Optimized Direct Power Control Strategy using Output Regulation Subspaces and Pulse Width Modulation

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Abstract – In this paper, a Direct Power Control (DPC) using Output Regulation Subspaces (ORS) for a three-phase two-level converter is presented. An optimal controller design and the use of optimized modulation techniques permit to improve the system performance minimizing the necessary grid-connection inductance. ORS plus proportional controllers are proposed for the generation of the reference vector to decrease active and reactive power errors and to achieve unity power factor. The computational cost of the proposed control is really low and all the calculations can be done online allowing its implementation in low cost microprocessors. Simulation and experimental results are shown in order to illustrate the good performance of the proposed control strategy.

I. INTRODUCTION

Power rectifiers are well-known power systems for industrial applications and the control of these power systems is actually one objective of researchers. One of the most efficient control strategies is Direct Power Control (DPC), this control strategy is based on power errors values and voltage vector position [1] or Virtual-Flux vector position [2]. Applications based on DPC have demonstrated that it is a simple and efficient control strategy achieving good dynamic performance and near unity power factor. However, grid inductances are still too large increasing the cost, size and weight of the total system and reducing dynamics and operation range of PWM rectifier [3]. A controller design based on Output Regulation Subspaces (ORS) can overcome this drawback optimizing the power system behaviour. In this paper, firstly the discrete model for a three-phase two-level rectifier is presented. Fig. 1 shows a three-phase, two-level power converter. Secondly, basic concepts of ORS are introduced and finally a DPC controller design based on ORS is presented.

II. DISCRETE MODEL

In this section a discrete model of the system is shown [4]. For a given DC-Link voltage the phase voltages in r, s and t are defined by the state of the power switches. The values \( u_r, u_s \) and \( u_t \) define the state of the power transistors in the r, s and t phase respectively. The value ‘1’ means upper switch of the phase is switched on and downer switch of the phase is switched off, and value ‘-1’ means upper switch of the phase is switched off and downer switch of the phase is switched on.

The equations of the system can be written compactly introducing the following vectors variables.

\[
y(t) = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}, \quad i(t) = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \quad \delta(t) = \begin{bmatrix} u_r \\ u_s \\ u_t \end{bmatrix},
\]

(1)

The equations that describe the three-phase power rectifier are (2) and (3) where the matrix B is defined by (4).

\[
y(t) = L \cdot \frac{di(t)}{dt} + B\delta(t) \cdot \frac{V_{dc}}{2},
\]

(2)

\[
C \cdot \frac{dV_{dc}}{dt} = \frac{1}{2} \cdot \delta(t)^T i(t) - i_{Load},
\]

(3)

\[
B = \begin{bmatrix} +2 & -1 & -1 \\ -1 & +2 & -1 \\ -1 & -1 & +2 \end{bmatrix}
\]

(4)

Now it can be supposed that the rectifier is connected to a resistive load \( R_L \). Using a mathematical manipulation, equation (3) is transformed to (5).

\[
C \cdot \frac{d}{dt} \left( \frac{V_{dc}^2}{2} \right) = \frac{V_{dc}}{2} \delta(t)^T i(t) - \frac{V_{dc}^2}{R_L},
\]

(5)

Finally the model can be transformed into stationary coordinates using the matrix transformation A (6), which has
a pseudo-inverse transformation defined by $A = A^\dagger$. Therefore equation (2) and (5) can be expressed using coordinates resulting equations (7) and (8).

$$A = \begin{bmatrix} \frac{2}{\sqrt{3}} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{2}{\sqrt{3}} & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{bmatrix},$$  \hspace{1cm} \text{(6)}$$

$$v_{\alpha\beta} = L \cdot \frac{di_{\alpha\beta}}{dt} + \frac{V_{dc}}{2} \delta_{\alpha\beta},$$  \hspace{1cm} \text{(7)}$$

$$C \cdot \frac{d}{dt} \left( \frac{V_{dc}^2}{2} \right) = \frac{V_{dc}}{2} \delta_{\alpha\beta} i_{\alpha\beta} - \frac{V_{dc}^2}{R_L},$$  \hspace{1cm} \text{(8)}$$

