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Shape phase transitions in odd-A nuclei

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Abstract.
We investigate shape phase transitions in odd nuclei within the Interacting Boson Fermion Model. Special attention is given to the case of the transition from the vibrational behaviour to the stable axial deformation. The odd particle is assumed to be moving in the three single particle orbitals j=1/2,3/2,5/2 with a boson-fermion Hamiltonian that leads to the occurrence of the $SU^{BF}(3)$ boson-fermion symmetry when the boson part approaches the $SU(3)$ condition. Both energy spectra and electromagnetic transitions show characteristic patterns similar to those displayed by the even nuclei at the corresponding critical point. The role of the additional particle in characterizing the properties of the critical points in finite quantal systems is investigated by resorting to the formalism based on the intrinsic frame.

Keywords: Shape phase transition. Interacting Boson Fermion Model. Critical point symmetries.
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Introduction

The study of shape phase transitions in finite nuclear quantal systems has recently been the subject of many investigations. Most of the work has been carried out for even-even nuclei, using either the Bohr Hamiltonian and the surface collective variables or algebraic approaches based on the use of interacting bosons. In the case of odd-even nuclei, where an odd particle is coupled to an even core undergoing a phase transition, attention has been put on the shape transition from sphericity to deformed gamma-instability. In correspondence to the critical point in the even core, characterized by the critical point symmetry E(5), two new boson-fermion critical point symmetries have been proposed, in the case of an odd particle moving in a single j=3/2 shell (E(5/4) symmetry [1]) or in the j=1/2,3/2,5/2 shells (E(5/12) symmetry [2]). Characteristic sequences of levels and ratios of electromagnetic transitions are predicted in both cases.

We consider here another leg of the Casten shape triangle, namely the transition from the spherical vibrational behaviour to stable axial deformation. This transition in the even-even case is normally named after the critical point symmetry X(5) [3]. We will couple along the transition the even-even core to an odd particle moving in the j=1/2,3/2,5/2 single-particle shells. This situation will be described within the framework of the IBFM (Interacting Boson Fermion Model), choosing a boson-fermion Hamiltonian that leads to the occurrence of the $SU^{BF}(3)$ boson-fermion symmetry when the boson part approaches the $SU(3)$ condition.

Our aim is twofold. From one side one we would like to show that spectra and transitions in the neighbour odd nuclei display characteristic features at the phase transition

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and offer therefore additional clear signatures of the phase transition. Our second point deals instead with the concept itself of critical point in finite quantal systems. For well deformed cases the contribution of the coupling to the additional odd particle (of the order of 1/N) does not change appreciably the position of the sharp minumum in the deformation parameter for the energy surface. At the critical point, however, the energy surface in the even core is known to be rather flat in the deformation parameter, a feature that precisely characterizes the transition point. This behaviour may imply that some of the energy surfaces characterizing the different odd nucleus states may be driven by the coupling in either direction (i.e. towards axial deformation, sphericity or triaxiality), effectively changing the position of the critical point.

The model

We will consider the spherical to deformed axially symmetric transition in the framework of the *IBFM*, when the odd particle can sit in a triplet of orbitals, namely $j = 1/2, 3/2$ and $5/2$. For this purpose we have chosen a Hamiltonian based on the one used for the corresponding $U(5)$ to $SU(3)$ transition in the *IBM* [4]

$$H^B = (1-x)n_d - \frac{x}{4N_B} Q_B \cdot Q_B \ , \tag{1}$$

where $Q_B = (s^\dagger \times \tilde{d})^{(2)} + (d^\dagger \times \tilde{s})^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger \times \tilde{d})^{(2)}$ is the quadrupole operator. The parameter x is a control parameter that drives the behaviour of the system. The Hamiltonian H^B can be written in terms of Casimir operators as

$$H^B = (1-x)\,\mathscr{C}_1(U^B5) - \frac{x}{8N_B}\left[\frac{3}{2}\,\mathscr{C}_2(SU^B3) - \frac{3}{8}\,\mathscr{C}_2(O^B3)\right] . \tag{2}$$

We will use a similar form for the Hamiltonian that describes the transition from spherical to axially deformed odd-even nuclei with *IBFM*, replacing the bosonic algebras with the corresponding Bose-Fermi algebras, i.e.

$$H^{BF} = (1-x)\,\mathscr{C}_1(U^{BF}5) - \frac{x}{8N_B}\left[\frac{3}{2}\,\mathscr{C}_2(SU^{BF}3) - \frac{3}{8}\,\mathscr{C}_2(O^{BF}3)\right] . \tag{3}$$

