Lightweight Conical Components for Rotational Parabolic Domes

Geometric Definition, Structural Behaviour, Optimisation and Digital Fabrication

Roberto Narváez-Rodríguez and José Antonio Barrera-Vera

R. Narváez-Rodríguez, J. A. Barrera-Vera
University of Seville, Spain
roberto@us.es
barrera@us.es

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

Abstract

Although initially intended for academic purposes, the research shown in this paper was drawn towards the development of hollow lightweight conical components to materialise rotational parabolic domes. The starting point is a projective interpretation of an Archimedean property of rotational paraboloid planar sections. This is used to discretise the parabolic surface with a set of tangent ellipses obtained via planar circle-packing algorithms. The ellipses are then materialised with components composed of three truncated conical surfaces, which may be composed of several laminar materials. The geometry and economy of the material, the good structural behaviour, the simple solution for fabrication and assembly, and the tests on a full-scale prototype prove this component to be an efficient self-supporting system for wide-span structures against the use of solid boundary rings, not only for rotational parabolic domes, but also for a possible translation to other types of surfaces.

Keywords:
rotational parabolic dome, Archimedes, computational design,
architectural geometry, design optimisation, power diagram

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

1. Introduction

The use of computation, in contrast to computerization (Terzidis 2003), in architectural design has definitely opened a new paradigm in architecture. The focus of this new design strategy has moved from the object to the process itself. Algorithms acquire the role of the new means of representation as the language that translates human thinking into the power of combination of computer-based processes, with implications both in architectural practice and in the academic sphere.

Although postgraduate courses on computational architectural design have become commonplace, many architecture schools still run a mainly conventional syllabus for their bachelor-degree courses, which forces students to initiate their training in one paradigm and to end up having to change it for the postgraduate training or professional practice, with the acute inconvenience that this may impose. This is the reason why ever more university lecturers and schools are becoming involved in the preparation of architectural geometry (Pottmann et al. 2007) courses to implement the computational paradigm at the beginning of the undergraduate period, which is exactly when students mould their way of thinking, conceiving, and expressing their architectural work (Menges and Ahlquist 2011).

Ideally, these courses should cover competences in mathematics, geometry, computation, algorithmics and digital fabrication in order to provide students with a meaningful workflow to foster their motivation and consequently their learning. However, integrating an introduction on these disciplines in a single geometry course for beginner university students is not an easy task. In addition to the drawbacks of working at this stage, a new problem arises for the faculty if the course is designed to be guided by an original and attractive task that consistently includes the desired competences and without assuming impossible-to-achieve goals. This constitutes the design of a project or the ideation of a system appropriate for the aforementioned purposes.

This paper is intended to show the research carried out within the described context in order to provide a consistent architectural system that matches the needs of the new conception of architectural design for the initial stages of the undergraduate training period by integrating the newly required competences with traditional geometrical contents which provide competences further to those being rendered under the use of the new digital tools.

In this search among the variety of possibilities for an architectural system that holds appropriate contents for first-year students, three themes stand out as playing a major role in the research results shown in this paper. Although they appear to be classic geometry topics to be included in a geometry course under the paradigm of computational design, they remain under consideration due to two main factors: their benefits in understanding architectural history and conventional compositions, and their possible contributions towards the new paradigm. These themes include:

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

Figure 1. Photograph from the interior of the Archimedean Pavilion prototype, composed of inclined rotational parabolic domes materialised with lightweight conical components.

- Working with classic surfaces, which provides essential contents and competences that enable the student to address surfaces of a more complex nature at a later stage. In the case of this paper, rotational paraboloids.
- Understanding projective relationships, not only for representational purposes, but also as a source of composition and as a method to generate simpler algorithms for solving three-dimensional problems. In this case, a discretisation method based on a projective interpretation of an Archimedean property of rotational paraboloids is used.
- Using developable surfaces to materialise double-curved surfaces and complex compositions. This includes the use of digital fabrication for planar sheets of material, subsequently manipulated to form the three-dimensional shapes. In this case, elliptical and irregular conical surfaces to generate a lightweight component to populate the parabolic surface over the previous discretisation method.

The combination of these three topics, originally intended as part of a pedagogical plan, resulted in the constructive system developed in this paper: the use of hollow lightweight components made up of conical surfaces to materialise a specific discretisation of rotational paraboloids. The aim is to get economy of material, good structural behaviour, and ease of assembly to conform an efficient system for wide-span structures against the use of solid boundary rings for rotational parabolic domes.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016

2. An Archimedean Property of Rotational Paraboloids

Archimedes of Syracuse (287–212 B.C.) provides the background to this work. In his work "On Conoids and Spheroids", which can be consulted on Heath's translation into English (Archimedes 1897), among other topics, Archimedes proves the properties of various planar sections of rotational paraboloids. A projective interpretation of a theorem included in this work provides the starting point.

2.1 Archimedes' Proposition as a Projective Property

In "Proposition 12" of "On Conoids and Spheroids" Archimedes states that the planar section of a paraboloid of revolution, produced by a plane neither parallel nor perpendicular to the axis, is an ellipse. He then identifies the position of the major axis and proves that the minor axis is equal to the perpendicular distance between the two lines, parallel to the axis of the paraboloid, which pass through the extremes of the major axis.

Although it obvious that Archimedes does not refer to the definition of the ellipse as a projective property of the paraboloid, he uses lines parallel to the axis of the paraboloid (rays) and the orthogonal distance between them to define the minor axis. In terms of normal projection, it could be stated that the axes of the ellipse, on the surface of the paraboloid, are projected onto a plane perpendicular to the axis, with the same length; that is, the projection of the ellipse is a circle (Gentil Baldrich 1997).