Equations (7) and (8) are the discrete model of the power converter in coordinates, in these equations has been defined as (9), and it is assumed that $V_{dc}$ is always positive. Additionally it is used the fact that $ABA^{-1} = I_2$, with $I_2$ a 2x2 identity matrix.

$$\delta_{\alpha\beta} = A\delta(t) = \begin{bmatrix} u_{\alpha} \\ u_{\beta} \end{bmatrix},$$  \hspace{1cm} \text{(9)}$$

The possible switch positions (the state vectors) in abc and alpha-beta frames are summarized in Table I and they are represented in the alpha-beta frame in Fig. 2.

![Switch positions in alpha-beta frame](image)

**Fig. 2.** Switch positions in alpha-beta frame

### III. OUTPUT REGULATION SUBSPACES (ORS)

The Output Regulation Subspaces (ORS) applied to a power converter was presented in [5]. The ORS split the alpha-beta frame in four quadrants taking into account the active and reactive instantaneous power first derivate sign [6]. In this paper, ORS are used to split the alpha-beta frame in regions where the active and reactive instantaneous power first derivate values are enclosed to certain constants. For this purpose the values of active and reactive instantaneous power are defined in (10) and (11) where matrix $J$ is (12).

$$y_1 = p = i_{\alpha\beta}^T \cdot v_{\alpha\beta}$$  \hspace{1cm} \text{(10)}$$

$$y_2 = q = i_{\alpha\beta}^T \cdot Jv_{\alpha\beta}$$  \hspace{1cm} \text{(11)}$$

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$  \hspace{1cm} \text{(12)}$$

Introducing (7) inside (10), equations from (13) to (16) are derived, and the active instantaneous power first derivate is expressed as (16).

$$L\dot{y}_1 = L i_{\alpha\beta}^T \cdot v_{\alpha\beta} + L i_{\alpha\beta}^T \cdot \dot{v}_{\alpha\beta},$$  \hspace{1cm} \text{(13)}$$

$$L\dot{y}_1 = v_{\alpha\beta}^T \left( v_{\alpha\beta} - \frac{V_{dc}}{2} \delta_{\alpha\beta} \right) + L i_{\alpha\beta}^T \cdot \omega Jv_{\alpha\beta},$$  \hspace{1cm} \text{(14)}$$

$$L\dot{y}_1 = v_{\alpha\beta}^T \left( v_{\alpha\beta} - \frac{V_{dc}}{2} \delta_{\alpha\beta} \right) + L\omega y_2,$$  \hspace{1cm} \text{(15)}$$

$$L\dot{y}_1 = v_{\alpha\beta}^T \left( 1 + L\omega y_2 \right) v_{\alpha\beta} - \frac{V_{dc}}{2} \delta_{\alpha\beta},$$  \hspace{1cm} \text{(16)}$$

Similar as below, introducing (7) inside (11), equations from (17) to (20) are derived, and the reactive instantaneous power first derivate is expressed as (20).

$$L\dot{y}_2 = L i_{\alpha\beta}^T \cdot Jv_{\alpha\beta} + L i_{\alpha\beta}^T \cdot J\dot{v}_{\alpha\beta},$$  \hspace{1cm} \text{(17)}$$

$$L\dot{y}_2 = \left( v_{\alpha\beta} - \frac{V_{dc}}{2} \delta_{\alpha\beta} \right) Jv_{\alpha\beta} + L i_{\alpha\beta}^T \cdot J\omega Jv_{\alpha\beta}$$  \hspace{1cm} \text{(18)}$$

$$L\dot{y}_2 = -\frac{V_{dc}}{2} \delta_{\alpha\beta} Jv_{\alpha\beta} - L\omega y_1,$$  \hspace{1cm} \text{(19)}$$

$$L\dot{y}_2 = v_{\alpha\beta}^T J^T \left( -\frac{V_{dc}}{2} \delta_{\alpha\beta} - L\omega y_1 \right) \frac{Jv_{\alpha\beta}}{v_{\alpha\beta}}.$$  \hspace{1cm} \text{(20)}$$
Now in order to calculate ORS, and ORS, (16) and (20) are transformed in (27) and (28) respectively.