In order to get physical insight into the problem, we rewrite the boson-fermion Hamiltonian (3) as,

$$H^{BF} = (1-x)\left(n_d + n_{\frac{3}{2}} + n_{\frac{5}{2}}\right) - \frac{x}{4N_B} Q_{BF} \cdot Q_{BF} \ , \tag{4}$$

where $Q_{BF} = Q_B + q_F$ is the total (boson-fermion) quadrupole operator. Note that the choice of the fermion space is such that one can profitably visualize the three angular momenta as arising from the coupling of a pseudo spin 1/2 with a pseudo angular momentum 0 or 2. Since the hamiltonian does not depend on the pseudo spin, this gives rise to repeated level doublets. This degeneracy might eventually be broken by introducing a term proportional to J^2 in the Hamiltonian.

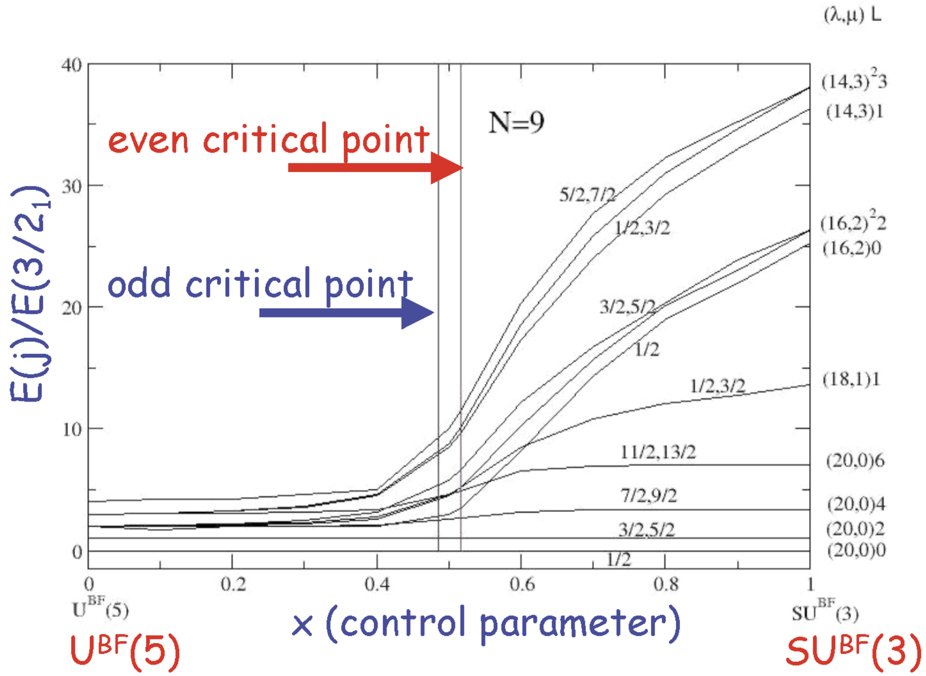

FIGURE 1. Energy levels (normalized to the energy of the first excited state) as a function of the control parameter x in the Hamiltonian (3). A number $N_B = 9$ of bosons has been assumed. Each state is characterized by the (λ, μ) asymptotic quantum numbers (strictly valid only for $x = 1$, at the $SU^{BF}(3)$ extreme) and L. The two vertical lines indicate the position of the critical value of x for the even-even and odd-even systems.

With our choice of the Hamiltonian we obtain for $x = 0$ the $U^{BF}(5)$ dynamic symmetry and for $x = 1$ the $SU^{BF}(3)$ one. In the latter case the levels are arranged in rotational bands, characterized by the quantum numbers (λ, μ) and K. Some of them can be identified with those of the even case with a number N_B+1 bosons, but others (not fully symmetric with μ odd) arise genuinely from the fermion-boson nature of the problem. In addition to the degeneracy coming from the pseudo spin coupling, we have degeneracies coming from the bosonic plus fermionic orbital parts.

Some of the degeneracies disappear outside the $SU^{BF}(3)$ extreme of the transition. This transition from $SU^{BF}(3)$ towards $U^{BF}(5)$ is more complicated than the one studied before [5] from $O^{BF}(6)$ to $U^{BF}(5)$. In that case there was a common subgroup, $O^{BF}(5)$, which supplied labels all over the transition. This is not the case any longer, and we can only use “asymptotic" quantum numbers (λ, μ) to label the states outside the extremes of the transition. One could invoke the concept of quasi dynamical symmetry to have a criterion on how to assign the asymptotic quantum numbers. Similarly the arrangement of the levels into bands is not always unique and the information from the electromagnetic matrix elements may be not always sufficient.

