z}{y} \right]_0^a$$

What gives

$$\frac{-1}{b} \left[\frac{1}{\sqrt{y^2+b^2}} \operatorname{arctg} \frac{a}{\sqrt{b^2+y^2}} - \frac{1}{y} \operatorname{arctg} \frac{a}{y} \right]$$

The second fraction solved by parts is:

$$\int_0^a \frac{1}{(\sqrt{z^2+y^2})^3} \operatorname{arctg} \frac{b}{\sqrt{z^2+y^2}}\, dz =$$

$$\left[\frac{z}{y^2\sqrt{z^2+y^2}} \operatorname{arctg} \frac{b}{\sqrt{z^2+y^2}} \right]_0^a + \int_0^a \frac{bz^2}{(z^2+y^2+b^2)\,(z^2+y^2)y^2}\, dz$$

The first term gives

$$\frac{a}{y^2\sqrt{a^2+y^2}} \operatorname{arctg} \frac{b}{\sqrt{a^2+y^2}}$$

The second term can follow a decomposition of the form

$$\frac{2b}{y^2} \left[\frac{dz}{(z^2+y^2+b^2)} - \frac{dz}{2(z^2+y^2)} + \frac{(b^2-y^2)\, dz}{2(z^2+y^2+b^2)\,(z^2+y^2)} \right]$$

as all the numerators can be considered constant, the precedent integrals have already been solved, and consequently we receive

$$\frac{2b}{y^2} \left[\frac{dz}{(z^2+y^2+b^2)} - \frac{dz}{2(z^2+y^2)} - \frac{1}{2}\left(\frac{dz}{(z^2+y^2+b^2)} - \frac{dz}{(z^2+y^2)} \right) + \frac{y^2}{2b^2} \left(\frac{dz}{(z^2+y^2+b^2)} - \frac{dz}{(z^2+y^2)} \right) \right]$$

Clearing the expression and adding the former results

$$\frac{b}{y^2} \int_0^a \frac{dz}{(z^2+y^2+b^2)} + \frac{a}{y^2\sqrt{a^2+y^2}} \operatorname{arctg} \frac{b}{\sqrt{a^2+y^2}}$$

That yields

$$\frac{b}{y^2}\ \frac{1}{\sqrt{y^2+b^2}}\ \text{arctg}\ \frac{a}{\sqrt{y^2+b^2}}\ +$$

$$+\ \frac{a}{y^2\sqrt{a^2+y^2}}\ \text{arctg}\ \frac{b}{\sqrt{a^2+y^2}}$$

And we arrive at the final result

$$E=\frac{L}{2}\left[\frac{b}{\sqrt{b^2+y^2}}\ \text{arctg}\ \frac{a}{\sqrt{b^2+y^2}}\ +\right.$$

$$\left.+\ \frac{a}{\sqrt{a^2+y^2}}\ \text{arctg}\ \frac{b}{\sqrt{a^2+y^2}}\right]$$

Thus the sought-for integral is the Higbie configuration factor for horizontal rectangular surfaces. Accordingly we can use the same procedure to find the configuration factor for sloped rectangular surfaces or nonrectangular surfaces. The configuration factor must be multiplied by the luminance from the surface to establish the illuminance at a definite point.

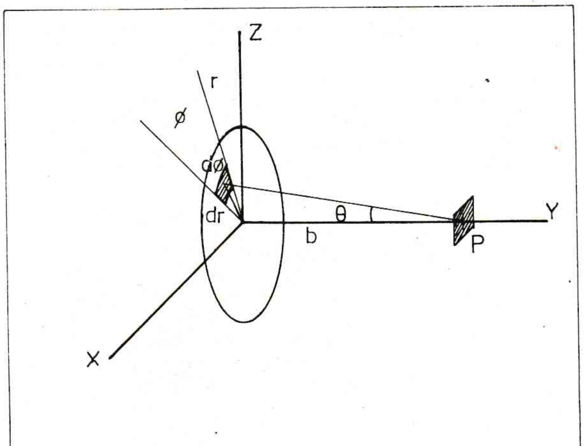

Fig 2. circular surface. polar coordinates

2.2. circular surface source

The station point lies directly under the centre of the circle. Using polar coordinates, as illustrated in figure 2, we have

$$dE=\frac{L*dS*\cos^2\theta}{l^2}$$

That yields

$$dE=\frac{L*r*dr*d\phi*b^2}{(r^2+b^2)^2}$$

Integrating over the circular domain

$$E=L\int_0^{2\pi}\int_0^{a}\frac{b^2*r*dr*d\phi}{(r^2+b^2)^2}$$

And then we obtain

$$E=L\int_0^{2\pi}\left[\frac{-b^2}{2(r^2+b^2)}\right]_0^a d\phi$$

Substituting

$$E=L\int_0^{2\pi}\frac{a^2\,d\phi}{2(a^2+b^2)}$$

And performing the outer integration we get

$$E=\frac{L\pi a^2}{a^2+b^2}.$$

Which is much simpler than the Higbie factor. However, for points other than the formerly described, the integral becomes complicated and is not suitable for the designer's use. Neverthless, the precedent formula shows that illuminance at the station point can be conceived as a ratio between the projection onto the horizontal plane of the solid angle subtended by the opening and the projection onto the same plane of the whole hemisphere taken as a unit. This is, in synthesis, the projected solid angle principle and many daylighting protractors are based on it though for re ctangular apertures.

