Boson-conserving one-nucleon transfer operator in the interacting boson model

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The boson-conserving one-nucleon transfer operator in the interacting boson model (IBM) is reanalyzed. Extra terms are added to the usual form used for that operator. These new terms change generalized seniority by one unit, as the ones considered up to now. The results obtained using the new form for the transfer operator are compared with those obtained with the traditional form in a simple case involving the pseudospin Bose-Fermi symmetry $U^B(6) \otimes U^F(12)$ in its $U^{BF}(5) \otimes U^F(2)$ limit. Sizeable differences are found. These results are of relevance in the study of transfer reactions to check nuclear supersymmetry and in the description of β decay within IBM.

The interacting boson model (IBM) has been extensively used in order to describe spectroscopic properties of even-even and odd-even nuclei [1,2]. One of the observables studied within IBM is the stripping and pick-up single-nucleon transfer spectroscopic intensity between even-even and odd-even nuclei. The form of the one-nucleon transfer operator in IBM is an open problem of current interest for several reasons: (i) some calculations indicate that the form commonly accepted is insufficient for detailed calculations [3], (ii) it is one of the key points for checking nuclear supersymmetry [4,5], and (iii) it is a basic ingredient for the study of β decay since in IBM it is described as a composed process of a neutron (proton) pick-up followed by a proton (neutron) stripping [6–8]. Several forms for the one-nucleon transfer operator in IBM have been proposed in the past based on different microscopic approaches. The original form, proposed by Scholten and Dieperink [9,10], was obtained in the generalized seniority $\tilde{v}$ scheme [11] by using the Otsuka-Arima-Iachello mapping method [12] and the number approximation [13]. This is still the operator implemented in the standard IBM codes and the most commonly used codes. Alternatively, other forms based on the nuclear field theory techniques [14], the use of random-phase approximation for quadrupole phonons and mapping the fermion space onto the fermion-quadrupole space [15], and the generalized Holstein-Primakoff mapping method [16,17] have been proposed but there have been few actual calculations using these operators.

In this paper we reanalyze the standard IBM boson-conserving one-nucleon transfer operator proposed originally by Scholten and Dieperink and improve that operator by including extra terms that change seniority by one unit and are the next order in *d-boson* creation and annihilation operator. We show that this new type of terms give rise to sizeable changes in the one-nucleon transfer spectroscopic intensities, and consequently, conclude that they must be included in the standard IBM operator.

The starting point is the IBM boson-conserving part of the one-nucleon transfer operator derived semimicroscopically by Scholten and Dieperink [9,10] using the generalized seniority formalism [11]. Consequently, it is only strictly valid for spherical nuclei where seniority is approximately a good quantum number. However, this operator has also been used with reasonable success for transfer reactions involving deformed nuclei [3,4]. Scholten and Dieperink constructed the interacting boson-fermion approximation (IBFA) image of the shell model single-nucleon creation operator, taking into account terms that change generalized seniority by one unit and using fermion states with $\tilde{v} \leqslant 2$. Under these approximations the transfer operator that does not change the number of bosons is

$$\mathcal{T}^{\dagger}_{jm} = u_j a^{\dagger}_{jm} - \sum_{j'} \frac{v_j}{\sqrt{N}} \beta_{j'j} \sqrt{\frac{10}{2j+1}} [(s^{\dagger} \times \tilde{d})^{(2)} \times a^{\dagger}_{j'}]^{(j)}_m, \tag{1}$$

where $a^{\dagger}_{jm}$ is the odd-nucleon creation operator in the IBFA boson-fermion space, v_j is the occupation probability of the single-particle orbit j, $u_j = \sqrt{1 - v_j^2}$, and N is the number of bosons. The summation on j' runs over the valence single-particle angular momenta and the normalized quantities $\beta_{j'j}$ are [9,10]

$$\beta_{j'j} = Q_{j'j}(u_{j'}v_j + v_{j'}u_j)/N_{\beta}, \tag{2}$$

where

$$Q_{j'j} = \langle j' || Y^{(2)} || j \rangle \tag{3}$$

and N_{β} is a normalization constant obtained from the condition

$$\sum_{jj'} (\beta_{jj'})^2 = 1, \tag{4}$$

where j and j' are the valence single particle angular momenta. The operators $\tilde{d}_m$, which behave appropriately under rotations, are $\tilde{d}_m = (-1)^m d_{-m}$.

