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Basic concepts of Lorentz symmetry and Minkowsky isogeometry by using MCIM model

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Abstract

At 1978, R.M. Santilli proposed to generalize the conventional Lie theory by using isotopies. To do it, he considered that the basic unit of any mathematical structure can depend on external factors (such as position, speed, acceleration, time, temperature or density): $\widehat{I} = \widehat{I}(x, \dot{x}, \ddot{x}, ..., \mu, \tau, ...)$. In this way, Santilli obtains a more general Lie theory, based on nonassociative, nonlinear and nonlocal-integral systems, which allows him to identify between themselves Euclidean, Riemannian and Minkowskian spaces, when working with extended particles or high energies into unusual physical conditions in exterior or interior dynamic systems. In the 80's and 90's, several mathematicians and physicists have investigated on Lie-Santilli's isotheory. Particularly, Lorentz symmetry and Minkowsky isogeommetry have been studied by Santilli and A.K. Aringazin in the 90's. We propose to revise some of these concepts by using the MCIM isotopic model studied at 2001. So, we will need to generalize the construction of isovectorspaces and the isodifferential calculus.

Introduction

Statement of the problem

The mathematics generally used in quantitative sciences in the 20-th century were based on ordinary fields with characteristic zero, a trivial unit $I = +1$ and an ordinary associative product $a \times b$ between generic quantities a, b of a structure $(E, \times)$, such as matrices, vector fields, etc. Such a mathematics is known to be **linear, local-differential** and **Hamiltonian**, thus solely representing a finite number of isolated point-particles with action-at-a-distance forces derivable from a potential. Such a mathematics was proved to provide an exact and invariant representation of planetary and atomic systems as well as, more generally, of all the so-called *exterior dynamical systems* in which all constituents can be well approximated as being point-like.

By contrast, the great majority of systems in the physical reality are **nonlinear, nonlocal** and **not entirely representable with a Hamiltonian in the coordinates of the experimenter**. This is the case for all systems historically called *interior dynamical systems,* such as the structure of: planets; strongly interacting particles (such as protons and neutrons); nuclei; molecules; stars; and other systems. The latter systems cannot be consistently reduced to a finite number of isolated point-particles. Therefore, the mathematics so effective for exterior systems is only approximate at best for interior systems.

Santilli's isotheory

Santilli's isotopies [5] (1978) constitute a new branch of mathematics characterized by axiom-preserving isotopic lifting of units, products, numbers, fields, topologies, geometries, algebras, groups, etc., with numerous novel applications in physics, chemistry and other quantitative sciences. It was specifically built for the invariant representation of nonlinear, nonlocal and non-Hamiltonian systems. Santilli's fundamental isotopy was that of the basic unit that was lifted from the trivial value $I = +1$ to a matrix or operator, nowhere singular, Hermitean and positive-definite, but that possesses an otherwise unrestricted, generally nonlinear, nonlocal and non-Hamiltonian functional depend on all needed local variables, such as time t, coordinates x, velocities v, density μ, temperature τ, etc.

$$I \to \widehat{I} = \widehat{I}(x, v, \mu, \tau, ...)$$

Jointly, Santilli lifted the conventional numbers and the associative product $a \times b$ with unit $I = +I$ into the new forms $\widehat{a} = a * \widehat{I}$ and $a \hat{\times} b = a \times T \times b$, with unit $\widehat{I} = T^{-I}$. So, the representation of interior systems via Santilli's isomathematics requires the knowledge of *two* quantities, the conventional; Hamiltonian H for the representation of conventional linear, local and potential forces, and the isounit $\widehat{I}$ for the representation of all nonlinear, nonlocal and non-Hamiltonian effects.

MCIM isotopic model

In 2001, Falcón and Núñez [2] generalized the isotopic model proposed by Santilli in 1978 although this generalization put stress on the use of several $*$-*laws* and isounits as operations existing in the initial mathematical structure. Such a model, which from now on will be called MCIM (*isoproduct construction model based on the multiplication*), was later generalized in [3] and [4].

