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Multi-criteria minimum cost spanning tree games

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Abstract

The minimum cost spanning tree game (mcst-game) is a well-known model within operations research games that has been widely studied in the literature. In this paper we introduce the multi-criteria version of the mcst-game as a set-valued TU-game. We prove that the extension of Bird's cost allocation rule provides dominance core elements in this game. We also give a family of core solutions that are different from the previous one; these solutions are based on proportional allocations obtained using scalar solutions of the multi-criteria spanning tree problem. Besides, we prove necessary and sufficient conditions ensuring that the preference core of this game is not empty.

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Keywords: Cooperative game theory; Multi-criteria optimization; Minimum cost spanning tree

1. Introduction

Optimization problems in which one or several decision-makers that consider one or several objective functions, analyze how to act in an optimal way constitute the essence of operations research models. Optimization theory analyzes situations in which one decision-maker faces an optimization problem with one or several criteria. If several decision-makers interact conventional game theory is a suitable framework. (See Owen, 1995 for further details.) Finally, when several decision-makers each one controlling several criteria interact it appears multi-criteria game theory. A methodological approach to cooperative games with vector-valued payoff can be seen in Fernández et al. (2002).

Traditionally, operations research focus on choosing the optimal alternatives and game theory focus on models of competition and cooperation. Nevertheless, recent developments in both disciplines have shown strong interplay between them. These models are called Operations Research Games (see Borm et al., 2001). In these models apart from the inherent optimization problem it arises the natural question of how to allocate the joint cost/benefit among the individual decision-makers. Recently, a new issue in game theory has been to consider the multi-criteria operations research games, see for instance Nishizaki and Sakawa (2001) and Fernández et al. (2001).

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31 In this paper we concentrate on the minimum cost spanning tree (mcst) game. These games arise from
 32 analyzing the problem of allocating the costs of a spanning tree in a graph among the users which are
 33 located at the nodes of a graph, with one node reserved for a common supplier which is not to participate in
 34 the cost sharing. This problem was first introduced by Claus and Kleitman (1973). Bird, in 1976, suggested
 35 a game theoretic approach to the problem and proposed a cost allocation scheme that consists of assigning
 36 to each player (node) the cost of the edge incident upon the node on the unique path from the source to the
 37 mentioned node in a minimum cost spanning tree. Since there can be more than one minimum cost
 38 spanning tree for a graph, this way of dividing the costs need not lead to a unique cost allocation. Later on,
 39 Granot and Huberman (1981) showed that the allocations arising from Bird's cost allocation scheme are
 40 always in the core of the mcst-game (see Curiel, 1997).

41 Situations in which the cost associated to an edge is a vector instead of a single number yields to multi-
 42 criteria minimum cost spanning tree games that we analyze in this paper.

43 The paper is organized as follows. Section 2 is devoted to define the Pareto-minimum cost spanning tree
 44 game as a set-valued transferable utility (TU) game. We include the necessary concepts about Graph
 45 Theory. In Section 3 we analyze two core concepts for set-valued TU-games. We prove that the extension of
 46 Bird's cost allocation rule provides dominance core elements in this game. We also give a family of core
 47 solutions that are different from the previous one.

48 2. The game

49 In general, a set-valued TU-game is a pair (N, V) , where $N = \{1, 2, \dots, n\}$ is the set of players and V is a
 50 function which assigns to each coalition $S \subseteq N$ a compact subset $V(S)$ of \mathbb{R}^k , the *characteristic set* of co-
 51 alition S , such that $V(\emptyset) = 0$.

52 Vectors in $V(S)$ represent the worth that the members of coalition S can guarantee by themselves. Notice
 53 that the characteristic function in these games are set-to-set maps instead of the usual set-to-point maps.

54 Consider a set of N users of some good that is supplied by a common supplier 0 ($N_0 = N \cup \{0\}$). There is
 55 a multi-criteria cost associated to the distribution system that has to be divided among the users. This
 56 situation can be formulated as a set-valued game with N players and a characteristic function that asso-
 57 ciates to each coalition S a set $V(S)$ that represent the Pareto-minimum cost of constructing a distribution
 58 system among the users in S from the source 0.

