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Granular avalanches: Deterministic, correlated and decorrelated dynamics

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Abstract. – A statistical analysis of granular avalanches in a half-filled and slowly rotated drum is presented. For large-sized grains the classical coherent oscillation is reproduced, *i.e.* we observe a quasi-periodic succession of regularly sized avalanches. As particle size is decreased, we see a crossover to a new complex dynamics characterized by long-range time correlations of local avalanches without a typical size, although the size distribution is not scale invariant. In the limit of large values of the time lag avalanches turn gradually to be decorrelated. The trend observed in the system dynamics as particle size is decreased is ascribed to the increase of cohesiveness which promotes bulk disorder. We argue that our experimental findings can be qualitatively predicted by theoretical models with adjustable parameters such as unquenched disorder, random critical slope, fluidization length, inertia and dissipation.

Avalanches in granular materials have attracted the attention of physicists from diverse fields and have been often paralleled to many nonlinear phenomena such as vortex avalanches in superconductors, breakdown in semiconductors, earthquake dynamics, geomagnetic field bursts, plasma edge turbulence in fusion devices, solar flares, X-rays emission from black holes, mass extinctions and forest fires [1]. Granular avalanches were early presented as the paradigm of the theory of self-organized criticality (SOC) which seeks to explain why the dynamics of these complex and dissipative systems lack characteristic time and length scales, exhibiting fluctuations that can be fitted to a power law statistics ($1/f^{\alpha}$ with $\alpha \approx 1$) [1]. The experimental work of Jaeger *et al.* demonstrated however that avalanches of large-sized spherical beads (particle size $d_{\mathrm{p}} \gtrsim 100\,\mu\mathrm{m}$) in a partially filled drum slowly rotated around its horizontal axis display a quasi-periodic oscillation and have a well-defined size incompatible with SOC [2]. Experimental evidence of SOC behavior was nevertheless claimed by Frette *et al.* [3], who investigated sandpile avalanches driven by the addition of grains at a low rate from above the pile. Only in the case of elongated grains the distribution of avalanche sizes could be fitted to a power law as predicted by SOC. It was hypothesized that in order to observe SOC, it is necessary to suppress inertial effects: sliding friction ruled the dynamics of long-shaped grains therefore inducing a higher effective friction that minimized inertia. An additional hypothetic requisite for SOC was that the surface of the pile should be rough, as was the case with elongated grains. A relevant parameter in geophysical applications such as snow avalanches, transport of fine powders in chemical industry, etc., is cohesion. Large beads ($d_{\mathrm{p}} \gtrsim 100\,\mu\mathrm{m}$) can be artificially made cohesive, for example, by adhering to their surface strongly attractive microparticles [4] or by moisturizing it [5]. The presence of cohesion produces rough surface profiles and furthermore contributes to diminish inertia [4, 5]. Experiments on the avalanching behavior of these cohesive systems revealed however a behavior inconsistent with SOC, showing instead a regular series of small precursors which led quasi-periodically to large relaxation events. On the other hand, agreement is found with the prediction of Mehta and Barker cellular-automaton model who took unquenched disorder into consideration [6]. According to Mehta and Barker, a memory of evolving disorder is built up in the internal structure of the disordered pile. Accumulated effects of small reorganizations result in quasi-periodic large avalanches which cause the breakdown of scale invariance. From this work it is clear that avalanches in real systems should be strongly influenced by memory effects that should be enhanced by configurational disorder. An interesting experimental program would be therefore to test for the memory of avalanche dynamics in granular systems with varying degree of disorder. But, how to tune up disorder in real experiments? The packing arrangement of particles is strongly influenced by their cohesive interaction. Cohesion becomes important when the interparticle attractive force balances particle weight, which for dry powders occurs for particle sizes typically below $100\,\mu\mathrm{m}$ [7]. With increasing fineness, the interparticle attractive force increases in comparison to the force of gravity, leading to the formation of stable arches that produce a rather heterogeneous structure and increase the average void fraction of the powder. Thus, we could look for configurational disorder effects by testing granular materials of varying cohesiveness. In the present paper we investigate avalanches in granular materials of decreasing particle size. The rescaled range analysis [8] will be presented as a powerful technique to test for long-term memory in the system dynamics.