In other words, by using the parallel projection defined by the direction of the axis of the paraboloid, any circle on a plane perpendicular to the axis of the paraboloid is projected onto the paraboloid's surface as an ellipse, which is a planar curve. This property, which is a particular case of a generalised theorem for rotational quadrics (Martin-Pastor, Narvaez-Rodriguez, & Hernandez-Macias 2016), means, among other things, that any circle-packing arrangement can be projected onto the paraboloid to obtain a discretisation of the surface based on planar elliptical faces tangent to each other at various points of the boundary.

2.2 Discretisation of Rotational Paraboloids

From the materialisation point of view, the previous theorem can be applied in several ways, not only for the whole circular base of the paraboloids, but also for any bounding area on the perpendicular plane. Additionally, there is also a variety of possible ways to materialise any initial discretisation: elliptical planar faces, polygonal planar faces, elliptical rings, elliptical arc-based frames, etc.

Whatever the arrangement chosen, it must be taken into account that ellipses increase their eccentricity the further away they are from the vertex of the

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

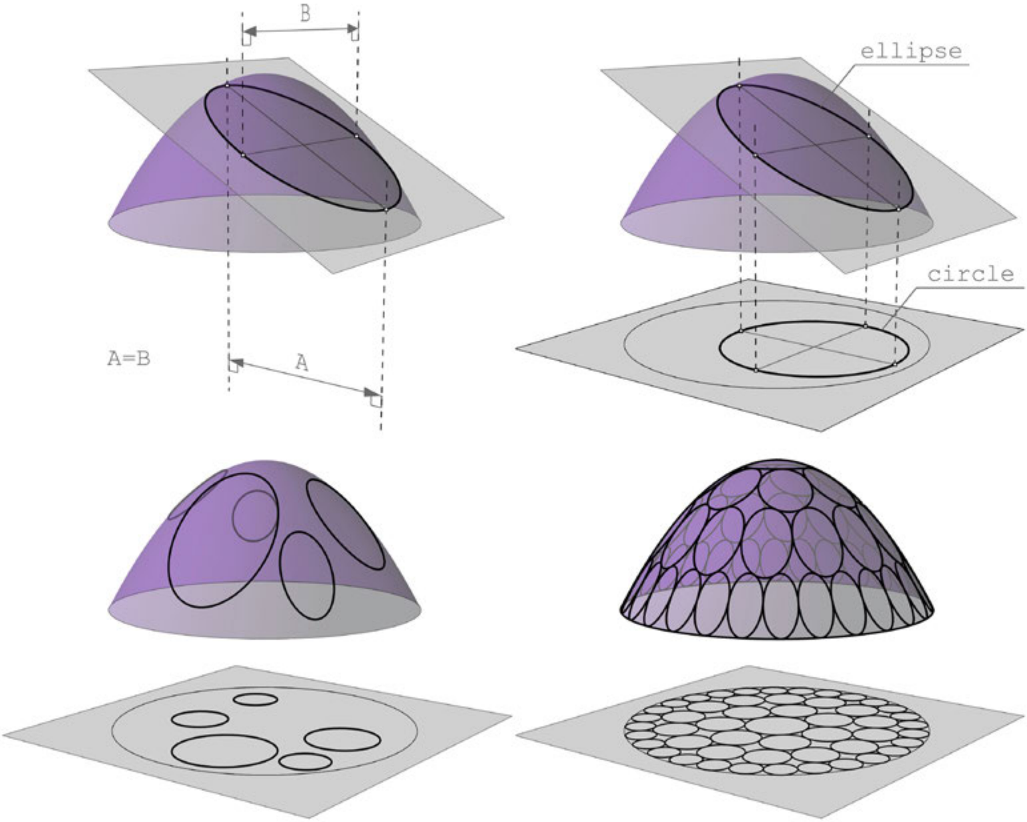

Figure 2. Top-left: Archimedes' definition of the minor axis of the ellipse (oblique planar section of the paraboloid) as the perpendicular distance between the lines parallel to the axis of the paraboloid passing through the extremes of the major axis. Top-right: Projective interpretation; the normal projection of the ellipse onto a plane perpendicular to the axis of the paraboloid is a circle. Bottom-left: with same normal projection, the projection of circles onto the paraboloid's surface as ellipses. Bottom-right: Discretisation of the paraboloid's surface with planar elliptical faces stemming from a circle-packing algorithm.

Figure 3. Left: Discretisation of the dome with elliptical rings and generation of the cones by extruding the ellipses to the centre of the base of the dome. Middle: Offset surface to trim the apex. Right: Thickening of the conical rings as an immediate solution to acquire rigidity.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016

http://vdf.ch/advances-in-architectural-geometry-2016.html

paraboloid due to the increase in the slope of the surface, and hence the algorithm used for packing must implement control of this deformation by selecting the appropriate radii for the circles, in accordance with their position with respect to the centre of the paraboloid and the material and constructive constraints of the project.

This paper focusses on the design and analysis of a specific materialisation of rotational parabolic domes, which uses the resultant elliptical rings as the starting point. From the possibilities of materialising these ellipses, the one chosen is based on the construction of conical surfaces by extruding all the ellipses to a common apex. This is a principle that was already used by other authors in applications to spherical surfaces, such as the Packed Pavilion (Leidi et al. 2010) (Beorkrem 2013). However, further to the discretisation method and the fact of working with parabolic surfaces, there exists another circumstance which differentiates this research from the Packed Pavilion: the way in which the resultant elliptical cones are rigidified to provide global stability to the structure.

The immediate solution to providing rigidity to a conical surface, regardless of its material, is to thicken the surface up to the desired strength. Nevertheless, this solution can be prohibitively expensive due to the considerable amount of material needed, and it would impose a major increase in weight in real architectural applications beyond the construction of prototypes. Therefore, the objective is to conceive a more efficient system to rigidify the conical surfaces, whose use remains feasible in real architectural practice.

3. Lightweight Conical Components

The methodology used for conceiving a lightweight conical component can be related in different stages. The starting point is the design itself of a rigid geometrical shape from the elliptical cones. Once designed, the feasibility must be proved from two points of view; first the structural behaviour, and second the fabrication and assembly constraints. Finally, an optimisation process to minimise the amount of material is desirable. The main considerations of this process, together with the results obtained, are explained in the following points.

