Direct versus sequential four-particle transfer in heavy ion collisions with superfluid nuclei: Sn+Sn reaction

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(Received 26 August 1987)

The direct transfer of four nucleons in reactions with superfluid systems is compared with the feeding of the same final channel by a successive transfer of correlated nucleon pairs. A quantitative analysis is carried out for the reaction ^{120}Sn+^{112}Sn which shows the overwhelming dominance of the direct multiparticle transfer at bombarding energies below the Coulomb barrier. Results of simple estimates—supplemented by full coupled-channel calculations—provide orientation for selecting the experimental conditions which best exhibit the specific characteristics of the superconductive phase. The possibility of detecting an interference pattern between the two processes is also discussed.

A macroscopic picture for analyzing the transfer of pairs of identical nucleons in the case of superfluid nuclei has been recently investigated. Earlier attempts[1] focused in the leading process which, for grazing collisions, is known to involve the transfer of a single nucleon pair. At this level, the weak character of the couplings allows for expressions which are formally identical to the ones previously exploited for pairing modes in the vicinity of closed shells.[2] A more general approach, however, requires the identification of a macroscopic form for the nuclear density as a function of the gauge angle ϕ. In Ref. 3 the expression

$$\rho(r,\phi)=\frac{\rho_0}{(1+\exp\{[r-R(\phi)]/a\})}$$

was proposed which follows naturally from the parametrization of the nuclear radius as a function of ϕ according to

$$R(\phi)=R_0\left[1+\frac{2\beta_p}{3A_0}\cos 2\phi\right] .$$

Here R_0 stands for the unperturbed nuclear radius, while the quantity β_p indicates the magnitude of the static deformation of the superfluid system in gauge space. This quantity thus plays a role entirely analogous to the β_p parameter in Ref. 2, which gave a measure of dynamical deviations in particle number for nuclei in their normal phase.

An interesting consequence of the macroscopic picture for pair rotational bands developed in Ref. 3 is the determination of transition densities for processes in which the particle number changes by an arbitrary amount $\Delta A = k$. These are simply identified with the Fourier components of the generalized density in the intrinsic frame $\rho(r,\phi)$, i.e.,

$$\delta_{\rho_k}(r)=\int \rho(r,\phi)e^{ik\phi}d\phi .$$

Multiparticle transfer data for collisions with superfluid systems are already available in the literature. Indeed, in experiments for the reaction ^{120}Sn + ^{112}Sn reported in Ref. 4, the transfer of four neutrons leading to residual ^{116}Sn nuclei has been recorded. A conventional view would associate these events with a two-step process involving the successive transfer of neutron pairs. Within the macroscopic approach, however, the contribution of a direct four-neutron coupling feeding the final ^{116}Sn + ^{116}Sn channel should also be considered.

In this Brief Report we make quantitative estimates of the relative importance of the two competing processes. We take as a specific example the reaction studied in Ref. 4. As it turns out, the results are strongly dependent on the choice of experimental conditions. We thus develop some guidelines to enhance the novel aspects in multiparticle transfer which are allowed by the superfluid character of the tin isotopes.

As a starting point we consider the reaction amplitudes for both the direct and sequential processes. Semiclassical expressions for the lowest orders are deemed appropriate since the regime of interest is restricted to grazing collisions induced by very weak couplings. We then take

$$a_{\rm dir}=\left[\frac{i}{\hbar}\right]\int_{-\infty}^{+\infty}F_4(t)e^{i\omega t}dt$$

and

$$a_{\rm seq}=\left[\frac{i}{\hbar}\right]^2\int_{-\infty}^{+\infty}F_2(t)e^{i\omega_2 t}dt\int_{-\infty}^{t}F_2(t')e^{i\omega_1 t'}dt'$$

for the one- and two-step processes, respectively. In the previous expressions $F_4(t)$ and $F_2(t)$ stand for the form factors for four- and two-particle transfer. These can be constructed from transition densities as obtained from the macroscopic picture. Using the ion-ion interaction as a reference for large distances one can write

$$F_2(r) \sim \left.\frac{\delta\rho_2(r)}{\delta\rho_0(r)}\right|_\infty U(r) ,$$

$$F_4(r) \sim \left.\frac{\delta\rho_4(r)}{\delta\rho_0(r)}\right|_\infty U(r) .$$

