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INERTIAL STABILIZATION CONTROLLER FOR A 2 DOF PLATFORM

C. Pérez and F.R. Rubio

Dept. Ingeniería de Sistemas y Automática
Escuela Superior de Ingenieros.
Universidad de Sevilla. Camino de los Descubrimientos s/n.
41092-Sevilla. SPAIN
Telf. + 34 954 48 73 50, Fax: + 34 954 48 73 40
e-mail: {cpf,rubio}@cartuja.us.es

Abstract: This paper presents the controller design for the stabilization of a platform of two degrees of freedom. The purpose of this application it is to control the velocities, which are measured in a inertial frame, rejecting the disturbances associated with moving components. A gain-scheduling controller based strategy is proposed, by way of which the desired specifications are accomplished .

Keywords: Gain Scheduling, platform control, Adaptive control

1. INTRODUCTION

The inertial stabilization of platforms shows large interest in the scope of the applications of aeronautical and navigational systems. These kinds of platforms are usually located in vehicles that have a variable orientation related to an inertial frame. These variations of the vehicle orientation are detected by gyroscopic sensors located in the edge of the platform.

This application must implement two operation modes. A non inertial one in which the positions are the controlled variables, and an inertial mode in which the inertial velocities of the platforms are controlled.

In this paper controllers for a large dimensions platform have been developed. These controllers must achieve strict specifications in the operation modes. In position mode, the precision must be greater or equal to 0.2mrad. and, in the inertial mode, must be greater than 0.5mrad with an inertial setpoint velocity equal to zero.

This paper is organized as follows: In section 2, a description of the platform model used in the development of the controller is given. Section 3 gives the adopted control structure. Section 4 presents some simulation results with the proposed controllers and finally, conclusions are given in section 5.

2. THE SYSTEM MODEL

Before designing the controller , it is necessary to obtain a system model of the platform. The dynamic equations of the system can be obtained from Lagrange's equations. The model used represents the 2-DOF platform shown in Fig.1. It is composed of two main bodies: the base, where the position is determined by the azimut angle ψ, and the main body where the coordinate is the elevation angle θ. The head of the platform is mechanically balanced with springs, so that the potential energy is invariant and hence discarded from the equations. The Lagrangian of the system is:

$$L = T - V = \frac{1}{2}(\dot{\psi}^2(I_{zz_1} + I_{xx_2} sin^2(\theta) + \\ + I_{zz_2} cos^2(\theta)) + I_{yy_2}\dot{\theta}^2) \qquad (1)$$

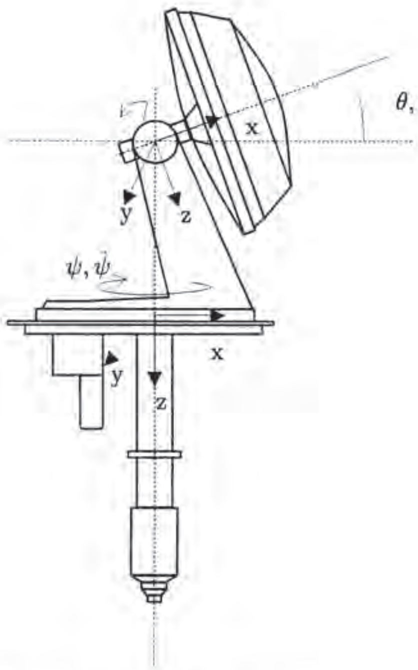

Fig. 1. Two degree-of-freedom platform.

Figure 2 shows a schematic diagram block of the 2 DOF platform model. This diagram shows how a gyroscopic measure model is attached to the output of the mechanic plant. The disturbances in the platform base and the internal dynamic of the gyroscopic sensors are considered in the gyro model.

