Determining routes for an auxiliary fleet in presence of emergencies on a railway line

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Abstract. Transportation on-demand represents a mode of transport in which a set of users formulates transport requests between pickup and delivery points, which must be served by means of vehicles of a given capacity. When the items to be transported are people, it is necessary to simultaneously consider a minimization of operating costs (perspective of transportation systems managers) and a reduction of the inconveniences for the user. An unexpected failure of the planned scheduling in the transport system (e.g. a failure in the engine of a train), may cause a service disruption to a large number of passengers who would remain isolated at origin stations. The result of that situation would be a scattered distributed population in points, with heterogeneous demand in terms of destinations and specific requirements of service. In this paper a model of transportation on-demand is formulated to address an incidence which happens in a transit line (metro, trains), forcing to reschedule the service in such transit line. The goal is to minimize the total cost associated to the delay experienced by users who have to be served by an auxiliary fleet to attend this incidence.

Keywords: Railways, disruptions, rescheduling.

1 Introduction

The management of interruptions and delays is a crucial task in the control of operations for any public transport company. In the setting of rapid transit systems like metro and local trains, an unexpected failure may leave without service to a large number of isolated users at their origin stations. Moreover, the lack of information about this incidence can lead to maintaining a regular rate of user arrivals to the stations and thickening the list of unattended users. Providing a protocol in case of contingencies in the system is a topic of interest profusely recognized in the specialized literature. The high cost involved in the implementation of an emergency system is of particular interest, so efficiency criteria must be considered in the design. In that sense, mathematical models and optimization methods are essential tools for decision-making, providing advice to minimize the negative consequences caused, such as the overall delay of the users and the economic impact to the system.

In metro and suburban systems, a widespread practice to address the occurrence of incidents or major disruptions (temporarily extended) is to establish special bus services that operate in the affected sections (Ortega et al., 2008). This technique is necessary when it is impossible to absorb the impact within the same network or with complementary supporting networks (Cepeda et al., 2006). However, despite its undoubted interest for operators, the analytical perspective of such problem has not received sufficient attention in the literature (Arriola et al., 2009).

The provision of an auxiliary fleet with several vehicles of different capacities can attend to a heterogeneous transport demand between different origins and destinations. Moreover, the efficiency of the auxiliary routes can be increased by including express services, omitting stops when a vehicle is running empty (deadheading), or making that a vehicle changes its travel direction before reaching its terminal station (short-turning). The simultaneous application of these strategies along of the itinerary of a transit line results in asymmetric routes forming short cycles (Tirachini et al., 2007) with the possible inclusion of intermediate express services (Mesa et al., 2009). Figure 1 shows an example of combined strategies, where the white dots indicate omitted stops along the line run.

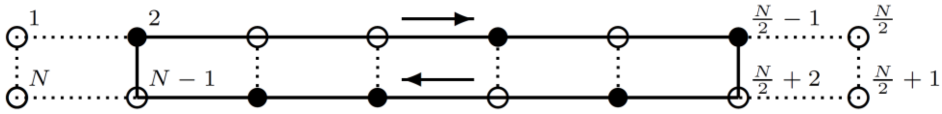

Figure 1: Asymmetric short cycle with express services in a transit line

In this work, an adaptation of the transport on-demand (TOD), known as "Pickup and Delivery" problems (PDP), is developed to manage disruptions in the service within a public transport system.

2 Pickup and delivery problems in the on-demand transport

PDPs (Desaulniers et al, 2002; Berbeglia et al, 2007) are a type of problems which consists of determining vehicle routes, where a set of objects or people have to be transported between certain origin-destination pairs. PDPs have been specially studied in the last years and are mainly related to logistics and the transportation on-demand for passengers (Cordeau, 2006; Melachrinoudis et al., 2007; Jørguensen et al., 2007).