\[
L\dot{y}_1 = k_1,
\]

\[
L\dot{y}_1 = v_{\alpha\beta}^T \left( 1 + \frac{L_\omega y_2}{v_{\alpha\beta}} \right) v_{\alpha\beta} - \frac{V_{dc}}{2} \delta_{\alpha\beta} = k_1,
\]

\[
L_\omega y_2 + \frac{V_{dc}}{2} v_{\alpha\beta}^T \delta_{\alpha\beta} = k_1,
\]

\[
v_{\alpha\beta}^T \delta_{\alpha\beta} = \frac{2}{V_{dc}} \left( L_\omega y_2 + \frac{V_{dc}}{2} \right)^2 - k_1,
\]

Equation (25) represents the set of values of \( y \) that makes \( \dot{y}_1 \) equal to the constant \( k_1 \), and splits the alpha-beta frame in two regions, values of above ORS, make \( \dot{y}_1 \) smaller than \( k_1 \), while values of below ORS, make \( \dot{y}_1 \) larger than \( k_1 \). In the same way, \( \dot{y}_2 \) can be determined and its equation is written in (26). Alpha-beta frame regions where the active and reactive instantaneous power first derivate are enclosed to certain values are shown in Fig. 3.

\[
ORS_{y_1}(\dot{y}_1 = k_1) = \left\{ \frac{2}{V_{dc}} \left( L_\omega y_2 + \frac{V_{dc}}{2} \right) v_{\alpha\beta} + c_1 J_{\alpha\beta} \right\}
\]

\[
ORS_{y_2}(\dot{y}_2 = k_2) = \left\{ -\frac{2}{V_{dc}} \left( L_\omega y_1 + k_2 \right) J_{\alpha\beta} + c_2 J_{\alpha\beta} \right\},
\]

Equation (25) represents the set of values of \( y \) that makes \( \dot{y}_1 \) equal to the constant \( k_1 \), and splits the alpha-beta frame in two quadrants as it is shown in Fig. 4. Each zone is characterized by the sign of the active and reactive instantaneous power first derivate, so when the system is working inside one of them the active and/or the reactive instantaneous power can increase or decrease according which selection area is. The intersection point between the two straight lines is calculated in (29). ORS represents the equilibrium point of the system in steady state due to the fact that in this point the active and reactive power demanded by the power converter to the power supply remains constants.

\[
ORS_{\alpha\beta} = \frac{2}{V_{c}} \left( 1 + \frac{\omega L_\omega y_2}{v_{\alpha\beta}^2} \right) v_{\alpha\beta} - \frac{2}{V_{c}} \left( \frac{\omega L_\omega y_1}{v_{\alpha\beta}^2} \right) J_{\alpha\beta},
\]

These expressions represent two straight lines that split the alpha-beta frame in four quadrants as it is shown in Fig. 4. The vector selection is made through a Look up Table (LUT) among the eight possible states in order to maintain the DC-Link voltage constant, and to keep the unity power factor. The vector selection is made through a Look up Table (LUT) where the input variables are the voltage grid vector position.