3.1 Geometry and Rigidity

Instead of adding material to the initial conical surface, sheets of material could be considered and a satisfactory three-dimensional shape could constitute an increase in the mass moment of inertia of the cross-section of the ring, and consequently provide a higher rigidity to the ring. In the search for a simple arrangement with the use of developable surfaces, the solution adopted consists of a set of three conical surfaces, which make up a triangular cross-section on the

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

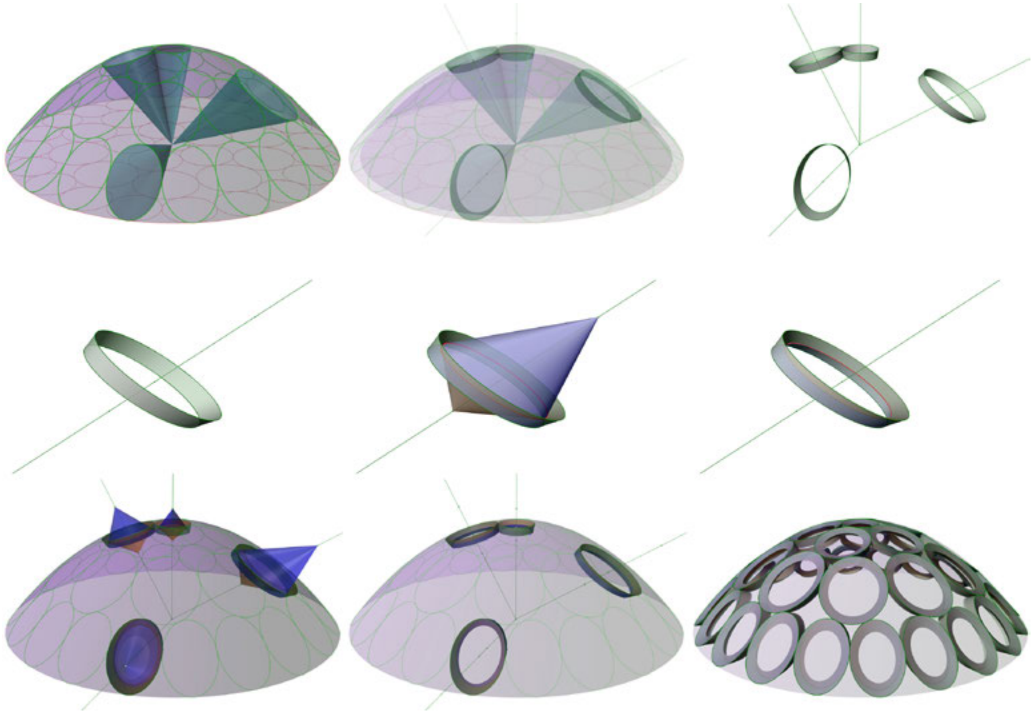

Figure 4. Top row: Initial discretisation, extrusion of four sample conical surfaces and the resulting truncated conical surfaces produced by the section with an offset surface. Middle row: Rigidification of the truncated conical surface by generating a component with two new cones with collinear vertices with the original cones. Bottom row: Reproduction of the previous rigidification for the four sample conical surfaces and the resulting dome by applying the process to all the elliptical rings.

Figure 5. Left: Photograph of the unrolled conical surfaces before assembly. Right: Result of the conical component after assembly and the emergence of the desired structural rigidity.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

ring; the initial lateral surface of the elliptical conical ring, and two new cones, whose vertices are collinear with the original cones, and whose bases are coincident with the two existing bases of the initial truncated conical surface.

There are two major considerations to bear in mind when designing the component. The first is that of the triangular cross-section of the ring, which can be parameterised in the model according to various factors, such as the inner span desired (for lighting or other purposes) among others, but the most important factor is that relative to the rigidity of the component and the stability of the whole structure. As the mass moment of inertia of the cross-section strongly affects the rigidity of the component, the first factor to take into account must be the forces acting over the ring and provision of the cross-section matching the resistance needs. The second consideration is the aforementioned increasing eccentricity of the rings as they are further from the vertex of the paraboloid. In addition to the aesthetic repercussions, this also has implications on the structural behaviour. Both considerations are analysed in the following point (structural behaviour and optimisation).

The simple bending and assembly of low-rigidity sheets of polyethylene provides an idea of the effectiveness of the system due to the emergence of a surprising rigidity of the component just after the assembly of unrolled surfaces, whose initial rigidity is not enough to avoid the deformations due to their own weight. As many other emergent phenomena in nature (Weinstock 2010), the emergence of a good structural behaviour, out of simple low-rigidity elements, is by itself a first index of the efficiency of a system. In addition, active bending is also a sign of economy of material use (Lienhard et al. 2013). However, in the next point this will be technically tested.

3.2 Structural Behaviour and Optimisation

Once the component is designed, the first property to prove from the structural point of view is its rigidity, and the amount of material necessary to acquire it, compared to the solution of a solid boundary ring resulting from the thickening of the initial conical surface. A finite element analysis of the two components was therefore carried out over two equal sets of rings composed of steel sheets, with different eccentricities and subject to the same loads. The results are clear and enough reason to reject the solid boundary ring for its use in many architectural applications:

- The first test assigns the same amount of material and the same axial force, 2 kN, to the two systems. In the case of the lightweight conical component, a sheet thickness of 0.5 mm keeps the deformation under 1 mm, whereas a 1.5 mm-thick solid boundary ring (the same amount of material) reaches inadmissible values, thereby producing the collapse of the rings.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