59 Let $G = (N_0, E)$ be the complete graph with set of nodes N_0 and set of edges (links) denoted by E . There is
 60 a vector of costs associated with the use of each link. Let $e^{ij} = e^{ji} = (e_1^{ij}, e_2^{ij}, \dots, e_k^{ij})$ denote the vector-valued
 61 cost of using the link $\{i, j\} \in E$. A tree is a connected graph which contains no cycles. A spanning tree for a
 62 given connected graph is a tree, with set of nodes equal to the set of nodes of the given graph, and set of
 63 edges a subset of the set of edges of the given graph connected and without cycles. A Pareto-minimum cost
 64 spanning tree for a given connected graph, with costs on the edges, is a spanning tree which has Pareto-
 65 minimum costs among all spanning trees (see Ehrgott, 2000).

66 **Definition 2.1.** A Pareto-minimum cost spanning tree game, associated to the complete graph $G = (N_0, E)$,
 67 is a pair (N, V) where N is the set of player and V is the characteristic function defined by:

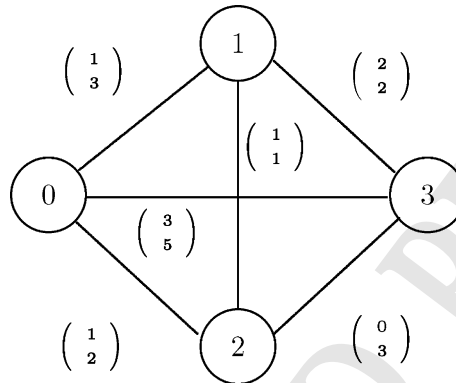
- 68 1. $V(\emptyset) = \{0\}$,
- 69 2. For each non-empty coalition $S \subseteq N$,

$$V(S) = v - \min_{T_{S_0} : \text{spanning tree}} \sum_{\{i,j\} \in E_{T_{S_0}}} e^{ij},$$

71 where $E_{T_{S_0}}$ is the set of edges of the spanning tree, T_{S_0} , that contains $S_0 = S \cup \{0\}$; and $v - \min$ stands for
 72 Pareto-minimization.

73 Remark that the resulting spanning tree T_{S_0} must contain S_0 but it may also contain some additional
 74 nodes.

75 **Example 2.1.** Consider the complete graph below.



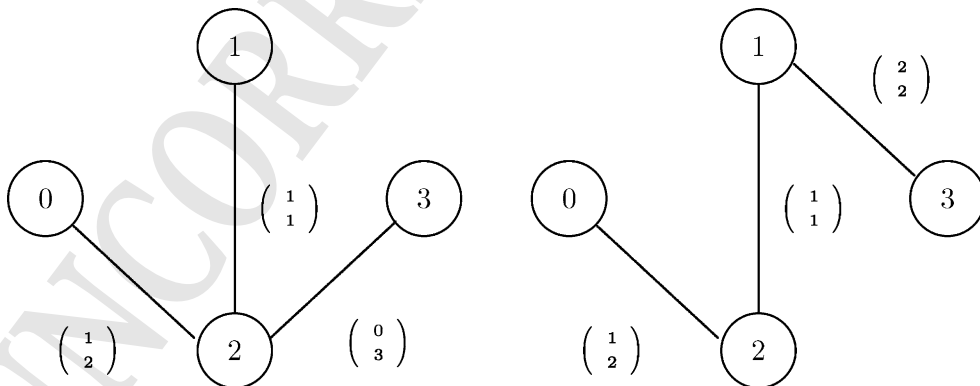
77 The bi-criteria Pareto-minimum cost spanning tree game associated to the graph is:

S	$\{1\}$	$\{2\}$	$\{3\}, \{2,3\}$	$\{1,2\}$	$\{1,3\}$	N
$V(S)$	$\left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \end{pmatrix} \right\}$

79 Note that $V(\{3\})$ is $(1,5)^t$ because the Pareto-minimum spanning tree that contains the nodes 0 and 3 is the
 80 subgraph induced by the edges $\{0,2\}$ and $\{2,3\}$.