Fig. 3. ORS representation in alpha-beta frame

When constants \( k_1 \) and \( k_2 \) are equal to zero, equations (25) and (26) are transformed in (27) and (28) respectively.
and the active and reactive instantaneous power errors. Using ORS concept, a DPC controller can be designed to overcome the main drawback of high inductance values. The basic selection algorithm is based on the ORS and the values of active instantaneous power error and reactive instantaneous power error defined as:

\[
p^{\text{error}} = p - p^\text{ref},
\]
\[
q^{\text{error}} = q - q^\text{ref},
\]  
(30)

where active instantaneous power reference \( p^\text{ref} \) is calculated in the external control loop through a PI controller and reactive instantaneous power reference \( q^\text{ref} \) is zero in order to achieve unity power factor. The power switches state are chosen among the discrete possible states inside a selection area, the ORS split the alpha-beta plane, and the selection area is chose as a function of power errors sign. Table II shows the selection area as a function of power errors sign.

<table>
<thead>
<tr>
<th>( p^{\text{error}} )</th>
<th>( q^{\text{error}} )</th>
<th>Selection Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>A3</td>
</tr>
<tr>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>A2</td>
</tr>
<tr>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>A4</td>
</tr>
<tr>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>A1</td>
</tr>
</tbody>
</table>

This basic selection algorithm has the following drawbacks:

- The high gain controller and consequently high inductance values necessary to achieve the system operation.
- There is more than one possible state vector inside the selection area.
- The power switches state is not always defined inside the selection area.

As an example, Fig. 5 shows a possible situation where in area A2 there are three possible state vectors and in area A4 there is not anyone.

IV. OPTIMIZED ORS USING PROPORTIONAL CONTROLLERS

The main drawback of DPC is the high gain of the controller and, as consequence, the values of the grid connection inductances have to be very large to attenuate the current ripple (usually around 10 mH) increasing the cost, size and weight of the system. In order to reduce these inductance values, [7] and [8] propose to connect the power converter through a LCL filter. This solution has the drawback of the filter resonance so has to be well studied.

Classical DPC controllers using ORS have the behaviour of a bang-bang controller and in this paper hysteresis bands to reduce the controller gain is used. The proposed controller has not a constant switching frequency and modulation techniques as Pulse Width Modulation (PWM) and Space Vector Modulation (SVM) with constant switching frequency have to be used [6][9][2][10].

The proposed DPC controller considers the sign and the magnitude of \( p^{\text{error}} \) and \( q^{\text{error}} \). The reference vector generated by the controller can be determined using (31) considering that \( k_1 \) and \( k_2 \) are always positive. The selection area where the reference vector is pointing to depends on the signs of \( p^{\text{error}} \) and \( q^{\text{error}} \) and the reference vector magnitude depends of \( k_1 \) and \( k_2 \) values.

\[
u_{dB} = ORS_{dB} + \text{sign}(p^{\text{error}}) \cdot k_1 \cdot v_{dB} + \text{sign}(q^{\text{error}}) \cdot k_2 \cdot Jv_{dB},
\]  
(31)

In order to optimize the control strategy, \( p^{\text{error}} \) and \( q^{\text{error}} \) magnitudes are taken into account and \( k_1 \) and \( k_2 \) expressions can be presented as proportional controllers as is shown in (32).

\[
\text{sign}(p^{\text{error}}) \cdot k_1 = k_p \cdot p^{\text{error}}
\]
\[
\text{sign}(q^{\text{error}}) \cdot k_2 = k_q \cdot q^{\text{error}},
\]  
(32)

Finally, the expression for the determination of the reference vector is presented in (33).

\[
u_{dB} = ORS_{dB} + k_p \cdot p^{\text{error}} \cdot v_{dB} + k_q \cdot q^{\text{error}} \cdot Jv_{dB},
\]  
(33)

Using these expressions, the reference vector is located in the appropriated selection area and it has the necessary magnitude in order to minimize active and reactive power errors. Once the reference vector is generated, classical bang-bang DPC strategy is avoided using PWM or SVM technique to modulate it. This fact can lead to use some state vector
located in other selection area, but the modulated vector is, in average over a switching period, in the appropriate selection area achieving the control objective. In this paper, PWM technique is used to generate the switching signals for the power devices.