We have tested monosized spherical beads of several particle sizes d_{p} and made of diverse materials such as steel ($d_{\mathrm{p}} = 110$, 80, 50, and $35\,\mu\mathrm{m}$, particle density $\rho_{\mathrm{p}} = 7.92\,\mathrm{g/cm^3}$), magnetite ($d_{\mathrm{p}} = 65$, 50, 40, and $35\,\mu\mathrm{m}$, $\rho_{\mathrm{p}} = 5.06\,\mathrm{g/cm^3}$), and ferrite ($d_{\mathrm{p}} = 50$ and $35\,\mu\mathrm{m}$, $\rho_{\mathrm{p}} = 4.83\,\mathrm{g/cm^3}$). Large glass beads ($d_{\mathrm{p}} \simeq 500\,\mu\mathrm{m}$, $\rho_{\mathrm{p}} \simeq 2.5\,\mathrm{g/cm^3}$) have been also tested. We have used a computer-controlled step motor with a velocity range from 0.06 to 3000 RPM and a resolution of 4000 steps/rev. A gear box reduces the motor speed by a factor of 1/25 and drives the movement of a half-filled polycarbonate drum (7.4 cm inner diameter and 2 cm depth). The length of the cylinder is large enough to avoid arching but sufficiently small to prevent 3D effects such as the development of incoherent avalanches at different places along the horizontal axis observed in long cylinders. The gear box is placed on an air table and joined to the motor through an elastic cardan to isolate the system from external vibrations. In our experiments the drum is rotated at 0.0074 RPM. A CCD camera interfaced to a computer and controlled with an image-processing software captures front images of the material surface profile at regular intervals of 6.44 s. The size resolution is about 1 bead/pixel. Each frame i is analyzed by a C++ edge detecting routine to calculate the average angle α_i of the slope which is stored in the computer. An avalanche is detected when the difference $\alpha_{i-1} - \alpha_i$ exceeds a prefixed threshold of 0.035 rad, small enough to detect the smallest avalanches but much larger than the typical indeterminacy in the slope due to the irregularities of the free surface. The volume fraction occupied by the grains is about 50% and in all the cases analyzed avalanches involved a fraction of powder close to the free surface, *i.e.* wall sliding of the whole powder was neither observed. Every time an avalanche is detected the computer stores the average maximum angle of stability α_{max}, the local maximum angle of stability $\alpha^{\mathrm{l}}_{\mathrm{max}}$, the average repose angle α_{rep}, the area of the avalanche A_{a}, and the time interval between successive avalanches. In each experimental run an order of ~ 5000 avalanches were recorded allowing us for a robust statistical analysis.

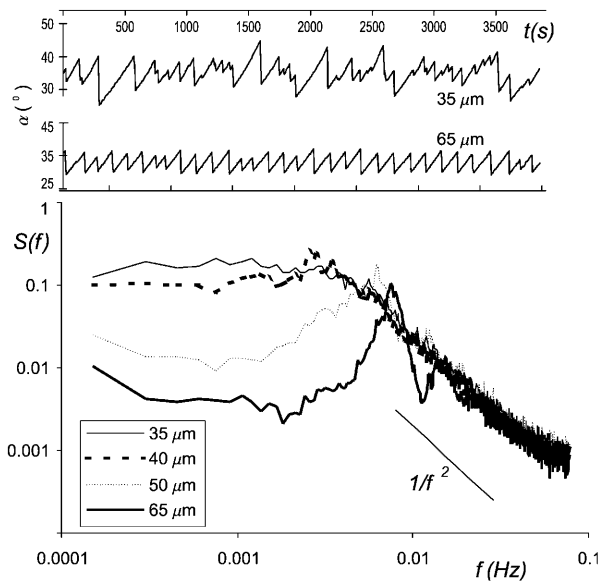

Fig. 1 – Top: typical sequences of the time evolution of the average angle of the slope for magnetite beads of different size ($35\,\mu$m top; $65\,\mu$m bottom). Bottom: power spectra of the time evolution of the average angle of the slope for samples of magnetite beads of several particle sizes (indicated). The straight line shows the $1/f^2$ behavior.

Figure 1(top) shows typical temporal sequences of the angle of the slope for two samples of the same material but with different particle sizes. The sample with larger particle size exhibits quasi-periodic avalanches of nearly uniform size as confirmed by inspection of the power spectra of the time series (fig. 1(bottom)) and the avalanche size statistical distribution (fig. 2). Moreover, the slope profile is rather smooth and flat as can be inferred from the close similarity between the statistical distributions of the average maximum angle of stability $\alpha_{\max}$ and the local maximum angle of stability $\alpha^{l}_{\max}$ (fig. 3). As particle size is decreased,