81 There are two Pareto-minimum cost spanning trees in the complete graph G for the grand coalition N .

82 The first one correspond to $(2, 6)^t \in V(N)$ and the second one correspond to $(4, 5)^t \in V(N)$:



84

85 The interesting question that arises when a multi-criteria mcst-game, or in general a set-valued TU-game,
 86 is played is how to allocate fairly an achievable vector $z^N \in V(N)$ among the players.

87 For set-valued TU-games an allocation consists of a matrix $X \in \mathbb{R}^{k \times n}$ whose rows are allocations of the
 88 criteria. The i th column, $X^i = (x_1^i, x_2^i, \dots, x_k^i)^t$ represents the payoffs of i th player for each criteria and the j th
 89 row, $X_j = (x_j^1, x_j^2, \dots, x_j^k)$ is an allocation among the players of the total amount obtained with respect to
 90 criterion j . The sum $X^S = \sum_{i \in S} X^i$ is the overall payoff obtained by coalition S .

91 The matrix X is an allocation of the game (N, V) if $X^N = \sum_{i \in N} X^i \in V(N)$. The set of all the allocations
 92 of the game is denoted by $I^*(N, V)$.

93 **3. Core concepts**

94 Apart from efficiency and using the individual and collective rationality principles, in what follows we
 95 are going to establish core concepts for multi-criteria mcst-games.

96 *3.1. Preference core*

97 It is reasonable to think that coalitions only accept allocations if they pay less than any of the worths
 98 given by the characteristic set. To simplify the presentation, by $X^S \leq V(S)$ we will be denoted that $X_j^S \leq z_j^S$,
 99 $\forall j = 1, 2, \dots, k, \forall z^S \in V(S)$.

100 This assumption leads us to introduce the concept of preference core.

101 **Definition 3.1.** The preference core of a game (N, V) is the set of allocations, $X \in I^*(N, V)$, such that
 102 $X^S \leq V(S) \quad \forall S \subset N$. We will denote this set as $C(N, v; \leq)$.

103 In order to characterize the non-emptiness of the preference core, consider a vector $\bar{z} \in \mathbb{R}^k$, not neces-
 104 sarily in $V(N)$, and the following k scalar games:

105 **Definition 3.2.** The scalar l -component minimum cost spanning tree game ($l = 1, 2, \dots, k$) associated to \bar{z} is
 106 a pair $(N, v_l^{\bar{z}})$ where N is the set of player and $v_l^{\bar{z}}$ is the characteristic function defined by:

- 107 1. $v_l^{\bar{z}}(\emptyset) = 0$.
 108 2. For each non-empty coalition $S \subset N$,

$$v_l^{\bar{z}}(S) = \min_{T_{S_0}: \text{spanning tree}} \sum_{\{i,j\} \in E_{T_{S_0}}} e_l^{ij},$$

110 where $E_{T_{S_0}}$ is the set of edges of the spanning tree, T_{S_0} , that contains the set of nodes $S_0 = S \cup \{0\}$.

- 111 3. $v_l^{\bar{z}}(N) = \bar{z}_l$. For each non-empty coalition, $S \subset N$, $v_l^{\bar{z}}(S)$ is the solution of the problem:

$$\begin{aligned} \min \quad & z_l^S, \\ \text{s.t.} \quad & z^S \in V(S), \end{aligned}$$

113 where $z_l^S, l = 1, 2, \dots, k$, is the l th component of vector z^S .

114 Notice that for a fixed coalition S , if an allocation X of the mcst-game, (N, V) , verifies $X^S \leq V(S)$ then
 115 $X^S \leq z^*(S)$, where $z^*(S) = (v_1^{\bar{z}}(S), v_2^{\bar{z}}(S), \dots, v_k^{\bar{z}}(S))$ denote the k -dimensional vector whose components are,
 116 respectively, the solutions of the above problems. Conversely, if $X^S \leq z^*(S)$ then $X^S \leq V(S)$.

