Core excitation contributions to the breakup of the one-neutron halo nucleus $^{11}$Be on a proton

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The effect of the core excitation in the breakup of a one-neutron halo nucleus is studied within two different reaction formalisms, namely, the core excited model and the single-scattering approximation of the three-body Faddeev–Alt-Grassberger-Sandhas equations with target-core potential allowing for the core excitation. As an example, we consider the breakup of $^{11}$Be on a proton target at 63.7 MeV/nucleon incident energy and calculate the semi-inclusive cross section in the excitation energy interval $E_x = 3.0–5.5$ MeV ($E_{rel} = 2.5–5$ MeV) containing the $3/2^+$ resonance with dominant contribution of the $^{10}$Be$(2^+)$ core excited state. The effect of the core excitation to the breakup cross section integrated around this resonance is found to be very significant. Moreover, when resonant and nonresonant contributions are added, the resulting semi-inclusive cross section is in reasonable agreement with the existing data, demonstrating the relevance of the core excitation mechanism for this observable. The present calculations also show the importance of incorporating the energy dependence of the core-target transition operators in the reaction formalism.

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I. INTRODUCTION

Halo nuclei are few-body weakly bound nuclear states that appear at the neutron drip line. A major source of information on the structure of these systems comes from reactions induced by exotic beams on stable targets. A powerful technique extensively used in the past years to extract information from these systems is the reconstruction of the excitation energy spectrum from the measurement in coincidence of the outgoing fragments following the dissociation of a weakly bound projectile. A meaningful interpretation of these experiments requires an accurate description of the reaction mechanism and a realistic structure model for the ground and the continuum states.

A recent example is the measurement of the breakup of the $^{11}$Be halo nucleus on a proton target at 63.7 MeV/nucleon [1]. Due to the experimental energy resolution, the measured semi-inclusive cross section contained integrated contributions from ranges of relative neutron-core energy. These observables have been recently analyzed within the Faddeev–Alt-Grassberger-Sandhas (Faddeev–AGS) scattering framework [2–4] extended to include the Coulomb force [5,6]. It was found [7,8] that these calculations reproduce well the shape of the cross-section observables containing integrated contributions in the energy range $E_x = 3.0–5.5$ MeV (relative neutron-$^{10}$Be energy range 2.5–5.0 MeV).

The origin of this discrepancy can not be ascribed to the choice of the proton-neutron or proton-core potentials. In fact, it was found in Ref. [8] that the calculated breakup observables are essentially independent of the choice of the proton-neutron potential, provided that one uses a realistic NN parametrization, and change by less than 10% for two different choices of the underlying proton-core potential. Thus, with the present $NN$ and $N$-core dynamic inputs, these disagreements remain up to now a puzzle and it is timely to understand the underlying physics of this discrepancy.

A possible source of the disagreement could be due to the structure model used in Ref. [8], in which both the ground and continuum states are assumed to be well described by pure single-particle configurations relative to the $^{10}$Be in its ground state. However, the energy range 3.0–5.5 MeV is likely to contain contributions from the second $^{11}$Be resonance at $E_x = 3.41$ MeV with spin assignment $3/2^+$ [9]. This resonance has a dominant $^{10}$Be$(2^+) \otimes 2S_1/2$ parentage with the core in its first excited state at $E_x = 3.368$ MeV and, hence, the population of this resonance involves the excitation of the $^{10}$Be core, a mechanism that can not be accounted for by the $^{10}$Be$(0^+)$+$n$ model assumed in the calculations of Ref. [8].

In this paper, we investigate the contribution of the $^{11}$Be $3/2^+$ resonance and hence the relevance of this core excitation mechanism in the description of the breakup cross section containing integrated contributions in the energy range $E_x = 3.0–5.5$ MeV measured in [1]. Two reaction mechanisms are used in our calculations. The first one is the core excited model (C-exc) pioneered by several works in the 1960s [10] and described below. The second is the single-scattering term (SST) of the three-body multiple scattering Faddeev–AGS expansion for the transition amplitude with the proton-core potential allowing for the core excitation. While the Faddeev–AGS framework for three-body nuclear reactions [6,11] has...
been used quite extensively in the last few years, it always assumed fixed masses and spins of the three involved particles. An extension including the dynamical coupling between different internal states of the particles still awaits to be done. In this paper, we present a step in this direction by calculating the Faddeev–AGS transition amplitudes in the SST approximation with interactions allowing for the excitation of the $^{10}\text{Be}$ core.

