Analytical expressions for the dispersive contributions to the nucleon-nucleus optical potential

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Analytical solutions of dispersion relations in the nucleon-nucleus optical model have been found for both volume and surface potentials. For the energy dependence a standard Brown-Rho function has been assumed for both the volume and surface imaginary contributions multiplied in this later case by a decreasing exponential function. The solutions are valid for any even value of the powers appearing in these functional forms.

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Mahaux and co-workers [1–5] have shown how the study of the nuclear mean field may benefit from the use of dispersion relations. These are mathematical expressions that link certain contributions to the real and imaginary components of the optical model potential (OMP). The constraint imposed by these dispersion relations helps in reducing ambiguities in the construction of phenomenological potentials from fits to the experimental data. We refer specifically to the so-called dispersive contribution ΔV, which adds dynamical content to the otherwise static (and real) Hartree-Fock potential term V_{HF}.

Under favorable conditions of analyticity in the complex E plane, the real part ΔV can be constructed from the knowledge of the imaginary part W of the mean field on the real axis through the dispersion relation

$$\Delta V(r,E)=\frac{\mathcal{P}}{\pi}\int_{-\infty}^{\infty}\frac{W(r,E')}{E'-E}dE', \tag{1}$$

where we have explicitly indicated the radial and energy dependence of these quantities. Assuming that $\Delta V(r,E=E_F)=0$, where E_F is the Fermi energy, Eq. (1) can also be written in the subtracted form

$$\Delta V(r,E)=\frac{\mathcal{P}}{\pi}\int_{-\infty}^{\infty}W(r,E')\left(\frac{1}{E'-E}-\frac{1}{E'-E_F}\right)dE'. \tag{2}$$

This transformation is difficult to implement in practice if the geometry of the dispersive potential depends on the energy. To simplify the problem, however, the shapes of the different components of the OMP are usually assumed to be energy independent and they are expressed in terms of a Woods-Saxon function f_{WS} or its derivative. In such case the radial functions factorize out of the integrals and the energy dependence is completely accounted for by two overall multiplicative strengths $\Delta V(E)$ and $W(E)$. Both of these factors contain, we note, volume and surface contributions.

It is customary to represent the variation with energy of the volume and surface components of the imaginary potential by functional forms that are suitable for an optical model analysis that exploits dispersion relations. An energy dependence for the imaginary volume term has been suggested by Brown and Rho in studies of nuclear matter [6],

$$W_V(E)=A_V\frac{(E-E_F)^n}{(E-E_F)^n+(B_V)^n}, \tag{3}$$

where A_V and B_V are constants. Brown and Rho proposed $n=2$, while Mahaux and Sartor [2] suggest, for the same expression, $n=4$. An energy dependence for the imaginary-surface term has also been investigated by Delaroche *et al* [7], who use the form

$$W_S(E)=A_S\frac{(E-E_F)^m}{(E-E_F)^m+(B_S)^m}\exp(-C_S|E-E_F|), \tag{4}$$

where $m=2,4$ and A_S, B_S, C_S are constants.

According to Eqs. (3) and (4) the imaginary part of the OMP turns out to be zero at $E=E_F$ and nonzero elsewhere. A more realistic parametrization of $W_V(E)$ and $W_S(E)$ forces these quantities to be zero in some interval around the Fermi energy. A reasonable range for such a region is measured by the average energy of the single-particle states E_P [4] and a new definition for the imaginary volume part of the OMP can thus be written as

$$W_V(E)=\begin{cases}0, & E_F<E<E_P\\ A_V\dfrac{(E-E_P)^n}{(E-E_P)^n+(B_V)^n}, & E\geqslant E_P.\end{cases} \tag{5}$$

Likewise, for the surface term we have

$$W_S(E)=\begin{cases}0, & E_F<E<E_P\\ A_S e^{-C_S|E-E_P|}\dfrac{(E-E_P)^m}{(E-E_P)^m+(B_S)^m}, & E\geqslant E_P.\end{cases} \tag{6}$$

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The symmetry condition

$$W(2E_F-E)=W(E) \tag{7}$$

defines the imaginary part of the OMP for energies below the Fermi energy.