Particularly, it is defined an **isotopy or isotopic lifting** as a correspondence (not necessarily a map, in general) between a mathematical structure $(E,+,\times)$ and another $(\overline{\widehat{E}},\overline{\widehat{+}},\overline{\widehat{\times}})$ (**isostructure**), such that it verifies the same properties, by using a general set $(V,\star,*)$ and a set of external factors $F=\{t,\mu,\tau,...\}$, such that:

$$I \to \widehat{I}; \qquad E \to \overline{\widehat{E}} : x \to \overline{\widehat{x}} = x * \widehat{I}(F_x)\ ; \qquad T = \widehat{I}^{-I};$$

$$V = E \cup \overline{\widehat{E}} \cup \{T\} \cup E_T; \qquad E_T = \left\{a_T = a * T : a \in \overline{\widehat{E}}\right\};$$

$$\Phi_\times : F \times F \to F : (F_\alpha, F_\beta) \to \Phi_\times(F_\alpha, F_\beta);$$

$$\left(a * \widehat{I}(F_a)\right) \overline{\widehat{+}} \left(b * \widehat{I}(F_b)\right) =$$

$$= \left[\left((a * \widehat{I}(F_a)) * T(F_a)\right) \star \left((b * \widehat{I}(F_b)) * T(F_b)\right)\right] * \widehat{I}\left(\Phi_+(F_a, F_b)\right);$$

$$\left(a * \widehat{I}(F_a)\right) \overline{\widehat{\times}} \left(b * \widehat{I}(F_b)\right) =$$

$$= \left[\left((a * \widehat{I}(F_a)) * T(F_a)\right) * \left((b * \widehat{I}(F_b)) * T(F_b)\right)\right] * \widehat{I}\left(\Phi_\times(F_a, F_b)\right);$$

where, fixed $F_0 \in F$:

$$\widehat{I}(F_0) * T(F_0) = I = T(F_0) * \widehat{I}(F_0)$$

$$\left[(a * \widehat{I}(F_0)) * T(F_0)\right] * \widehat{I}\left(\Phi_\circ(F_0, F_0)\right) = a * \widehat{I}(F_0)$$

By the other way, an isotopic lifting of the structure E will be **injective** if $a = b$, for all $a, b \in E$ such that $\overline{\widehat{a}} = \overline{\widehat{b}}$ and it will be **compatible with respect to** a law $\circ$ on E if for all $a * \widehat{I}(F_a), b * \widehat{I}(F_b) \in \overline{\widehat{E}}$ we have:

$$\left(a * \widehat{I}(F_a)\right) \overline{\widehat{\circ}} \left(b * \widehat{I}(F_b)\right) = (a \circ b) * \widehat{I}(\Phi_\circ(F_a, F_b))$$

All this construction must be made for every mathematical structure in use.

Minkowsky isogeometry (I)

The first significant application of the isotopies of the unit was that for the liftings of conventional numbers and fields. The second one was the lifting of the conventional vector and metric spaces, first presented in paper [6] of 1983. The Euclidean and Minkowskian isogeometries were studied in details in monograph [9].

However, all these concepts must be revised if we use the MCIM isotopic model:

Isovectorspaces

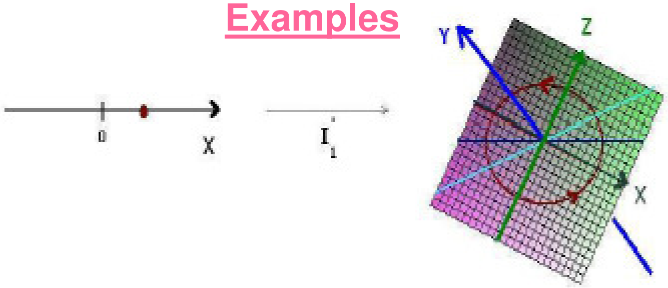
Let be given a $(\mathbb{R}, +\times)$-space $(U, +, \bullet)$. By using the MCIM isotopic level, we can obtain an isofield $(\overline{\widehat{\mathbb{R}}}, \overline{\widehat{+}}, \overline{\widehat{\times}})$, with the elements $\star, \widehat{S}, *$ and $\widehat{I} = \widehat{I}(F)$. Similarly, we can use other elements $\diamond, \widehat{S}', \Box, \diamondsuit, \widehat{I}' = \widehat{I}'(F') = T'^{-I'}, \Phi_+$ and $\Phi_\bullet$, such that:

$$\overline{\widehat{X}} = X \diamondsuit \widehat{I}'(F'_X);$$

$$\left(X \diamondsuit \widehat{I}'(F'_X)\right) \overline{\widehat{+}} \left(Y \diamondsuit \widehat{I}'(F'_Y)\right) = (X \diamond Y) \diamondsuit \widehat{I}'(\Phi_+(F'_X, F'_Y));$$

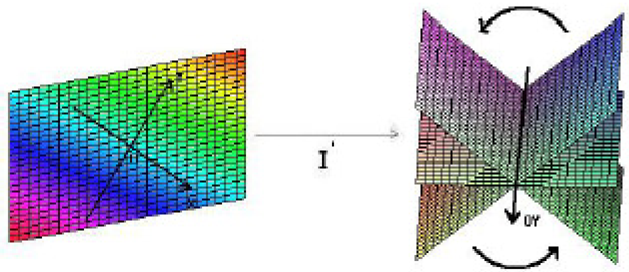
$$\left(a * \widehat{I}(F_a)\right) \overline{\widehat{\bullet}} \left(X \diamondsuit \widehat{I}'(F'_X)\right) = (a \Box X) \diamondsuit \widehat{I}'(\Phi_\bullet(F_a, F'_X)).$$

Examples



$$F' = [0, 2\pi)$$

$$x \diamondsuit \widehat{I}'(x, t) = (x\ cos(t), 0, x\ sen(t))$$



$$F' = [0, 2\pi)$$

$$(x, y) \diamondsuit \widehat{I}'((x, y), t) = (x\ cos(t), y, x\ sen(t))$$

Minkowsky isogeometry (II)

Isometric spaces

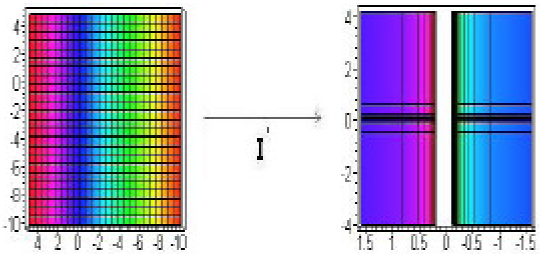
Let consider a $\mathbb{R}$-space U n-dimensional, endowed with a metric $g \in (\mathbf{M}_n(\mathbb{R}), +, \cdot)$. We fixe a isofield $\overline{\widehat{\mathbb{R}}}$ and so, we have $\mathbf{M}_n(\overline{\widehat{\mathbb{R}}}, \overline{\widehat{+}}, \overline{\widehat{\cdot}})$ too. If we want to get a $\overline{\widehat{\mathbb{R}}}$-space $\overline{\widehat{U}}$ endowed with a metric $g' \in \mathbf{M}_n(\overline{\widehat{\mathbb{R}}})$ depending on external factors F', we will find in the first place an isounit $\widehat{I}'$ such that $g' = g \diamondsuit \widehat{I}'(F') = \overline{\widehat{g}}$. (Santilli proposed $g' = T \cdot g$ in [6]).

