Abstract—This work presents a Model Predictive Control (MPC) strategy for a Voltage Source Inverter (VSI) used in a three-phase Uninterruptible Power Supply (UPS) for critical loads. An MPC using continuous variables is proposed for solving this problem and the output of the controller is used as the reference voltage to be generated by a PWM modulator. The solution of this unconstrained MPC gives rise to an explicit solution that can be computed beforehand, allowing the prediction horizon to be easily extended. Therefore, the effect of the length of the prediction horizon over the system performance is also evaluated in the paper. This study addresses how this parameter should be chosen to minimize the error between the actual and the desired output voltage. The proposed control strategy has been tested on a simulated model of a UPS supplying a three-phase resistor load. This model has been developed using MATLAB/Simulink with PSIM software. The simulation results show that proposed continuous MPC controller achieves high performance and high degree of robustness.

I. INTRODUCTION

Several industrial and critical applications include loads that need constant power supply to ensure the correct working of the system, [1]. These critical loads are usually supplied by Uninterruptible Power Supplies (UPSs), [2], where a Voltage Source Inverter (VSI) provides a high quality output sinusoidal voltage to the loads. To produce the output voltage without high order harmonic components the loads are connected to the VSI through an LC filter that removes these components. As a counterpart, this filter increases the complexity of the system controller design. Several control strategies have been developed to generate the output voltage of the VSI, including PI, resonant, repetitive, deadbeat and predictive controllers [3]-[13].

Model Predictive Control (MPC) is a well-known control strategy that has been used in several fields of engineering. Although MPC was born in the framework of industrial process control, nowadays it is been applied to a wide variety of fields such as energy, bioengineering, robotics or aerospace. MPC presents several advantages over other methods, [14]. The MPC paradigm is based on selecting the best amongst all feasible input sequences over a future horizon according to some criterion. The first input of this sequence is applied to the plant and the scheme repeated in a receding horizon fashion at every sampling time, when new state information is available. MPC is possibly the most general way of posing the optimal control problem in the time domain. MPC designates an ample range of control methods which make explicit use of a model of the system to obtain the control signal by minimizing an objective function. MPC presents several features that makes it suitable for the control of power converters. Apart from being intuitive and easy to understand, constraints, nonlinearities and the multivariable case can easily be included in the formulation. Since an open-loop optimal problem is solved at each sampling instant, the computational cost is high compared to a classic control scheme. This point is of crucial important in the case of power converters, therefore different varieties of MPC have been proposed in literature to cope with this problem [15].

One way of reducing the computational effort needed to solve the optimal problem on-line is to take advantage of the inherent discrete nature of power converters, which have a finite number of switching states. This gives rise to the approach known as Finite Control Set MPC (FS-MPC) [16], where the possible control actions (switching states) are finite. This method reduces the MPC optimization problem to the prediction of the system behaviour only for those possible switching states. This approach is also known as finite alphabet MPC and it has been successfully applied to a wide range of power converters [17] [18].

This extended approach for implementing MPC for power converters is currently limited to short horizons (usually 1 or 2) due to the fact that the solution is obtained evaluating a cost function that typically measures the absolute error between the predictions and the reference. The evaluation of the cost function with the finite number of control actions (n) will lead to n different costs. The method, therefore, uses the control action leading the minimum cost to control the converter.

In the case of a three-phase UPS inverter, the voltage (control action) can take 7 different values at each sampling instant (n = 7), and therefore the prediction of the system output along the horizon must be done considering that this voltage can change along the future, giving rise to multiple possibilities that can generate a combinatorial explosion if the horizon is too long. This problem is clearly depicted in [16]. Instead of evaluating the objective function for all possible values of the voltage along the control horizon, another possibility of tackling the issue is by formulating the MPC
as a hybrid problem. In this way, the problem to be solved combines continuous variables (output voltages and currents) with discrete variables (those that take only discrete values, such as switches). Hybrid MPC is difficult to implement for fast systems, since Mixed Integer Programming (MIP) must be used [19]. There are several applications of Hybrid MPC to electronic converters, as in the case of DC/DC boost converters [20] but there are only few applications of Hybrid MPC to inverters, [21]. In this case, the number of decision variables is higher and other approach is needed.