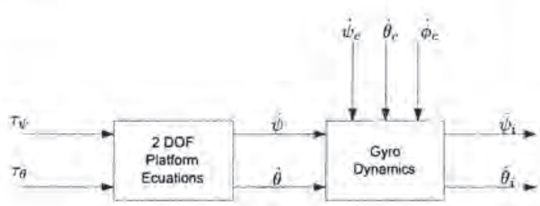

Fig. 2. Block diagram of the system model

The 2 DOF platform model dynamics equations can be expressed from the lagrangian formulation results:

$$\begin{cases} \tau_\psi = & \ddot{\psi}\left(I_{zz_1} + s_\theta^2 I_{xx_2} + c_\theta^2 I_{zz_2}\right) \\ \quad + & \dot{\psi}\dot{\theta} s_{2\theta}\left(I_{xx_2} - I_{zz_2}\right) + F_\psi(\dot{\psi}) \\ \tau_\theta & = \ddot{\theta} I_{yy_2} - \frac{1}{2}\dot{\psi}^2 s_{2\theta}\left(I_{xx_2} - I_{zz_2}\right) + F_\theta(\dot{\theta}) \end{cases} \tag{2}$$

where the following variables are used:

- ψ: Platform orientation.
- θ: Platform elevation.
- $\dot{\theta}_i$: Elevation velocity of the platform.
- $\dot{\psi}_i$: Orientation velocity of the platform.
- τ_θ: Applied torque in elevation.
- τ_ψ: Applied torque in orientation.
- I_{ij_1}: Inertia matrix of the orientation axis.
- I_{ij_2}: Inertia matrix of the elevation axis.
- $\dot{\theta}_c, \dot{\phi}_c, \dot{\psi}_c$ angular velocities of the platform base.

The functions $F_\psi(\dot{\psi})$ and $F_\theta(\dot{\theta})$ introduce friction torque in the orientation and elevation axes respectively. A static nonlinear relation between torque and shaft velocity as a friction model is used. The expression that compute the friction torque τ_f for a generic degree of freedom q is:

$$\begin{aligned} \tau_f = & \Big[F_v^+|\dot{q}| + F_c^+ + \\ & + (F_s^+ - F_c^+)e^{-B_1|\dot{q}|}\Big] sgn^+(\dot{q}) + \\ & + \Big[F_v^-|\dot{q}| + F_c^- + \\ & + (F_s^- - F_c^-)e^{-B_2|\dot{q}|}\Big] sgn^-(\dot{q}) \end{aligned} \tag{3}$$

where:

- F_c^+, F_c^-: Coulomb friction torques.
- F_s^+, F_c^-: Stribek friction torques.
- B_1, B_2: Coefficients for the stiction phenomena.
- F_v^+, F_c^-: Viscous friction coefficients.

and the sign functions are defined as follows:

$$sgn^+(\dot{q}) = \begin{cases} 1 & (\dot{q} \geq 0) \\ 0 & (\dot{q} < 0) \end{cases}$$

$$sgn^-(\dot{q}) = \begin{cases} 0 & (\dot{q} \geq 0) \\ -1 & (\dot{q} < 0) \end{cases}$$

In order to implement an inertial control of the platform, the gyro dynamics should be incorporated (Li and Hullender, May 1998). The gyro model can be described by a second order transfer function.

$$G_g(s) = \frac{\omega_g^2}{s^2 + 2\delta_g\omega_g s + \omega_g^2} \tag{4}$$

3. CONTROLLER DESIGN

The platform stabilization attempt that the variables $\dot{\theta}_i$ y $\dot{\psi}_i$ converge to the given references $r_{\dot{\theta}_i}$ y $r_{\dot{\psi}_i}$, independently of the present disturbances. Moreover, the maximum error of the inertial position with a zero velocity setpoint must be minor or equal to 0.5mrad. as another specification which must be achieved.

3.1 *Position Control*

The equations (2) show how the system is coupled due to Coriolis and inertia terms. However, as a result of the torque is being applied to the joint axis through gear reductions, the coupling is decreased and an independent joint control technique can be applied.

The design specification imposes the system must achieve, as soon as possible, a position in a region close to the reference. Moreover, the amplitude of the limit cycle that is created by the hunting phenomena must be bounded. In order to accomplish this, two controllers are designed. The first controller brings the states near the setpoints and the second controls the states around the setpoints. This control strategy is shown in figure 3.