Therefore, we have:

$$\overline{\widehat{X}} = X \diamondsuit \widehat{I}'(F'_X) = X \diamondsuit g^{-I'} \diamondsuit \overline{\widehat{g}}; \qquad \overline{\widehat{X} \widehat{\cdot g \cdot} \widehat{Y}} = \left(\overline{\widehat{X}}_T \overline{\widehat{g}}_T \overline{\widehat{Y}}_T\right) \diamondsuit \widehat{I}'(\Phi.(F_X, F_Y)).$$

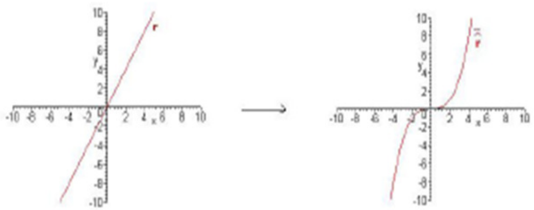
Particularly, if $\diamondsuit$ is associative, we will have the isometric $\overline{\widehat{g}}(\overline{\widehat{X}}, \overline{\widehat{Y}}) = \overline{\widehat{g(X,Y)}}$, which depends on external factors.

Example



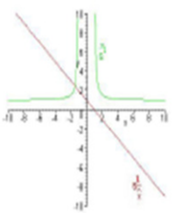
$$\overline{\widehat{x}} = \left\{ \begin{array}{l} 0, \text{ si } x = 0 \\ \frac{1}{x}, \text{ si } x \neq 0 \end{array} \right\}; \qquad \overline{\widehat{y}} = \left\{ \begin{array}{l} 0, \text{ si } y = 0 \\ \frac{1}{y^3}, \text{ si } y \neq 0 \end{array} \right\}.$$

$$(x, y) \diamondsuit \widehat{I}'(x, y) = (\frac{1}{x}, \frac{1}{y^3}))$$



$$r \equiv y = 2x; \qquad \overline{\widehat{r}} \equiv y = \frac{x^3}{8}$$

Straight isoline



1-dimensional isospheres

Minkowsky isogeometry (III)

Minkowsky isogeometry

Let $(M(x, \eta, \mathbb{R}))$ be a Minkowskian $\mathbb{R}$-space, with local chart $x = (x^k) = (r, x^4)$ (being $r \in \mathbb{R}^3$ and $x^4 = c_0 t$) and n-dimensional metric $\eta = diag_n(1, 1, 1, -1)$. It will be $x^2 = x^{1^2} + x^{2^2} + x^{3^2} - x^{4^2}$.

In the absence of gravity, space-time is determined as a smooth flat manifold endowed with the Minkowsky metric η. Any modification of η requires non canonical transformations, by using non invariant spacial-temporal units.

To solve it, Santilli proposes in [6] the **Minkowskian $\widehat{\mathbb{R}}$-isospace** $(\widehat{M}(\widehat{x}, \widehat{\eta}, \widehat{\mathbb{R}}))$, by using:

$$\widehat{\eta} = \widehat{\eta}(x, v, \mu, \tau, ...) = diag\,(\widehat{\eta}_{11}, \widehat{\eta}_{22}, \widehat{\eta}_{33}, \widehat{\eta}_{44}) = T(x, v, \mu, \tau, ...) \cdot \eta;$$

$$x^{\hat{2}} = \sum_{x=1}^{4} x^i \widehat{\eta}_{ii} x^i.$$

It represents all modifications of the Minkowsky metric as encountered, by example, in particle physics, conventional exterior gravitational line elements with $\widehat{\eta} = \widehat{\eta(x, v, \mu, \tau, ...)}$ (such as the full Schwarzschild line element [10]), all its possible generalizations for the interior problem, ...