An interesting solution method is multi-parametric programming, which solves the MPC problem off-line for every system state in a bounded set and thus only requires a look up operation at runtime [22], [23]. However, the obtained explicit solution may be excessively complex for medium to large scale systems, in which case approximate explicit solutions can be computed. An application of this technique to a PWM Inverter With an LCL Filter is presented in [24]. The authors propose piecewise affine models that account for the switched behavior of the converter. Based on these improved models, an explicit MPC scheme is derived in order to provide a fast response, making it very suitable for applications, such as active filtering, where a large bandwidth is required.

When the explicit solution is not appropriate, on-line optimization methods can be used, the two main proponents being Interior Point and Active Set methods. Dedicated implementations of these methods for MPC exist, for instance, [25] reports a fast implementation of an interior point method where a significant speed-up is gained by exploiting the structure of the involved matrices as well as by early stopping and warm-starting from a solution obtained at the previous time-step.

The paper [26] presents a practical implementation of the fast gradient method for the control of an AC-DC power converter, with computation times as small as a few tens of μs making the approach ideal for power electronics applications. Unlike explicit MPC, the exact constrained finite horizon control problem can be solved taking into account the time varying nature of the input constraints. An application of real-time optimization for boost converters can be found in [27]. This method applies results of recently developed hybrid optimal model predictive control to determine switching through real-time minimization of a user-defined performance index. Similar results for a Cuk converter is shown in [28].

In spite of these promising results, the on-line solution of the hybrid optimization problem is still an open issue. The reduction of the computational effort is the biggest challenge, especially for long horizons. However, the problem can be tackled from another point of view, which can lead to a simpler solution, as shown in this paper. The method proposed here provides an explicit solution of the unconstrained MPC strategy, whose complexity (an therefore computing time) is almost independent of the prediction horizon. Notice that most of the computation of this explicit solution can be done beforehand and the computational cost of the control law is small, as will be shown in section III.

\[ x_{\alpha\beta} = A^{\alpha\beta} x_{\alpha\beta} + B^{\alpha\beta} \pi_{I,\alpha\beta} + B^{\alpha\beta} T_{O,\alpha\beta} \]

\[ \pi_{I,\alpha\beta} = \{ v_{I,\alpha\beta} 0 \}^T \]

\[ i_{O,\alpha\beta} = \{ 0 0 i_{O,\alpha\beta} \}^T. \]

**TABLE I**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{C,abc} )</td>
<td>Output filter capacitor voltage vector</td>
</tr>
<tr>
<td>( i_{L,abc} )</td>
<td>Output filter inductor current vector</td>
</tr>
<tr>
<td>( v_{I,abc} )</td>
<td>VSI output voltage vector</td>
</tr>
<tr>
<td>( i_{O,abc} )</td>
<td>Output load current vector</td>
</tr>
<tr>
<td>( S_{abc} )</td>
<td>Switching vector</td>
</tr>
<tr>
<td>( S_p )</td>
<td>Switching functions</td>
</tr>
<tr>
<td>( L )</td>
<td>Output filter inductance</td>
</tr>
<tr>
<td>( C )</td>
<td>Output filter capacitance</td>
</tr>
<tr>
<td>( \mu_{dc} )</td>
<td>dc-link voltage</td>
</tr>
</tbody>
</table>

**II. SYSTEM DESCRIPTION**

A three-phase two-level power converter used as a voltage source inverter (VSI) is depicted in Fig. 1. A critical load is connected to the VSI through a LC filter in order to remove the high order harmonic components in the converter output voltage and provide a high quality sinusoidal voltage to the load. The system parameters and variables are described in Table I.

The behavior of the system is defined by the dynamic equations of the output filter inductor currents and the output filter capacitor voltages. Using the Clake’s transformation, the equations can be expressed in the stationary \( \alpha\beta \) frame as (1) and (2) respectively.

\[ i_{L,\alpha\beta} = C \frac{dv_{C,\alpha\beta}}{dt} + i_{O,\alpha\beta} \]

\[ v_{I,\alpha\beta} = L \frac{di_{L,\alpha\beta}}{dt} + v_{C,\alpha\beta}. \]