In a recent work we have presented a numerical solution of the dispersion integral relations between the real and the imaginary parts of the nuclear optical potential [8]. In this contribution we obtain analytic solutions of Eq. (2) for the particular functional form of the imaginary potential $W(E)$ given above by Eqs. (5)–(7). Following Ref. [9], we adopt a notation where the offset energy is $E_0=E_P-E_F$, the excitation energy $E_x=E-E_F$ and introduce the convenient quantities $E_+=E_x+E_0$, $E_-=E_x-E_0$.

For the surface potential $W_S(E)$ given by Eqs. (4) and (7), we can write the dispersive integral (2) for even m as

$$\begin{aligned}\Delta V_S(E)&=\frac{E_x}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{W_S(E')}{(E'-E)(E'-E_F)}dE'\\&=A_S\frac{E_x}{\pi}\mathcal{P}\int_0^{\infty}\frac{U^m\exp(-C_SU)}{(U^m+B_S^m)(U-E_-)(U+E_0)}dU\\&\quad+A_S\frac{E_x}{\pi}\mathcal{P}\int_0^{\infty}\frac{U^m\exp(-C_SU)}{(U^m+B_S^m)(U+E_+)(U+E_0)}dU.\end{aligned} \tag{8}$$

The integrand can then be replaced by its expression in terms of poles and residues [10], and therefore we can also write

$$\begin{aligned}&A_S\frac{E_x}{\pi}\frac{U^m}{(U^m+B_S^m)(U\mp E_\mp)(U+E_0)}\\&\quad=A_S\frac{1}{\pi}\left\{\sum_{j=1}^{m}\frac{\mathrm{Res}(p_j)}{U-p_j}+\frac{\mathrm{Res}(\mp E_\pm)}{U\mp E_\mp}+\frac{\mathrm{Res}(E_0)}{U+E_0}\right\}.\end{aligned} \tag{9}$$

In the previous expression p_j are the m zeros of $U^m+B_S^m$ and $\mathrm{Res}(p_j)$ represent their corresponding residues,

$$p_j=B_S\exp\left(i\,\frac{2j-1}{m}\,\pi\right), \tag{10}$$

$$\mathrm{Res}(p_j)=\frac{E_x}{m}\,\frac{p_j}{(p_j\mp E_\mp)(p_j+E_0)}. \tag{11}$$

Here $\pm E_\mp$ and $-E_0$ are the poles of $U\mp E_\mp$ and $U+E_0$, whereas $\mathrm{Res}(\pm E_\mp)$ and $\mathrm{Res}(-E_0)$ are their residues,

$$\mathrm{Res}(\pm E_\mp)=\pm\frac{(E_\mp)^m}{(E_\mp)^m+B_S^m}, \tag{12}$$

$$\mathrm{Res}(-E_0)=\mp\frac{(E_0)^m}{(E_0)^m+B_S^m}. \tag{13}$$

As was pointed out by Raynal [10], the contribution of each complex pole p_j to the surface dispersive integral (8) is

$$\begin{aligned}\int_0^{\infty}\frac{\mathrm{Res}(p_j)e^{-C_SU}}{U-p_j}dU&=\mathrm{Res}(p_j)e^{-C_Sp_j}\int_{-C_Sp_j}^{\infty}\frac{\exp(-z)}{z}dz\\&\equiv\mathrm{Res}(p_j)e^{-C_Sp_j}E_1(-C_Sp_j),\end{aligned} \tag{14}$$

where $E_1(z)$ is the *exponential integral function* E_1 [11]. The contribution of the real poles corresponding to the second term in the right-hand side of Eq. (9) is