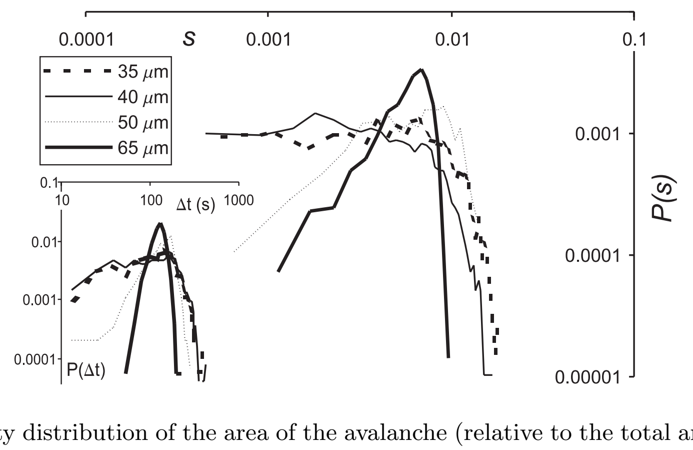

Fig. 2 – Probability distribution of the area of the avalanche (relative to the total area of the powder) for magnetite beads of different particle size (indicated). The inset shows the probability distribution of the time interval between successive avalanches.

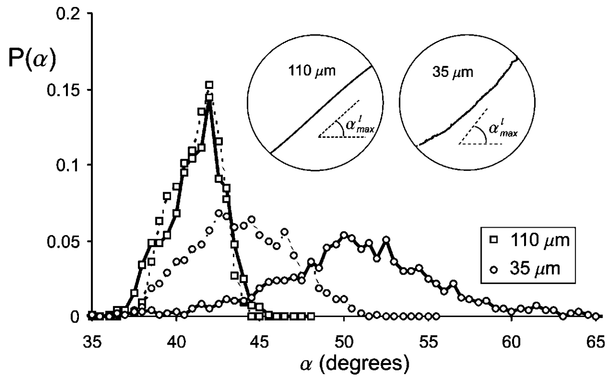

Fig. 3 – Probability distribution of the local (continuous line) and average (discontinuous line) maximum angle of stability for steel beads of different particle size (indicated). The inset illustrates typical examples of surface profiles at the stability limit for the coarse and fine beads.

we observe that the peak in the power spectra flattens and eventually disappears for fine enough samples. This is seen, for example, for the $35\,\mu$m sized magnetite beads (fig. 1) and is a common feature of all the materials tested below a given particle size. In those cases the power spectrum $S(f)$ becomes constant for $f < 1/T_0$ and crossover to a power law $1/f^2$ for high frequencies, matching the shape predicted by SOC theory for the flow down the slope of a sandpile [9]. Furthermore, the pronounced difference between the distributions of the local and average maximum angle of stability (fig. 3) indicates a bumpy profile of the slope as seems to be a hallmark of granular flows exhibiting SOC. These observations could drive us to *wrongly* interpret that the flow down the slope of our finest grains adjusts to SOC behavior paralleling the behavior of elongated grains reported by Frette *et al.* [3]. For the coarser particles the avalanche size probability distribution is sharply peaked around the average value. As particle size is decreased, the distribution broadens and for the finest powders we see a broad flat shoulder ($P(s) \sim 1/s^0$), corresponding to size-independent probability of events, followed by a roll-off rapid decay ($\sim 1/s^4$) for $s > s_0$, where s_0 (cutoff size) is a characteristic scale that involves the whole system. A similar shape is found for the distributions of time interval between avalanches (fig. 2 inset). Clearly, the flat distribution of avalanche sizes and time interval between consecutive avalanches obtained here (fig. 2) *deny* the existence of SOC. These distributions are not scale invariant, they do not adjust to the power law characteristic of SOC, and therefore are inconsistent with SOC. This behavior contrasts, however, with the previously reported dynamics of artificially made cohesive large grains, where well-defined small precursor avalanches and large avalanches were clearly distinguishable within our time resolution following a cyclic behavior [4, 5].

Our analysis demonstrates that a naive Fourier analysis can be a very blunt tool with which determining whether or not a system manifests SOC and may lead to spurious conclusions on the underlying physical mechanisms of avalanche flow. It can be shown analytically that a perfectly periodic signal of triangular pulses has a $1/f^2$ tail at high frequencies. Jaeger *et al.* [2] found in their spectra a crossover to a $1/f^3$ power law because they had a combination of triangular and rectangular pulses in the time signals of the avalanche flow-induced change in capacitance. Thus, the universal crossover in our experimental spectra to a $1/f^2$ fall-off, which is found even for the larger beads, should be entirely due to the sawtooth shape of the signals and *not* to SOC behavior. An alternative technique to gain further insight into the