For a better mathematical consistency, the MCIM isotopic model proposes the isometric

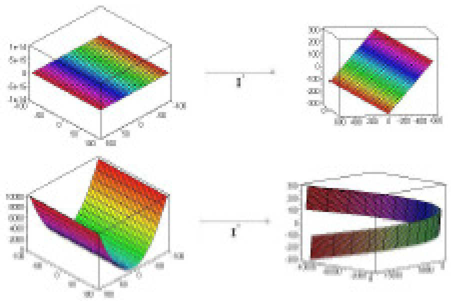
$$\overline{\widehat{\eta}} = \overline{\widehat{\eta}}(x, v, \mu, \tau, ...) = \eta \cdot \widehat{I'}(x, v, \mu, \tau, ...); \qquad x^{\bar{\hat{2}}} = \overline{\widehat{x \lozenge T'}} \cdot \overline{\widehat{\eta}} \cdot \overline{\widehat{x \lozenge T'}} = (x \lozenge T')^2_{\eta}$$

It allows to use the law $\bar{\hat{\cdot}}$ instead of $\cdot$ and to obtain more general structures. However, it is necessary to change the definition of Santilli's isodifferential calculus [10] ($\widehat{dx}^k = \sum_{i=1}^{4} T^i_k dx^i$) in the following form:

$$\overline{\widehat{dx}}^k = \sum_{i=1}^{4} dx^i * \widehat{I'}^k_i.$$

Example

$$\overline{\widehat{\eta}} = \widehat{I'} = \begin{pmatrix} 1 & -3 & 2 & 0 \\ -3 & 3 & -1 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$



$$\overline{\widehat{(x^1, x^2, x^3)}} = (x^1 - 3x^2 + 2x^3, -3x^1 + 3x^2 - x^3, 2x^1 - x^2)$$

$$\overline{\widehat{dx}}^1 = dx^1 - 3dx^2 + 2dx^3; \quad \overline{\widehat{dx}}^2 = -3dx^1 + 3dx^2 - dx^3;$$

$$\overline{\widehat{dx}}^3 = 2dx^1 - dx^2; \quad \overline{\widehat{dx}}^4 = dx^4.$$

Lorentz isosymmetry

Transformation group of the space-time which leaves the space-time interval, $ds^2 = dx^i \eta_{ij} dx^j$, invariant is called Lorentz group. Besides, the Lorentz symmetry is one of the fundamental symmetries of physical theories. It is correct into ordinary conditions, but not with extended particles, high energies or unusual physical conditions [1], such as non-linear dependence.

To solve it, we can use the Minkowsky isogeometry. Non Minkowskian part of the isometric is assumed to describe geometrically local physical properties of the space-time, such as non homogeneity, deformations, resistance, anisotropy and velocity dependence.

In this way, Lorentz isosymmetry was introduced by Santilli in [6] and studied in detail in monograph [7], at the classical level, and in monograph [9], at the operator level. It was conceived to study the global symmetries of gravity.

Particularly, he defines the **isolorentz symmetries** as the **isotransformations group**:

$$x' = \widehat{\Lambda} x = \widehat{\Lambda} * T * x;$$

$$\widehat{\Lambda}^t \widehat{\eta} \widehat{\Lambda} = \widehat{\eta}^{-1} = \widehat{\Lambda} x = \widehat{\Lambda} * T * x.$$

