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IDENTIFYING ACOUSTICALLY COUPLED VOLUMES IN THE CATHEDRAL OF TOLEDO, SPAIN

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ABSTRACT

Cathedrals are large monuments combined with great geometrical complexity due to different architectural styles, commonly featuring modifications resulting from additions and adaptations to liturgical practice in various historical periods. In the so-called "Spanish mode", the choir is situated in the centre of the main nave and generally constitutes a more absorbent volume connected by an acoustically transparent opening with the rest of the cathedral. This space, together with the presbytery and the lateral chapels, can present the phenomenon of acoustic coupling. Taking advantage of a recent acoustic survey carried out in Toledo cathedral, the presence of such non-uniform acoustic energy distribution is investigated. By means of standardised parameters and the application of Bayesian methods, an analysis is performed of the non-linearity of the energy decay curves of the impulse responses registered in the cathedral.

RESUMEN

Las catedrales son grandes monumentos con una gran complejidad geométrica debido a los diferentes estilos arquitectónicos, consecuencia de las modificaciones resultantes de adiciones y adaptaciones a diferentes períodos históricos y a la evolución de la práctica litúrgica. En el llamado "modo español" el coro está situado en el centro de la nave principal y, generalmente, constituye un volumen más absorbente conectado por una abertura acústicamente transparente con el resto de la catedral. Este espacio, junto con el presbiterio y las capillas laterales, pueden presentar el fenómeno del acoplamiento acústico. Aprovechando una reciente campaña acústica llevada a cabo en la catedral de Toledo, se investiga la presencia de dicho fenómeno. El análisis se realiza calculando parámetros normalizados y aplicando métodos bayesianos para caracterizar la no linealidad de las curvas de decaimiento de energía derivadas de las respuestas al impulso registradas en la catedral.

INTRODUCTION

Multiple-sloped decay, which is inherent to coupled volume geometries, has received its greatest attention in recent decades for the acoustic design of modern performance venues [1]. Acoustic coupling presents a method of providing variable acoustics and non-exponential sound-energy decay, which can control the two competing perceptual attributes of the performance hall: reverberance and clarity. One of the most complex conditions that can be found in sound propagation in worship places is that of acoustic coupling between their different sub-volumes [2]. In the case of the acoustics of extensive internal spaces, representation of the space as a system of acoustically coupled volumes has contributed towards explaining the experimental results of various acoustic parameters. Thus, the original two-room model based on statistical-acoustics assumptions [3] was extended by means of matrix notation to a larger number of sub-rooms. Such is the case of the numerical model of St Paul's Cathedral, London, created by Anderson and Bratos–Anderson [4], who divide the cathedral into 70 acoustical subspaces, which are coupled together by the interchange of sound energy. Taking advantage of the use of Bayesian analysis [5] and the correction introduced by Summers et al. [6] in the statistical model, Martellotta [7], with a similar methodology and with experimental measures, analyses the effects of surface absorption and acoustic coupling on reverberation in the Basilica of St. Peter. Subsequently, this author extends the study to encompass all the Papal Basilicas in Rome [8]. Likewise, the hypothesis by Chu and Mak [9] corresponding to a church model composed of a set of coupled spaces instead of one single diffuse space, presents good agreement with the experimental results of two Christian churches in Hong Kong.

Recently, for a better understanding of this acoustic manifestation, Xiang et al. [10] have used an experimental scale-model of two coupled rooms combined with an automated scanning system of high-spatial resolution, and Martellotta et al. [11] investigate the origin of the multi-rate decay by both the Bayesian methods and directional intensity maps based on the use of B-format impulse responses. A major consequence of this phenomenon is that, within the same church, different acoustic conditions can be experienced, depending on the relative position of the sound source and receiver. In cathedrals arranged in the Spanish mode, an area that shows significant differences is that of the choir, since the presence of large quantities of absorbent material and the isolation of that portion of space due to the presence of high stalls propitiates the conditions for a non-uniform energy distribution within such spaces [12]. Several quantifiers of the double-sloped effect or ratios between portions of decay are used in the literature [13, 14] to describe the slope variation as a function of time. However, the introduction of Bayesian probability theory into the evaluation of decay time in acoustically coupled spaces has proved itself a useful framework for the analysis of Schroeder decay functions [5]. In this work, by taking advantage of a recent acoustic survey carried out in Toledo cathedral, the presence of such a non-uniform acoustic energy distribution in all the areas of the cathedral is investigated by using standardised parameters and Bayesian analysis.