As mentioned before, the expression (1) was obtained under the restrictions of changing generalized seniority by one unit and using fermion states with $\tilde{v} \leqslant 2$. The term $[(d^{\dagger} \times \tilde{d})^{(\lambda)} \times a^{\dagger}_{j'}]^{(j)}_m$ changes generalized seniority in one unit as $[(s^{\dagger} \times \tilde{d})^{(2)} \times a^{\dagger}_{j'}]^{(j)}_m$, and does not appear in the traditional expression of the one-particle transfer operator. The reason is that the derivation of the corresponding coefficient implies the use of fermion states with $\tilde{v} > 2$. However, this kind of

terms are obtained naturally when using nuclear field theory techniques [14], or the generalized Holstein-Primakoff mapping method [16]. For this reason it is natural to wonder which is the influence of this kind of terms. Taking into account these terms, the new transfer operator reads[1]

$$T_{jm}^{\dagger\prime} = T_{jm}^{\dagger} + \sum_{j',J} \phi_{jj'}^{J} [(a_{j'}^{\dagger} \times d^{\dagger})^{(J)} \times \tilde{d}]_{m}^{(j)}. \qquad (5)$$

This is the operator to be used in IBFA if no distinction between protons and neutrons is made. When working with versions of the IBM IBFA in which the proton-neutron degrees of freedom are treated explicitly, all the operators in the preceding equation will have an index $\pi(\nu)$ if the transferred nucleon is a proton(neutron). In this work the coefficients $\phi_{jj'}^{J}$ will be obtained by requiring that the matrix elements of the fermion operator $C_{j}^{\dagger}$ and the IBFA operator $T_{j}^{\dagger\prime}$ between states of $\tilde{v}=2$ and $\tilde{v}=3$ in the corresponding spaces are equal. For that purpose, first one has to construct the $\tilde{v}=3$ shell model states that, in general, will not be orthonormal. From this set of nonorthonormal fermion states a mapping procedure onto the $\tilde{v}=3$ orthonormal boson states is needed. In this paper we use the so-called *democratic* mapping [18]. This mapping is based on the diagonalization of the shell model overlap matrix. In our case, the method starts from a set of n fermion states that are linearly independent but not necessarily orthonormal $|F,(i);JM\rangle (i=1,\ldots,n)$ and a set of n orthonormal boson states $|B,(i);JM\rangle (i=1,\ldots,n)$. The *democratic* mapping follows the following steps:

(1) One has to construct the initial fermion states $|F,(i);JM\rangle (i=1,\ldots,n)$ and the orthonormal boson states $|B,(i);JM\rangle (i=1,\ldots,n)$.

(2) One has to construct the matrix Θ^{J} with the overlaps of the fermionic states for each total angular momentum J

$$\Theta_{ik}^{J} = \langle F,(i);JM|F,(k);JM\rangle; \quad i,k=1,\ldots,n. \qquad (6)$$

(3) The diagonalization of the matrix Θ^{J} provides its eigenvectors and eigenvalues. Let us call C^{J} the $n \times n$ square matrix containing the eigenvectors (in columns) fixed by the condition $\Sigma_{k}(C_{ik}^{J})^{2}=1$ and λ^{J} the diagonal matrix containing the eigenvalues λ_{i}^{J}. The following relations hold in matrix notation

$$\Theta^{J} \cdot C^{J} = C^{J} \cdot \lambda^{J} \qquad (7)$$

and

$$(C^{J})^{T} = (C^{J})^{-1}. \qquad (8)$$

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[1] We prefer to write the operator in this way but with the appropriate angular momenta recoupling it can be written as

$$T_{jm}^{\dagger\prime} = T_{jm}^{\dagger} - \sum_{j',J,\lambda} (-1)^{\lambda} \hat{J}\hat{\lambda} \begin{Bmatrix} 2 & 2 & \lambda \\ j & j' & J \end{Bmatrix} \phi_{jj'}^{J} [(d^{\dagger} \times \tilde{d})^{(\lambda)} \times a_{j'}^{\dagger}]_{m}^{(j)}.$$