In both cases, we assume that the $^{11}\text{Be}$ ground state and $3/2^+$ resonance are well described by the pure single-particle configurations $^{10}\text{Be}(0^+) \otimes \nu 2s_{1/2}$ and $^{10}\text{Be}(2^+) \otimes \nu 2s_{1/2}$, respectively. Under this assumption, the $3/2^+$ resonance becomes a bound state of neutron and $^{10}\text{Be}(2^+)$ with 0.46-MeV binding energy and the corresponding transition is calculated as an inelastic scattering leading to the two-cluster final state, not as a true three-body breakup. Aside from the $3/2^+$ resonance, we add the three-body breakup contributions calculated in Ref. [8] from all partial waves with the core being in its ground state $^{10}\text{Be}(0^+)$, which can be regarded as a nonresonant background. Since we ignore possible admixtures of $^{10}\text{Be}(2^+)$ and $^{10}\text{Be}(0^+)$ in the $^{11}\text{Be}$ states, the resonant and nonresonant contributions can be added incoherently.

It is worth mentioning that the same problem was recently studied using the XCDCC formalism [12], an extension of the continuum discretized coupled channels (CDCC) method [13], which incorporates core excitation effects. Estimations of core excitation effects using this reaction tool were found to be small, in particular in the case of our working example [14].

II. FORMALISMS

We shall start by summarizing the C-exc model. Consider the excitation of a projectile nucleus in its ground state $J_0$ to a final state $J$. Although the model can be formulated in a more general way, in this paper we assume that both the initial and final states are well described by pure single-particle configurations relative to a given state of the core, i.e.,

$$|\Psi_i\rangle = |\phi_{i_0} \otimes \psi_i\rangle, \quad |\Psi_f\rangle = |\phi_f \otimes \psi_f\rangle,$$

where $|\phi_{i_0}\rangle$ ($|\phi_f\rangle$) describes the internal state of the core and $\psi_i$ ($\psi_f$) is the single-particle neutron-core bound-state wave function in the initial (final) state. The indices $i$ and $f$ comprise the set of quantum numbers $\ell, s, j$ corresponding to the orbital angular momentum, the spin, and their sum, and any additional label required to specify the state. In our case study, the considered transition is from the $^{11}\text{Be}(1/2^+)$ ground state to the $^{11}\text{Be}(3/2^+)$ resonant state ($E_r = 3.41$ MeV), where the core is excited from a $J_0 = 0^+$ initial state to a $I = 2^+$ final state, and the valence neutron remains in a $2s_{1/2}$ single-particle configuration.

Within the distorted-wave Born approximation (DWBA), the transition matrix for the excitation from the initial state $|\Psi_i\rangle$ to the final state $|\Psi_f\rangle$ is given by

$$A_{if} = \langle \chi_{f}^{(+)}(\vec{R}) | \Psi_f(\vec{R}, \vec{\xi}) \rangle V_{ct}(\vec{R}_{ct}, \vec{\xi}) + V_{ct}(\vec{R}_{ct}, \vec{\xi})$$

$$- U_{\text{aux}}(\vec{R}) \langle \chi_{i}^{(-)}(\vec{R}) | \Psi_i(\vec{\xi}) \rangle,$$