TOLEDO CATHEDRAL

The cathedral of Toledo (Fig. 1) is located in the historic centre of the city. The first stone was laid in 1226, and, in 1493, with the completion of the final vaults, this great construction was concluded. The church has a clear French influence, due to the Gallic origin of its first architect. During the sixteenth to eighteenth centuries, work was carried out in accordance with the new styles. The interior space, 120 m long and 60 m wide, is composed of 5 naves (Fig. 1), supported by 88 columns and 72 vaults. The side aisles, somewhat wider than the other two, extend behind the Main Chapel surrounding the chancel and create a U-turn with a double semi-circular aisle (Fig. 2). The 15 chapels of the ambulatory are of various sizes and, over time, certain reforms altered the layout of several chapels. Its large retrochoir is of remarkable beauty. The approximate interior volume of this cathedral is 125,000 m^3 and has a maximum height of 31 m.


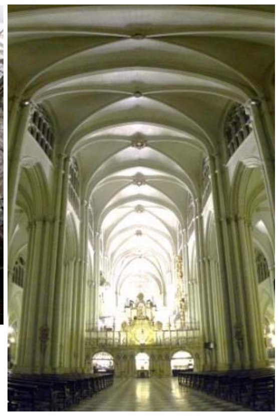

Figure 1. Toledo Cathedral: (left) view of the presbytery; (centre) view of the choir; (right) view of the retrocoir.

METHODS: MEASUREMENTS AND ANALYSES

Room impulse responses (RIR) have been obtained in the unoccupied cathedral, at night, following the recommendations of the ISO 3382-1:2009 [15] and other guidelines specific to churches [16]. As a general procedure in all Spanish cathedrals, 5 source positions have been considered: high altar, transept, pulpit, choir, retrochoir; and other specific positions in the most relevant lateral chapels (in this cathedral: the organ, the Santiago chapel, and the San Ildefonso chapel) [17]. At each source position, the microphone positions have been studied for the set of reception points where direct sound is received. This is reflected in Figure 2 for each of the subspaces of the cathedral.

The process of generation, acquisition, and analysis of the acoustic signal was performed by using the EASERA v1.2 programme, through an AUBION x8 multichannel sound card. At each reception point, located 1.20 m from the floor, the RIR were registered by exciting the enclosure with sine-swept signals, in which the scanning frequency increases exponentially with time. The frequency range, the level, and the duration of the excitation signal were adjusted so that the frequency range would cover the octave bands from 63 to 16000 Hz, and the impulse to noise ratio (INR) would be at least 45 dB in each octave band to guarantee accuracy of certain parameters, such as T_{30}. The signal generated was emitted through an AVM DO-12 dodecahedral sound source with a B&K 2734 power amplifier. At each reception point, RIR were captured by means of an Audio-Technica AT4050/CM5 microphone in its omnidirectional and figure-of-eight configurations connected to Sound Field SMP200 polarisation source and preamplifier.

In order to investigate acoustic coupling, the energy decay curves for the representative source-receiver configuration in each subspace of the cathedral were analysed to study their degree of linearity in accordance with the parameters defined in Annex B of the ISO 3382-2: 2008 standard [18]: C and ξ. This procedure has also been used by Fernández et al. [19] in the study of acoustic coupling in the same cathedral. The curvature parameter, C, expresses the percentage deviation between the reverberation times T_{20} and T_{30}:

$$C = 100\left(T_{30}/T_{20} - 1\right) \quad (\%) \tag{1}$$

C values between 0% and 5% denote linearity in the decay, while values above 10% indicate a decay curve that digresses away from a straight line. The non-linearity parameter, ξ, is expressed as:

$$\xi = 1000\left(1 - r^2\right) \quad (‰) \tag{2}$$

where r is the coefficient of linear correlation between the line of best fit and the curve of energy decay. Values of ξ between 0‰ and 5‰ denote linearity in the decay, while values above 10‰ indicate a decay curve that cannot resemble a straight line.