The orthonormal eigenvectors are

$$|F,k;JM\rangle_{\perp} = \frac{1}{\sqrt{\lambda_{k}^{J}}} \sum_{i=1}^{n} C_{i,k}^{J} |F,(i);JM\rangle; \quad k=1,\ldots,n. \qquad (9)$$

(4) The boson states are transformed to a new orthonormal set according to the same transformation as the fermion states

$$|B,k;JM\rangle_{\perp} = \sum_{i=1}^{n} C_{i,k}^{J} |B,(i);JM\rangle; \quad k=1,\ldots,n. \qquad (10)$$

It is easy to see that these new boson states are orthonormal using Eq. (8).

(5) Finally, the mapping is done between the new fermion, Eq. (9) and boson, Eq. (10), states

$$|F,k;JM\rangle_{\perp} \Leftrightarrow |B,k;JM\rangle_{\perp}; \quad k=1,\ldots,n. \qquad (11)$$

In the following, we proceed with these steps and derive semimicroscopically, in the same spirit as Scholten and Dieperink, the coefficients $\phi_{jj'}^{J}$.

First, it is necessary to construct properly the $\tilde{v}=3$ states in the shell model space. In order to get this set, we start with the following unnormalized shell model states

$$|S^{N-1}(Dj);JM\rangle = [C_{j}^{\dagger} \times D^{\dagger}]_{M}^{J} (S^{\dagger})^{N-1}|0\rangle, \qquad (12)$$

which, in general, contain $\tilde{v}=1$ and $\tilde{v}=3$ components. $C_{j}^{\dagger}$ represents the creation operator of one nucleon in the single particle orbit j and the fermion pair creation operators $S^{\dagger}$ and $D^{\dagger}$ are [9]

$$S^{\dagger} = \sum_{j} \alpha_{j} \sqrt{\frac{\Omega_{j}}{2}} (C_{j}^{\dagger} C_{j}^{\dagger})^{(0)} \qquad (13)$$

and

$$D^{\dagger} = \sum_{jj'} \beta_{jj'}' \sqrt{\frac{1}{2}} (C_{j}^{\dagger} C_{j'}^{\dagger})^{(2)}, \qquad (14)$$

where the structure coefficients α_{j} and $\beta_{jj'}'$ are

$$\alpha_{j} = \sqrt{\frac{\Omega_{e}}{N}} v_{j}, \quad \beta_{jj'}' = \frac{\beta_{jj'}}{u_{j}u_{j'}}, \qquad (15)$$

where $\beta_{jj'}$ are defined in Eq. (2), $\Omega_{j}=(2j+1)/2$ and $\Omega_{e} = \Sigma_{j}\alpha_{j}^{2}(2j+1)/2$ is the effective shell degeneracy [9].

Unnormalized $\tilde{v}=3$ states are constructed subtracting the $\tilde{v}=1$ components in Eq. (12) in the following way:

$$|\tilde{v}=3;S^{N-1}(Dj);JM\rangle$$

$$=\frac{|S^{N-1}(Dj);JM\rangle}{u_j\sqrt{\dfrac{(N-1)!\Gamma(\Omega_e+1)}{\Gamma(\Omega_e+1-(N-1))}}}-\chi_j^J|\tilde{v}=1,N;JM\rangle,$$

(16)

where the denominator in the first term on the right-hand side is introduced for later convenience in order to simplify the expressions of some quantities given below. $|\tilde{v}=1,N;JM\rangle$ are properly normalized seniority $\tilde{v}=1$ states. χ_j^J is obtained requiring that the overlap between the states (16) and the states of seniority one is zero. The expression obtained is

$$\chi_j^J=-\sqrt{\frac{10}{2J+1}}\frac{v_j}{u_j}\beta_{Jj},$$

(17)

if J is equal to one of the allowed angular momenta and zero otherwise. In the above derivation the number approximation [13] $\langle 0|S^N(S^\dagger)^N|0\rangle=[N!\Gamma(\Omega_e+1)]/[\Gamma(\Omega_e+1-N)]$ has been used.