The large displacements controller can be obtained imposing a closed loop dynamic. If a PD controller is used:

$$u(t) = K_p e(t) - K_d \dot{q} \tag{5}$$

where K_p and K_d are the controllers gains, $e(t)$ is the tracking error and and $\dot{q}$ is the velocity of the axis. By neglecting nonlinear terms in (2), we can design the linear controller with the plant:

$$G_p(s) = \frac{K}{s(s+a)} \tag{6}$$

with $K = K_\tau / J_{max}$ and $a = F_{v_{min}} / J_{max}$, so a closed loop transfer function can be obtained in the form:

$$G_{bc} = \frac{KK_p}{s^2 + (a + KK_d)s + KK_p} \tag{7}$$

where K and a are the gain and the pole of the open loop transfer function respectively. The goal is to bring the system to a place near the reference, the disturbance created by the Coulomb torque produces an error that can be bounded with the proportional gain. The steady state error becomes:

$$e_{ss} = \frac{F_c}{K_p} \tag{8}$$

where F_c is the Coulomb friction. This condition may be more restrictive than the one imposed by the rise time of the close loop response. The controller parameter K_d is computed by forcing the dynamics in equation (7) to a critically damped second order response.

$$K_d = \frac{2\sqrt{KK_p} - a}{K} \tag{9}$$

In order to accomplish the commutation, an elliptic limit is designed. This limit is given by the equation:

$$\frac{e^2}{e_{lim}} + \frac{\dot{q}^2}{q_{lim}} = 1 \tag{10}$$

where e_{lim} y q_{lim} are the diameters of the ellipse. Once the system is inside the ellipse, the controller gains are modified in order to control the amplitude of the limit cycle. The amplitude A_{lc} and the period T_{lc} can be estimated by the expressions (C. Canudas de Wit, 1999):

$$A_{lc} \approx \frac{F_s - F_c}{2K_p} \quad T_{lc} \approx \frac{4K_p}{K_i} \frac{F_s + F_c}{F_s - F_c} \tag{11}$$

hence some second gains may be obtained for the gain scheduling controller.

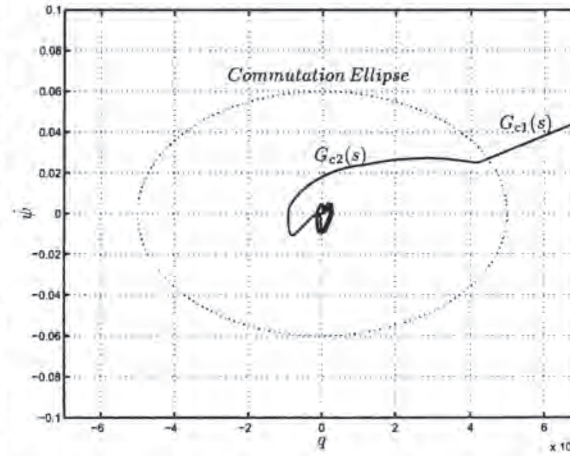

Fig. 3. Commutation ellipse between controllers gains

3.2 *Velocity control*

Given the required specification and the dynamics of the gyro measure, the controller cannot be implemented in a single loop form. This is due to the high controller gain required in order to reject friction disturbances which produces an unstable closed loop when the gyro dynamics is introduced. In order to solve this problem, a

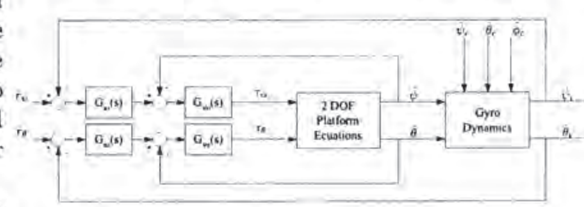

Fig. 4. Cascade control structure

master-slave structure of controllers is proposed in figure 4. An inner velocity loop is created with the measure of the tachometer and another outer one is closed with the measure of the gyro. With this structure the nonlinear disturbances of the plant are rejected by the inner loop, as long as the outer loop rejects the disturbances created by the platform base motion.