In order to investigate the non-linear behaviour of the decay curves, Bayesian analysis [5, 20] represents the most powerful and reliable tool for the detection and quantification of multiple slopes in the RIR. In fact, Bayesian analysis allows the exact determination of both the decay constant and the relative amplitude of each of the exponential decays that characterise the space. In order to ensure the greatest accuracy in the identification of the different slopes, the fully parameterised approach proposed by Xiang et al. [5, 20] was implemented in a MATLAB GUI, which facilitates data entry and output of both numerical and graphical results. Although the procedure can be applied to an unlimited number of slopes, only double and triple slopes were investigated for the RIR analysed (in the latter case, only in limited circumstances due to the heavy computational load). The Bayesian information criterion (BIC) was employed to evaluate the most appropriate model (selection of energy decay orders). In a first term, this parameter expresses the degree to which the model was adjusted to the data minus a second term that represents the penalty for the over-parameterised models, since the over-parameterised models result in a greater value associated with the first term. In order to select the best decay model, that which yields the highest BIC value can be considered the most concise, since it ensures a good fit to the decay function data without the use of over-parameterised solutions.

According to Bayesian analysis, any decay curve can be expressed in terms of a multi-rate decay process as:

$$F(A,B,t_k) = \sum_{i=1}^{n} A_i e^{-B_i t_k} + A_0 (L - t_k) \qquad 0 \le t_k \le L \qquad (3)$$

where n is the decay order, t_k is the time step, L is the total duration of the decay, A_i is the "linear parameter" representing the amplitude of the i-th decay, B_i is the "non-linear parameter" equal to 13.8/T_i, with T_i as the i-th reverberation time to be estimated. Finally, A_0 is the amplitude of the background-noise component of measured RIR. The scope of Bayesian analysis involves determining the set of A_i and B_i parameters that best describes the measured decay curve.

RESULTS AND DISCUSSION

Parametric Analysis

The analysis of the sound field of the various areas of the cathedral, through the spectral behaviour of the monaural acoustic parameters defined in ISO 3382-1, starts in Figure 2(b) with the spatially averaged reverberation times as a function of frequency, for each source position. The spatial averages have been made considering all the receivers linked to a source position that receive direct sound, that is to say: S1 (presbytery) 16; S2 and S3 (pulpits without and with canopy, respectively) 16 in each; S4 and S5 (choir) 8 positions in each, S6 (transept) 20; S7 (retrochoir) 8; S8 (Santiago chapel) 6; S9 (St. Ildefonso chapel) 6; S10 (organ) 12. In Figure 2(b), it can be seen that T_{30} remains practically unchanged in its spectral behaviour at all source positions except for the two chapels (S8 and S9).

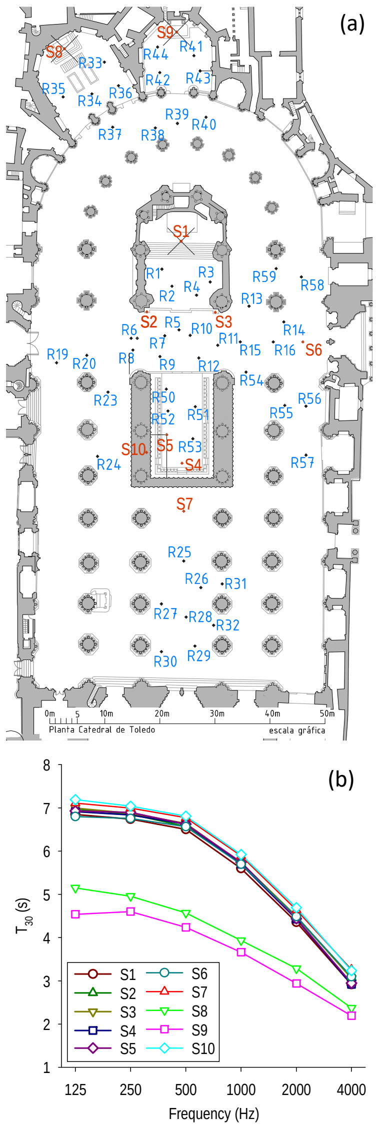

Figure 2. (a) Ground plan of the cathedral of Toledo with source (S1-S10) and receiver locations (R1-R59) indicated. (b) Spectral reverberation times spatially averaged at the various source positions.