We recall that the relative magnitude of the transition densities becomes independent of the distance in the tail region and is essentially determined by the value of β_p.[3] The quantities ω_1, ω_2 appearing in the sequential amplitude correspond to the excitation energy associated with the two-particle transfer channels ^{120}Sn(^{112}Sn,^{114}Sn)^{118}Sn and ^{118}Sn(^{114}Sn,^{116}Sn)^{116}Sn. These are both positive Q-value transitions which, in our case, yield $\hbar\omega_1 = -2.34$ MeV and $\hbar\omega_2 = -0.83$ MeV. In the amplitude a_{dir} enters, on the other hand, the total balance of energies in the four-particle channel ^{120}Sn(^{112}Sn,^{116}Sn)^{116}Sn, i.e., $\omega = \omega_1 + \omega_2$.

The evaluation of these integrals can be easily performed exploiting the sharp exponential character of the couplings. Expanding the trajectory of relative motion arounds its turning point r_0 one gets

$$a_{\mathrm{dir}} = \left[\frac{i}{\hbar}\right] F_4(R_0) e^{-(r_0 - R_0)/a} \int_{-\infty}^{+\infty} dt\, e^{-(t^2/2\sigma^2) + i\omega t} ,$$

$$a_{\mathrm{seq}} = \left[\frac{i}{\hbar}\right]^2 F_2(R_0)^2 e^{-2(r_0 - R_0)/a}$$

$$\times \int_{-\infty}^{+\infty} dt\, e^{-(t^2/2\sigma^2) + i\omega_2 t} \int_{-\infty}^{t} dt' e^{-(t'^2/2\sigma^2) + i\omega_1 t'} ,$$

where $\sigma = \sqrt{a/\ddot{r}_0}$ is related to an effective collision time τ through $\tau = 8(\log 2)\sigma$. The acceleration $\ddot{r}_0$ at the distance of closest approach can be accurately estimated from the Coulomb orbit at energies below the barrier E_B. For these bombarding energies the largest cross sections are found at back angles and correspond to head-on collisions. As the energy is gradually increased over E_B the relevant partial wave (and the scattering angle) must be adjusted so as to maintain the grazing character of the process. In this regime $r_0 \sim r_B$ while $\ddot{r}_0$—and consequently the effective collision time—remains a moderate function of the bombarding energy (e.g., Ref. 5).

In Fig. 1(a) the ratio $|a_{\mathrm{dir}}/a_{\mathrm{seq}}|^2$ is displayed as a function of the bombarding energy for a characteristic value $\beta_p = 52$. This quantity can be used to measure the relative magnitude of the direct and sequential processes. Neither the ratio nor even the absolute cross sections are expected to have a strong dependence for energies above the barrier E_B. In fact, the coupling form factors are essentially probed in this regime at a constant radius $r \sim r_B$. There is, on the other hand, a sharp qualitative change in the ratio as the energy drops below the barrier. This part of the curve—more reliable because it is not affected by uncertainties in the optical potential—shows a dramatic gain of the direct multiparticle transfer over its sequential counterpart.

The larger values of the ratio $|a_{\mathrm{dir}}/a_{\mathrm{seq}}|^2$ are of course obtained as both numerator and denominator rapidly decrease with the bombarding energy. It is important to note, however, that the substantial gain in orders of magnitude is achieved at an acceptable cost in the size of the cross sections for direct transfer. To illustrate this point we show in Fig. 1(b) the quantities $|a_{\mathrm{dir}}|^2$ and $|a_{\mathrm{seq}}|^2$ as a function of the bombarding energy, normalizing their values to one for $E_{\mathrm{c.m.}} = E_B$. The curves in Fig. 1(b) can thus be used to estimate the loss in counting rate to be expected as the energy is lowered below the barrier to favor the direct multiparticle transfer. We note that measurements of one- and two-nucleon transfer have been recently extended to energies well below the barrier by the introduction of new detection techniques.[6]