For circular openings or the like, the problem reduces to intersection of a symmetric elliptical cone and a sphere with common centre and plane of symmetry. This is a usual question of descriptive geometry which can be easily solved over the very desing drawings. Unfortunately, mathematical proofs show that the projection of such intersection is often a fourth degree curve whose area is difficult to determine via analytical methods but, for practical purposes, we can assimilate it to an ellipse. In this way, the illuminance due to direct component of daylighting canbe found graphically with a high degree of accuracy. All this is illustra ted in figure 3.

3. INTERIOR REFLECTED ILLUMINANCE FROM CURVED GEOMETRIES

3.1 The integrating sphere formula

With the internally reflected component we could invoke other approaches as, for instance, the integrating sphere formula /4/. There, the average interior reflected illuminance depends on an infinite number of reflections inside the sphere and then

$$IRE = E\ \frac{W}{A}\ (R+R^2+R^3+...+R^n)$$

Where W is the total window area, A is the internal surfae of the sphere and R its mean reflectance. The sum of this series gives

$$\lim_{n\to\infty} \frac{R^{n+1}-1}{R-1} - 1 = \frac{R}{1-R}$$

Whence

$$IRE = E \frac{W * R}{A(1-R)}$$

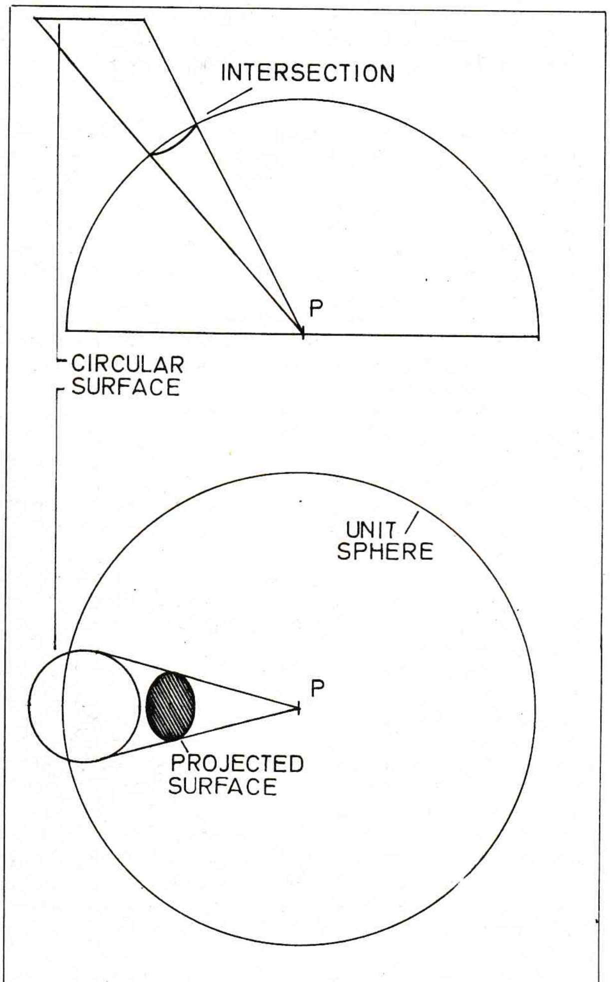

Fig.3 circular surface. determination of direct component of daylighting

4. ILLUMINANCE FROM DIRECT SUNLIGHT IN CURVED ROOMS

4.1 The secondary source approach

With direct sunlight we can work as if we had two circular surface sources. In fact, the cylinder defined by beam daylighting through a circular aperture depicts onto the interior s urfaces a curvilinear figure often assimilable to an ellipse or circle. Of course, the luminance of this second source depends upon the intensities of solar radiation and its relative position changes with the sun as well. Also the sky and internal components have tobe modified to take into account the circumsolar brightness and the internal reflection of direct sunlight.

5. CONCLUSIONS

Further generalization of the method with the help of Computer Aided Design will considerably enlarge the possibility of analyzing complex systems in the design stage. Also, curvilinear finite elements will be available. Another important conclusion is the fact, that under symmetric skies, apertures posessing axial symmetry yield a vectorial illumination field with axial symmetry as well. This can be a step forward in the aim to create a new environment in the daylighting field.

6. REFERENCES

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/3/ Wittkower, R. 1972. Vittone. Atti del Covegno internazionale dell'Academia delle Scienze di Torino.

/4/ Phillips, G. and Littlefair, P. 1988. Average Daylight Factor Under Rooflights. Proc CIBS National Lighting Conf., Cambridge.

$(R^3-1) = (R^2+R)(R-1) = (R^3-R^2+R^2-R)$

$(R^3-1) = (R^2-1)(R-1) = (R^3$

$\frac{(R^3-1)}{R-1} - 1 = R^2$

$(R^2+1)(R-1) = R^3-R^2+R-1$

$(R^2-1)(R-1) = R^3$

$n=0 \quad S=0$

$n=1 \quad S=R$

$n=2 \quad S=R+R^2$

$n=3 \quad S=R+R^2+R^3$

$\frac{R^3-1}{R-1} - 1 = \frac{R^3-1-R+1}{R-1} = \frac{R^3-R}{R-1}$

$\frac{R^3-1-R+1}{R-1} = \frac{R(R^3-1)}{R-1}$

$= \frac{R(R^2-1)}{R-1} = \frac{R(R+1)(R-1)}{R-1} = R^2+R$

$(R+R^2+R^3)(R-1) = (R^2+R^3+R^4-R-R^2-R^3)$