The inner loop must be designed in such way that it has a faster behavior than the external loop, therefore the controllers can be independently designed. A PI controller is designed for the inner loop, where the controller is given by the equations:

$$K_p = \frac{2\delta\omega_o\tau - 1}{K} \tag{12}$$

$$K_i = \frac{\omega_o^2 \tau}{K} \tag{13}$$

where ω_o and δ are respectively the natural frequency and the damping factor of the closed loop created by the inner loop. K and τ are respectively the gain and the time constant of the linear part of the system.

Due to the nonlinear influence of the actuator saturation, the system has an undesirable overshoot when large reference changes are given. Hence an anti-windup strategy is added to the controller as show in figure 5.

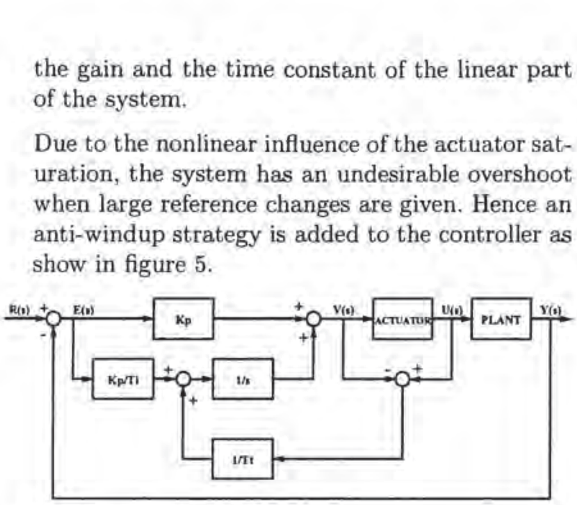

Fig. 5. Anti-windup strategy

A gain scheduling controller has been implemented for the inertial velocity outer loop (Aström and Wittenmark, 1989), this way it is possible to fulfill the specifications in several work zones. The margin phase for a PI controller associated to the proportional and integral parameters (K_{pi}, K_{ii}) is shown in figure 6. The limit stability can be symbolically computed, the limit is defined by the equations:

$$0 = \left(\omega_g^2 a + K_p \omega_g^2 K\right) - \\ - \frac{\alpha^2 K \omega_g^2 K_i}{\left(\alpha\left(\omega_g^2 + 2\delta_g \omega_g a\right) - \omega_g^2 a - K_p \omega_g^2 K\right)} \\ \alpha = 2\delta_g \omega_g + a \tag{14}$$

The operational region of the controller has

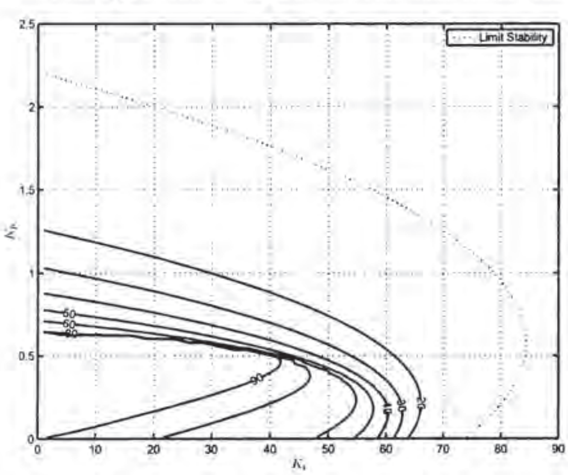

Fig. 6. Margin phase of the system according to the selected controller gains K_p y K_i

been classified according to the magnitude of the setpoint and the velocity of the shafts. The system must basically operate in two regions, a high reference value mode and another for a small or zero reference. In the second mode, the platform must move slowly in order to compensate the motion of the base. In this mode the integral gain is modified according to the velocity axis. This produces an improvement of stabilization. The operation regions of the outer loop are shown in figure 7.