These dynamics are functions of the VSI output voltages that depend on the power semiconductors switching functions. To develop the control algorithm the system equations are expressed in the state space as (3), taking as state variables the output filter inductor currents and capacitor voltages (4) and defining new vectors for the VSI voltages and load currents (5)-(6).
where the matrices $A_{\alpha\beta}$, $B_{\alpha\beta}$ and $B_d^{\alpha\beta}$ are calculated through (7)-(9)

$$A_{\alpha\beta} = \begin{bmatrix} 0_{2x2} & -\frac{1}{C}I_{2x2} \\ 0_{2x2} & 0_{2x2} \end{bmatrix}$$

$$B_{\alpha\beta} = \begin{bmatrix} \frac{1}{C}I_{2x2} & 0_{2x2} \\ 0_{2x2} & 0_{2x2} \end{bmatrix} ;
B_d^{\alpha\beta} = \begin{bmatrix} 0_{2x2} & 0_{2x2} \\ 0_{2x2} & -\frac{1}{C}I_{2x2} \end{bmatrix}$$

$$I_{2x2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ;
0_{2x2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$ (9)

### III. Control Strategy

In order to control the converter, a two-stage approach is used. Instead of solving a hybrid problem (that implies a great computational effort), the solution is obtained in two steps: first a continuous MPC problem is solved, which computes the continuous values of the voltage that must be provided by the converter. Then, PWM modulation is used to give the discrete values (ON/OFF) of the switches that generate the desired input voltage to the LC filter. The first stage implies a dynamic optimization while the second one is just a static relationship.

The control strategy needed for the first step uses the model of the system and tries to minimize a quadratic function that penalizes both the voltage tracking error and the control effort, using the typical MPC cost function:

$$J = \sum_{j=1}^{N}(\hat{y}(t+j|t) - w(t+j))^2 + \sum_{j=1}^{N_u}(\lambda(j) (\Delta u(t+j-1))^2).$$ (10)

Where $\hat{y}$ are the predicted outputs (voltage), $w$ are the references to be tracked, $u$ are the control actions and $\lambda$ is the control weighting factor, that penalizes abrupt changes in the control actions. $N$ is the prediction horizon and $N_u$ is the control horizon. Since in this paper operational constraints are not considered, the solution of the dynamic optimization problem can be obtained in an explicit form [14]:

$$u = (G^T G + \lambda I)^{-1} G^T (w - F),$$ (11)

being $G$ the dynamic matrix and $F$ the free response, as described in [30].

Notice that, although the computation of the control action implies a matrix inversion, this can be done off-line, since the matrix values are fixed and known. The size of the matrix to be inverted is $N_u \times N_u$, so the influence of the prediction horizon ($N$) on the computational burden is negligible. Therefore, the horizons can be as long as necessary and the controller can be tuned to see the effect of horizons and control effort on the system performance, as done in section IV.

An input-output formulation of MPC has been used. Therefore, a discrete input-output model of the system described in the previous section has been obtained.

Once the dynamic optimization has been performed and therefore the continuous values of the input voltages are known, the switch positions (ON/OFF) are obtained by a static transformation in the PWM.

### IV. Simulation Results

The proposed MPC strategy has been simulated using Matlab/Simulink together with PSIM to depict the system for a balanced resistive load. A sinusoidal reference phase voltage

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Output filter inductance</td>
<td>1.2 mH</td>
</tr>
<tr>
<td>$C$</td>
<td>Output filter capacitance</td>
<td>20 $\mu$F</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Sampling period</td>
<td>50 $\mu$s</td>
</tr>
<tr>
<td>$T_{sw}$</td>
<td>Switching period</td>
<td>100 $\mu$s</td>
</tr>
<tr>
<td>$R$</td>
<td>Output load</td>
<td>20 $\Omega$</td>
</tr>
<tr>
<td>$v_{dc}$</td>
<td>dc-link voltage</td>
<td>700 V</td>
</tr>
</tbody>
</table>

Fig. 2. Simulations results for $N= 3$ and $\lambda = 0$. From top to bottom: a) Output and reference voltages. b) Output load and filter inductor currents.

Fig. 3. Simulations results for $N= 6$ and $\lambda = 0$. From top to bottom: a) Output and reference voltages. b) Output load and filter inductor currents.
of 220 V RMS and 50 Hz is applied to the system. A resistive balanced load is connected to the output of the converter. The rest of system parameters used to perform the simulation are summarized in Table II.