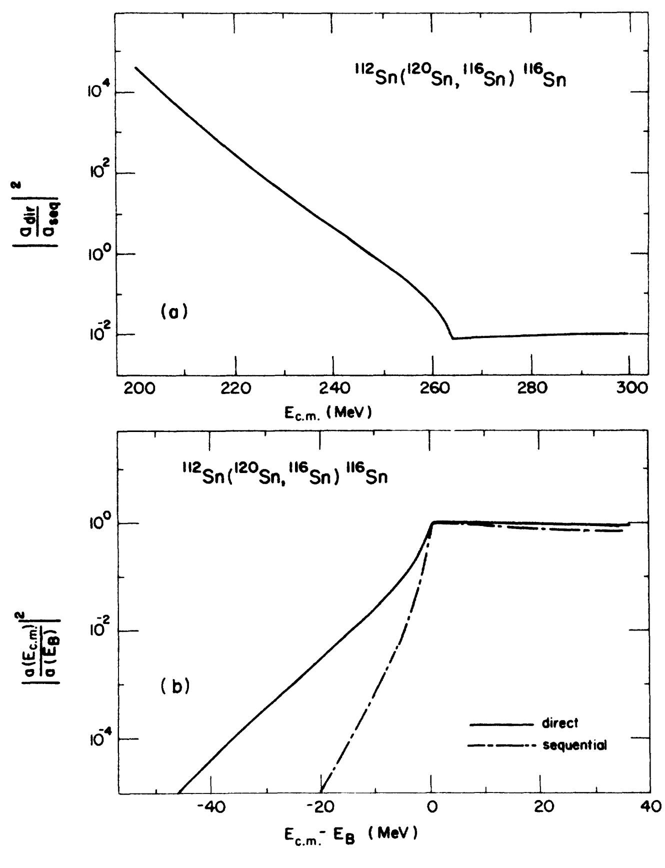

FIG. 1. (a) Ratio $|a_{\mathrm{dir}}/a_{\mathrm{seq}}|^2$ for the four-neutron transfer process in the reaction ^{112}Sn(^{120}Sn,^{116}Sn)^{116}Sn as a function of the bombarding energy. The value $\beta_p = 52$ has been assumed for the pairing deformation parameter. (b) Transfer probabilities $|a_{\mathrm{dir}}|^2$ and $|a_{\mathrm{seq}}|^2$ as a function of the bombarding energy. Both quantities are normalized to unity for $E_{\mathrm{c.m.}} = E_B$.

TABLE I. Differential cross section at maximum angle (columns two and three) and total cross sections (columns four and five) obtained in the coupled channel calculation for the reaction $^{112}Sn(^{120}Sn,^{116}Sn)^{116}Sn$ at different bombarding energies. The labels indicate whether the results have been obtained for the direct or the sequential process.

$E_{c.m.}$ (MeV)	$d\sigma_{dir}/d\Omega$ (mb/sr)	$d\sigma_{seq}/d\Omega$ (mb/sr)	σ_{dir} (mb)	σ_{seq} (mb)
200	1.5×10^{-6}	7.8×10^{-12}	2.8×10^{-6}	7.0×10^{-12}
220	1.9×10^{-4}	1.3×10^{-7}	3.9×10^{-4}	1.3×10^{-7}
240	7.4×10^{-3}	3.4×10^{-4}	1.9×10^{-2}	4.0×10^{-4}
260	2.6×10^{-2}	1.6×10^{-2}	1.4×10^{-1}	6.0×10^{-2}

The results in Fig. 1 correspond to a given value of the parameter β_p. In the very weak regime of two- or four-particle couplings (we recall the equivalent deformation parameters for inelastic excitations would be in the order of 10^{-2}) the choice of β_p is not, however, critical. In fact, the second-order character of the sequential transfer compensates the variation in the ratio (F_2/F_4) so that the results displayed in Fig. 1 are actually representative for other typical values of β_p. We also point out that a different balance between the direct and sequential processes could be obtained by allowing for a $\cos 4\phi-$ term in the macroscopic expression of the density (cf., e.g., Ref. 3).