In fig. 1 we show the full evolution of the energy of some selected levels, normalized to the energy of the first excited state, along the transition, in the case of $N_B = 9$. The states are labelled all over with the $SU(3)$ quantum numbers (λ,μ) and L, which are strictly valid only for $x = 1$. The total angular momenta come from the coupling of the $O^{BF}(3)$ quantum number L to the $SU_s^F(2)$ quantum number S. In general the states are doubly degenerate except for $L = 0$. The general behaviour of the energy levels is rather smooth close to the dynamic symmetry limits, changing more rapidly in the neighbourhood of the critical point for the even core, indicated by a vertical line. It is clear from the figure that by using only energies (and transition rates) is rather difficult to single out the precise position of the critical point. This can be done by resorting in the next section to the concepts of intrinsic frame and energy surfaces.

The intrinsic frame description and the critical point

A usuful way of looking at phase transitions is to resort to the concept of intrinsic states and associated energy surfaces. In the case of the Interacting Boson Model for even nuclei one introduces a ground state intrinsic state of the form

$$\Phi_{gs}(\beta,\gamma) = \frac{1}{\sqrt{N_B!}} (b_{gs}^\dagger(\beta,\gamma))^{N_B} |0\rangle \tag{5}$$

where the basic boson creation operator is given in the form

$$b_{gs}^\dagger(\beta,\gamma) = \frac{1}{\sqrt{1+\beta^2}} \left(s^\dagger + \beta \cos\gamma d_0^\dagger + \frac{\beta}{\sqrt{2}} \sin\gamma (d_2^\dagger + d_{-2}^\dagger) \right) \tag{6}$$

and β and γ play a role similar to the intrinsic collective variables in the Bohr Hamiltonian. The ground state energy surface is obtained as expectation value of the boson Hamiltonian (1) in the intrinsic state, i.e. $E_{gs}(\beta,\gamma) = \langle\Phi_{gs}(\beta,\gamma)|H_B|\Phi_{gs}(\beta,\gamma)\rangle$. In our specific case, for any value of the control parameter x the energy surface has a minimum, as a function of the parameter γ, for $\gamma = 0$. In other words, our boson hamiltonian can never lead to a stable triaxial shape. As far as the β dependence is concerned, for small values of x the system finds more convenient a spherical shape (β_{min}=0), while after the critical point the second (and deformed) minimum becomes lower in energy. This first-order transition takes place at $x = 16N_B/(34N_B - 27)$, i.e. for x=0.516 in the case of $N_B = 9$ bosons.

The corresponding intrinsic frame states and energy surfaces for odd systems can be constructed by coupling the odd single-particle states (with each angular momentum j and magnetic component k) to the intrinsic states of the even core. The lowest odd states are expected to originate from the coupling to the intrinsic ground-state $\Phi_{gs}(\beta,\gamma)$. One first constructs the coupled states

$$\Psi_{jk}(\beta,\gamma) = \Phi_{gs}(\beta,\gamma) \otimes |jk\rangle \tag{7}$$

and diagonalize in this basis (for each value of β and γ) the total boson-fermion Hamiltonian, giving a set of energy eigenvalues $E_n(\beta,\gamma)$, n being a running index to count

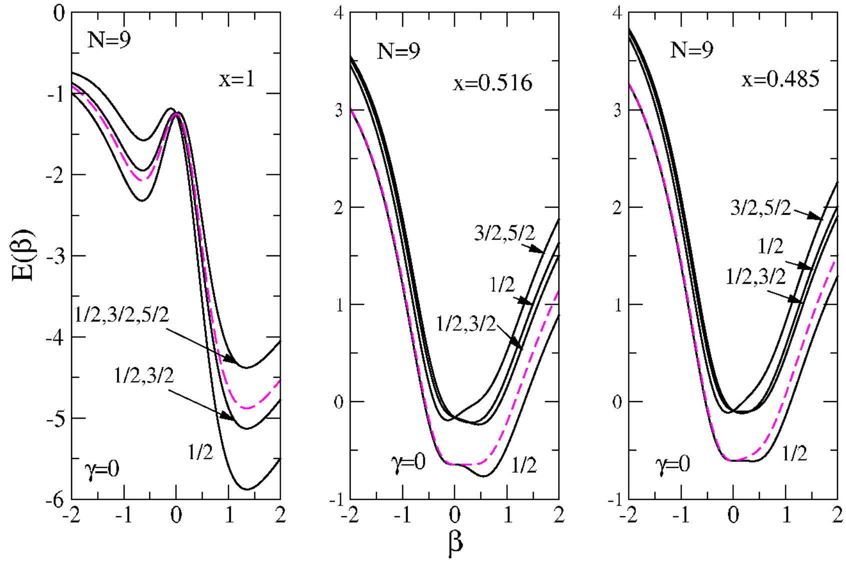

FIGURE 2. Energy surfaces for odd systems. The left frame refers to the case of x=1 ($SU^{BF}(3)$ dynamical symmetry). The central frame has been obtained for x=0.516, corresponding to the critical value of the even core, while the right frame correspond to the "effective" odd critical point, i.e. x=0.485. Dashed lines give the ground state energy surface for the even core.