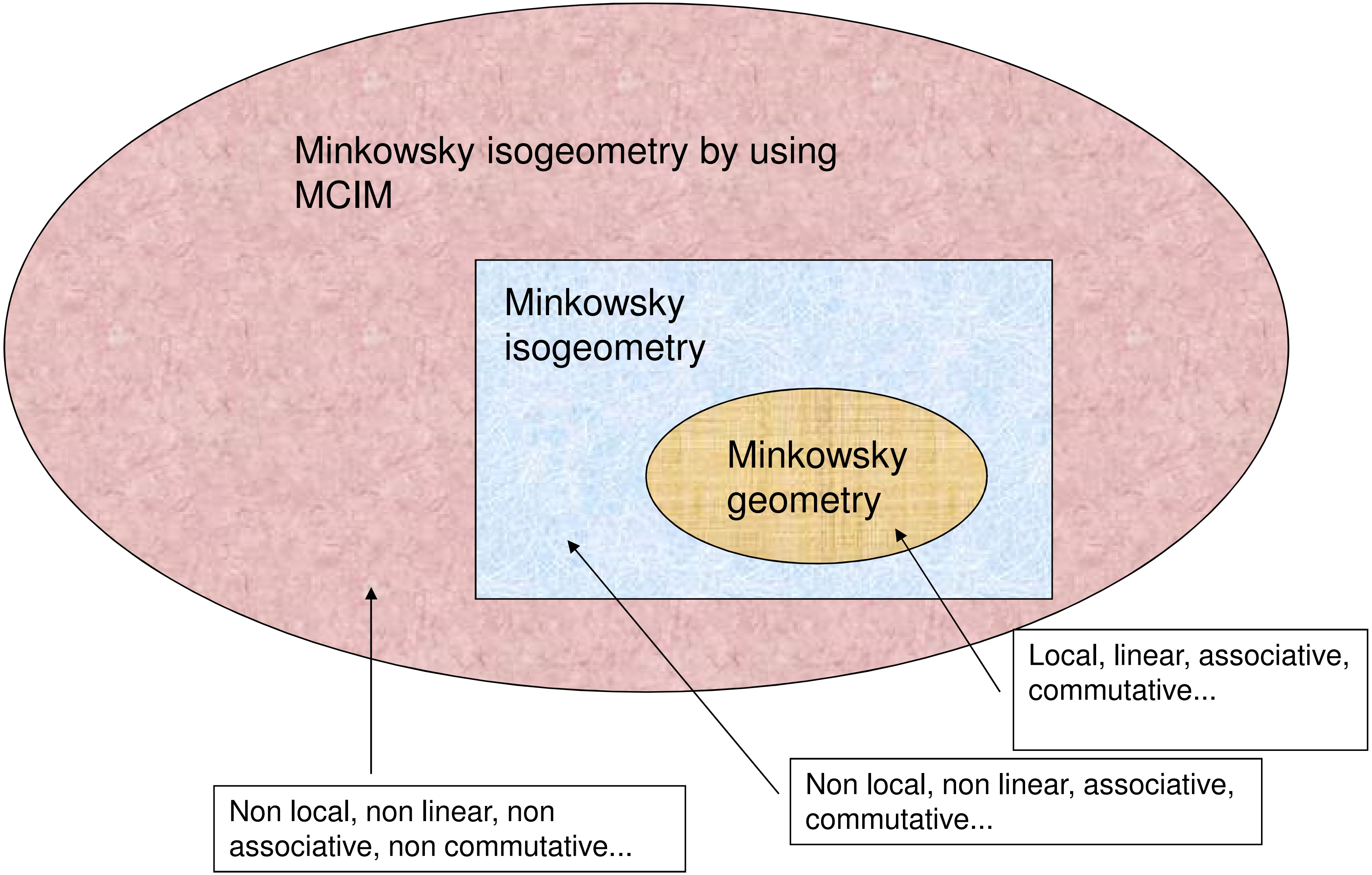
These transformations are formally linear and local on the Minkowskian isospace $\widehat{M}$, but generally non-linear and non-local on the conventional space M. They provides methods for the explicit construction of (generally nonlinear but local) symmetries of conventional gravitational metrics, such as Schwarzschild's metric. Besides, Einstein's gravitation or any other gravitational theory (not necessarily Riemannian) with metric $\widehat{\eta} = T\eta$ $(det(T) > 0)$, admits the conventional Lorentz symmetry as a global isotopic symmetry.

If we use the MCIM isotopic model, isotransformations group become the following:

$$x' = \overline{\widehat{\Lambda}} \overline{\widehat{\cdot}} x = \overline{\Lambda * \widehat{(x * T)}}.$$

It allows to obtain more general structures, because the law $*$ can be non associative or non commutative.

Summary


Minkowsky isogeometry by using MCIM
Minkowsky isogeometry
Minkowsky geometry
Local, linear, associative, commutative...
Non local, non linear, associative, commutative...
Non local, non linear, non associative, non commutative...

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