The main difference, compared to other PDPs applied to transport of objects, is that in the TOD the minimization of operational costs must share in importance with reducing user inconveniences. Therefore, under conflicting objectives (e.g., to maximize the number of total served requests versus to minimize the operational costs), the human perspective (minimize user's discomfort) heavily weighs on the model, either in the combined formulation of the global objective or in their importance in the list of restrictions on the feasibility of solutions. One of the priorities in the quality of service claimed by the user is to restrict the time interval in which each point of picking up must be visited. Moreover, in front of what happens in many PDPs associated with the transport of objects (such as small mail volumes), the vehicle capacity is a priority when it comes to transport passengers who travel with their luggage items.

A well-known example of TOD is the public transport on demand in rural areas, whose profile of potential passengers are elderly population, disabled people or children students, and where the implementation of a fixed transit line of public character is not feasible or expensive. However, the economic and social interest of TOD can be not limited to troubleshoot service to small communities or passengers with special requirements. An appropriate clustering of passengers in groups (Dumas et al., 1989; Ioachim et al., 1995) and the incorporation of transfer nodes in the model (Mitrović-Minić and Laporte, 2006; Cortés et al., 2009) allow to transform the problem into a real case of planning a mass transit system.

Although there have been proposed various exact methods for solving TOD problems with multiple vehicles (see Berbeglia et al., 2007), however, the high complexity of the problem cannot be avoided, and the solution must be limited to small-scaled scenarios. This has led to the need of providing efficient algorithms to solve large-scaled instances. The practical management of on-demand transportation involves decision making in 3 areas: clustering of requests, routing of vehicles and scheduling of line-runs. Clustering requests is devoted to create groups of petitions within a station, to be served by the same vehicle. The grouping is made according to criteria of pickup time and proximity to the destinations. Once formed these clusters, the routing stage consists of deciding in what order the corresponding nodes will be visited for the available vehicles. Finally, the scheduling phase will determine the exact times to visit each node. Each of these phases is strongly linked to the other, so that a good solution will require a simultaneous minimization of them. Then a formulation is proposed to determine the optimal service required to meet, with vehicles of different size and origin, a set of requests where the usual transport service that runs along a transit line has been cancelled.

3 Formulation

Let $G = (V, A)$ be a directed graph where the vertex set is composed of three subsets $V = O \cup P \cup D$. Set $O = \{0, 2n+1\}$ represents nodes associated to initial and final depots, $P = \{1, \ldots, n\}$ is the node set of pickup points of the *n* requests and $D = \{n+1, \ldots, 2n\}$ is the node set of destination points of those requests.

A point of a pickup request $i \in P\ (i \leq n)$ will have associated another node of delivery request, labeled by $n+i$. Requests from N users are concentrated in stations of set S that collect groups of q_i passengers wishing to travel to other stations of S. The affected line run had scheduled a timetable given in advance; hence, we denote $\overline{u_i}$ the desired time in which the request *i* should complete their journey. We denote as follows the following indexes and parameters:

k	identifier of available vehicle for attending the emergency $(k \in K)$
t_{ij}^k	travel time of vehicle *k* between nodes *i* and *j*. If travel times, incurred when vehicles move from node *i* to node *j*, are not depending on train type then $t_{ij}^k \equiv t_{ij}$.
a_i^k	time of service / waiting required by the vehicle *k* at node *i*
q_i	weight / load associated to the node *i*
Q_k	maximum load allowable for vehicle *k*
e_i	earliest time for picking request at node *i*
l_i	latest time for picking request at node *i*
L	maximum allowable time for a request delay
c_f^k	fixed cost for renting vehicle *k*
c_v^k	fixed cost of transport associated to vehicle *k* per unit of time
c_r	fixed cost per unit of time due to delay in response to a request
c_d	fixed cost of compensation for each passenger due not attend their request

Additionally, the model uses a set of binary variables to establish the routing throughout edges in the graph that are used by each vehicle, and two sets of variables to respectively control the starting time of service and the load on each vertex:

x_{ij}^k	binary variable which equals 1 only if the vehicle *k* crosses the edge (i, j)
u_i^k	time when vehicle *k* begins its service at vertex *i*
w_i^k	load of vehicle *k* when goes out from vertex *i*

The measures taken in front of incidents can be of two types:
- Use a reserve fleet of vehicles similar to which suffered the incidence (new train units) or different (buses, taxis).
- Renounce the use of rolling stock and proceed to compensate passengers affected by the incident.