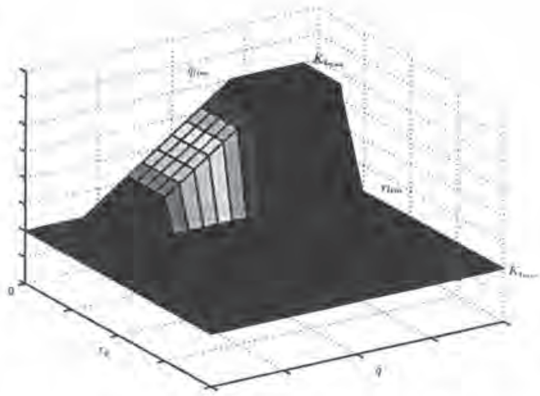

Fig. 7. Adaptation for the K_i parameter in the external loop

4. COMPUTER SIMULATIONS

The computer simulations of the controller have been implemented with the whole model from the equations (2). A static and asymmetric friction model is used for the position and velocity controllers simulations.

In order to accomplish the simulation a parameter set has been used from the following table:

Parameter	Azimut	Elevation	Ud.
Izz	16	13	kg/cm^2
Ixx	25	-	kg/cm^2
Iyy	20	-	kg/cm^2
F_v^+	61	30	$Nm/(rad/s)$
F_v^-	39	59	$Nm/(rad/s)$
F_{s1}^+	6	30	Nm
F_{s1}^-	52	60	Nm
F_{c1}^+	5	15	Nm
F_{c1}^-	50	31	Nm
R	108	202	
K_τ	1.3350	0.81	Nm/V

Figures 8 and 9 show the step responses in the orientation and elevation axis. When a large setpoint change is produced, the controller produces a saturated signal control and the system is accelerated and decelerated quickly. Next the second controller corrects the steady state and the system is situated in a limit cycle that fulfills the required precision.

Figures 10 and 11 show how the inertial velocity controller works when a reference equal to zero is given. In this simulation the disturbances created by the motion the of the platform base have been included. The signals used are the following:

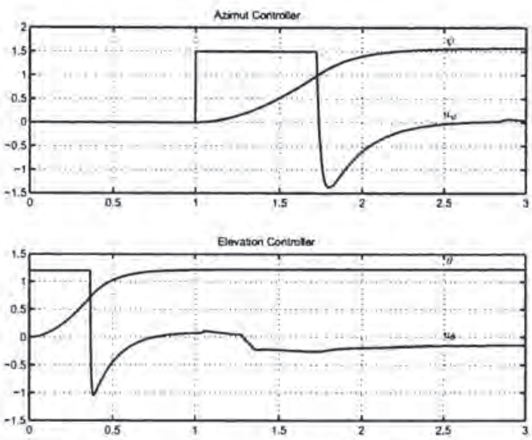

Fig. 8. Step responses of the controlled system (rads.)

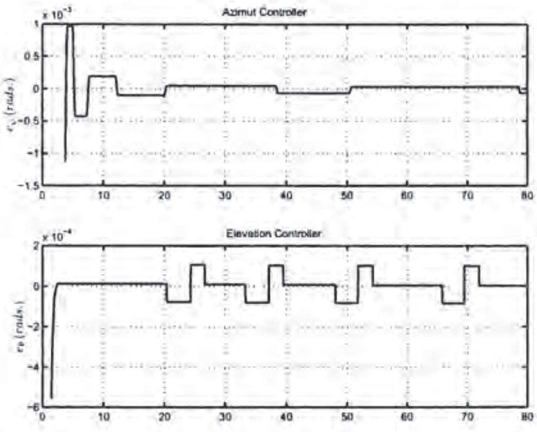

Fig. 9. Limit cycles of the controlled system

$$\psi_c = \phi_c = \theta_c = 0.083sin(0.08t - \frac{\pi}{2})$$
$$\dot{\psi}_c = \dot{\phi}_c = \dot{\theta}_c = 0.007cos(0.08t - \frac{\pi}{2}) \qquad (15)$$

Some small disturbances have been chosen, hence the platform must perform motion at low velocities where the friction is more relevant. Figure 10 shows how the inertial velocity remains close to zero, only some velocity peaks can be observed when the motor velocity has a zero velocity transition. At this moment, the motor shaft is stopped due to the friction torque and the disturbances cannot be rejected.