the solutions. In our specific $U^F(12)$ algebra we have a total of 12 components, but restricted to 6 because of the symmetry $k \leftrightarrow -k$. In addition, since the Hamiltonian does not depend on the pseudo spin, the active role is played by the orbital angular momenta $l = 0$ (with $k_l = 0$) and $l = 2$ (with $k_l = 0, \mp 1, \mp 2$). We expect therefore for the general Hamiltonian (4) four different intrinsic states and consequently four different bands. For γ=0 the hamiltonian preserves the quantum number k and the diagonalization is independently done for each value of k. In this case we obtain two states with $k = 1/2$, one degenerate pair with $k = 1/2, 3/2$ and the last degenerate pair with $k = 3/2, 5/2$.

We first show in the left frame of Fig. 2 the boson-fermion energy surfaces for a well deformed case ($x = 1$, leading to the $SU^{BF}(3)$ dynamical symmetry). For a better comparison we also include, as dashed line, the energy surface corresponding to the core ground state. We can see that all energy surfaces display the minimum for the same value of the deformation parameter β. In this well deformed case, therefore, the addition of the extra particle is not changing the features of the system and each odd state can be put in correspondence with one of the rotational bands appearing in the spectrum. The situation is different around the critical point of the even core (central frame). We see from the figure that in this case the odd particle drives the system towards larger or smaller deformation (according to the different odd states). This gives rise to an effective shift of the critical point which is different for each given state. For example, for the ground band, the critical point moves to x=0.485, where in fact the corresponding energy surface becomes flat (right frame).

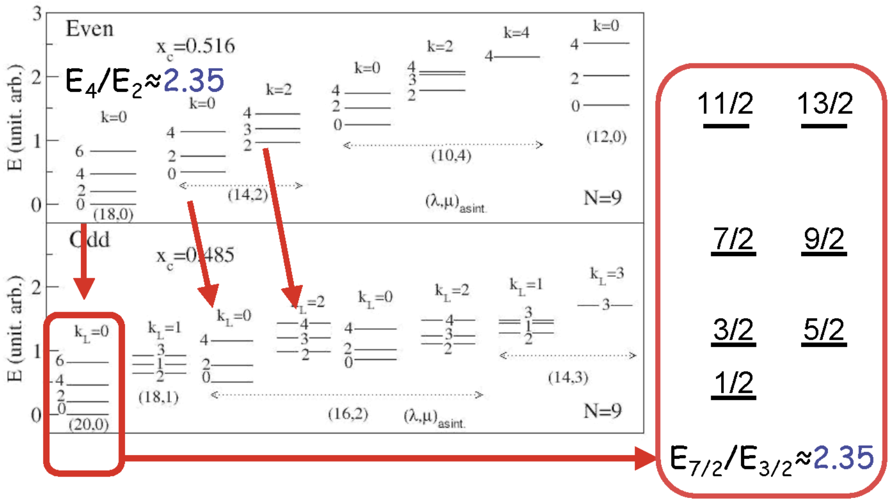

FIGURE 3. Spectra obtained at the critical points in the case of N_B=9 bosons. The upper part display the spectrum in the even case (x_{crit}=0.516), the lower part the spectrum in the neighbour odd boson-fermion case (x_{crit}=0.485). In the odd case the value of the orbital angular momentum (without the coupling to the pseudo spin) is shown, but for clarification the ground band is enlarged and the total angular momenta are shown.

Spectrum and transitions at the critical point

The resulting spectrum at the critical point is finally shown in Fig. 3. The sequence of levels are arranged in bands, on the basis of the transition intensities. For a better comparison, in the upper part of the figure we show the corresponding spectrum for the even core at the corresponding value for the critical point. The odd spectrum clearly resembles the corresponding even one, aside from the presence of bands which are not allowed in the fully symmetric boson case.

Additional important information comes from the electromagnetic transitions. The corresponding intensities will be reported in Ref.[6]. Also in this case the general behaviour for the odd case closely follows the one for the even case.

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