117 A necessary and sufficient condition for the non-emptiness of the preference core is given in the next
 118 result. This condition is based on the balancedness concept of standard scalar cooperative games. For a
 119 definition of balanced games the reader is referred to Owen (1995).

120 **Theorem 3.1.** *The preference core is non-empty if and only if there exists at least one $z^N \in V(N)$ such that all*
 121 *the scalar l -component games $(N, v_l^{z^N})$ are balanced.*

122 **Proof.** If every scalar l -component game $(N, v_l^{z^N})$ is balanced, consider any allocation, X_l , in the core of
 123 $(N, v_l^{z^N})$, $l = 1, 2, \dots, k$. Then, the $k \times n$ -matrix X whose rows are X_l , $l = 1, 2, \dots, k$, is an allocation asso-
 124 ciated with z^N . Moreover, for each $S \subset N$, $X^S \leq z^*(S)$ and $X^S \leq V(S)$.

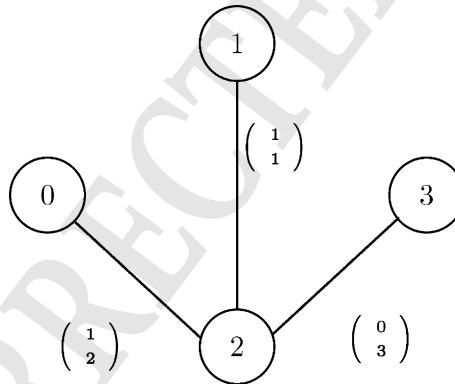
125 Conversely, let X be an allocation in the preference core such that $X^N = z^N \in V(N)$. Then $X^S \leq V(S)$,
 126 $\forall S \subset N$ and $X_l^S \leq v_l^{z^N}(S)$, $\forall S \subset N$, $\forall l = 1, 2, \dots, k$. Therefore, X_l is an allocation in the core of the game
 127 $(N, v_l^{z^N})$. \square

128 In scalar mcst-game there exists a simple rule to allocate costs among the users in the game. This al-
 129 location, called Bird rule (Bird, 1976), is given by:

130 “Each player supports the cost of the edge incident upon it on the unique path between 0 and the player’s
 131 node, in the corresponding minimum spanning tree.”

132 This rule can be extended to the multi-criteria mcst-game by allocating to each player the cost vector of
 133 the edge incident upon it on the unique path between 0 and the player’s node, in the corresponding Pareto-
 134 minimum spanning tree.

135 **Example 2.1 (Continued).** In the example above, we can allocate $(2,6)^t \in V(N)$ by the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix}$
 136 that is in the preference core. This allocation has been obtained applying Bird’s rule to the Pareto-minimum
 137 tree given in the following figure.



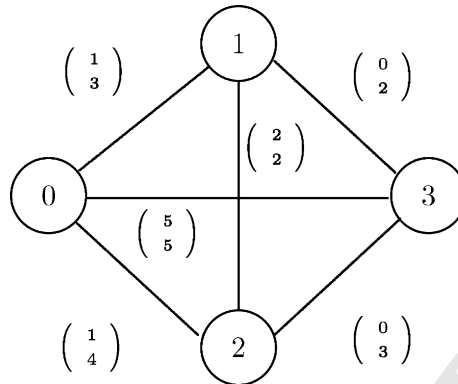
139 Nevertheless we can not divide among the players the vector $z^N = (4,5)^t \in V(N)$ by an allocation in the
 140 preference core because the game $(N, v_1^{z^N})$ given by:

S	$\{1\}$	$\{2\}$	$\{3\} \{2,3\}$	$\{1,2\}$	$\{1,3\}$	N
$v_1^{z^N}(S)$	1	1	1	2	2	4

142 is not balanced.

143 Unfortunately extended Bird’s cost allocation scheme is not, in general, a way to obtain allocations in
 144 the preference core as we show in the following example.

145 **Example 3.1.** Consider the complete graph below.