Once we have the nonorthonormal fermion states, Eqs. (16) and (17), the matrix of the overlaps Θ^J, can be constructed, obtaining

$$\Theta_{mr}^J=\langle\tilde{v}=3;S^{N-1}(Dj_m);JM|\tilde{v}=3;S^{N-1}(Dj_r);JM\rangle$$

$$=\delta_{mr}+10\left(\sum_{i=1}^{n}\delta_{j_i,J}\right)\sum_{k=1}^{n}\beta_{j_k,j_r}\beta_{j_k,j_m}\begin{Bmatrix}j_r & 2 & j_k\\ j_m & 2 & J\end{Bmatrix}.$$

(18)

Note that this expression tells us that the states (16) are already orthonormal when J does not coincide with one of the valence single particle angular momenta. For the general case in which there are several single particle angular momenta j that contribute to a given J one has to follow the procedure sketched above. First, the matrix Θ^J is diagonalized obtaining its eigenvectors C^J and eigenvalues λ^J. Then, orthonormalized fermion and boson seniority three states are constructed as given in Eqs. (9) and (10)

$$|F,\tilde{v}=3,k;JM\rangle_\perp=\frac{1}{\sqrt{\lambda_k^J}}\sum_{i=1}^{n}C_{i,k}^J|\tilde{v}=3;S^{N-1}(Dj_i);JM\rangle,$$

$$k=1,\ldots,n.$$

(19)

and

$$|B,\tilde{v}=3,k;JM\rangle_\perp=\sum_{i=1}^{n}C_{i,k}^J|s^{N-1}(dj_i);JM\rangle,\quad k=1,\ldots,n.$$

(20)

We are now in a position to map the $\tilde{v}=3$ shell model states into the $\tilde{v}=3$ boson-fermion states of the IBFA

$$|F,\tilde{v}=3,k;JM\rangle_\perp\Leftrightarrow|B,\tilde{v}=3,k;JM\rangle_\perp,\quad k=1,\ldots,n.$$

(21)

Since now the $\tilde{v}=3$ states are known both in the fermion space and in the boson-fermion space, the coefficients $\phi_{j_kj_{k'}}^J$ will be obtained by requiring that the matrix elements of the fermion operator $C_{j_k}^\dagger$ and the IBFA operator $T_{j_k}^{\dagger'}$ between states of $\tilde{v}=2$ and $\tilde{v}=3$ in the corresponding spaces are equal

$$_\perp\langle F,\tilde{v}=3,l;J||C_{j_k}^\dagger||S^{N-1}D;2\rangle$$

$$=_\perp\langle B,\tilde{v}=3,l;J||T_{j_k}^{\dagger'}||s^{N-1}d;2\rangle.$$

(22)

With this requirement the following expression for the coefficients $\phi_{j_kj_{k'}}^J$ is obtained

$$\phi_{j_kj_{k'}}^J=(-1)^{j_k-j_{k'}}u_{j_k}\frac{\hat{J}}{\hat{j}_k}[\delta_{j_kj_{k'}}-\{C^J\sqrt{\lambda^J}(C^J)^T\}_{kk'}],$$

(23)

where $\hat{j}=\sqrt{2j+1}$ and $\sqrt{\lambda^J}$ is a diagonal matrix whose elements are the square root of of the corresponding ones in λ^J (the square root of the eigenvalues of Θ^J). The transfer operator given in Eq. (5) with the coefficients in Eq. (23) is the form proposed in this work for a particlelike transfer. For the case of a holelike transferred nucleon the $u's$ and $v's$ have to be interchanged [in addition it should be noted that the reduced matrix elements given in Eq. (3) change sign if the transferred nucleon is holelike [19]]. This new form for the boson-conserving one-nucleon transfer operator has been implemented in the IBM codes [20] that are available under request.

We have performed calculations both in IBM-1 and IBM-2 with these new terms in order to show their relative importance and as a result sizeable differences are found. Below we present a simple example.

The observable we calculate is the spectroscopic intensity defined as

$$I_j(J_i\rightarrow J_f)=|\langle J_f||T_j^{\dagger'}||J_i\rangle|^2.$$

(24)

J_i is a state in the even-even nucleus and J_f a state in the odd even. It is easy to show that $I_j(J_i\rightarrow J_f)=I_j(J_f\rightarrow J_i)$ and this fact allows us not to specify which is the initial nucleus in the table presented below.