Regarding the behaviour of EDT in Figure 3(a), it can be observed how the group corresponding to the source positions

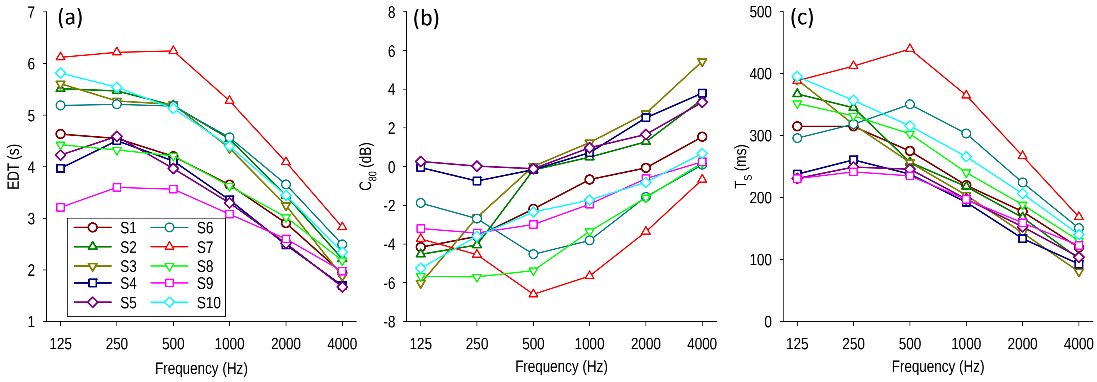

Figure 3. Spatially averaged monaural parameters versus frequency for all source positions studied in the cathedral of Toledo.

S1, S4, S5, S8, and S9 (the latter decreasing the values even more) have shorter values at mid frequencies (of the order of 1.1 s, 4 JND) than the rest of the positions of the sound source in the cathedral. The behaviour of parameter C_{80} as a function of frequency and sound sources, Figure 3(b), is more complex, but greater values of the parameter can be observed for the positions of the source S2, S3, S4, S5, and values close to those of S1. Similar behaviour is observed in D_{50} parameter, omitted herein for reasons of space. As for the results of T_s, Figure 3(c), there is a group of results corresponding to sources S4, S5, S9, and values close to those of S1, with shorter values (around 100 ms, 4 JNDs, 8.5% of 300 ms) at low frequencies.

As a consequence of acoustic coupling, when source and receivers are in the same subspace and this subspace is less reverberant than the whole church, then a double slope can be observed in the energy decay, with a steeper initial decay and a late part that reverberates according to the coupled volume. Under these conditions, shorter early decay time (EDT) and greater clarity (C_{80}) and speech intelligibility (D_{50}) can be observed: this is the case of the behaviour of these monaural parameters in this cathedral. Especially in cathedrals arranged in the Spanish mode, the choir, due to the presence of large quantities of absorbent material and the isolation of that portion of space by the presence of high stalls, propitiates the conditions for a non-uniform energy distribution within such spaces.

The analysis begins with the non-linearity parameters defined in Annex B of the ISO 3382-2: 2008 [18]. In the case of the non-linearity parameter, ξ, the corresponding values for T_{10}, T_{20}, and T_{30} have been calculated for different sections of the decay curves. The results corresponding to the range between -5 and -15 dB (T_{10}) are not presented here due to their insignificance: their derived non-linearity is better related to the variations of the pattern of first reflections from one receiver to another than with the possibility of coupling spaces. Since the values for T_{20} and T_{30} intervals are very similar, results corresponding to T_{30} have been selected for presentation. Table 1 summarises the results of the non-linearity analysis for the four source positions S1, S4, S5, and S9 and their receivers linked within their own subspaces.

