Discrete breathers collisions:
An overview and some recent results

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Outline

- Overview of previous studies:
  - Collisions in FPU lattices
  - Collisions in dissipative Klein–Gordon lattices
  - Collisions in DNLS lattices

- New results on DNLS lattices with saturable nonlinearity:
  - Properties of SDNLS lattices
  - Collisions in SDNLS lattices

- Future challenges
Y. Doi, PRE 68, 066608 (2003)

β-FPU model is considered

Collisions of breathers with the same/different energies and the same/different phase

Observed regimes:

- Reflection
- Fusion
- Symmetry breaking (even if same energy and same phase)

The energy exchange is throughoutly studied

Transferred energy may be very sensitive to phase difference
FPU lattices

Fusion and symmetry breaking regimes

(a)

(b)

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Dissipative Klein-Gordon lattices


Frenkel–Kontorova model with dissipation and external force
Collisions of breathers with the same energy and phase

Observed regimes:
- Reflection
- Destruction of one/both breathers
- Bound state:
  - Pair of breathers subject to the force of the emitted phonons
  - The distance between breathers is related to the velocity

The final states are attractors of the system
Dissipative Klein-Gordon lattices

Different regimes
Dissipative Klein-Gordon lattices

- A bound state
Nonintegrable DNLS lattices


Collisions of breathers with the same energy and the same/different phase

Observed regimes (in-phase breathers):

△ Reflection
△ Bound state
△ Spontaneous symmetry breaking (due to numerical errors):
   △ Bound state moves with a well-defined value of the velocity
   △ Mutual bounce after multiple collisions

High Peierls-Nabarro barrier → Strong differences between on-site and inter-site collisions.
Nonintegrable DNLS lattices

Several regimes
DNLS lattices with saturable nonlinearity


Discrete version of the Vinetskii-Kukhtarev equation (SDNLS equation):

\[
\dot{u}_n - \beta \frac{u_n}{1 + |u_n|^2} + (u_{n+1} - 2u_n + u_{n-1}) = 0.
\]

Models 1D waveguide arrays of photorefractive materials → SBN61 (Sr\(_{0.61}\)Ba\(_{0.39}\)Nb\(_2\)O\(_6\))

Main feature: Bounded Peierls-Nabarro barrier → High power moving breathers can be found.
DNLS lattices with saturable nonlinearity

Peierls-Nabarro barrier in 1D chains

Saturable Nonlinearity

Kerr Nonlinearity
Collisions in SDNLS lattices

J. Cuevas and J.C. Eilbeck, ArXiv:nlin.PS/0501050

Collisions of breathers with the same energy and phase

Observed regimes (in-phase breathers):

- Reflection
- Bound state
- Bound state + Reflection (for high powers)
- Bound state + spontaneous symmetry breaking

Symmetry breaking due to numerical errors
Collisions in SDNLS lattices

Reflection and bound state regimes

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Collisions in SDNLS lattices

Reflection + bound state and symmetry breaking regimes
Collisions in SDNLS lattices

Different regimes

Inter-site collisions

On-site collisions
Collisions in SDNLS lattices

Velocity difference (Incoming-Outgoing)

Inter-site collisions

On-site collisions
Conclusions

- In all cases, reflection and bound state formation is observed.
- Symmetry breaking is also observed, with different origins:
  - DNLS: Numerical errors
  - Dissipative KG: Attractor of the system
  - FPU: Unexplained
- Saturable nonlinearity allows high-power breathers
- Future challenge: Study the case of Hamiltonian Klein-Gordon lattices.