$$\begin{aligned}&\int_0^{\infty}\frac{\mathrm{Res}(\mp E_\pm)e^{-C_SU}}{U\mp E_\mp}dU\\&\quad=\mathrm{Res}(\mp E_\pm)e^{\mp C_SE_\mp}\mathcal{P}\int_{\mp C_SE_\mp}^{\infty}\frac{\exp(-x)}{x}dx\\&\quad\equiv-\mathrm{Res}(\mp E_\pm)e^{\mp C_SE_\mp}Ei(\pm C_SE_\mp),\end{aligned} \tag{15}$$

where $E_i(x)$ is the *exponential integral function* E_i [11]. Finally, the contributions from the third term on the right-hand side of Eq. (9) in integral (8) cancel.

For the volume potential $W_V(E)$ given by Eqs. (5) and (7), the dispersive integral for even n can be written as

$$\begin{aligned}\Delta V_V(E)&=\frac{E_x}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{W_V(E')}{(E'-E)(E'-E_F)}dE'\\&=A_V\frac{E_x}{\pi}\mathcal{P}\int_0^{\infty}\frac{U^m}{(U^m+B_V^m)(U-E_-)(U+E_0)}dU\\&\quad+A_V\frac{E_x}{\pi}\mathcal{P}\int_0^{\infty}\frac{U^m}{(U^m+B_V^m)(U+E_+)(U+E_0)}dU.\end{aligned} \tag{16}$$

According to Eq. (9), the contribution of each pole p (either real or imaginary) in integral (16) diverges as

$$\int_0^{\infty}\frac{dU}{U-p}=\lim_{U\to\infty}\ln(U-p)-\ln(-p). \tag{17}$$

Obviously, their sum is a finite quantity, which is calculated by taking its limit.

We quote, below, exact expressions for the surface and the volume dispersive integrals for any even value of m and n in the potentials. These forms have no limitations regarding their range of validity. The dispersive contribution of the surface imaginary potential $W_S(E)$, according to Eqs. (6) and (7), is

$$\begin{aligned}\Delta V_S(E)=\frac{A_S}{\pi}\Bigg\{&\sum_{j=1}^{m}Z_je^{-p_jC_S}E_1(-p_jC_S)\\&-\mathrm{Res}(-E_+)e^{C_SE_+}Ei(-C_SE_+)\\&-\mathrm{Res}(E_-)e^{-C_SE_-}Ei(C_SE_-)\Bigg\},\end{aligned} \tag{18}$$

where Z_j comes from the sum of the residues $\mathrm{Res}(p_j)$ in the two integrals (8) and is given by

$$Z_j=\frac{E_x}{m}\frac{p_j(2p_j+E_+-E_-)}{(p_j+E_0)(p_j+E_+)(p_j-E_-)}. \tag{19}$$

For the dispersion relation corresponding to the volume imaginary potential $W_V(E)$, calculated with Eqs. (5) and (7), the real contribution yields

$$\Delta V_V(E)=-\frac{A_V}{\pi}\left\{\sum_{j=1}^{n} Z_j\ln(-p_j)+\mathrm{Res}(-E_+)\ln E_+ + \mathrm{Res}(E_-)\ln|E_-|\right\}, \tag{20}$$

where p_j is calculated according to Eq. (10) using B_V and n instead of B_S and m and Z_j is calculated by Eq. (19) using n instead of m.

A computer code to calculate the analytical expressions (18) and (20) has been recently published by the authors [12].

In conclusion, we have found analytical solutions of the dispersion relations for the volume and surface terms of the OMP when they are parametrized in the form given by expressions (5)–(7). The formulas are compact and easy to implement in current codes for the optimum parameter search defining the nucleon-nucleus OMP. Usually these searches for elastic scattering data are performed by adjusting simultaneously the real and imaginary parts of the OMP. In particular, the fact of having available a functional form of $\Delta V(E)$ in terms of the parameters that define the imaginary potentials, makes it possible to implement a convenient alternative to the ordinary search procedures by adjusting only volume and surface real parts of the OMP.

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