V. SIMULATION AND EXPERIMENTAL RESULTS

Simulations using PSCAD® have been carried out in order to illustrate the good performance of the proposed control technique. It is considered the system represented in Fig. 1 where the electrical parameters for the simulation model are shown in Table III.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>ELECTRICAL PARAMETERS FOR THE SIMULATION MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid inductance</td>
<td>3 mH</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>10 KHz</td>
</tr>
<tr>
<td>DC-Link capacitor</td>
<td>5500 µF</td>
</tr>
<tr>
<td>Resistive load</td>
<td>50 ohm</td>
</tr>
<tr>
<td>DC-Link Reference Voltage</td>
<td>750 V</td>
</tr>
<tr>
<td>Phase-to-neutral Voltage</td>
<td>230 V</td>
</tr>
</tbody>
</table>

In the simulation experiment, the power system is suddenly connected to the resistive load. Dynamics for the DC-Link voltage and for the phase voltage and current are included in the simulation results. Firstly, classical DPC technique using ORS but not including the proportional controllers and using LUT for the state vector selection is assumed. Simulation results for this control design are shown in Fig. 6. The same simulation experiment has been carried out considering the proposed DPC controller design obtaining the results shown in Fig. 7. It must be noticed that using the proposed DPC controller the current ripple is drastically minimized achieving similar dynamics in the DC-Link voltage obtaining near unity power factor in both cases. With the proposed DPC controller, the high gain present in classical DPC is made smooth obtaining as result a current ripple reduction that allows reducing the inductance value for the grid connection.

Experimental results have been obtained using a 30kW two-level three-phase rectifier using a control board with TMS320VC33 DSP. The electrical parameters for the experiment are shown in Table IV. Simulation results are also shown in order to validate the comparison between classical DPC and the proposed DPC carried out in Fig. 6 and Fig. 7. Simulation results considering the experimental electrical parameters of Table IV are shown in Fig. 8 and experimental results for the same parameters are shown in Fig. 9 and Fig. 10 showing the DC-Link voltage dynamics and the phase current and phase voltage. The experiment is carried out suddenly connecting the resistive load to the controlled rectifier. Proposed DPC technique achieves the DC-Link voltage control resulting phase currents with very low ripple using 0.8mH of grid inductance and even improving the results of classical DPC using 3mH grid inductance.

Finally, the harmonic content of the phase currents is shown in Fig. 11. It must be noticed that THD factor is equal to 3%.

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>ELECTRICAL PARAMETERS FOR THE EXPERIMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid inductance</td>
<td>0.8 mH</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>11.2 KHz</td>
</tr>
<tr>
<td>DC-Link capacitor</td>
<td>7050 µF</td>
</tr>
<tr>
<td>Resistive load</td>
<td>60 ohm</td>
</tr>
<tr>
<td>DC-Link Reference Voltage</td>
<td>750 V</td>
</tr>
<tr>
<td>Phase-to-neutral Voltage</td>
<td>230 V</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS

DPC strategies are simple and efficient control strategies applied to power systems as power rectifiers allowing the use of low cost microprocessors for their implementation. However, some drawbacks associated to the high grid connection inductance value are present in this type of control techniques. The proposed DPC strategy based on ORS and using proportional controllers permits to make smooth the high gain of previous DPC techniques and consequently, permits to decrease the grid connection.
inductance value reducing the cost, size and weight of the total system and increasing the operation range of the power converter and improving the dynamics of the system. Simulations have been carried out to illustrate the good performance of the proposed control technique.

Fig. 8. Simulation results using proposed DPC considering electrical parameters presented in Table IV

Fig. 9. Experimental results showing the DC-Link voltage dynamics when resistive load is connected using proposed DPC

Fig. 10. Experimental results showing the phase current (CH1 10A/div) and phase voltage (CH2 100V/div) using proposed DPC

Fig. 11. Obtained harmonic content of the phase currents (dB)

VII. REFERENCES