As a simple example we present here one-nucleon particlelike transfer intensities between two nuclei that follow particular dynamical symmetries of the IBM-1 and IBFA-1. The even-even nucleus has just one boson, is the boson core for the odd-even nucleus. The odd-even nucleus has one boson plus one particle and is described within the pseudospin Bose-Fermi dynamical symmetry $U^{BF}(5)\otimes U^F(2)$ [21] obtained from the product $U^B(6)\otimes U^F(12)$, where the superscripts B and F stand for boson and fermion. In the odd-even nucleus, the odd particle has been allowed to occupy the $j=1/2,3/2$, and $5/2$ single particle states. Under these conditions, once the number of bosons is fixed, the wave functions

TABLE I. Calculated spectroscopic intensities for one-nucleon transfer between an even-even nucleus with $N=1$ and an odd-even $U^{BF}(5)$ nucleus with $N=1$ plus one fermion. Results for transferred $j=1/2, j=3/2$, and $j=5/2$ are shown. Calculations made with Eqs. (1) and (5) are displayed under the labels gs and $gs+$, respectively. Only transfers involving the state $|2_B\rangle=|[1]\langle 1\rangle(1)2\rangle$ in the even-even core nucleus are included, since those involving the state $|0_B\rangle=|[1]\langle 0\rangle(0)0\rangle$ in the even-even core have no contribution from the added terms to the transfer operator. The quantum numbers in the first column specify the state in the odd-even nucleus.

| $[N_1,N_2]\langle n_1,n_2\rangle(v_1,v_2)L;J$ | $j=1/2$ | | $j=3/2$ | | $j=5/2$ | |
odd-even state	gs	$gs+$	gs	$gs+$	gs	$gs+$
$[1,1]\langle 1,1\rangle(1,1)1;1/2$			0.900	0.477	0.800	1.542
$[2,0]\langle 0,0\rangle(0,0)0;1/2$			0.539	0.539	0.000	0.000
$[2,0]\langle 2,0\rangle(0,0)0;1/2$			0.600	0.980	1.200	0.844
$[1,1]\langle 1,1\rangle(1,1)1;3/2$	0.000	0.007	0.900	1.638	2.800	2.181
$[1,1]\langle 1,0\rangle(1,0)2;3/2$	3.007	2.526	0.217	0.067	0.000	0.003
$[2,0]\langle 1,0\rangle(1,0)2;3/2$	0.071	0.015	0.217	0.451	0.000	0.003
$[2,0]\langle 2,0\rangle(2,0)2;3/2$	0.000	0.053	2.100	1.659	1.200	2.164
$[1,1]\langle 1,0\rangle(1,0)2;5/2$	3.546	5.168	0.031	0.220	0.000	0.027
$[1,1]\langle 1,1\rangle(1,1)3;5/2$	0.000	0.142	3.600	3.217	1.200	2.441
$[2,0]\langle 1,0\rangle(1,0)2;5/2$	0.321	0.916	0.031	0.014	0.000	0.027
$[2,0]\langle 2,0\rangle(2,0)2;5/2$	0.000	0.000	0.900	1.662	4.800	4.921
$[1,1]\langle 1,1\rangle(1,1)3;7/2$			0.600	0.600	7.200	7.200
$[2,0]\langle 2,0\rangle(2,0)4;7/2$			5.400	5.400	0.800	0.800
$[2,0]\langle 2,0\rangle(2,0)4;9/2$					10.000	10.000

are fixed by the symmetry, independent of the parameters used in the Hamiltonian. The occupation probabilities that appear in the transfer operator are not provided by the symmetry. In the example presented here the values used are $v_{1/2}^2=0.5, v_{3/2}^2=0.25$, and $v_{5/2}^2=0$ that fulfil $\Sigma_{j_k}(2j_k+1)v_{j_k}^2=2N$.

The states in the even-even core nucleus are identified by the labels of the irreducible representations of the group chain $U^B(6)\supset U^B(5)\supset O^B(5)\supset O^B(3):|[N]\langle n_d\rangle(v)L\rangle$. In our case, with just one boson, there are only two states, the ground state $|0_B\rangle=|[1]\langle 0\rangle(0)0\rangle$ with one s boson and the excited state $|2_B\rangle=|[1]\langle 1\rangle(1)2\rangle$ with one d boson. The states in the odd-even nucleus are labeled by the labels of the irreducible representations of the group chain $U^{BF}(6)\supset U^{BF}(5)\supset O^{BF}(5)\supset O^{BF}(3)\supset Spin^{BF}(3):|[N_1,N_2]\langle n_1,n_2\rangle(v_1,v_2)L;J\rangle$.