In order to obtain the lowest output voltage error, several values of prediction horizon N has been tested for the same control weighting factor \( \lambda \). Besides, the control horizon \( N_u \) is set to 1 for all simulations, this reduces the computational cost of the control algorithm. [30] Fig. 2a shows the reference voltages and the actual values provided by the VSI for \( \lambda = 0 \) and \( N = 3 \), while Fig. 2b plots the load and filter inductor currents associated to these output voltages. The same waveforms are represented in Fig. 3a and Fig. 3b for \( \lambda = 0 \) and \( N = 6 \). It can be noticed that in both cases the system works properly. However, when the output voltages are analyzed in detail, it can be observed that for a prediction horizon length of \( N = 3 \) the output voltages present higher ripple compared with voltages achieved with \( N = 6 \). This can be clearly appreciated in Fig. 4 where it is presented a zoom of the voltages generated by the VSI using the proposed MPC controller with these parameter values.

The influence of the prediction horizon on the performance of the proposed controller has been analyzed. For this purpose, Fig. 5 shows the error between the reference and actual voltages for different values of prediction horizon. It can be noticed that system performance depends on the tuning of the parameter \( N \). From the simulations developed the best results have been achieved with \( N = 6 \). This is related to the fact that the value of the reference along the prediction horizon is included in the cost function, which helps the controller to reduce future errors. When the value of \( N \) is too large, the controller will try to track a "mean" value of the sinusoidal reference, increasing the tracking error.

Several tests have been performed to check the robustness of proposed controlled. Two different simulations are developed, the first one considers a mismatching between the actual load and the load value used in the system model. The second one includes an Additive White Gaussian Noise (AWGN) added to the output voltage measures used to compute the control algorithm.

Fig. 6 and Fig. 7 show the results to the first situation. In Fig. 6 actual output resistor is 30% higher than parameter used in the model, whereas in Fig. 7 an increment of 100%
MPC controller achieves high performance and robustness. through simulations results, showing that proposed continuous performance of the proposed control strategy has been assessed (MPC) strategy for a Voltage Source Inverter (VSI) used in the measurements, showing the robustness of the proposed system.

load and filter inductor currents.

has been considered. In both cases, it can be noticed that proposed controller presents high performance and the output voltages track their references. In Fig. 6b and Fig. 7b can be observed how the load currents have been reduced compared with Fig. 3b due to the increased load values.

The results for the second set of simulations are presented in Fig. 8 and Fig. 9. The AWGN added to the output voltage measures have a mean value of 1% and 10% of reference peak voltage, respectively. This corresponds with approximately 3 V for Fig. 8 and 30 V for Fig. 9. It can be observed that system performance is diminished compared with Fig. 3, but it can be considered good enough taking into account the error in the measurements, showing the robustness of the proposed MPC controller.

V. CONCLUSION

This paper presents a continuous Model Predictive Control (MPC) strategy for a Voltage Source Inverter (VSI) used in a three-phase Uninterruptible Power Supply (UPS). The performance of the proposed control strategy has been assessed through simulations results, showing that proposed continuous MPC controller achieves high performance and robustness.

The proposed controller output is a continuous voltage that is used as the reference voltage to be generated by a PWM modulator. This allows to achieve constant switching frequency of the power converter and makes the design of the output LC filter easier. On the other hand, the proposed strategy permits to extend easily the prediction horizon length N. The choice of this parameter value has been also analyzed. Simulation results have revealed that this parameter affects directly to the performance of the system. It has been shown that inclusion of the values of the reference along the prediction horizon increases the performance of the controller for certain values of the prediction horizon. However, higher values of N do not necessarily improve the system performance, since they will force the controller to track reference values that are far in the future. The proposed controller robustness has been also tested. Two different cases have been evaluated, mismatching between model and actual system parameter values and behavior under system measures with noise. Simulation results show that proposed MPC strategy provides good performance under both situations, guaranteeing a robustness operation of the system.
ACKNOWLEDGMENT
The authors gratefully acknowledge the contribution of the Spanish Ministry of Science and Innovation under grant DPI2010-21589-C05-01 and Andalusian Government Research Council under project P07-TIC-02991.

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