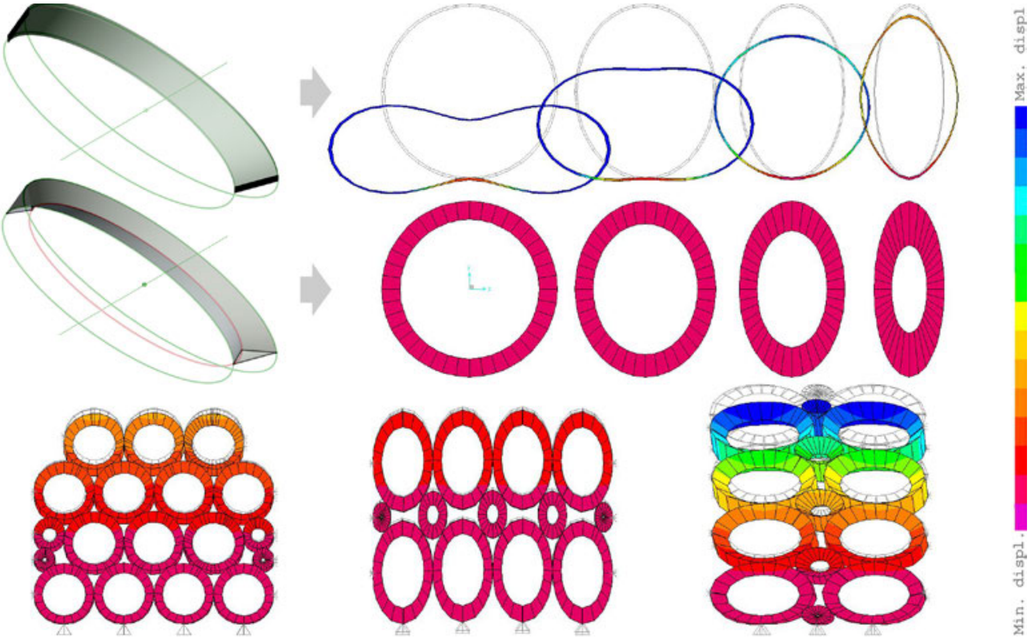

Figure 6. Top: Comparison of the solid boundary ring with the lightweight conical component (triangular cross-section). The rings analysed are made of steel and have a diameter of 1.2 m. Against the same loads, when the amount of material is the same, the deformation of the lightweight conical component is insignificant compared to the solid boundary ring. To perform the same deformation, the lightweight conical component needs a sheet of 0.5 mm thick whereas the solid boundary ring needs a plate of 30 mm thick (about 20 times more material). Bottom row: Comparison of the behaviour in groups of the lightweight conical component according to the eccentricity and the orientation of the ellipses.

- The second test strives towards finding the necessary amount of material such that both systems perform the same deformation against the same axial force of 2 kN. The lightweight conical component suffices with the thickness of 0.5 mm, whereas the solid boundary ring needs a thickness of 30 mm to match the same deformation. This contains up to 20 times more material than the lightweight conical component, thus having a weight twenty times heavier.

After testing the effectiveness and economy of the material of the lightweight conical component, the optimisation of the design is desirable in order to render the system even more efficient. There are numerous factors that carry influence, some of which are intrinsic to every component, due to their geometry, while others are produced by circumstances external to the component, such as the value and the direction of the loads acting upon it, the position of the component within the whole structure, the number of tangency points with the adjacent components, etc. All these factors should be taken into account for every project in which the system is used, and it is always necessary to perform a customised optimisation to obtain the best results. Nevertheless, in order to facilitate the initial design and to take certain variables into account in the parameterisation, an additional analysis was carried out to facilitate the following conclusions being drawn about three important factors illustrated with the examples of deformation diagrams and the Pareto curves for the multi-objective optimisation (minimum material and minimum deformation):

- *Eccentricity of the components and loads*. Due to the constant existence of eccentricities of the initial ellipses, it is important to analyse the main influence of this factor in the structural behaviour. The diagrams show how the behaviour differs along the two different axes of the elliptical components. Two conclusions can be drawn in this sense: (1) The more eccentricity there is involved, the more widely different is the behaviour along each axis. (2) The loads parallel to the major axis are better supported than the loads perpendicular to this axis and are also better supported by components of a more eccentric nature than by those less eccentric with the same diameter.
  Taking into account that the most eccentric ellipses are located near the boundary of the base and linked to the ground, due to the nature of the discretisation method itself, and as long as the axis of the dome is vertical, then this factor turns out to be beneficial for the spreading of the forces. Since the axial forces concentrate as they approach the ground, this is where the most eccentric components lie.

- *Cross-section of the components.* The triangular section of the components is crucial for the success of the system and has a determining influence on the optimisation of the structure. On the one hand, there are three tests, each of which has a set inner radius of the component, a set area of the cross-section and a range of different proportions of the cross-section. The result in the optimum section, minimum amount of material and minimum deformation, varies depending on the inner span, although the values range around the equilateral section, for which the same amount of material supports a greater load than in the other cases. However, not only does the optimum depend on the proportion of the section, but also on the ratio between the area of the triangle and the inner span of the component measured in a perpendicular direction to the cross-section considered.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