It can be seen that, in the majority of cases, the values of C remain outside the interval [0%, 5%] and those of ξ_{T30} outside the interval [0‰, 5‰], many of which even exceed 10% (or 10‰), which clearly shows the non-linearity of the decays and therefore the clear possibility of acoustic coupling between this subspace and the rest of the cathedral. In the case of presbytery, chapels, and choir, the analysis of parameter values for receivers located outside the source space, but in the vicinity of the aperture, reveals no signs of non-linearity in the decays, which, for reasons of space, have not been shown. Likewise, the analyses of the C parameter for the other sound sources remain inside the interval [0%, 5%] and those of ξ_{T30} inside the interval [0‰, 5‰].

Table 1. Values of *C* and ξ_{T30} for the receivers in each space of the cathedral. Results in italics display values between 5% and 10%, and between 5‰ and 10‰; results in bold correspond to values greater than or equal to 10% and 10‰, for each parameter, respectively.

		125		250		50		1000		2000		4000	
		C (%)	ξ_{T30} (‰)	*C* (%)	ξ_{T30} (‰)	*C* (%)	ξ_{T30} (‰)	*C* (%)	ξ_{T30} (‰)	*C* (%)	ξ_{T30} (‰)	*C* (%)	ξ_{T30} (‰)
S1	R01	**10.8**	*6.0*	*7.2*	4.0	**15.8**	**12.0**	**15.7**	**12.0**	**28.6**	**23.9**	**34.6**	**29.8**
	R02	4.3	*6.0*	**20.7**	**15.9**	**12.6**	**10.0**	**19.3**	**14.0**	**18.5**	**15.9**	**29.1**	**23.9**
	R03	*9.0*	*6.0*	**14.8**	**10.0**	**18.1**	**12.0**	**12.7**	**10.0**	**17.1**	**14.0**	**24.9**	**19.9**
	R04	3.1	4.0	*9.5*	*6.0*	*8.3*	*6.0*	**18.0**	**14.0**	**16.6**	**12.0**	**21.0**	**15.9**
S4	R50	**13.4**	**12.0**	*5.2*	2.0	2.6	2.0	*7.1*	4.0	**12.2**	*8.0*	**18.1**	**12.0**
	R51	0.4	*6.0*	2.6	2.0	4.1	4.0	**10.3**	*6.0*	**13.8**	**10.0**	**22.7**	**15.9**
	R52	4.9	*6.0*	1.1	2.0	*5.8*	4.0	*7.9*	*6.0*	**15.4**	**12.0**	**22.7**	**17.9**
	R53	**23.1**	**19.9**	**14.0**	*8.0*	**10.7**	**10.0**	**15.7**	**14.0**	**22.3**	**17.9**	**31.2**	**25.8**
S5	R50	*7.7*	4.0	*7.4*	2.0	*5.0*	2.0	*6.1*	4.0	*9.1*	*6.0*	3.8	**14.0**
	R51	**12.4**	**10.0**	*6.4*	4.0	*5.5*	4.0	*7.5*	*6.0*	**13.1**	**10.0**	2.3	**15.9**
	R52	**11.1**	*8.0*	**11.8**	*6.0*	**12.2**	**10.0**	**16.3**	**14.0**	**17.7**	**17.9**	**26.6**	**23.9**
	R53	**19.3**	**14.0**	**10.7**	*8.0*	*9.3*	*8.0*	**15.2**	**14.0**	**18.5**	**14.0**	**51.3**	**23.9**
S9	R41	4.7	4.0	3.5	2.0	**10.2**	4.0	*7.7*	4.0	4.3	2.0	2.2	2.0
	R42	4.7	4.0	**11.2**	*6.0*	*7.7*	4.0	*6.6*	4.0	*6.1*	2.0	*5.7*	2.0
	R43	**11.5**	*8.0*	*7.5*	*6.0*	*6.4*	4.0	*5.9*	4.0	*5.0*	2.0	4.6	2.0
	R44	4.5	*6.0*	*7.7*	4.0	4.1	2.0	4.3	2.0	**10.2**	4.0	3.9	2.0

