Localized oscillations in nonlinear hamiltonian Klein-Gordon lattices. Breathers and Anderson modes

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Introduction

- There are two different sources of localization in discrete lattices:
  - Anderson modes in disordered harmonic lattices [1]
  - Discrete breathers in homogeneous nonlinear lattices [2]

Objective

- Study of the conditions for which localized modes exist in disordered anharmonic lattices
- We undertake the problem studying the possibility of connection of discrete breather with Anderson modes.

Model

\[ H = \sum_{n=1}^{N} \left( \frac{1}{2} m_n \dot{u}_n^2 + V(u_n) + \frac{1}{2} C (u_n - u_{n+1})^2 \right) \]

\[ V(u_n) = \frac{1}{2} \omega_n^2 u_n^2 - su_n^2 \]

\( s=0 \): Linear disordered limit (Anderson modes)
\( s=1 \): Nonlinear ordered limit (discrete breathers)

\[ \omega_n = 1 + \rho(s) \frac{r_n}{2} \quad (r_n \text{ random vector}) \]

\[ \rho(s) = 1 - s^q, \quad q > 0 \quad (\text{path function}) \]

Connection of discrete breathers and Anderson modes

A solution in one of the limits is calculated and continued to the other limit keeping the action (phase space area) constant.
- The number of discrete breathers is huge compared to the number of Anderson modes.
- This fact suggest that the bifurcations in the path from breathers to Anderson modes should be turning points and pitchforks.
- It also appears period doubling bifurcations
- The Anderson modes of highest and lowest frequency are connected
- It has also been found the existence of isolas in the last case
- The random vector takes its values in a discrete random distribution

Broken pitchfork in the q=1/4 path (2d)

References