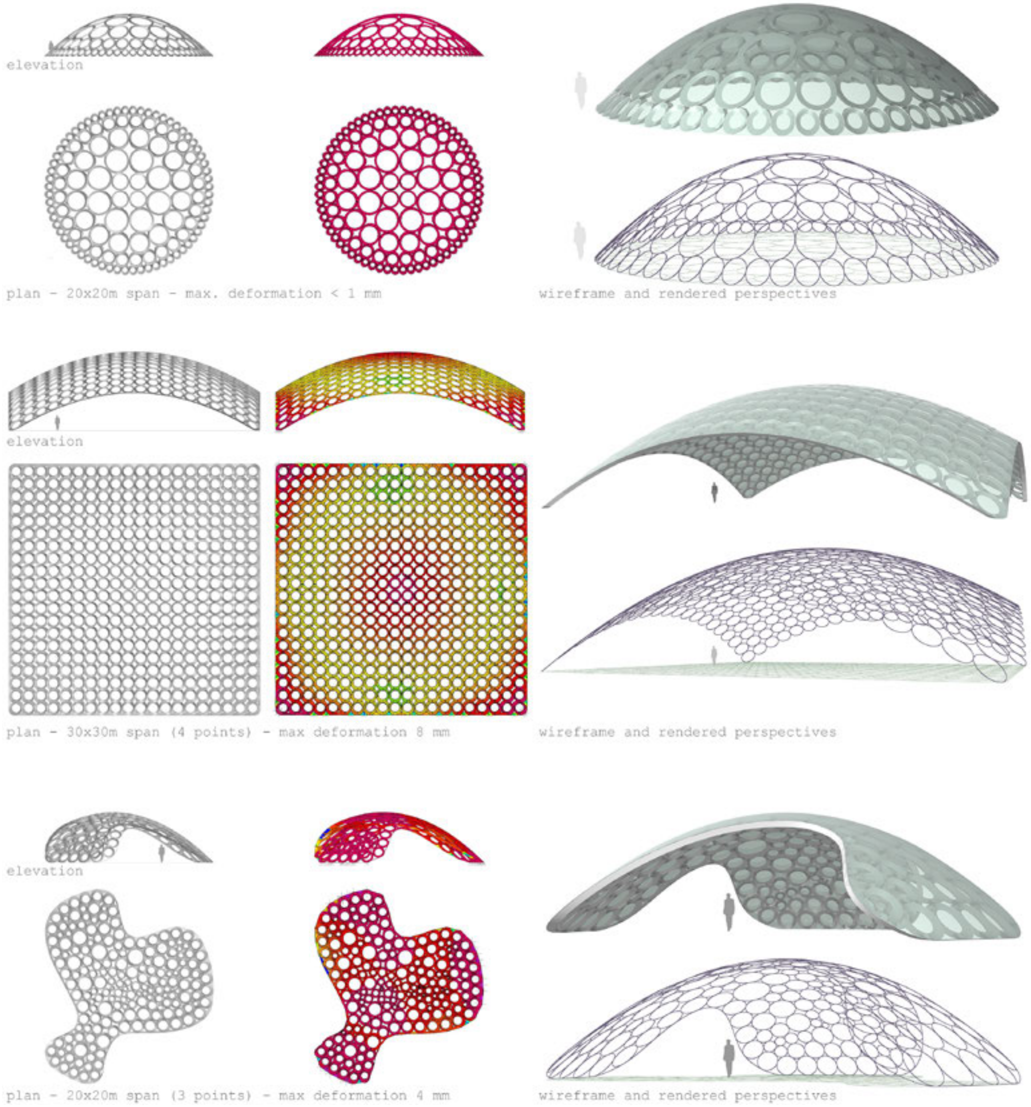

Figure 7. Feasibility of the system with three different wide-span design examples stemming from rotational paraboloids. The maximum deformations of the component system shown (finite element analysis) are for lightweight conical components executed with 0.5-mm-thick steel sheets and subject to the dead load of its own structure. Top row: Dome with a diameter of 20 m with a maximum deformation smaller than 1 mm. Middle row: Roof of 30 m x 30 m only resting on its four corners, with a maximum deformation of 8 mm. Bottom row: Pavilion with a diameter of 20 m resting on three boundary segments, with a maximum deformation of 4 mm.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

The Pearson correlation coefficient between that ratio and the deformation of the component for a set external force is -0.98. Hence, for a set triangular section, there is better behaviour if the perpendicular inner span of the component is shorter. Since the triangular section of an eccentric component is variable along the boundary, the determination of the optimum turns out to be more complex than it may seem at first sight. Nevertheless, for a circular component with a constant equilateral cross-section, the optimum ratio between the height of the triangle and the inner span of the component is about 1/4.5.

- *Number of connections with adjacent components.* This factor seems to be obvious, but is still worth mentioning; the more connection points there are evenly spaced along the boundary, the more stable the component becomes. This is a factor that can be controlled via the initial circle-packing algorithm, in an effort to avoid non-triangular gaps between components.

3.3 Fabrication and Assembly Constraints

As far as digital fabrication and assembly of isolated components are concerned, this system, based on the use of laminar materials, is simple to execute and is normally fabricated from standard sheets or panels cut with a laser cutter or CNC milling machines. As in many other systems, the success of the constructive system relies on achieving consistency between the model for the structural analysis and the real execution of the components. In this sense, after the experience of constructing a full-scale prototype (the Archimedean Pavilion) there are three types of joints to execute with special care:

- *Surface seam.* When preparing the detailing algorithm for the digital fabrication, the unrolled development of the three conical surfaces involved must accomplish two important conditions. (1) The origin surface seam must be placed at a common location for the three surfaces to have a common origin for all the issues occurring on all three surfaces. (2) Additionally, the location of the seam must not coincide on high-curvature segments and should finally be executed obliquely to the generator of the surface. This ensures the continuity of the surface curvature after bending the sheet. The joint itself could be executed in many ways, but in this prototype an internal joint strip with rivets is the chosen solution.