In this paper, both strategies are presented in the same model to evaluate the effectiveness of possible measures to be implemented.

There are cost- and passenger-oriented components in all models for the line planning problem. In our model, cost-oriented components are the fixed cost for renting vehicle and the fixed cost of transport associated to vehicle k per unit of time. On the other hand, passenger-oriented costs are the fixed cost per unit of time due to delay in response to a request and the fixed cost of compensation for each passenger

due to not attend their request. A calibration of these parameters would be required for a better adjust of the results.
The proposed goal is to minimize the costs derived from using the available fleet (rent and transportation costs), assumed by the system operator, and simultaneously, to minimize the inconvenience costs to the user (formulated in terms of delay cost and compensation). This short list of criteria is additively expressed by means of:

$$\text{Minimize} \quad \sum_{k \in K} (c_f^k + (u_{2n+1}^k - u_0^k) c_v^k) \sum_{j \in V} x_{0j}^k + c_r \sum_{i \in P} q_i \sum_{k \in K} \sum_{j \in V} (u_{n+i}^k - \overline{u_i}) x_{n+i\,j}^k + c_d \sum_{i \in P} q_i \, (1 - \sum_{k \in K} \sum_{j \in V} x_{n+i\,j}^k)$$

Subject to constraints:

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k \leq 1; \quad i \in P \cup D \tag{1}$$

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k \leq 1; \quad j \in P \cup D \tag{2}$$

$$\sum_{i \in P} x_{0i}^k = \sum_{i \in D} x_{i\,2n+1}^k \leq 1; \quad k \in K \tag{3}$$

$$\sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{n+i\,j}^k = 0; \quad i \in P, k \in K \tag{4}$$

$$\sum_{j \in V} x_{ji}^k - \sum_{j \in V} x_{ij}^k = 0; \quad i \in P \cup D, k \in K \tag{5}$$

$$u_j^k \geq \left(u_i^k + a_i^k + t_{ij}^k\right) x_{ij}^k; \quad i \in V, j \in V, k \in K \tag{6}$$

$$e_i \sum_j x_{ij}^k \leq u_i^k \sum_j x_{ij}^k \leq l_i \sum_j x_{ij}^k; \quad i \in P, k \in K \tag{7}$$

$$\sum_{k \in K} \sum_{j \in P \cup D} u_{i+n}^k \, x_{i+n\,j}^k - \overline{u_i} \leq L; \quad i \in P \tag{8}$$

$$w_j^k \geq \left(w_i^k + q_j\right) x_{ij}^k; \quad i \in V, j \in V, k \in K \tag{9}$$

$$\max\{0, q_i\} \leq w_i^k \leq \min\{Q_k, Q_k + q_i\}; \quad i \in V, k \in K \tag{10}$$

$$x_{ij}^k \in \{0,1\}; \quad i \in V, j \in V, k \in K \tag{11}$$

$$u_i^k \; , \; w_i^k \geq 0; \quad i \in V, \; k \in K \tag{12}$$

Constraints (1) - (5) force the feasibility for the considered routes (constraint block of routing), while restrictions (6) - (8) control the feasibility of that service in a temporal context (constraint block of scheduling). Constraints (9) and (10) require that the load restrictions that were established on vehicles cannot be violated.

If we analyze the provided model we can see that constraints (1) - (5), together with restriction (11), correspond to a classical Vehicle Routing Problem (VRP). The inclusion of restrictions (9) and (10) leads to a capacitated version of the VRP. Finally, once restrictions (6) - (8) are added to the formulation, the model leads to a variant of the Capacitated Vehicle Routing Problem with Time Windows, where linearity property has disappeared, because binary variables of routing appear multiplying to other variables that control the time of service and the vehicle capacity in both objective function and constraints (6) - (9). Since the resulting optimization model is nonlinear and it is based on a set of discrete independent variables, the problem is NP-hard and, to solve such inherently intractable problem on large-size real instances, some algorithm must be specifically developed for that purpose.