where $U_{\text{aux}}(\vec{R})$ is an auxiliary optical potential that produces the distorted wave $\chi_{i}^{(-)}(\vec{R})$ for the incoming channel. In our calculations, the distorted wave for the outgoing channel $\chi_{f}^{(+)}(\vec{R})$ will be also generated with the same auxiliary potential. In this expression, $V_{ct}(\vec{R}_{ct})$ and $V_{ct}(\vec{R}_{ct}, \vec{\xi})$ are the valence-target and core-target interactions, respectively. To allow the core excitation mechanism, the latter depends on the internal states of the core ($\vec{\xi}$). The valence-target and the auxiliary optical potential can not induce transitions of the core and can therefore be omitted from the previous expression. In Eq. (2), the initial and final distorted waves are evaluated at the projectile-target coordinate ($\vec{R}$), whereas the interaction $V_{ct}(\vec{R}_{ct}, \vec{\xi})$ is evaluated at the core-target coordinate. However, in our core excited model approach, we make the approximation $\vec{R} \approx \vec{R}_{ct}$ in Eq. (2), which means that both distorted waves and the potentials $V_{ct}$ are evaluated at the $\vec{R}_{ct}$ coordinate.

As a consequence of this approximation, the core-valence coordinate disappears from the transition operator, remaining only in the initial and final states, and the scattering amplitude factorizes into the product of two terms, one depending on the valence wave functions and the other depending on the core. Under these approximations, the projectile-target scattering amplitude is directly related to a core-target amplitude describing the excitation of the core. By working out the angular-momentum algebra as given in detail in Ref. [15], one gets the following expression of the differential cross section:

$$\frac{d\sigma}{d\Omega}(J_0 \rightarrow J) = VW|\langle \psi_f|\psi_i\rangle|^2 \delta_{J_0\delta_{Jf}, J\delta_{Jf}} \frac{d\sigma_{ct}}{d\Omega}(I_0 \rightarrow I).$$

with $d\sigma_{ct}/d\Omega$ the core-target inelastic differential cross section and $W$ the geometric factor

$$W = (2J_0 + 1)(2I + 1)W(\lambda J I J_0 \delta_{I0} I_0 J_0)^2.$$

The core excited model can be deduced from the XCDCC approach [12] by calculating the transition in the Born approximation, neglecting the valence-target interaction and core-recoil effects, and taking a central valence-core potential.

The core-target interaction can be expressed in terms of a multipole expansion

$$V_{ct}(\vec{R}_{ct}, \vec{\xi}) = 4\pi \sum_{\lambda, \mu} V_{ct}(\vec{R}_{ct}) Y_{\lambda\mu}(\vec{R}_{ct}) Y_{\lambda\mu}^*(\vec{\xi}).$$

Note that, in the present case, only the term $\lambda = 2$ contributes. In our approach, we adopt a simple axially symmetric rotational model (with band head $K = 0^+$) assuming that the core has a permanent quadrupole deformation characterized by the deformation parameter $\beta_2$. By deforming the central potential $V_{ct}(R)$ and expanding it in terms of the deformation parameter $\lambda = 2$, one gets the leading term

$$V_{ct}(R) = \frac{1}{\sqrt{4\pi (2\lambda + 1)}} \delta_2 \frac{dV_{ct}(R)}{dR},$$

with the deformation length $\delta_2 = \beta_2 R_0$ for the multipolarity $\lambda$, where $R_0$ is an average radius of the nucleus.

We now briefly describe the three-body Faddeev–AGS reaction approach. This is a nonrelativistic three-body multiple-scattering framework that has been used [6–8,11] to calculate
various observables of three-body nuclear reactions. In this paper, we make steps toward extending it to include the interactions with the core excitation. We use here the odd-man-out notation for the three interacting particles (1, 2, 3), which means, for example, that the potential between the pair (2, 3) is denoted as $v_1$. We take particle 1 as the proton target, and particles 2 and 3 as the valence neutron and the core of the projectile, respectively. The transition amplitudes are given by the on-shell matrix elements of the operators $U^{\alpha\mu}$. These operators are obtained by solving the three-body AGS integral equations [3,4] that can be found by iteration leading to

$$U^{\alpha\mu} = \delta_{\mu\nu} G_0^{-1} + \sum \delta_{\mu\nu} t_\gamma \delta_{\gamma\alpha}$$

$$+ \sum \delta_{\mu\nu} t_\gamma G_0 \sum \delta_{\gamma\xi} t_\kappa \delta_{\kappa\alpha} + \cdots,$$  \hspace{1cm} (7)