- *Joints between the component's surfaces.* The most important property in the theoretical model that must be preserved in the execution is the continuity of this joint. In practice, this joint should be welded, in the case of weldable materials, or performed to work continuously. Maybe in the case

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

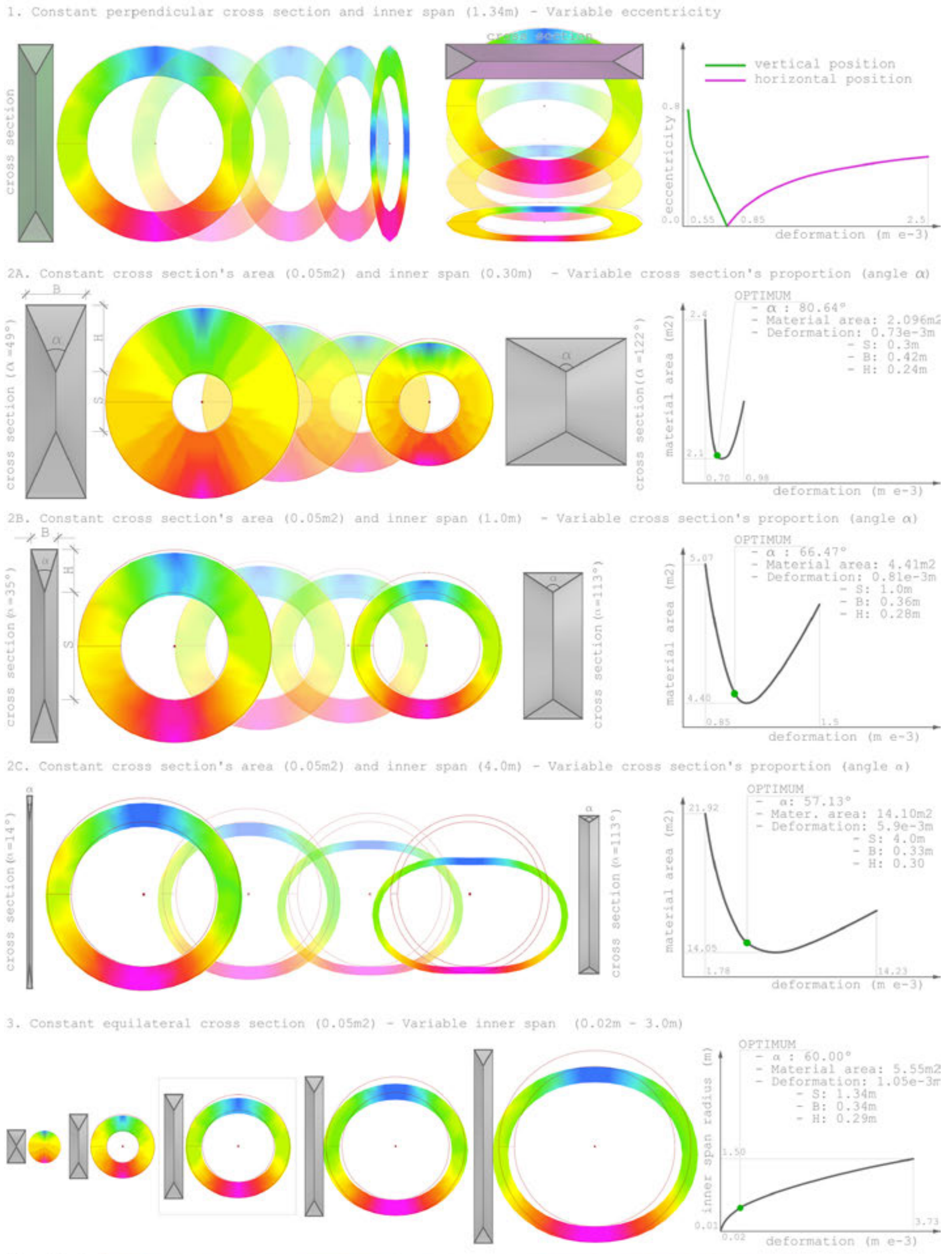

Figure 8. Finite element analysis to draw conclusions about the structural behaviour. Top row (1): Eccentricity tests and deformation graphs according to the position of the components. Three middle rows (2): Variable cross-section tests for three different inner spans and the graphs with the Pareto curves to determine their optimum. Bottom row (3): Tests on the constant equilateral cross-section with variable inner span. The optimum is a ring with equilateral cross-section and with an inner span of between 4 and 5 times the height of the triangle.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016

http://vdf.ch/advances-in-architectural-geometry-2016.html

of wooden panels, a fingerjoint could provide a good solution, although it has yet to be tested by the team for these components.

- *Connections between various components*. Although the theoretical model is analysed with a continuous joint along the tangency line between adjacent cones, the practice has proved that this continuity is not as important as the previous continuity. It may be substituted by two bolts with nuts at the extremes of the tangency line, as long as there is a rigidifying element between the two points and the screws are tightened sufficiently to provoke a strong contact and friction between the two components.

It is also worth mentioning that glazing or panelling is feasible for the system. The plane containing the external ellipse of the components can be extended until it meets its neighbours, thus facilitating the creation of a 3D irregular mesh of planar faces which also has projective relationships with the initial circle packing; the projection of the mesh edges onto the circle packing plane results in a 2D Voronoi power diagram (Aurenhammer 1987).

3.4 Full-Scale Prototype

The teaching experience mentioned at the beginning of this paper was finally concluded with the installation of a full-scale prototype: the Archimedean Pavilion. Although the design remains part of the research carried out by the lecturers, students have participated in the process since the onset. They solved, with the help of their tutors, all the geometrical problems: parametric definition, through propagation-based systems (Aish & Woodbury 2005), to explore the design space composed of inclined rotational paraboloids; parametric definition to discretise the parabolic surfaces; parametric definition for the detailing and digital fabrication, assembly of rings, and the setting up of the installation.