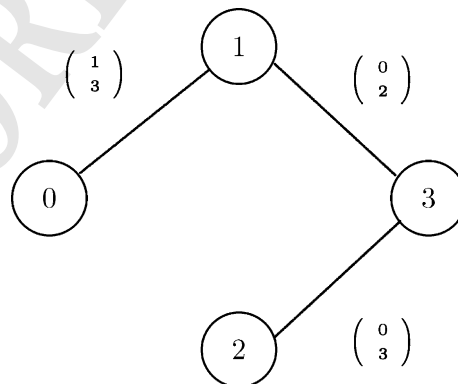
147 The bi-criteria minimum cost spanning tree game associated with the graph is:

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	N
$V(S)$	$\left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 1 \\ 7 \end{pmatrix} \right\}$	$\left\{ \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \end{pmatrix} \right\}$

149 Consider $z^N = (1, 8)^t \in V(N)$. Applying Definition 3.2 we obtain that the scalar component games $(N, v_l^{z^N})$,
 150 $l = 1, 2$ are given by:

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	N
$v_1^{z^N}(S)$	1	1	1	2	1	1	1
$v_2^{z^N}(S)$	3	4	5	5	5	7	8

153 Notice that both games are balanced (Owen, 1995). Therefore the vector $(1, 8)^t$ can be divided among the
 154 three players by allocations in the preference core. For instance, $X = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 3 \end{pmatrix} \in C(N, V; \leq)$. However
 155 Bird's tree allocation, $X = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 3 & 2 \end{pmatrix}$ associated with the following Pareto-minimum spanning tree



158 is not in the preference core because the coalition $\{1,2\}$ obtains $X^{\{1,2\}} = (1, 6)^t$ and $X^{\{1,2\}} \not\leq V(\{1,2\})$ does
 159 not hold.

160 3.2. Dominance core

161 To impose that coalitions only accept allocations in which they pay less than any of the worths given by
 162 the characteristic set is too strong. Now suppose that each coalition S will not accept to pay a total cost
 163 greater than any of the guaranteed costs in $V(S)$. This will be denoted in the following by $X^S \not\geq V(S)$ and
 164 means $X^S \not\geq z^S \quad \forall z^S \in V(S)$, that is, there does not exist $z^S \in V(S)$ such that $X_j^S \geq z_j^S \quad \forall j = 1, 2, \dots, k$,
 165 $X^S \neq z^S$.

166 **Definition 3.3.** The dominance core of (N, V) is the set of allocations, $X \in I^*(N, V)$, such that
 167 $X^S \not\geq V(S) \quad \forall S \subset N$. We will denote this set as $C(N, V; \not\geq)$.

168 Bird's cost allocation scheme leads always to an element in the scalar core. In the following result we
 169 prove that any vectorial Bird's cost allocation belongs to the dominance core.

170 **Theorem 3.2.** Let T_N be a Pareto-minimum cost spanning tree of a complete graph with associated cost vector
 171 $z^N \in V(N)$. Then the corresponding vectorial Bird's cost allocation is in the dominance core.

172 **Proof.** Let X be the vectorial Bird's allocation of the Pareto-minimum cost spanning tree T_N . It is clear that
 173 $X^N = z^N \in V(N)$ and therefore $X \in I^*(N, V)$. For a non-empty coalition $S \subset N$ let T_S be a Pareto-minimum
 174 cost spanning tree on the graph G that contains $S \cup \{0\}$. Construct a spanning tree \hat{T}_N for N_0 as follows.
 175 Add all the nodes in $N \setminus S$ to T_S and for each $i \in N \setminus S$ add the edge incident upon i on the unique path from
 176 0 to i in T_N . Then \hat{T}_N , constructed in this way is a spanning tree for N_0 . So, if $z^S(T_S) \in V(S)$ is the vector of
 177 costs associated to T_S , $z(\hat{T}_N) = z^S(T_S) + X^{N \setminus S}$ is the vector of costs associated to the spanning tree \hat{T}_N . Then
 178 $z^N \not\geq z(\hat{T}_N)$, indeed, $z^N = X^N = X^S + X^{N \setminus S} \not\geq z^S(T_S) + X^{N \setminus S}$. Then $X^S \not\geq z^S(T_S)$. As T_S is any minimum cost
 179 spanning tree on G for S_0 , we can conclude that $X^S \not\geq V(S)$. \square

180 **Example 2.1 (Continued).** In this example, we show that vector $(4, 5)^t \in V(N)$ can not be allocated among
 181 the players by an allocation in the preference core. Nevertheless, as we have seen above, it can be allocated
 182 by Bird's cost allocation that is an element of the dominance core: $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix} \in C(N, V; \not\geq)$.