with $\delta_{\gamma\alpha} = 1 - \delta_{\gamma\alpha}$ and the pair transition operator

$$t_\gamma = v_\gamma + v_\gamma G_0 t_\gamma,$$  \hspace{1cm} (8)

where $G_0$ is the free resolvent $G_0 = (E + i0 - H_0)^{-1}$ and $E$ is the total energy of the three-particle system in the center of mass (c.m.) frame. Much like the proton-core potential $v_\gamma$, the corresponding transition operator $t_\gamma$ couples different states of the core; it is calculated without any approximations in close analogy with Ref. [16].

In these calculations, we will consider only the leading term of the series (7) that will be referred to hereafter as SST. Note that the single-scattering term $t_1$, where the proton scatters from the neutron, does not contribute to the core excitation. Thus, we shall retain only the proton-core single-scattering term $t_2$ of the multiple-scattering expansion.

The SST amplitude for the transition from the initial $|k_1 \Psi_1\rangle$ to the final $|k'_1 \Psi_f\rangle$ three-body state is given by

$$\langle k'_1 \Psi_f | t_2 | k_1 \Psi_1 \rangle = \int d^3q_{23} \langle \Psi_f | t_2 | q_{23} \rangle \times \langle \Psi_f | t_2 | q_{23} \rangle \times \langle \Psi_f | t_2 | q_{23} \rangle$$

$$\times \langle \Psi_f | t_2 | q_{23} \rangle \times \langle \Psi_f | t_2 | q_{23} \rangle$$  \hspace{1cm} (9)

with $q_{ij} = m_i k_i - m_j k_j$ and relative pair (1, 3) energy $\omega_{13} = E - [q_{13}^2 - (m_1 + m_3)k^2]^{1/2}$ from [23]. As usual, $k_1 (k'_1)$ is the initial (final) relative momentum between particle 1 and the center-of-mass of pair (2, 3). The SST involves a full folding integral of a product of a transition matrix evaluated at a variable energy $\omega_{13}$ and a transition density form factor with $q_{23}$, $q_{13}$, and $q_{23}$ being uniquely determined by $k_1$, $k'_1$, and $q_{23}$. The SST only assumes a factorized form if additional approximations are made [17].

III. RESULTS

In this paper, we analyze the $p^{(11}\text{Be},^{10}\text{Be} n)p$ breakup semi-inclusive cross section containing integrated contributions for excitation energies $E_x = 3.0$–5.5 MeV measured in [1]. The initial and final states of the $^{11}\text{Be}$ projectile are generated using a Woods-Saxon potential with radius $R_0 = 1.25 \times 10^{-3}$ fm, diffuseness $a = 0.67$ fm, and the depth adjusted in each case to yield the appropriate separation energy [18,19].

The wave function of the ground state of $^{11}\text{Be}$ can be written as

$$|^{11}\text{Be}_g\rangle = |^{10}\text{Be}(0^+_1) \otimes 2s\rangle + \beta |^{10}\text{Be}(2^+_1) \otimes 1d\rangle + \cdots,$$  \hspace{1cm} (10)

where $\alpha^2$ and $\beta^2$ represent the spectroscopic factors for each component. The precise relative weight of each component is still fairly uncertain. The spectroscopic factors for the component where the core is in its ground state, obtained from theoretical calculations [20–25] that can be found in the literature, or extracted from experimental data [26,27], range from 0.55 to 0.92.

Nevertheless, in the case of the $^{11}\text{Be}$ ground state, the component where the core is in its ground state is, in most predictions, clearly the dominant one. For the case of the second resonance, the core is dominantly in the first excited state.

For the purpose of estimating the effects of the transition to the second $^{11}\text{Be}(3/2^+)$ resonance to the breakup, we have assumed pure single-particle configurations: For the initial state, we assumed that the valence neutron is in the $2s_{1/2}$ pure single-particle configuration relative to the $^{10}\text{Be}$ core in its ground state, whereas for the $3/2^+$ resonance, we also assumed the $2s_{1/2}$ configuration, but coupled to the $^{10}\text{Be}$ in its first excited ($2^+_1$) state.