Due to budget limitations, facilities available, and safety reasons, the material chosen for the prototype was high-density polyethylene (HDPE) in sheets of 1 mm thickness. The main advantages of this material include the laminar behaviour for the assembly process, similar to that of metal sheets or thin wooden panels in real applications, the cost, the lightness, the ease, and safety of use, since the manipulation and processing was wholly carried out by students and lecturers, and finally the possibility of testing all the assembly and setting up processes on a 1:1 scale upon a pavilion of about 10 x 6 m. On the other hand, this material presents certain properties that render it as non-appropriate for installations intended to last: non-linear deformation, variable behaviour according the temperature, and, mainly, creep or cold flow, which causes it to continue deforming when exposed to high levels of stress over long periods.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

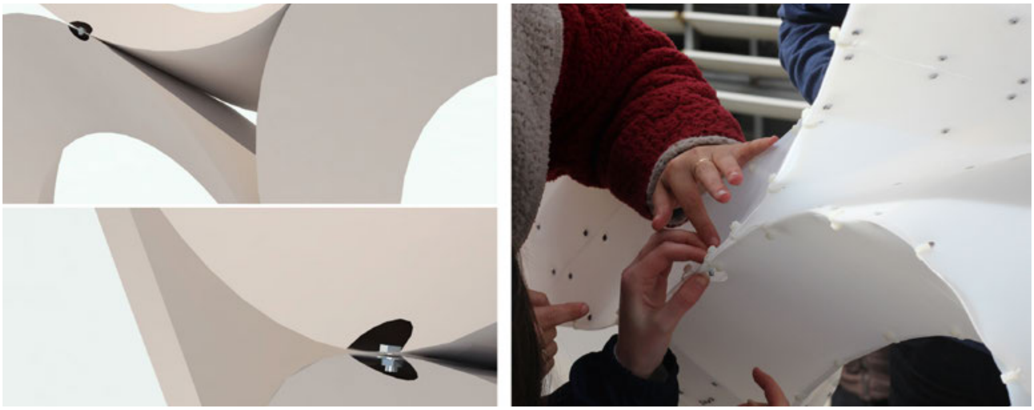

Figure 9. Two pictures and a photograph of the connections between components, including the drilling for the screw and the opening to operate on it.

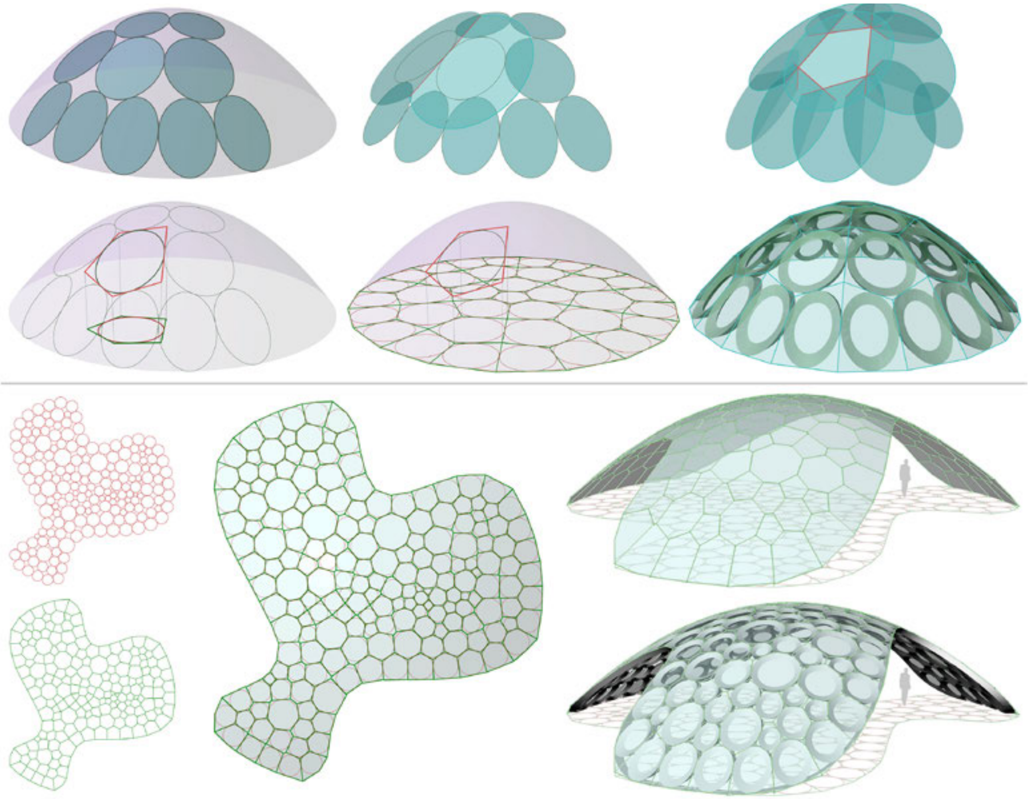

Figure 10. Generation of a mesh of planar faces containing the external ellipse of the components for glazing or panelling. Top: Geometrical operations to generate the mesh, projective relationship and the resulting glazing over the components. Bottom: Top view of another example of the same process including circle packing (red), 2D Voronoi power diagram (green), and 3D mesh over the circle packing (middle). Pictures of the result without and with the components.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

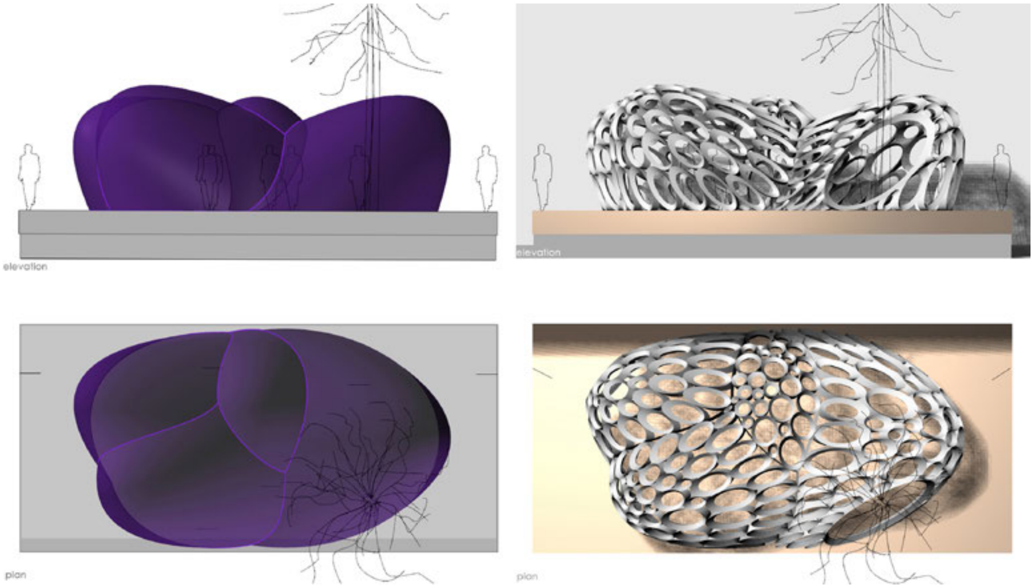

Figure 11. Left: Pictures of the composition with inclined rotational paraboloids which make up the space of the Archimedean Pavilion. Right: Picture of the resulting discretisation and materialisation with lightweight conical components.