183 Apart from Bird's cost allocations, there are many other allocations in the dominance core. The question
 184 that arises is whether we can provide a method to obtain, easily, some of them and whether all the vectors
 185 of costs in $V(N)$ can be allocated with this method.

186 A way to deal with this problems is using topological orders in \mathbb{R}^k . As was shown in Ehrgott (2000),
 187 every Pareto optimal spanning tree of a graph is the conventional mcost using the appropriate topological
 188 order. Unfortunately, any topological order may not result in a Pareto optimal tree. Nevertheless, re-
 189 stricting to topological orders induced by an increasing linear utility function, the mcost obtained from the
 190 weighted graph is a Pareto optimal tree.

191 In what follows we are going to consider only topological orders defined by an strictly increasing linear
 192 utility function $u : \mathbb{R}^k \rightarrow \mathbb{R}$. In this situation a player or coalition prefers a vector of cost a to another vector
 193 b if $u(a) \leq u(b)$.

194 In order to find a condition that permits to divide among the players a total cost $z^N \in V(N)$ accordingly
 195 with a given strictly increasing linear utility function, u , we will define the following scalar game (N, v_u) :

$$v_u(\emptyset) = 0, \quad v_u(S) = \min_{z^S \in V(S)} u(z^S), \quad \forall S \subseteq N, S \neq \emptyset. \quad (1)$$

197 Using Bird's rule in the scalar game (N, v_u) , we can construct dominance core allocations for some
 198 $z^N \in V(N)$.

199 Let $x = (x^1, \dots, x^n)$ be the Bird's allocation of the game (N, v_u) . This vector allows us to give a pro-
 200 portional allocation of $z^N \in V(N)$ defined by

$$X = (X^1, \dots, X^n), \quad \text{where } X^i = \frac{x^i}{u(z^N)} z^N \quad \forall i \in N. \tag{2}$$

202 The following theorem states a condition ensuring that these allocations belong to the dominance core.

203 **Theorem 3.3.** *If $v_u(N) = u(z^N)$, $z^N \in V(N)$ then the proportional allocation X defined in (2) belongs to the*
 204 *dominance core of (N, V) .*

205 **Proof.** As u is a linear function, (N, v_u) is the mcst-game associated to the graph G with scalar cost on his
 206 edges, $u(e^{ij})$. Let x be the allocation of $u(z^N)$ obtained by Bird's rule applied in (N, v_u) . Then x is a core
 207 allocation for the scalar game. Let $X \in \mathbb{R}^{k \times n}$ be the proportional allocation defined in (2). It is straight-
 208 forward that $X^N = \sum_{i=1}^n \frac{x^i}{u(z^N)} z^N = z^N$ and then $X \in I^*(N, V)$. Moreover, if we assume that $X \notin C(N, V; \neq)$
 209 then, there exists a coalition $S \subset N$ and a vector $w^S \in V(S)$ such that $X^S \geq w^S$, $X^S \neq w^S$. Then, as u is linear
 210 strictly increasing utility function and as x belongs to the core of the game (N, v_u) ,

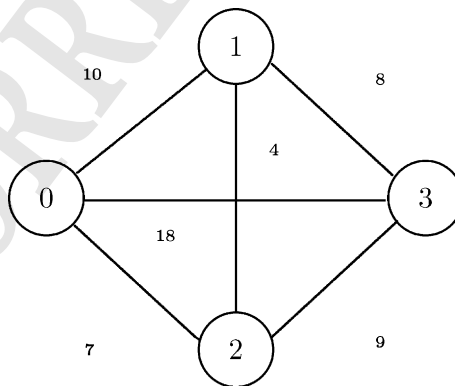
$$\min_{z^S \in V(S)} u(z^S) \leq u(w^S) < u(X^S) = \sum_{i \in S} u(X^i) = \frac{\sum_{i \in S} x^i}{u(z^N)} u(z^N) = x^S \leq v_u(S) = \min_{z^S \in V(S)} u(z^S).$$

