Analysis of leakage in multilayered microstrip lines using complex images

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Abstract
The Mixed Potential Integral Equation (MPIE) is combined with the complex image method to analyze the complete spectrum of multilayered printed transmission lines. A relevant contribution of this method is its ability to study both the bound and leaky regimes in a very simple and efficient way. Since the analysis is carried out in the spatial domain, this work also makes it possible to analyze the leakage phenomenon for structures with nonzero thickness conductors.

1 Introduction

The propagation characteristics of guiding structures in layered media are very efficiently computed by means of integral equation methods. For the layered geometry, the required Green's functions are only known in closed-form in the spectral domain. If a spatial domain formulation were used, the spatial-domain Green's functions can be obtained from their spectral versions through Fourier transform inversion [1]. Strictly zero-thickness printed transmission lines can be solved both in the spectral and the spatial domains with rather low computational effort. Non-planar conductors are better accounted for by means of the spatial-domain version of the integral equation [2]. During the last decade, a powerful tool —the Discrete Complex Images Technique (DCIT)— was developed to obtain the spatial Green's functions for layered structures [3]. This technique has been predominantly applied to 3D planar circuits and antennas problems. Recently this method has been adapted to deal with 2D guiding problems, where the relevance of a fast generation of the Green's functions is even more significant than in the 3D context. Thus, planar strip-like [4] and slot-like [5] structures have been solved following that procedure. However, the important question posed by the existence of both surface and space leaky-wave solutions has not been considered in those works where only bound modes were treated. Leaky-waves have been comprehensively studied in the frame of the Spectral Domain Analysis (SDA) [6]. Working in SDA implies to perform numerical integration along a properly chosen path in the spectral variable complex plane. Various physical and mathematical arguments have been employed to determine the features of this path. In this paper we will show how to adapt the method in [4] to include the leaky regime. This new approach has the advantage of being very simple and efficient and the capability of dealing with non-planar conducting structures.

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2 Analysis

The problem of the determination of the propagation constants of a multistrip system embedded in a layered substrate can be posed in terms of a Mixed Potential Integral Equation as shown in [4]. The Green's functions for the scalar and vector potentials, K^{Φ} and K^{A}_{xx}, must be obtained from their spectral versions. This implies to perform Fourier transform inversions of the type:

$$G(x, x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jk_x|x-x'|} \widetilde{G}(k_\rho)\, dk_x \tag{1}$$

where $k_\rho = \sqrt{k_x^2 + k_z^2}$, k_x being the transverse wavenumber and $k_z = \beta - j\alpha$ the assumed propagation constant.

The DCIT can be advantageously used in this problem to avoid the tedious numerical integration involved in (1). The underlying idea is to expand the spectral function as a sum of complex exponential functions whose spatial-domain counterpart is known thanks to a 2D Sommerfeld identity [4]. However, this technique was only applied to characterize bound modes. Next we will explain the guidelines of the method along with the modifications that must be introduced to generalize the approach to account for leaky modes.

It is well known that the spectral functions $\widetilde{G}(k_x)$ show two types of singularities: branch points at $k_x = \pm\sqrt{k_0^2 - k_z^2}$ and poles. They are respectively related to the radiation into free space and to the modes of the background waveguide. Since the complex images expansion can only fit analytical functions, the above singularities must be properly handled. This is done by first introducing a change of variable ($u_0 = \sqrt{k_\rho^2 - k_0^2}$) that eliminates branch points and second extracting out in analytical form the significant poles (in practice, those poles associated with above cutoff waveguide modes). The quasi-static contribution is also explicitly extracted out. Thus, the spectral domain Green's functions are split into three contributions:

$$\widetilde{G} = \widetilde{G}_0 + \widetilde{G}_P + \frac{1}{u_0}\widetilde{G}_I \tag{2}$$

where $\widetilde{G}_0$ is the quasi-static term ($\widetilde{G}_0 = \lim_{k_\rho \to \infty} \widetilde{G}$), $\widetilde{G}_P$ represents the contribution of the surface wave terms and $\frac{1}{u_0}\widetilde{G}_I$ is the remaining that is suitable for complex exponential expansion.

The spatial domain version of $\widetilde{G}_P$ turns out to be [4]:

$$G_P(|x - x'|) = \sum_{p=1}^{N_0} \frac{R_p k_{\rho p}}{\delta_p} e^{-\delta_p|x-x'|} \tag{3}$$

where N_0 is the number of significant poles, $k_{\rho p}$ is the location of the p-th pole, R_p its residue and $\delta_p^2 = k_z^2 - k_{\rho p}^2$. In the bound regime ($k_z = \beta > k_{\rho p}$) this expression yields exponentially decaying fields in the transverse direction. If we are interested in the leaky regime associated to a particular surface-wave pole, it can be proved that it suffices to change the sign of δ_p for that particular pole in (3).