4. Conclusion

The implementation of the paradigm of computational design in architecture at the beginning of the undergraduate training period, integrated into a geometry course, fosters the search for simple and ingenious geometrical solutions which may result in discovering applications for architectural practice in the real world.

The discretisation of rotational parabolic domes, based on the projective interpretation of the Archimedean property of oblique planar sections of the paraboloid, provides an appropriate framework to materialise this type of dome through the use of hollow lightweight conical components and facilitates the subsequent glazing or panelling.

In general, lightweight conical components provide an efficient system to materialise rotational parabolic domes. However, each component is rendered still more efficient under the following summarised conditions:

- When the axis of the rotational paraboloid is vertical.

- When the main loads are parallel to the major axis of the initial ellipses.

Figure 12. Photographs of the assembly and result of the Archimedean Pavilion.

- When the cross-section of the component is similar to an equilateral triangle and the inner span of the component, measured perpendicular to the cross-section, is approximately between four and five times the height of the triangular section.

- When there are several connections with adjacent components and these are evenly spaced along the boundary of the component.

In the fabrication and assembly process of the components, three types of joints must be carefully and accurately executed in order to match the theoretical conditions of the analysis, and consequently to ensure their expected behaviour; the seam of the surface, the joints between the surfaces of the component, and the connections between different components.

The materialisation system based on lightweight conical components provides economic use of material, good structural behaviour, and ease of assembly to form an efficient system for wide-span structures as opposed to the use of solid boundary rings, not only for parabolic domes, but also for a possible translation to other types of surface discretisation.

Acknowledgements

We would like to thank the sponsors of this research: University of Seville, through ETSIE, Department of Graphic Engineering, Vicerrectorado de Estudiantes and Vicerrectorado de Investigación, and Dow Building Solutions (a division of The Dow Chemical Company). Thanks to Shuming Wang Zhu for his collaboration and to the group of students, lecturers, FabLab Sevilla and Ehcofab for their support and collaboration throughout the development, fabrication and assembly of the Archimedean Pavilion (http://archimedeanpavilion.blogspot.com.es).

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016
© 2016 vdf Hochschulverlag AG an der ETH Zürich, DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html

References

Aish, Robert, and Robert Woodbury. 2005. "Multi-Level Interaction in Parametric Design." *Smart Graphics*, 151–62. doi: 10.1007/11536482_13

Archimedes. 1897. *The Works of Archimedes*. Edited by T. L. Health. First edit. London: Cambridge University Press Warehouse. Accessed at https://archive.org/stream/worksofarchimede029517mbp#page/n9/mode/2up

Aurenhammer, Franz. 1987. "Power Diagrams: Properties, Algorithms and Applications." *SIAM Journal on Computing* 16, 1: 78–96. doi: 10.1137/0216006

Beorkrem, Christopher. 2013. *Material Strategies in Digital Fabrication*. Edited by Wendy Fuller. New York: Routledge (Taylor and Francis Group). Accessed at http://www.amazon.com/Material-Strategies-Fabrication-Christopher-Beorkrem/dp/041553366X

Gentil Baldrich, José María. 1997. *Sobre La Intersección de Cuádricas de Revolución de Ejes Paralelos*. Seville (Spain): Departamento de Expresión Gráfica Arquitectónica. Universidad de Sevilla.

Leidi, Michele, Dominik Zausinger, Min-Chieh Chen, and Tom Pawlofsky. 2010. "Packed Pavilion." Accessed at http://packed-pavilion.blogspot.com.es/

Lienhard, Julian, Holger Alpermann, Christoph Gengnagel, and Jan Knippers. 2013. "Active Bending, A Review on Structures Where Bending Is Used as a Self-Formation Process." *International Journal of Space Structures* 28, 3: 187–96. doi: 10.1260/0266-3511.28.3-4.187

Martin-Pastor, Andres, Roberto Narvaez-Rodriguez, and Daniel Hernandez-Macias. 2016. "A New Theorem about Quadrics Intersections. Speculations Around a Property of Rotational Paraboloids Stated by Archimedes." *Forthcoming publication*.

Menges, Achim, and Sean Ahlquist. 2011. *Computational Design Thinking*. Chichester: Wiley. Accessed at http://fama.us.es/record=b2487754~S5*spi

Pottmann, Helmut, Michael Hofer, Andreas Asperl, and Axel Kilian. 2007. *Architectural Geometry*. Exton, PA: Bentley Institute Press. Accessed at http://www.bentley.com/en-US/Training/Products/Resources/Books/Architectural Geometry.htm

Terzidis, Kostas. 2003. *Expressive Form: A Conceptual Approach to Computational Design*. New York: Spon Press – Taylor & Francis Group. doi: 10.4324/9780203586891

Weinstock, Michael. 2010. *The Architecture of Emergence: The Evolution of Form in Nature and Civilisation*. Edited by Helen Castle, Miriam Swift, and Calver Lezama. Chicester, UK: John Wiley & Sons Ltd.

S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, M. Pauly (eds.): Advances in Architectural Geometry 2016

DOI 10.3218/3778-4_25, ISBN 978-3-7281-3778-4
http://vdf.ch/advances-in-architectural-geometry-2016.html