Bayesian Analysis

By employing a tool implemented in MATLAB to attain information of a more detailed nature, the decay curves have been analysed through the application of Bayesian methods in order to identify both the various decay constants, if any, and their respective amplitudes. The BIC enables identification of the best approach to describing each of the decays with the lowest computational cost. Although descriptions with three slopes have been attempted, in nearly all cases the results have revealed that the application of two slopes provides sufficient approximation. In Table 2, the results of the Bayesian analysis are synthesized for the S1-R02 (presbytery) and S4-R50 (choir) combinations at the six octave bands, while in Figure 4, the measured decay curves are compared with those calculated using several models with the tool implemented in MATLAB, for the various octave bands in the S1-R02 (presbytery) combination. It can be observed that the 2-slope model was better than the 1-slope model in fitting the experimental decay curve and that the addition of a 3rd slope failed to greatly improve the accuracy; the greatest values of BIC in the 2-slope model also confirmed that this model provides the most concise and best description of the decay curves for all octave bands.

In an attempt to identify the significance of the values of T21 and T22, Figure 5 shows a comparison of the reverberation times measured across the whole cathedral and the corresponding values of T_{30}, T_{10} and EDT measured in each subspace with T21 and T*22* values from Bayesian analysis, for three source-receiver combinations, in the presbytery, in the choir, and finally in St. Ildefonso chapel, respectively, at the six octave bands. In the first two cases, the values of T21 are very similar to the values of T_{30} averaged for all source-receiver positions of the cathedral,

Table 2. Summary of decay constants, BIC, and ratios of amplitudes resulting from the application of Bayesian analysis for double-slope model to two source-receiver combinations in Toledo cathedral.

	S1-R02, presbytery						S4-R50, choir					
Freq. (Hz)	125	250	500	1000	2000	4000	125	250	500	1000	2000	4000
T1 (s)	5.74	4.65	4.86	3.84	2.94	1.71	5.35	6.32	5.87	4.67	3.45	2.04
T21 (s)	7.15	7.03	6.77	6.04	4.84	3.53	7.32	7.25	6.99	6.20	4.97	3.40
T22 (s)	3.38	1.94	1.15	1.54	1.60	1.32	2.17	2.83	2.47	2.59	1.98	1.34
BIC2-BIC1	17.02	42.51	35.68	44.39	33.37	34.50	34.03	20.64	22.33	25.76	34.64	21.46
$10\,Log\,{}^{A_2}/_{A_1}\,(dB)$	0.8	4.6	4.3	6.4	7.1	11.3	2.9	-0.3	-0.5	1.7	4.1	6.5

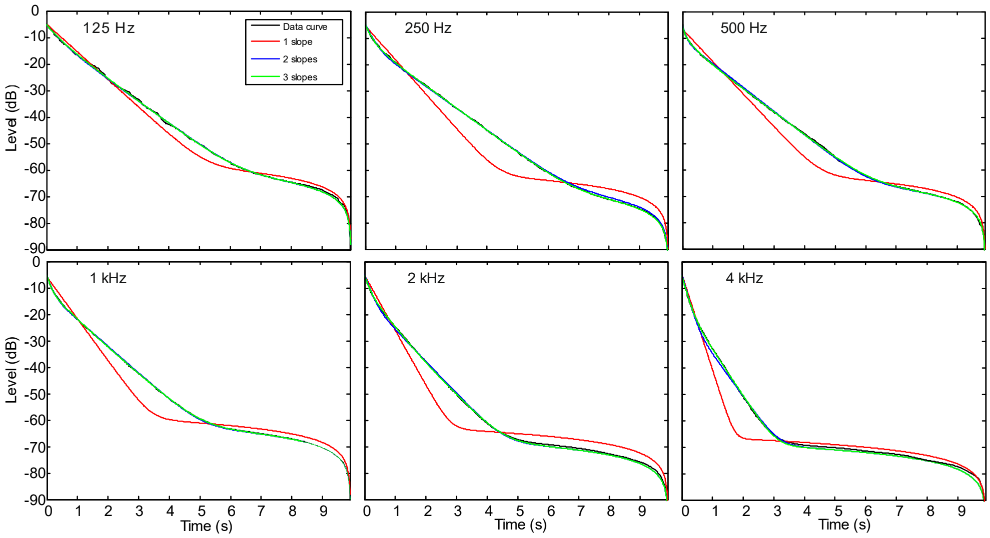

Figure 4. Comparison of experimental decay curves and different decay models with 1 to 3 slopes for S1-R02 (presbytery) at the six octave bands.