Specifically, the algorithm provided in Mesa et al. (2014) allows to accumulate at each node the penalization costs by delay and compensation due to not having attended with the corresponding pickup request. In this way, the number of requests $|P|$ can be reduced by considering clusters of request which share a common preferred departure time. Let *P'* be the new set of clustered requests that are identified by means of index *j*, weighted by factor q_j and that have associated a preferred interval for the departure time $\overline{u_j} \in [e_j, l_j]$.

The following routine describes how set *P'* of clustered requests can be constructed:

- For each individual request $i \in P$, we must check if there already exists a clustered request $j \in P'$ such that $\overline{u_i} \in [e_j, l_j]$, $u_{i+n}^k - \overline{u_i} \leq L$ and, additionally, inequalities $q_j - q_i \leq Q_k$ are held for $k \in K' \subseteq K$. In that case, request $i \in P$ is aggregated to clustered request $j \in P'$; otherwise, a new cluster of requests is initialized starting from item $i \in P$.

Next, in order to heuristically solve our model, we apply the following procedure:

1. Solve the model (1)-(12) for variables x_{ij}^k (the routing sub-problem) by considering the set of clustered requests *P'* instead of *P*. Let $\underline{x} \equiv (\underline{x}_{ij}^k)$ be the returned solutions.

2. Solve the model (1)-(12) for variables w_i^k and u_i^k (the scheduling sub-problem with capacities) by considering the initial set of requests *P* and maintaining variables x_{ij}^k with fixed values obtained from the previous step. Let $\underline{u} \equiv (\underline{u}_i^k)$ and $\underline{w} \equiv (\underline{w}_i^k)$ be the sets of solutions that determine the timetables of line runs.

3. Solve the model (1)-(12) for variables x_{ij}^k when variables w_i^k and u_i^k are fixed, taking values equal to $\underline{w}_i^k$ and $\underline{u}_i^k$, respectively. Let $\underline{x} \equiv (\underline{x}_{ij}^k)$ be the returned set of solutions of line runs.
4. Repeat steps 2-3 until the number of iterations reaches a pre-defined number.

4 Model application

Suppose a transit line running between four railway stations. There are six origin-destination (OD) pairs to distribute the trips corresponding to the 300 users of this line. In this scenario, assume that train V1 of Figure 2 (left polygonal) suffers an incident and cannot operate normally, leaving a set of users without transport service along the line. The above model was programmed in GAMS and, given the small size of the experiment, the computation time has been considered as not relevant.

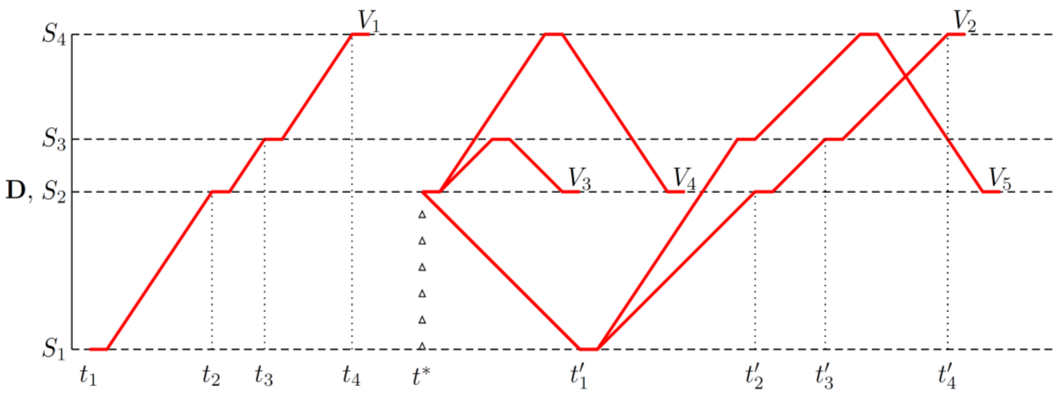

Figure 2: Different options of scheduling in response to an incidence on train.