212 This is a contradiction. \square

213 **Example 2.1** (Continued). Suppose that the strictly increasing linear utility function, u , used to compare the
 214 worth of the coalitions consist of giving triple importance to the second criterion, that is, the utility of
 215 vector a is $u(a) = a_1 + 3a_2$. Then, the scalar game (N, v_u) is:

S	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	N
$v_u(S)$	10	7	6	11	18	16	19

217 In this case, $v_u(N) = u((4, 5)')$, the mcst for the weighted graph is the Pareto-optimal tree associated to
 218 $z^N = (4, 5)'$ and (N, v_u) is the mcst-game associated to the weighted graph.



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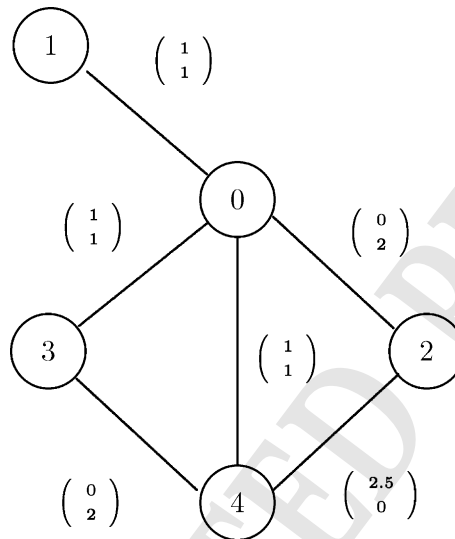
221 Therefore Bird's cost allocation $x = (4, 7, 8)$ is in the core of (N, v_u) . Then the proportional allocation

222 $X = \begin{pmatrix} \frac{16}{19} & \frac{28}{19} & \frac{32}{19} \\ \frac{20}{19} & \frac{35}{19} & \frac{40}{19} \end{pmatrix} \in C(N, V; \neq).$

223 Notice that the allocations obtained with this method are different from those obtained applying the
 224 vectorial Bird's rule to the corresponding Pareto > mcst-game.

225 Unfortunately is not always possible to find a topological order, defined through an increasing linear
 226 utility function, for every $z^N \in V(N)$.

227 **Example 3.2.** Consider the complete graph where all the links $\{i, j\}$ not drawn below have costs $(5, 5)^t$:



229 Three Pareto-minimum cost spanning trees for this graph are

$$T_1 = \{\{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 4\}\}, \text{ with associated cost } (3, 5)^t,$$

$$T_2 = \{\{0, 1\}, \{2, 4\}, \{0, 3\}, \{5.5, 3\}\}, \text{ with associated cost } (5.5, 3)^t,$$

$$T_3 = \{\{0, 1\}, \{2, 4\}, \{0, 3\}, \{3, 4\}\}, \text{ with associated cost } (4.5, 4)^t.$$

233 Then $z^N = (4.5, 4)^t$ is a solution of $\min_{z^N \in V(N)} u(z^N)$ for no increasing linear utility function u .

234 **4. Final remarks**

235 This paper contains the methodological developments of the multi-criteria mcst-games through the
 236 analysis of their dominance and preference cores. However, there is enough room for further research. An
 237 interesting avenue of research should aim to construct new core concepts being consistent with the pref-
 238 erence structure of the decision-maker. In particular, this scheme would lead to construct interactive
 239 procedures, which could be used by the decision-maker in decision support systems when the original core
 240 sets do not reduce to a singleton.

241 Another issue that deserves future research is the application of this model to real problems. There are
 242 challenging economical models in the new Europe that clearly fall into this category: design and operation
 243 of a common oil pipeline network, operation of existing inter-Europe electrical distribution network, ...
 244 These topics are currently under research and will be considered in a follow up paper.

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