The $\widetilde{G}_I$ function can be appropriately expanded as a sum of complex exponential functions:

$$\widetilde{G}_I \approx \sum_{i=1}^{N_m} a_i e^{-\gamma_i u_0} . \tag{4}$$

The space domain contribution of the terms in the above expansion can be analytically obtained by using the following identity:

$$\int_{-\infty}^{\infty} \frac{e^{-\gamma u_0}}{4\pi u_0} e^{-jk_x|x-x'|}\, dk_x = \begin{cases} \frac{1}{4j} H_0^{(2)}\left(\xi\sqrt{k_0^2-k_z^2}\right) & \mathrm{Im}\sqrt{k_0^2-k_z^2} > 0 \\ \frac{1}{2\pi} K_0\left(\xi\sqrt{k_z^2-k_0^2}\right) & \mathrm{Im}\sqrt{k_0^2-k_z^2} < 0 \,, \end{cases} \tag{5}$$

where $\xi = \sqrt{(x-x')^2+\gamma^2}$, K_0 is the zero order modified Bessel function of the second kind and $H_0^{(2)}$ the zero order Haenkel function of the second kind. The first option gives place to growing fields so accounting for leaky regime whereas the second option provides the bound solutions. In [6, 7] the space-wave leaky modes are obtained by imposing an alternative integration path in the k_x complex plane for the numerical evaluation of the spectral integrals (1). This calculation is superseded by the present approach. The $\widetilde{G}_0$ term in (1) can be considered a particular case of (5).

To summarize, the proposed method allow us to analytically obtain the space domain kernel of the integral equation in bound and leaky regimes. Once the space domain kernels have been obtained, the integral equation is solved by using the Galerkin method with adequate basis functions leading to quasi-analytical evaluation of matrix entries [4].

3 Numerical results

Our method has been validated by comparing our results with the ones obtained by means of a well established SDA code [7] developed for zero-thickness strip-like structures. The agreement in both bound and leaky regimes is excellent for all the cases considered. As an example, Figure 1 shows propagation constant data for a space-wave leaky mode supported by a microstrip line. For the zero-thickness case, this figure compares SDA results versus our results for β/k_0 and α/k_0. An excellent agreement is observed. This structure was also analyzed in [1], showing our results good agreement with the data reported in [1]. Although the extension of our approach to nonzero thickness conductors requires further mathematical steps than those presented in this paper, this task has been done and some numerical results have been include in the figure.

4 Conclusions

The MPIE approach combined with the complex image method has been adapted for analyzing microstrip transmission lines in leaky regime by the first time. Both surface-wave and space-wave leaky modes are incorporated to the analysis in a very simple way. The complex image technique provides high accuracy and efficiency to the analysis. Working in the spatial domain allows for an extension to nonplanar conductors. Although this generalization is not straightforward, the theory reported in this paper offers the necessary background.

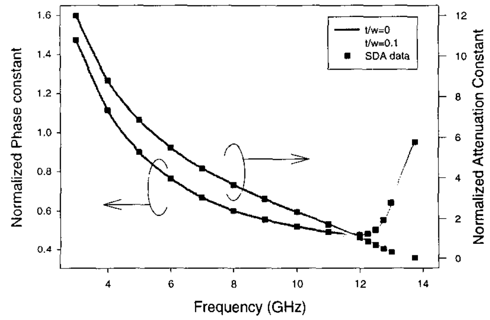

Figure 1: Normalized propagation (β/k_0) and attenuation (α/k_0) constants for a space-wave leaky mode in a microstrip line: $\varepsilon_r = 9.8$, $w = 3$ mm, $h = 0.635$ mm, $t =$ strip thickness.

References

[1] K. A. Michalsky and D. Zheng, "Rigurous analysis of open microstrip lines of arbitrary cross section in bound and leaky regimes," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 2005–2010, Dec. 1989.

[2] C-I.G.Hsu, R.F.Harrington, K.A.Michalski and D.Zheng "Analysis of multiconductor transmission lines of arbitrary cross section in multilayered uniaxial media," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 70–78, Jan. 1993.

[3] D.G. Fang, J.J. Yang and G.Y. Delisle, "Discrete image theory for horizontal electric dipoles in a multilayered medium," *Proc. Inst. Elec. Eng.*, vol. 135, pp. 297–302, Oct. 1988.

[4] J.Bernal, F.Medina, R.R.Boix and M.Horno, "Fast full wave analysis of multistrip transmission lines based on MPIE and complex images," *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp.445–452, Mar. 2000.

[5] E. A. Soliman, P. Pieters, E. Beyne and G.A.E. Vandenbosch, "Numerically efficient spatial-domain moment method for multislot transmission lines in layered media — aplication to multislot in MCM-D technology," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 1782–1787, Sep. 1999.

[6] N.K. Das and D.M. Pozar, "Full-wave spectral-domain computation of material, radiation and guided wave losses in infinite multilayered printed transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 54–63, Jan. 2000.

[7] R. Marqués, F. Mesa, "Spectral domain analysis of higher order leaky modes in microstrip lines: a new spectral-gap effect," *Journal of Electromagnetic Waves and Applications*, vol. 11, pp. 1367–1384, 1997.