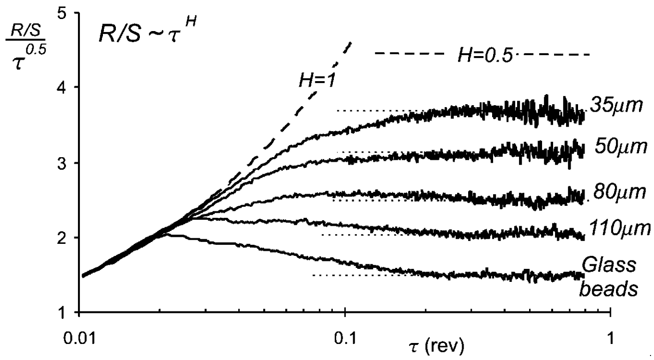

Fig. 4 – Rescaled range R/S divided by the square root of the time lag τ as a function of τ (measured in revolutions units) for steel beads of several particle sizes d_{p} (indicated). Data for glass beads ($d_{\mathrm{p}} \simeq 500\,\mu$m) are also shown for comparison. The discontinuous lines illustrate a deterministic trend ($H = 1$) and a fully decorrelated trend ($H = 0.5$).

system dynamics is the rescaled range (R/S) analysis. Let us apply it to our experimental time series of the angle of the slope ($\alpha(t)$, $t \in [0, T]$). In the R/S analysis, the time series is partitioned into nonoverlapping adjacent intervals of length $\tau < T$. Then we calculate the cumulative deviation from the mean, $X(t,\tau) = \sum_{u=1}^{t\leq\tau}(\alpha(u) - \langle\alpha\rangle_\tau)$, and the range $R(\tau) = \max\{X(t,\tau)\} - \min\{X(t,\tau)\}$, with $1 \leq t \leq \tau$, representing the "distance" covered between the two most separate points in the lag time τ. Finally, the rescaled range R/S is obtained, where S is the standard deviation of the time series in the lag time τ. We start with a lag time τ covering the whole sequence of data, and then reduce it by a factor of two in successive steps. Values of R/S for the same τ are averaged. If R/S scales with τ by the power law $R/S \sim \tau^H$ over a long range of τ, the signal is self-similar over that range pointing to a universal character of the dynamics. $H \in [0, 1]$ is the Hurst exponent, often called self-similar parameter. $H = 0.5$ describes uncorrelated white noise such as Brownian motion, otherwise the system presents an underlying nonlinear dynamics. If $H < 0.5$, the process shows a trend reversing or antipersistent behavior, meaning that successive values are likely to alternate around the mean. If $H > 0.5$, the process exhibits long-range time correlations or persistent behavior, meaning that increasing (decreasing) trends in the past imply on average increasing (decreasing) trends in the future, with stronger persistence strength as H approaches 1. For $H = 1$ the process is deterministic. In fig. 4 we show the R/S analysis of the avalanche dynamics for steel beads. Note that we have plotted $(R/S)/\tau^{0.5}$ *vs.* τ. For large-sized beads we identify the signature of the quasi-periodic avalanching process previously reported for large glass beads [2, 4] (results of the R/S analysis for large glass beads are also plotted for comparison). As expected, $H \simeq 1$ for τ smaller than the typical time between avalanches, whereas for longer-lag values $H < 0.5$, meaning that after an avalanche the slope will be at repose likely below the average slope angle. In this case, the predominant large surface flows produce regularly sized avalanches that disrupt and level off most of the surface. The angle of the slope oscillates then between the maximum angle of stability and the angle of repose in a quasi-perfect cyclic behavior. As τ increases further, the small noise in the avalanche size leads however to long-time decorrelated behavior ($H \to 0.5$). For decreasing particle size, the transition from fully correlated ($H = 1$) to antipersistent behavior ($H < 0.5$) gets more diffuse, meaning that the cyclic behavior is blurred due to the increased noise in the avalanche size caused by the increased structural disorder. For $d_{\mathrm{p}} < 50\,\mu$m, we cannot distinguish traces of periodicity anymore and the initial deterministic behavior ($H = 1$) turns to a long-ranged correlated behavior ($1 > H > 0.5$). For these systems the dynamics is strongly influenced by small rearrangement events that only alter a small part of the free surface, thus it is likely that after a number of small noisy events the angle of the slope keeps above the average. In the asymptotic limit of long time lags the persistence fades away gradually and turns over to a decorrelation of avalanches ($H \to 0.5$).

