On the Rees algebra of a bipartite graph

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Abstract

Let $G$ be a bipartite graph and let $I$ be its edge ideal. First it is shown that if $A$ is the incidence matrix of $G$, then adding a row of 1's to $A$ preserves total unimodularity. Then as a consequence it is shown that the toric ideal of the Rees algebra $R(I)$ of $I$ is generated by square-free binomials. As another consequence we give a simple proof of the fact that $R(I)$ is normal. We prove that the facets of the Rees cone are in one to one correspondence with the minimal covers of the graph. Thus as a byproduct we obtain a method to compute the minimal covers using linear programming. We are able to prove that the a-invariant of $R(I)$ as a standard graded algebra over a field $K$ is equal to $-\beta_0$, where $\beta_0$ is the independence number of $G$.

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