In figure 11 the inertial position of the platform when the reference is zero velocity is presented. The position oscillates due to the imperfect cancellation of the disturbances. But with gain scheduling strategy, the amplitude of the oscillation can be bounded and the specifications are fulfilled.

The inertial velocity controller has been compared with a friction compensation based controller (Altpeter, 1999), without the cascade structure. In this case, a LuGre model is used for the plant and a static model is used for the compensation. The LuGre friction model (C. Canudas de Wit and

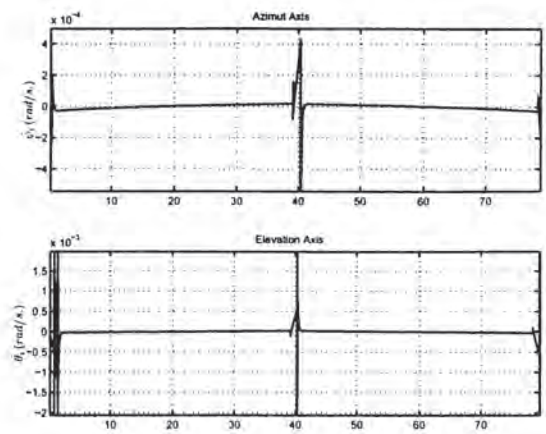

Fig. 10. Inertial velocity response for external disturbances and $r = 0$

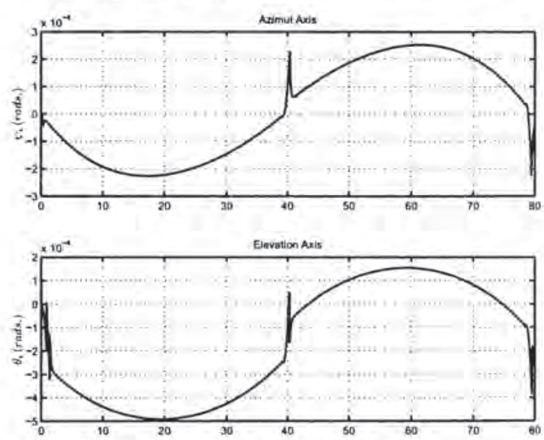

Fig. 11. Inertial position response for external disturbances and $r = 0$

Lischinscky, March, 1995) has been implemented in an asymmetric form as the static model.

$$\begin{aligned}
\frac{dz}{dt} &= \dot{q} - \frac{\sigma_0}{g(\dot{q})} z|\dot{q}| \\
g(\dot{q}) &= \left[F_c^+ + (F_s^+ - F_c^+)e^{-B_1|\dot{q}|}\right] sgn^+(\dot{q}) + \\
&\quad + \left[F_c^- + (F_s^- - F_c^-)e^{-B_2|\dot{q}|}\right] sgn^-(\dot{q}) \\
\tau_f &= \sigma_0 z + \sigma_1 \frac{dz}{dt} + \\
&\quad + \left(F_v^+ \dot{q}\right) sgn^+(\dot{q}) + \left(F_v^- \dot{q}\right) sgn^-(\dot{q}) \qquad (16)
\end{aligned}$$

where:

- z: Average deflection for the bristle friction model.
- σ_0: Stiffness of the material .
- σ_1: Damping coefficient.

The two different models allow us to test the friction compensation when there are unmodelled dynamics. Similar results are obtained using both control strategies. The cascade control has inherent benefits, the system stability is not compromised and the control shows better robust properties when model deviation occurs.

5. CONCLUSIONS

This paper presents a inertial controller for a two degree of freedom platform. The mechanical model corresponds to a large dimension platform where a strict operation specification are required where a gyro sensor has been added.

Some gain scheduling algorithms have been designed for position control of the motors and the inertial velocities of the platform. These controllers have shown an acceptable performance, achieving all the required specification.

The contents reveal how the control of inertial velocities of the platform can be solved by simple control structures, despite the fact that the system has important nonlinear elements. Therefore this fact produces more robust and reliable controllers.

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