Since the ground state of the even-even nucleus has no d bosons the only term of the transfer operator that contributes to the transfers from this state to any state in the final odd-even nucleus is the $a_{j_k}^\dagger$ term. We have checked that our results in this case coincide with those obtained analytically in Ref. [21]. The calculated spectroscopic intensities between the state $|2_B\rangle$ in the even-even nucleus and all possible states in its odd-even counterpart are presented in Table I. In the first column the quantum labels corresponding to the state in the odd-even nucleus are specified. Two calculations are shown, the first one under the label gs for the traditional operator given in Eq. (1) and the second one under the label $gs+$ for the full operator proposed in this work, Eqs. (5) and (23). Important differences between both calculations can be seen for some transfers. Let us discuss briefly the transfers involving the state $|2_B\rangle$ in the even-even nucleus and the

states $|[2,0]\langle 0,0\rangle(0,0)0;1/2\rangle$ and $|[1,1]\langle 1,1\rangle(1,1)1;1/2\rangle$ in the odd-even nucleus. The state $|[2,0]\langle 0,0\rangle(0,0)0;1/2\rangle$ has just one component $|(0_B\times a_{1/2}^\dagger)^{(1/2)}\rangle$ (see Table XIII in Ref. [21]), thus the connection with the state $|2_B\rangle$ in the even-even nucleus can only be done through the terms $(s^\dagger\widetilde{d}a_{1/2}^\dagger)^{(3/2,5/2)}$ in the transfer operator. The new terms proposed in this work give no contribution to these transfers as it is seen in Table I. The value zero for the $j=5/2$ transfer is due to the value $v_{5/2}^2=0$ selected. On the other hand, the state $|[1,1]\langle 1,1\rangle(1,1)1;1/2\rangle$ has two components (see Table XIII in Ref. [21]): $\sqrt{3/5}|(2_B\times a_{3/2}^\dagger)^{(1/2)}\rangle+\sqrt{2/5}|(2_B\times a_{5/2}^\dagger)^{(1/2)}\rangle$. The connection with the state $|2_B\rangle$ in the even-even nucleus can be done only by the $a_{j=3/2,5/2}^\dagger$ and the new terms. The values under the label gs are those coming just from $a_j^\dagger$. The values under the label $gs+$ include the contributions from $a_j^\dagger$ and those coming from the proposed terms $(d^\dagger\widetilde{d}a^\dagger)$. It can be seen that these contributions are important, constructive in one case and destructive in the other case.

It can be observed in Table I that some forbidden transfers with the traditional operator given in Eq. (1) become allowed with the new one, Eq. (5). We have performed calculations with larger number of bosons with IBM-1 and IBFA-1 and with the two types of bosons (neutron and proton) (IBM-2, IBFA-2) and in every case sizeable differences are observed.

In conclusion, we have calculated extra $\Delta\widetilde{v}=1$ terms in the bosonic expansion of the transfer operator that connects states of IBM with states of IBFA. Through the comparisons between the transfer intensities calculated using the transfer operator with and without these terms in the $U^{BF}(5)$ limit of the pseudospin symmetry $U^{BF}(6)\otimes U^F(2)$ [21], we have shown that the added terms are not negligible when com-

pared to the ones taken into account up to now and expect improvements in the description of the one-nucleon transfer intensities using this modified operator. Calculations in the Pt region are in progress and will be published somewhere else [22].

The influence of this modified operator for the calculation of β decay within the interacting boson model remains to be studied. The matrix elements needed in this case are of the same kind as the ones needed in order to calculate spectroscopic intensities. However, in β decay all transfers connecting an initial state in an odd-even nucleus (not necessarily the ground state) to all possible states in the even-even intermediate nucleus and then all transfers from these intermediate states to the final state (not necessarily the ground state too) in the even-odd nucleus have to be calculated. Thus, many transfers contribute and it is important to study the changes induced by the new transfer operator in its description.

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