The remaining polygonal lines of Figure 2 show diagrams of possible timetables to be implemented by supporting decisions that attend the incidence suffered by train V1 by means of vehicles located at depot S2. The illustrated actions correspond to the strategies outlined in Section 1. Train V2 repeats the itinerary of the cancelled line run V1. Vehicle 3 (V3) stops at stations S2 and S3 and makes a *short-turning* before returning to depot S2. Train V4 makes an *express service* between stations S2 and S4 before returning to the depot. Vehicle 5 runs in mode *deadheading* between stations S1 and S2, and covers trips between group of stations S1-S3, S1-S4, S1-S3-S4 and S1-S2.

Table 1 summarizes the results obtained in different scenarios determined by the available number (K) of rescue vehicles without load restrictions and by different values assigned to the four types of costs considered.

Table 1: Optimum results in a context of absence of load restrictions.

c_v^k	c_f^k	c_v^k	c_d	K	Rent	Delay	Comp.	Users	Obj.	Trains
70	400	1	30	1	1800	3000	0	300	4800	V2
40	400	1.15	30	2	2160	2100	0	300	4260	V2, V3+V4
40	400	1.25	30	2	2310	2310	0	300	4620	V2, V3+V4
20	300	1.35	20	2	1990	2835	0	300	4825	V2, V3+V4

In Table 2, the optimum results, achieved when there is a restriction of maximum capacity of 60 users per vehicle, are exposed. In contrast to the previous scenario, it is possible to obtain an optimal result in our model whose answer is "not serving and to indemnify", when the limited capacity of the vehicles forces to do many and costly shipments (case $c_v^k = 70)$. In the opposite side, when the shipping cost is low (case $c_v^k = 20)$, it is also possible to recommend that destinations are served by means of express services, taking the shortest possible travel times (Figure 3).

Table 2: Optimum results in a context that includes load restrictions

c_v^k	c_f^k	c_v^k	c_d	K	Rent	Delay	Comp.	Users	Obj.	Trains
70	400	1	30	1	0	0	9000	0	9000	Not serving
40	400	1.15	30	1	1050	1121	6000	100	8171	V3+V4
40	400	1.25	30	2	1640	760	4500	150	6900	V2, V3+V4
20	300	1.35	20	3	2237	3225	0	300	5462	see Figure 3

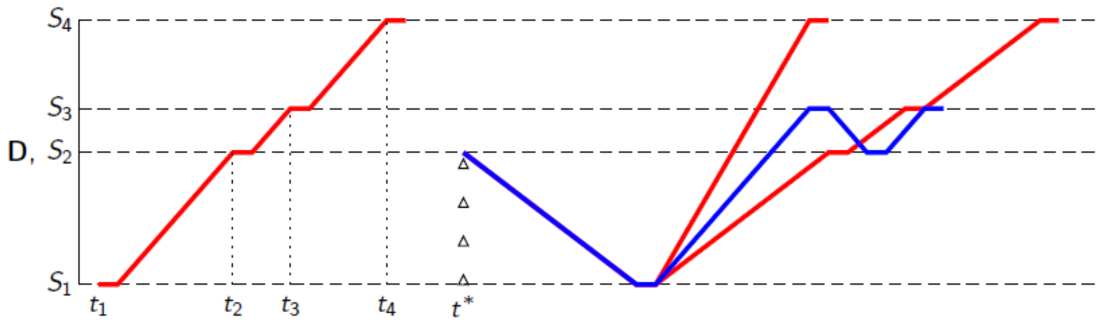

Figure 3: Optimum vehicle scheduling in response to failure of train V1 for scenario of last row in Table 2

5 Conclusion

In this work, an optimization model has been formulated to solve a routing problem in case of an emergency situation caused by the failure of a train that runs along a line. The response action is based on the availability of support vehicles with different capacities and different costs previously established. The proposed model has been applied to a simple case, in order to make a sensitivity analysis for the parameters, providing information on the total operator costs and on the number of served users. The size of the experiment has allowed to obtaining exact solutions, showing the strong dependence of the results with respect to the parameter values, an issue of particular interest in the evaluation of real contexts. In case of large-size real scenarios, heuristic procedures may be required for circumventing the undoubted computational complexity of the model.

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