The R/S analysis of the sandpile cellular automaton (CA) model introduced by Bak, Tang, and Wiesenfeld (BTW) [1] as a paradigm of SOC is characterized by a self-similar correlation ($H \simeq 0.8$) over infinite time range [10]. Note that we always observe a decline of correlations in the long term ($H \to 0.5$), indicating that our systems behavior cannot be described by SOC, in agreement with our previous conclusion. In the BTW model, sandpiles are built up by randomly adding sand to the pile until the local height (or slope) reaches a predetermined critical value and sand is redistributed to nearest-neighbor sites. Yet we might think that the critical slope for instability in any physically motivated CA model cannot be deterministic. Interparticle contact forces are heterogeneously distributed and mostly transmitted through strong chains with a correlation length $\lesssim 100$ grains. The critical slope should be sensitive to the evolving structural disorder of the force network. Moreover, random variations in particle shape, size, etc. must contribute to an inherent noise in the local critical slope. Barker and Mehta included in their CA model a tuneable critical slope depending on pile preparation that reproduces some realistic patterns of behavior such as the uphill propagation of avalanches (seen also in our experiments) [11]. Chapman *et al.* [12] and Dendy and Helander [13] have used randomized critical slopes in their CA model. If a probabilistic rule instead of a deterministic rule is used for the critical slope, the power law in the avalanche size distribution vanishes [13] as seen in our experiments. Additionally, nonlocal transport was accounted for by Chapman *et al.* [14, 15] incorporating a controllable fludization length $L_{\rm f}$ which represents an amount of sand that avalanches behind the unstable cell. The model then yields different regimes of avalanche dynamics ranging from a regular succession of homogeneously sized events to an irregular succession of noisy events as $L_{\rm f}$ is increased, resembling closely the evolution observed in our experiments as particle size is decreased (compare fig. 1(top) with figs. 9-11 of ref. [14] and figs. 2-4 of ref. [15]). Cohesive powders are typically characterized by a correlation length involving clusters of individual particles whose size increases as particle size decreases [16]. It is therefore reasonable to assume in our case a coherence length $L_{\rm f}$ inversely related to particle size, thus the phenomenology observed in our experiment would be reproduced by Chapman *et al.*'s model. We find also a qualitative agreement between our experimental results and the theoretical analysis by Barker and Mehta [17], who proposed a coupled-map lattice model wherein both surface flow and internal restructuring were included as relaxation mechanisms. Internal restructuring is linked to powder consolidation by filling of voids larger than grain size, which is most likely in fine powders where cohesion favors the formation of large internal voids. For weakly consolidating systems with large inertia, as would be the case of our large noncohesive grains, Barker and Mehta find that the surface flow is dominant, leading to regularly sized and quasi-periodic large avalanches. In the opposed limit of small inertia and strongly consolidating systems, restructuring events predominate and the model predicts a succession of localized and small avalanches lacking a characteristic size. For moderate degree of compressibility and inertia, both small and large events are predicted and the size distribution is flat shaped as seen for our finest powders. This type of size distribution has been also predicted by Barker and Mehta in the intermediate disorder regime of a disordered sandpile CA model [11].

To sum up, it has been experimentally demonstrated that the increase of bulk disorder enhances memory effects, strongly influencing the avalanche behavior as early suggested by the theory of Mehta and Barker [6]. The presence of cohesiveness implies the magnification of local heterogeneities and thus of configurational disorder. As cohesiveness is increased by decreasing particle size d_p, we see a gradual crossover in the system dynamics. The behavior is almost deterministic for low cohesive systems ($d_\mathrm{p} \gtrsim 100\,\mu\mathrm{m}$): intermittent avalanches of a well-defined size succeed to each other at a quasi-periodic pace. For smaller particle sizes the distributions of avalanche size and time interval between avalanches flatten, *i.e.*, although there is not a characteristic scale, the distributions are *not* scale invariant as in SOC systems. The flat shape of these distributions contrasts however with the two-peak shape seen in other cohesive systems for which small precursor and large events were clearly distinguishable. For sufficiently small particles ($d_\mathrm{p} \lesssim 50\,\mu\mathrm{m}$) the nearly periodic behavior is completely erased, the avalanche size distribution is broad and the system dynamics turns to be correlated over long time scales, indicating the inefficiency of the likely small events in erasing the memory of the system. Only in the asymptotic limit of quite long time lags memory is lost and avalanches turn to be fully decorrelated. In the light of our experimental results we have seen that the incorporation of randomized critical slope, variable fluidization length, or controllable disorder, to cellular-automaton models is necessary to reproduce the phenomenology observed in real avalanches. We have shown also that the effects observed in the dynamics of granular avalanches when particle size is decreased (*i.e.* cohesion increased) could be reproduced by a theoretical approach consisting of including bulk reorganization with tuneable compressibility and grain inertia to a coupled-map lattice model.

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