Absolute values of the transfer cross sections can be obtained by multiplying the probability for the different processes by the elastic cross sections at the corresponding angles. To provide an independent check, however, we have carried out more complete calculations within a coupled channel approach,[7] supplying the strength of the couplings according to the macroscopic prescription as discussed above. The ion-ion potential of Ref. 8 was used, adding an imaginary component of the same geometry and half its strength. Even though this parametrization may be somewhat uncertain, the choice of the optical potential should not be critical in the region of interest, i.e., well below the barrier. Results of calculations performed for four bombarding energies are collected in Table I. These support the conclusions drawn from the semiclassical estimates. In particular, the different rates at which the individual direct and sequential cross sections drop for the lowest energies.

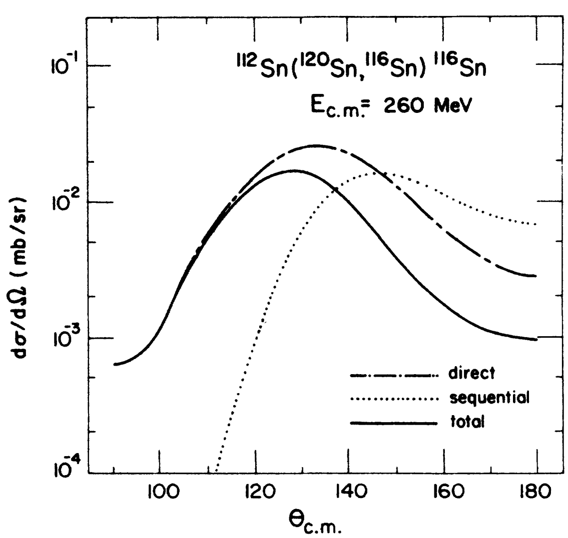

FIG. 2. Angular distribution (solid line) obtained in the coupled channel calculation for the reaction $^{112}Sn(^{120}Sn,^{116}Sn)^{116}Sn$ at bombarding energy $E_{c.m.}=260$ MeV, including both direct and sequential contributions. Also shown are the angular distributions obtained for the direct (dotted line) or sequential (dashed line) processes.

The energy scale in Table I may have to be adjusted somewhat in order to compare the calculations to experimental data like the ones reported in Ref. 4 for higher energies. In fact, the absolute values of the cross sections are extremely sensitive to the actual position of the Coulomb barrier. We note that the value $E_B\sim260$ MeV resulting from the potential of Ref. 8 may differ by as much as 10–20 MeV from systematic parametrizations of the Coulomb barrier like the one given in Ref. 9. Thus, an eventual comparison with data would best be formulated in terms of the variable $E_{c.m.}-E_B$.

Judging by the results reported above, one may conclude that $E_{c.m.}\sim E_B$ represents the optimal conditions at which direct and sequential processes compete. This raises the possibility of detecting an interference between the two coherent ways of populating the four-particle transfer channel. While stressing the speculative character of this proposition we show in Fig. 2 angular distributions for the $^{116}Sn+^{116}Sn$ channel obtained for $E_{c.m.}=260\ \mathrm{MeV}\sim E_B$. There is an appreciable difference in the angular distributions for the direct and sequential processes, leading to a discernible interference pattern. In particular, one may note the accelerated rate in which the sequential angular distributions drop towards the forward angles. This is again a manifestation of the difficulties encountered by the two-step process to keep up with the direct one for the larger radial distances. In a sense, the trend which emerges as one moves into more forward angles (which require the contribution of larger partial waves) can be put in correspondence with the pattern obtained for fixed backscattering as the bombarding energy is lowered.

This work was supported in part by grants from the Bundesministerium fur Forschung und Technologie, Federal Republic of Germany (under Grant No. LM 06 177 II) and the Comisión Asesora de Investigación Científica y Técnica, Spain (under Grant No. 2868-83).

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