The central part of the $p^{11}\text{Be}$ interaction was taken from the global parametrization of Watson et al. [18]. The transition potential describing the excitation of the core was generated according to Eq. (6) using a deformation parameter $\beta = 0.67$ from Ref. [23] and the $^{10}\text{Be}$ matter radius $R_0 = 2.94$ fm from Ref. [19].

Since we assume pure single-particle states for $^{11}\text{Be}$, the integrated cross section will be given as an incoherent sum of the single-particle (s.p.) breakup cross section (that is, with the $^{10}\text{Be}$ in its ground state) and the inelastic cross section for the transition to the $3/2^+$ state of $^{11}\text{Be}$.

The s.p. breakup cross section is taken from Ref. [8], where it was calculated using the Faddeev–AGS scattering framework with the realistic CD-Bonn $NN$ potential. The inelastic cross section was estimated using both the SST and C-exc models [Eq. (3)]. In the latter case, the geometric factor is $W = \xi$, and the extra factor due to the overlap of the single-particle wave functions is $\langle \psi_f | \psi_1 \rangle = 0.9487$.

In Fig. 1, we compare our results for the breakup cross-section angular distribution $d\sigma/d\Omega_{\text{c.m.}}$ with the experimental data. The dashed curve represents the s.p. contribution that fails in reproducing magnitude and shape of the data. The thin solid and dashed-dotted curves correspond to the inelastic cross-section results of the C-exc model and the three-body SST, respectively, demonstrating that the effects of the core excitation are significant. The SST differs from the C-exc results, being lower at $\theta_{\text{c.m.}} < 15^\circ$ and higher at $\theta_{\text{c.m.}} > 15^\circ$. In order to explore the origin of this difference, we have fixed in the SST approach the energy of the proton-core pair in the transition amplitude to its maximum value, i.e., $\omega_{13} = E$. These results, represented with circles, are almost indistinguishable from the C-exc results. Thus, it is important to incorporate a proper treatment of the energy dependence.
cross section for the breakup \( p^{(11}\text{Be}, p)^{10}\text{Be} n \) at 63.7 MeV/u with contributions where the relative core-neutron energy is integrated over the energy range \( E_{\text{rel}} = 2.5\text{–}5.0 \text{MeV} \).

of the two-body \( t \) matrix into the reaction framework. The incoherent sum of the s.p. and SST contributions is given by the thick solid curve and is in reasonable agreement with the data except for the first data point, which is overestimated due to the size of the s.p. contribution.

Our result is in contrast to those presented in Ref. [12], where estimations of core excitation effects using the extension of the CDCC approach (XCDCC) were found to be negligible. The XCDCC includes the interference between possible multichannel components of the projectile as well as dynamical excitation of the core within the projectile during the reaction, and therefore has perturbation effects beyond the DWBA. The detailed multichannel effects are presently an open question. In the core excited and SST reaction approaches described in the text, structure and dynamics effects are clearly delineated. Further work to include multichannel components within these reaction frameworks is underway.

IV. SUMMARY AND CONCLUSIONS

In summary, we have studied the breakup of \( ^{11}\text{Be} \) on the proton target at 63.7 MeV/nucleon incident energy. We estimated the effects of the core excitation \( p + ^{10}\text{Be}(0^+_1) \rightarrow p + ^{10}\text{Be}(2^+_1) \) on the semi-inclusive cross section around the second resonance of \( ^{11}\text{Be} \) with excitation energy \( E_x = 3.41 \) MeV and spin \( 3/2^+ \). The inelastic cross section was evaluated using the proton-core single-scattering term of the three-body Faddeev–AGS equations as well as the C-exc model; in both cases, the core excitation provides a significant contribution to the breakup cross section. In addition, our calculations show that it is important to handle properly the energy dependence of the two-body \( t \) matrices in the reaction formalism.

An extension of the Faddeev–AGS framework to include higher-order core excitation terms is highly desirable to make a more accurate evaluation of these contributions.

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