and those of T22 present major correspondence with the values of EDT in their own subspace. In the case of the S9-R44 combination in the St Ildefonso chapel, the lowest values of T21 with respect to T_{30} in the cathedral suggest a more complex decay with 3 slopes possibly involved. Bayesian analysis identified in all receptors in this chapel three slopes with the greatest BIC values for all octave bands for this model, for example for S9-R44 decay times of T31=5.969 s, T32=3.18 s, and T33=1.778 s, at 1000 Hz are obtained. Thus, results suggest that, for this chapel, the contribution in the decay is not only from the cathedral as a whole, but also from the ambulatory.

CONCLUSIONS

Various techniques of analysis have been applied for the evaluation of the non-linearity of the experimental decay curves in several volumes of the cathedral of Toledo and to ascertain whether this non-linearity is due to acoustic coupling phenomena. Double slopes in the decay curves of impulse responses have been identified in receivers in the presbytery and choir subspaces, and triple slopes in the St. Ildefonso chapel; linear decays in the remaining areas (transept, retrochoir, lateral naves, and Santiago chapel). These multiple slopes have been detected with three analysis procedures of the energy decay curves: by analysis of the spectral behaviour of the spatially averaged monaural acoustic parameters in each subspace ISO 3382-1: 2009 [15]; by using the standardised parameters defined in ISO 3382-2: 2008 standard [18]; and with Bayesian methods [20].

In the case of Bayesian analysis, whereby a model with two slopes sufficiently describes the measured curves in the presbytery and choir subspaces, it must be highlighted that the first decay is characterised by a value of T21 similar to that of the T_{30} of the whole cathedral, and the second decay is characterised by T22, similar to the value of EDT measured in the subspace. These results are in agreement with the conclusions reached by Fernández et al. [19] who employed a different procedure to obtain the RIR and a different mathematical method of determination of the double slope. In this cathedral, acoustic coupling has been detected in the presbytery, unlike other Spanish cathedrals studied with this same methodology, such as the Cathedral of Seville [12] and that of Murcia [21]. The subsequent challenge is to proceed to an acoustic simulation-auralisation of these subspaces that reproduces these acoustic phenomena, whereby these acoustic improvements of several JNDs in the parameters of acoustic quality, especially in the choir, are perceptible in the auralisations.

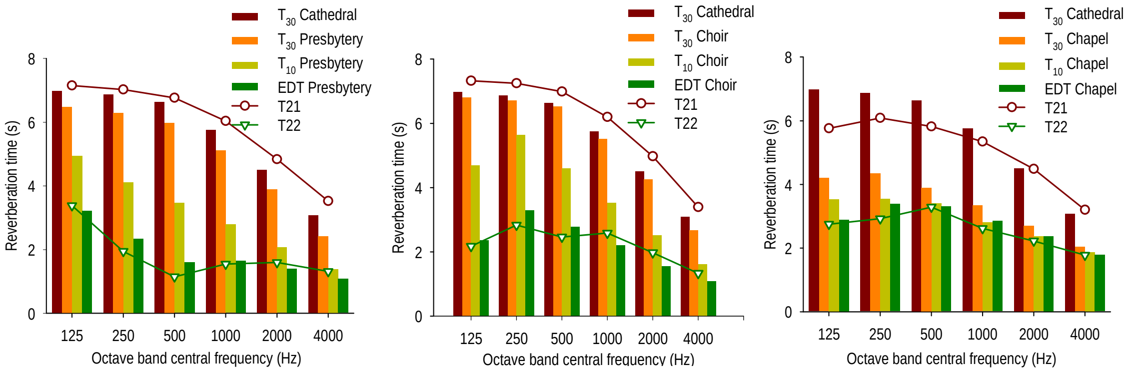

Figure 5. Acoustic parameters measured in S1-R02 of the presbytery (left), in S4-R50 in the choir (middle), and in the St. Ildefonso chapel S9-R44 (right), and the average calculated over all source-receiver combinations in the cathedral and in each subspace. T21 and T22 are the reverberation times of decays resulting from Bayesian analysis.

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